Mathematics 103 — Spring 2000

Table of integrals

Elementary integrals

All of these follow immediately from the table of derivatives. Implicit in every one of the indefinite integrals is an integration constant. These should be memorized.

- $\int cf(x)\,dx = c\int f(x)\,dx$
- $\int f(x)+g(x)\,dx = \int f(x)\,dx + \int g(x)\,dx$
- $\int c\,dx = cx$
- $\int x^r\,dx = \dfrac{x^{r+1}}{r+1} \quad (r \neq -1)$
- $\int \dfrac{1}{x}\,dx = \log|x|$
- $\int e^x\,dx = e^x$
- $\int \sin x\,dx = -\cos x$
- $\int \cos x\,dx = \quad \sin x$
- $\int \dfrac{1}{x^2+1}\,dx = \arctan x$
- $\int \dfrac{1}{\sqrt{1-x^2}}\,dx = \arcsin x$

A selection of more complicated integrals

These begin with the two basic formulas, change of variables and integration by parts.

- $\int f(g(x))g'(x)\,dx = \int f(u)\,du$ where $u = g(x)$ (change of variables)
- $\int f(g(x))\,dx = \int f(u)\dfrac{dx}{du}\,du$ where $u = g(x)$ (different form of the same change of variables)
- $\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx$ (integration by parts)
- $\int f\,dg = fg - \int g\,df$ (different form of integration by parts)
- $\int e^{cx}\,dx = \dfrac{1}{c}e^{cx}$
- $\int a^x\,dx = \dfrac{1}{\log a}a^x$ (for $a > 0$)
- $\int \log x\,dx = x\log x - \log x$
- $\int \dfrac{1}{x^2+a^2}\,dx = \dfrac{1}{a}\arctan\dfrac{x}{a}$
- $\int \dfrac{1}{x^2-a^2}\,dx = \dfrac{1}{2a}\log\left|\dfrac{x-a}{x+a}\right|$
- $\int \dfrac{1}{\sqrt{a^2-x^2}}\,dx = \arcsin\dfrac{x}{a}$
- $\int \dfrac{1}{\sqrt{x^2\pm a^2}}\,dx = \log\left|x+\sqrt{x^2\pm a^2}\right|$
- $\int \dfrac{1}{x^2+bx+c}\,dx$? It depends, essentially, on the nature of the roots of $x^2+bx+c=0$, but not explicitly. We first complete the square to write

$$\begin{aligned} x^2+bx+c &= x^2+bx+\frac{b^2}{4}+c-\frac{b^2}{4} \\ &= \left(x+\frac{b}{2}\right)^2+c-\frac{b^2}{4} \end{aligned}$$

  If $c - b^2/4 > 0$, set it equal to a^2; if < 0 equal to $-a^2$; and if $= 0$ forget it. In any event you will arrive after a change of variables at one of the three integrals

$$\int \frac{1}{x^2+a^2}\,dx, \quad \int \frac{1}{x^2-a^2}\,dx, \quad \int \frac{1}{x^2}\,dx,$$

- $\int \sqrt{x^2\pm a^2}\,dx = \dfrac{1}{2}\left(x\sqrt{x^2\pm a^2} \pm a^2\log\left|x+\sqrt{x^2\pm a^2}\right|\right)$
- $\int x^n e^{cx}\,dx = \dfrac{x^n e^{cx}}{c} - \dfrac{n}{c}\int x^{n-1}e^{cx}\,dx$ etc. This is to be used repeatedly until you arrive at the case $n = 0$, which you can do easily.