Mathematics 102 — Fall 1999 — Bill Casselman's section

First homework — due Friday, September 16

1. (a) What is the sum of all the odd numbers from 1 through 1651?

(b) What is a formula for the n-th term in the sequence

$$1, \ 5, \ 9, \ 13, \ \ldots?$$

Is 1651 ever going to appear in this list? Find a formula for the sum of the first n terms. Draw a picture to explain your formula.

2. A ball is dropped from initial height h_0 and falls to the ground. The height of the ball at time t is observed to be $h(t) = h_0 - ct^2$. (a) Find the time at which the ball hits the ground. (b) It is known that the value of the constant c is $c = 4.9\,\mathrm{ms}^{-2}$. If the ball hits the ground at $t = 1$ second, what was its initial height? (c) Sketch a graph showing the height of the ball every 0.2 seconds along its trajectory.

3. Graph the following straight lines:

(a) $y = -1 + 3x$ for $x = -10$ to 10
(b) $y = -1 - 3x$ for $x = -10$ to 10
(c) $y = -1$ for $x = -10$ to 10

Explain in your own words and a picture what is meant geometrically by a negative slope.

4. Find the equations of the following straight lines:

(a) The line has slope -1 and y intercept 2
(b) The line goes through the points $(2, 1)$ and $(-5, -2)$
(c) The line has y-intercept 3 and x-intercept 4
(d) The line has slope 2 and goes through the point $(3, 3)$

5. The following two data points (t, g) are recorded for blood glucose following a meal:

$$(0, 90) \quad (15, 30)$$

(where t is in minutes and g is in mg per cent). Plot these two points on a graph. Use linear interpolation to estimate the concentration of glucose in the blood at $t = 8.5$ min. Plot the interpolated point as well.

6. The planet Krypton is small and dense, and the way in which falling objects behave is not quite as simple as it is on Earth. An object dropped from $h = 5$ metres on Krypton is observed every 0.2 seconds, and the points (t, h) are recorded as follows:

$$(0, 5) \quad (0.2, 4.9510) \quad (0.4, 4.8030) \quad (0.6, 4.5530) \quad (0.8, 4.1955) \quad (1.0, 3.7221) \quad (1.2, 3.1197)$$

(a) Plot the data on a graph and connect the plotted points. Estimate as carefully as you can where the object is at time $t = 1.05$; $t = 0.99$; $t = 1.01$; (b) Use the results to estimate the velocity of the object at $t = 1$. Give reasons for your estimate. (c) In one sentence: how can you tell this is not taking place on Earth?

7. It is known that the *rate*, v at which a certain chemical reaction proceeds depends on the *concentration* of the reactant, c according to the formula

$$v = \frac{Kc^2}{a^2 + c^2}$$

where K, a are some constants. When the chemist plots the values of the quantity $1/v$ (on the "y" axis) versus the values of $1/c^2$ (on the "x" axis), she finds that the points are best described by a straight line with y-intercept 2 and slope 8. Use this information to find the values of the constants K and a? (Hint: rewrite the formula so that only $1/c^2$ appears.)

8. (Properties of a sphere) The volume V and surface area S of a sphere of radius r are given by the formulae
$$V = \frac{4\pi}{3}r^3, \quad S = 4\pi r^2$$

We say that "V is a function of r" (and also "S is a function of r").
 (a) Sketch V as a function of r and S as a function of r on the same set of axes. Which one grows faster as r increases?
 (b) What is the ratio of the volume to the surface area; that is, what is V/S and how does it depend on r? Sketch a graph of V/S as a function of r.
 (c) The formulae above tell us the volume and the area of a sphere of a given radius. But suppose we are given either the volume or the surface area and asked to find the radius. Find the radius as a function of the volume (i.e. express r in terms of V).

Find the radius as a function of the surface area. Use your results to find the radius of a balloon whose volume is 1 litre. (1 litre = 10^3 cm^3). Find the radius of a balloon whose surface area is $10\,\text{cm}^2$.

9. Write down the difference and sum sequences, and also the difference and sum sequences corresponding to these in turn, corresponding to the sequence

$$3,\ 5,\ 8,\ 12,\ 17,\ 23,\ 30,\ \ldots$$

Do 7 terms of each.

10. Try to figure out how each of the following sequences was generated, and write down the next 5 terms.
$$1,\ 1,\ 2,\ 3,\ 5,\ \ldots$$
$$1,\ 3,\ 4,\ 7,\ 11,\ \ldots$$

Plot 10 terms of the first sequence on a graph. (Hint: Write down the first few terms of the difference sequences as well.)