

**Mathematics 103 — section 203 — Spring 2000****Fifth homework — due Friday, February 25**

**Exercise 1.** Evaluate the following integrals. You must show all your work to get credit.

(a)  $\int \frac{1}{1-y} dy$

(b)  $\int_0^T te^{-2t} dt$

(c)  $\int \frac{2}{4+x^2} dx$

(d)  $\int_2^p \frac{1}{1-y^2} dy$

(e)  $\int_1^p \frac{1}{2+2y+y^2} dy$

(f)  $\int_0^\pi x \sin\left(\frac{x}{2}\right) dx$

(g)  $\int_1^S \frac{k_1}{k_2-n} dn$  ( $k_2$  outside the range  $[1, S]$ )

**Exercise 2.** Find the average value of the function

$$f(x) = \sin(\pi x/2)$$

over the interval  $[0, 2]$ .

**Exercise 3.** The intensity of light cast by a streetlamp at a distance  $x$  (in meters) along the street from the base of the lamp is found to be approximately  $I(x) = 20^2 - x^2$  in arbitrary units for  $-20 < x < 20$ . (a) Find the average intensity of the light over the interval  $-5 < x < 5$ . (b) Find the average intensity over  $-7 < x < 7$ . (c) Find the value of  $b$  such that the average intensity over  $[-b, b]$  is  $I_{\text{av}} = 10$ .

**Exercise 4.** In November 1999, the rain in Vancouver fell at the rate  $R(t) = 4((1 + t \sin(\pi t/30)))$  where  $t$  is time in days and  $R(t)$  is in cm/day. Find the total amount of rain that fell and the average rate of rainfall over the first 10 days of the month ( $0 \leq t \leq 10$ ) and over the whole month ( $0 < t < 30$ ).

**Exercise 5.** Consider a distribution function  $y = f(x) > 0$  defined on some interval  $[a, b]$ . The **median** of  $f$  is defined to be a value of the independent variable,  $x$ , say  $x = m$  which splits the area under  $f(x)$  into two equal portions, i.e. such that

$$\int_a^m f(x) dx = \int_m^b f(x) dx = \frac{1}{2} \int_a^b f(x) dx$$

Use this definition to find the median of the following functions on the indicated interval.

(a)  $f(x) = 1 - x^2$ , ( $-1 \leq x \leq 1$ )

(b)  $f(x) = |1 - x|$ , ( $-1 \leq x \leq 1$ )

(c)  $f(x) = 5 - x$ , ( $0 \leq x \leq 5$ )

(d)  $f(x) = \sin(2x)$ , ( $0 \leq x \leq \pi/4$ )

[ Remark: it will help to sketch the given function and interval and use considerations of symmetry for some of these examples.]