

Mathematics 102 — Fall 1999

Second homework — due Friday, September 24

This assignment will require much calculation, but not much thought. As for the calculations of tables, we shall discuss this in class how to make the work less tedious.

Exercise 1. (a) Plot the polynomial $P(x) = x^2 - x - 1$. Do this by calculating values of $P(x)$ for $-1.0, -0.9, -0.8, \dots, 1.0$; and then plotting carefully the data points $(x, P(x))$ you get; and finally connecting these by line segments. Make a table of your calculations.

(b) Tell in what interval in this sequence a root of $P(x)$ lies (i.e. a number a such that $P(a) = 0$).

(c) Plot 11 values of $P(x)$ inside this interval (on a different scale) by doing the same thing. So you are now scanning at the interval $\Delta x = 0.01$. Tell which of these intervals contains a root.

(d) One more time, now at the resolution $\Delta x = 0.001$.

Note that when you are through, you will have calculated about 30 values of $P(x)$, and you will have drawn 3 graphs at various scales. Persist! Keep your work neat!

Exercise 2. Calculate and plot values of $P(x) = x^3 - x + 1$ for $x = -2, -1.8, -1.6, \dots, 1$ (about 15 in all). Tell in which of these intervals roots of $P(x)$ lie. Specify intervals of width $1/1000$ containing all roots. Explain your answer.

Exercise 3. Use the data from the previous exercise to plot in a bar graph the differences of the table you computed there. Use these data to give (very rough) upper and lower bounds for the slope of the graph $y = P(x)$ at $x = -1$.

Exercise 4. Sketch the graphs of (a) $y = x^2$ and $y = 2x^2$ (b) $y = x^3, y = (x-1)^3, y = (x+1)^3$; (c) $y = -x^4, y = -x^4 + 1$; (d) $y = x^2 + 1, y = (x/2)^2 + 1$. for the range $x = -4$ to 4 . Scale things so all graphs are about the same size, and indicate the scale carefully. Do each group on the same graph. For each group, make a comment in English about what the moral is.

Exercise 5. Normally, the level of medicine in a patient's blood stream starts increasing from the moment he takes it, as his body absorbs it. Blood samples taken at times $t = 0.0, 0.2, 0.4, 0.8, 0.9, 1.0$ reveal concentrations of $0, 0.139, 0.259, 0.450, 0.490, \text{ and } 0.527$. Plot these on a graph. Then graph roughly by a bar graph at what **rate** the concentration is increasing at $t = 0.1$, etc. Think about what is going on; roughly what would you expect to happen to the concentration as time goes on?

Exercise 6. Let $P(x) = x^4$. Calculate $P(1+h)$ exactly for $h = 0.1, 0.01, 0.001, 0.0001$. The same for $P(1-h)$.

Exercise 7. Let

$$f(x) = \frac{1}{x}$$

Calculate $f(1+h)$ for the same range of h as in the previous exercise, to 6 decimal figures.

Exercise 8. The same with $f(x) = \sqrt{x}$.

Exercise 9. Calculate

$$\left(1 + \frac{0.05}{n}\right)^n$$

for $n = 1, 2, 3, \dots, 12$. Plot these on a bar graph.