Mathematics 103 — Spring 2001

Table of integrals

Part I — Elementary integrals

All of these follow immediately from the table of derivatives. Implicit in every one of the indefinite integrals is an integration constant. This table should be memorized.

- $\displaystyle\int cf(x)\,dx = c\int f(x)\,dx$
- $\displaystyle\int f(x)+g(x)\,dx = \int f(x)\,dx + \int g(x)\,dx$
- $\displaystyle\int c\,dx = cx$
- $\displaystyle\int x^r\,dx = \frac{x^{r+1}}{r+1} \quad (r \neq -1)$
- $\displaystyle\int \frac{1}{x}\,dx = \log|x|$
- $\displaystyle\int e^x\,dx = e^x$
- $\displaystyle\int \sin x\,dx = -\cos x$
- $\displaystyle\int \cos x\,dx = \quad \sin x$
- $\displaystyle\int \frac{1}{x^2+1}\,dx = \arctan x$
- $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x$

Part II - A selection of more complicated integrals

These begin with the two basic formulas, change of variables and integration by parts. Note that some of the formulas do not apply when a denominator is 0.

- $\displaystyle\int f(g(x))g'(x)\,dx = \int f(u)\,du$ where $u = g(x)$ (change of variables)
- $\displaystyle\int f(g(x))\,dx = \int f(u)\frac{dx}{du}\,du$ where $u = g(x)$ (different form of the same change of variables)
- $\displaystyle\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx$ (integration by parts)
- $\displaystyle\int f\,dg = fg - \int g\,df$ (different form of integration by parts)
- $\displaystyle\int e^{cx}\,dx = \frac{1}{c}\,e^{cx}$ $(c \neq 0)$
- $\displaystyle\int a^x\,dx = \frac{1}{\log a}\,a^x$ $(a \neq 1, a > 0)$
- $\displaystyle\int \log x\,dx = x\log x - x$
- $\displaystyle\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\arctan\frac{x}{a}$
- $\displaystyle\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|$
- $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arcsin\frac{x}{a}$
- $\displaystyle\int \sqrt{a^2-x^2}\,dx = \frac{a^2}{2}\,\arcsin(x/a) + \frac{x}{2}\,\sqrt{a^2-x^2}$
- $\displaystyle\int \frac{1}{\sqrt{x^2\pm a^2}}\,dx = \log\left|x+\sqrt{x^2\pm a^2}\right|$
- $\displaystyle\int \frac{1}{x^2+bx+c}\,dx$? It depends, essentially, on the nature of the roots of $x^2+bx+c=0$, but not explicitly. We first complete the square to write

$$\begin{aligned} x^2+bx+c &= x^2+bx+\frac{b^2}{4}+c-\frac{b^2}{4} \\ &= \left(x+\frac{b}{2}\right)^2+c-\frac{b^2}{4} \end{aligned}$$

  If $c - b^2/4 > 0$, set it equal to a^2; if < 0 equal to $-a^2$; and if $= 0$ forget it. In any event you will arrive after a change of variables at one of the three integrals

$$\int \frac{1}{x^2+a^2}\,dx, \quad \int \frac{1}{x^2-a^2}\,dx, \quad \int \frac{1}{x^2}\,dx,$$

- $\displaystyle\int \sqrt{x^2\pm a^2}\,dx = \frac{1}{2}\left(x\sqrt{x^2\pm a^2} \pm a^2\log\left|x+\sqrt{x^2\pm a^2}\right|\right)$
- $\displaystyle\int x^n e^{cx}\,dx = \frac{x^n e^{cx}}{c} - \frac{n}{c}\int x^{n-1}e^{cx}\,dx$ etc. This is to be used repeatedly until you arrive at the case $n = 0$, which you can do easily.