Mathematics 103 — section 203 — Spring 2000

Sixth homework — due Monday, March 6

Exercise 1. *More practice with integration:*

(a) $\int xe^{-2x}\,dx$

(b) $\int \tan(x)\,dx$

(c) $\int x^2\ln(x)\,dx$

(d) $\int \frac{1}{x}\ln(x)\,dx$

Exercise 2. Center of Mass: *The density of a beam is given by the function* $\rho(x) = x^r$ *where* $r \geq 0$ *and* $0 \leq x \leq 1$*. (a) Find the center of mass* $\bar{x}$*. (b) What happens to the center of mass if* r *is very large ?*

Exercise 3. *For each of the following cases, find the mean and the median of the given function. Comment on the cases in which the mean and median come out to be the same. What property of the distribution makes this happen?*

(a) $p(x) = 1, \quad -0.5 \leq x \leq 0.5$

(b) $p(x) = 1 - x/2, \quad 0 \leq x \leq 2$

(c) $p(x) = (3/4)(1 - x^2), \quad -1 \leq x \leq 1$

Exercise 4. *Find the* x *coordinate of the centroid of the following shapes: (a) A triangle with vertices at* $(0,0), (p,0), (q,r)$*. (b) A parallelogram with vertices at* $(0,0), (p,0), (q,r), (q+p,r)$

Exercise 5. *A bar whose length is 1 meter is made of a non-uniform mixture of alloys so that it has a mass density distribution given by* $\rho(x) = 20(1-x)$ *in units of gm/cm length where* x *is distance in cm from one end of the bar. (a) Find the total mass of the bar. (b) Find the position* $x = x_m$ *at which the bar should be cut in order to produce two pieces of equal mass. (c) Find the center of mass of the bar.*

Exercise 6. *Core samples of the earth are obtained by drilling vertically and removing a long cylindrical sample of soil. These can then be analyzed for pollen, or other organic material, revealing how the climate on earth may have changed over thousands of years. In one particular sample, a pollen density distribution given by* $p(x) = 5x(10-x)$ *(in millions of pollen particles per meter length) was found, where* $0 \leq x \leq 10$ *is distance from the top surface of the core sample in meters.*

(a) Let $F(x)$ *be the total number of pollen particles found in the section* $[0,x]$ *(i.e. in the first* x *meters of the core). Find this function. How is it related to* $p(x)$ *?*

(b) Find the total number of pollen particles in the core sample. How can you find this number using the function $F(x)$ *?*

(c) What is the average density of pollen particles over the first 4 meters of the sample (i.e. over the interval $0 \leq x \leq 4$*) ? Over the next 6 meters (i.e. over* $4 \leq x \leq 10$*) ?*

(d) Where in the core is the greatest density of pollen?

(e) Find a position, $x = x_p$ *along the core sample such that the amount of pollen in* $0 \leq x \leq x_p$ *is the same as the amount in* $x_p \leq x \leq 10$*.*

(f) What is the **mean** *of this distribution ?*

Exercise 7. *Gel electrophoresis is an experimental technique used in molecular biology to separate proteins (or other molecules) according to their molecular weights and charges. Suppose that in such an experiment, the distribution of protein along a 1 dimensional strip of this gel is found to be* $g(x) = xe^{-x/2}$ *where* $0 \leq x \leq 5$ *is distance in cm from the end of the strip and* $g(x)$*, which is related to the density of the protein per unit distance is in arbitrary units.*

(a) Sketch a graph of this distribution.

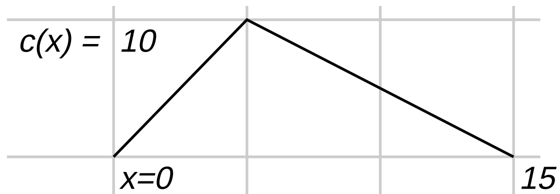
(b) Determine the location $x = x_c$ *where the density of the protein is greatest. (Indicate on graph and find using calculus.)*

(c) Find the total amount of protein in the region $x_c - 1 \le x \le x_c + 1$.

(d) Using your sketch, locate the approximate position of the median of the protein distribution (i.e. the location x_m *such that half of the protein is to the left and half to the right.) Set up an equation that could be used to find the value of* x_m *(Do not solve the equation.)*

(e) Find the mean of the distribution, i.e. the average x *coordinate of the protein distribution.*

Exercise 8. *The distribution of cars along a highway near the scene of an accident is shown below. (*$c(x)$ *stands for the number of cars per 100 meters). Find the center of mass of the car distribution (i.e. the average* x *coordinate of the cars) and the median of the distribution.*



Exercise 9. *(a) The distribution of grades on the first midterm was found to be* $m(x) = \frac{1}{10}x$ *where* $0 \le x \le 10$. *Find the average mark and the median. What does this function mean? Was it an easy test? What fraction of the students scored better than the average mark? better than the median?*

(b) On the second midterm, the distribution was found to be $m(x) = \frac{1}{10}$ *where* $0 \le x \le 10$. *Was this test easier or harder than the first one? Find the average mark and the median. What fraction of the students scored better than the average mark? better than the median?*