

Advice on problem solving in mathematics

Mathematics is not just an amalgamation of formulas which can be applied in all circumstances without much thought. No matter how many formulas you learn, there will always be circumstances in which you will have to work something out for yourself to some extent. Learning how to do this is an important part of a general scientific education. In this note I want to offer some suggestions for a **strategy for solving problems**, and in particular problems in mathematics. These ideas certainly won't cover all situations, but they ought to help you out occasionally. Even if you don't go along with the particular ideas I propose, the idea of organizing a strategy should prove useful—maybe a simple checklist you might run through when you are facing an assignment question whose solution isn't completely obvious to you.

- (1) *Be sure you understand the problem.* Here a good idea is to rewrite the question in your own words. If strange terms appear, look them up before you begin. This might seem like a lot of work at first, but as you go along this will seem natural and easy.
- (2) *If the problem looks hard, try identifying what makes it hard.* Try to make up an easier problem of the same type. This will help you in being sure you understand the problem, too.
- (3) *Recall where you have seen related problems in your notes or in other reference material.* Sometimes it pays to set all such places in front of you, even though the relationship might seem distant. Then look over all the things you have assembled to see if they combine.
- (4) *Can you break down the problem into simpler ones?* Especially, simpler ones you already know how to solve. I'll repeat this, because it is **the principal trick to solving mathematics problems of any kind: reduce a difficult problem to a chain of simpler ones.**
- (5) As you go along, you might not understand everything you are doing. Sometimes this is important, but more often not, as long as you seem to be advancing. *Do not clutter up your mind with irrelevant things.* Or from the other side: try to keep in mind only things which are clearly important in what you are working on at the moment. *Don't look back.*
- (6) *Ask yourself occasionally: is it that I don't know how to solve this problem at any price, or is it that I could solve it, but in just some ridiculous time-consuming manner?* Often, once you have some solution in mind no matter how difficult it would be to carry it out in practice, you can start to simplify it to something more practical.
- (7) *When you get an answer, ask yourself if you have any way to check it.* This is an extremely important step. Especially because often the answer you get will be wrong for very silly reasons which checking will fix immediately. Whenever you solve a problem which is not trivial, you should check your solution for consistency with everything you know that's relevant. For problems with numerical answers, this often just means checking for answers of ridiculous size. Another check is to be sure you have been using consistent systems of units.

We shall often see explicit examples of how you can use these ideas. They are not the only tricks to keep in mind in this course, but systematic use of them might help you to avoid wasting time.