Mathematics 103 — section 203 — Spring 2000

Fourth homework — due Friday, February 11

Exercise 1. *(a) Find the following indefinite integrals using the substitution rule:*

$$(i) \quad \int \sin(3x)\,dx$$

$$(ii) \quad \int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$$

$$(iii) \quad \int x^3\sqrt{x^4+1}\,dx$$

$$(iv) \quad \int \frac{4}{1+2x}\,dx\ .$$

(b) Find the following definite integrals using the substitution rule:

$$(i) \quad \int_0^5 \sqrt{3+2x}\,dx$$

$$(ii) \quad \int_0^{\pi/4} \sin(4t)\,dt$$

$$(iii) \quad \int_0^4 \frac{dx}{(x-2)^3}$$

$$(iv) \quad \int_0^{\sqrt{\pi}} x\cos(x^2)\,dx\ .$$

Exercise 2. *A hot, wet summer is causing a mosquito population explosion in a lake resort area. The number of mosquitos is increasing at an estimated rate of $2000 + 2te^{0.5t^2}$ per week (where t is time in weeks). By how much does the mosquito population increase between the beginning of the fifth and the end of the ninth week of summer?*

Exercise 3. *Two cars, labeled 1 and 2, start side by side and accelerate from rest. The figure shows a graph of their velocity functions. (The vertical scale is in multiples of 10 km per hour, the horizontal one in minutes. These cars are pretty slow.)*
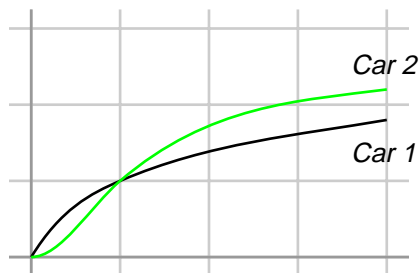

(a) At what time(s) do the cars have equal velocities?
(b) At what time(s) do the cars have equal accelerations?
(c) Which car is ahead after one minute?
(d) Which car is ahead after 3 minutes?
(e) When does one car overtake the other?

Exercise 4. *The level of glucose in the blood depends on the rate of intake from ingestion of food and on the rate of clearance due to glucose metabolism.*
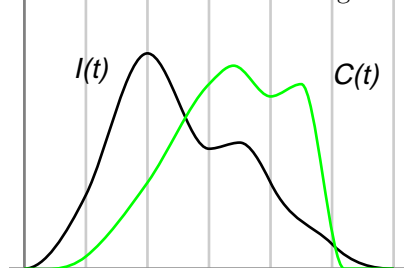
Shown in the figure at left are two functions, $I(t)$ and $C(t)$ for the intake and clearance rates over a period of time. Suppose that at time 0 there is no glucose in the blood. (a) Express the level of blood glucose as a definite integral. (b) At what time was the intake rate maximal? (c) At what time was the clearance rate maximal? (d) At what time was the blood level of glucose maximal?

Exercise 5. *In a previous homework problem, we found the volume of a cone of height h and base radius r by summing up volumes of disk-shaped slices, and taking a limit as the thickness of the slices went to 0. Here we use calculus to find a shortcut to write down an integral representing this volume.*
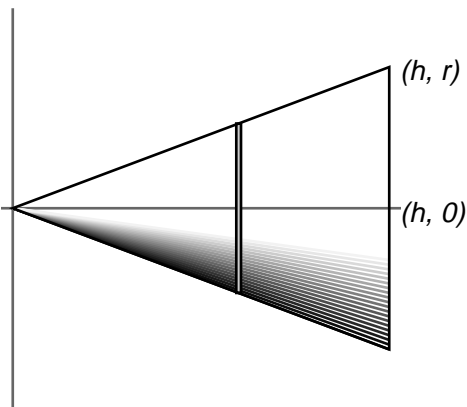
We will place the cone on its side, as shown in the figure, and let x represent position along its axis. The straight line shown will generate the surface of the cone by revolving about the x axis. (a) What is the equation of this line? (b) Slice the cone perpendicularly to the x-axis by slices distributed along the x axis (i.e. into vertical disks) with very small slice thickness Δx. What is the radius, and hence the volume of a slice at position x? (c) Write down the integral which calculates the volume of the cone and find this volume.

Exercise 6. (a) Use an integral to estimate the sums $\sum_{k=1}^{N} \sqrt{k}$. (b) For $N = 4$ draw a sketch in which the value computed by summing the four terms is compared to the value found by the integration. (Your sketch should show the graph of the appropriate function and a set of steps that represent the above sum.)

Exercise 7. In this problem you will find the volume of a pyramid with a square base of width w and with a height h.
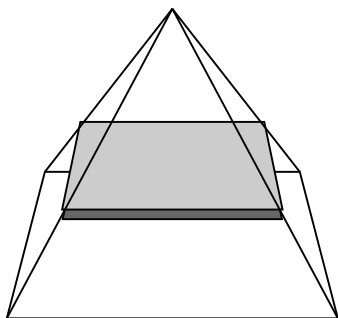
Let the variable z stand for distance **down** the axis of the pyramid with $z = 0$ at the top, and consider slicing the pyramid perpendicularly to this axis (thus into horizontal slices). This will produce square slices (having area $A(z)$ and some thickness dz). Calculate the volume of the pyramid as an integral by figuring out how $A(z)$ depends on z and integrating it.

Exercise 8. Find the volume of water that can be held in a container of height h shaped like a paraboloid—i.e. generated by the curve $y = x^2$.

Exercise 9. The distribution of sediment in a lake is stratified by height along the water column—i.e. the density of sediment (in mass per unit volume) is a function of the form $s(z) = C(40 - z)$ where z (in meters) is vertical height from the point at the bottom of the lake. Suppose the lake has depth 40 meters and a bowl-like shape (generated by the curve $z = x^2/1000$). Find a formula for the total mass of sediment in the lake.

Exercise 10. The trunk of a tree grows incrementally, so that its cross-section (as is well known) consists of tree-rings. Suppose that over a period of 10 years, the growth rate of the radius of the tree trunk (in cm/year) is given by the function $f(t) = 1.5 + \sin(\pi t/5)$. (a) If the radius was initially r_0 at time $t = 0$, what will the radius of the trunk be t years later? Suppose the height of the tree (and its trunk) is approximately constant over this ten year period. (b) What will be the ratio of its final and initial mass? (c) Suppose that $r_0 = 1$. Over which year will the net increase of the mass of the tree trunk be largest? Same question with $r_0 = 100$.