

Mathematics 103 — Spring 2000 — Section 203

Second homework assignment — due Friday, January 21

In these problems, keep in mind that

$$\int_a^b x^n dx = \frac{b^{n+1} - a^{n+1}}{n+1}.$$

That should get you through all of these problems, with a little careful thought. And some calculations of anti-derivatives.

Exercise 1. Find the area of the finite plane region bounded by the parabolas $y = 6 - x^2$ and $y = x^2 - 4x$.

Exercise 2. (a) Find the area enclosed between the graphs of the functions $y = x^3$ and the function $y = x^{1/3}$ in the first quadrant. (b) Find the area enclosed between the graphs of the functions $y = x^3$ and $y = x$ in the first quadrant.

Exercise 3. (a) The temperature of a cup of juice is observed to be $T(t) = 25(1 - e^{-0.1t})$ degrees Celsius where t is time in minutes. Find the change in the temperature of the juice between $t = 1$ and $t = 5$. (b) The rate of change of temperature of a cup of coffee is observed to be $f(t) = 80e^{-0.2t}$ degree Celsius per minute where t is time in minutes. What is the total change in the temperature between $t = 1$ and $t = 5$ minutes?

Exercise 4. The rate of flow of blood through the heart can be described approximately as a periodic function of the form $F(t) = A(1 + \sin(0.15t))$ where t is time in seconds and A is a constant in units of cubic cm per second. Find the total volume of blood that flows through the heart between $t = 0$ and $t = 1$. (Express your answer in terms of A .)

Exercise 5. The density of a band of protein along a one-dimensional strip of gel in an electrophoresis experiment is given by $p(x) = 2(x - 1)(2 - x)$, for $1 \leq x \leq 2$ where x is in cm and p is in protein mass per cm. (a) Graph the density p as a function of x . (b) Find the total mass of the protein in the band for $1 < x < 2$. [Hint: simplify the function first.]

Exercise 6. An intravenous infusion delivers a flow rate of $y = 100(1 - t^3)$ cm³/hr over a period of 1 hour, where t is time in hours. The infusion contains a drug at concentration 0.1mg/cm³. Find the total volume of fluid and the total amount of drug delivered to the patient over a 1 hour period from $t = 0$ to $t = 1$.

Exercise 7. The density of bacteria growing in a circular colony of radius 1 cm is observed to be $b(r) = 1 - r^2$, $0 \leq r \leq 1$ (in units of one million cells per square centimeter), where r is the distance (in cm) from the center of the colony. (a) Graph the density as a function of distance from the center of the colony. (b) What is the total number of bacteria in the colony?

Exercise 8. A test-tube of height 10 cm and radius 2 cm contains a density gradient of glucose (i.e. the concentration varies from the top to the bottom). The concentration of glucose as a function of the depth, x is $c(x) = (x/10)$ (in units of gm/cm³). (Note: $x = 0$ at the top of the tube). What is the total amount of glucose in the tube (in gm)?

Exercise 9. (a) Estimate the area under the graph of $f(x) = \sqrt{x^2 + 1}$ from $x = -1$ to $x = 2$ using three rectangles and right endpoints. Then improve your estimate by using 6 rectangles. Sketch the graph and the rectangles.

(b) Repeat part (a) using left endpoints.

(c) Repeat part (a) using midpoints.

(d) From your sketches in parts (a), (b) and (c), which appears to be the best estimate you can make without doing any more nasty arithmetic?

Exercise 10. (a) Compute the area of the region $y \geq x^2$, $1 \leq y \leq 2$. (Hint: reduce to an area you already know.)

(b) Let V be the volume of the solid obtained when the same region is rotated rigidly about the y -axis. Compute V by first approximating V by appropriate slices and then taking the limit.

Exercise 11. For each of the following functions, find the area under the graph of the function on the interval $[0, 10]$. (a) $y = 1$, (b) $y = 1 + x$, (c) $y = 1 + x^2$.

Exercise 12. Compute the area of the finite region bounded by the curves $x = 0$, $x = 3$, $y = x + 2$, $y = x^2$.