## Mathematics 255—January 30, 1996

## Third homework solutions

**Exercise 1.** *(a) Use a spread sheet and the RK4 method to calculate and graph the solution of the differential equation plus initial conditions in the range indicated:*

$$y' = xy + 1, \quad y(0) = 1, \quad 0 \le x \le 1$$

*Use intervals* $\Delta x = 0.25, 0.125, 0.0625, .03125$.

*(b) Estimate the error in each of the values you get for* $y(1)$.

(a) Here are the final values you get for $y(1)$.

| $N$ | $y(1)$ |
|---|---|
| 4 | 3.059370459991 |
| 8 | 3.059405177012 |
| 16 | 3.059407270692 |
| 32 | 3.059407397109 |

(b) See the notes for a discussion of error.

**Exercise 2.** *(a) For each of the three methods we know, estimate as well as you can without any further calculation what a reasonable choice of step size would be to obtain 16-decimal accuracy in the previous question.*

*(b) Same differential equation. Assuming an initial trial step size of* 0.001*, for each of the three methods explain what step size you would use on the first step, and what trial step size for the second step, assuming an overall accuracy of* 16 *decimals.*

(a) 'Without further calculation' here means without doing any more runs of any of the methods. I'll not give answers for Euler's and Improved Euler's. Look at the notes for $RK4$.

(b) This problem is unfortunately not feasible, since $10^{16}$ is too small for most machines to handle. Look at the attached notes for a similar problem.

**Exercise 3.** *The same as 1.(a) for the equation*

$$y' = x^2 + y^2, \quad y(0) = 1$$

*in as much of the range* $[0, 1]$ *as you can. Produce estimates of the error rate as you go with interval values* $\Delta = 0.0625, 0.03125$.

Here are the runs with $N = 8$ and a few values for 16, 32.

| | |
|---|---|
| 0.000000 | 1.000000000000 |
| 0.125000 | 1.143554250865 |
| 0.250000 | 1.339461911072 |
| 0.375000 | 1.623490951463 |
| 0.500000 | 2.066907120386 |
| 0.625000 | 2.840716522023 |
| 0.750000 | 4.497305933693 |
| 0.875000 | 10.358093619568 |
| 1.000000 | 327.756411349109 |

| | |
|---|---|
| 0.000000 | 1.000000000000 |
| 0.062500 | 1.066750753781 |
| 0.125000 | 1.143555910802 |
| 0.187500 | 1.233231656922 |
| 0.250000 | 1.339468526171 |
| ... | |
| 0.750000 | 4.501320555043 |
| 0.812500 | 6.316986900386 |
| 0.875000 | 10.503360937999 |
| 0.937500 | 29.611243722547 |
| 1.000000 | 13239.165391148194 |

| | |
|---|---|
| 0.000000 | 1.000000000000 |
| 0.031250 | 1.032268401032 |
| 0.062500 | 1.066750793136 |
| 0.093750 | 1.103737430973 |
| 0.125000 | 1.143556017842 |
| ... | |
| 0.875000 | 10.520318399994 |
| 0.906250 | 15.705088595889 |
| 0.937500 | 30.742968970501 |
| 0.968750 | 224.508894214360 |
| 1.000000 | 352905874235.927917480469 |

Of course we know something goes wrong before $x = 1$.

**Exercise 4.** *Use Euler's method for second order equations with* 4, 8, 16, 32, 64 *steps to graph approximate solutions of*

$$y'' + y' + y = 0, \quad y(0) = 0, \quad y'(0) = 1$$

*in the interval* $[0, 5]$.

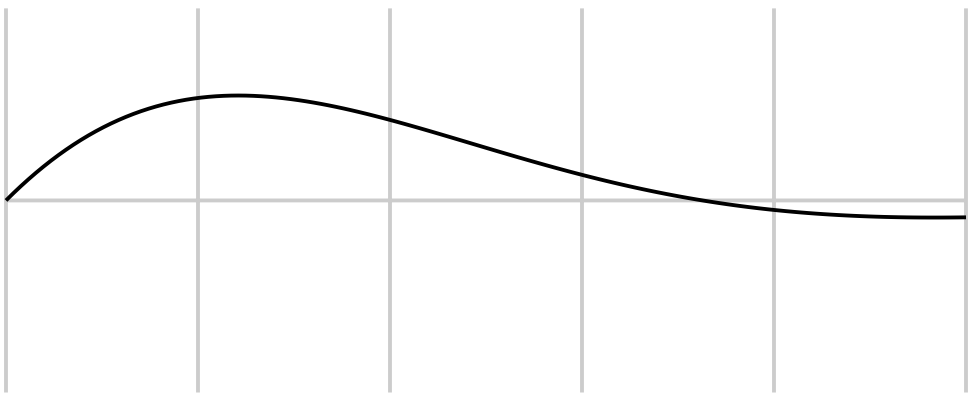
Here are a few values:

| $x$ | $y$ | $v$ |
|---|---|---|
| 0.0000000000 | 0.0000000000 | 1.0000000000 |
| 0.2500000000 | 0.2500000000 | 0.7500000000 |
| 0.5000000000 | 0.4375000000 | 0.5000000000 |
| 0.7500000000 | 0.5625000000 | 0.2656250000 |
| 1.0000000000 | 0.6289062500 | 0.0585937500 |

| | | |
|---|---|---|
| 0.0000000000 | 0.0000000000 | 1.0000000000 |
| 0.1250000000 | 0.1250000000 | 0.8750000000 |
| 0.2500000000 | 0.2343750000 | 0.7500000000 |
| 0.3750000000 | 0.3281250000 | 0.6269531250 |
| 0.5000000000 | 0.4064941406 | 0.5075683594 |
| 0.6250000000 | 0.4699401855 | 0.3933105469 |
| 0.7500000000 | 0.5191040039 | 0.2854042053 |
| 0.8750000000 | 0.5547795296 | 0.1848406792 |
| 1.0000000000 | 0.5778846145 | 0.0923881531 |

| | | |
|---|---|---|
| 0.0000000000 | 0.0000000000 | 1.0000000000 |
| 0.0625000000 | 0.0625000000 | 0.9375000000 |
| 0.1250000000 | 0.1210937500 | 0.8750000000 |
| 0.1875000000 | 0.1757812500 | 0.8127441406 |
| 0.2500000000 | 0.2265777588 | 0.7509613037 |
| ... | | |
| 0.7500000000 | 0.4989881258 | 0.2962099643 |
| 0.8125000000 | 0.5175012485 | 0.2465100837 |
| 0.8750000000 | 0.5329081288 | 0.1987593754 |
| 0.9375000000 | 0.5453305897 | 0.1530301564 |
| 1.0000000000 | 0.5548949745 | 0.1093826098 |

| | | |
|---|---|---|
| 0.0000000000 | 0.0000000000 | 1.0000000000 |
| 0.0312500000 | 0.0312500000 | 0.9687500000 |
| 0.0625000000 | 0.0615234375 | 0.9375000000 |
| 0.0937500000 | 0.0908203125 | 0.9062805176 |
| 0.1250000000 | 0.1191415787 | 0.8751211166 |
| ... | | |
| 0.8750000000 | 0.5224885287 | 0.2057656954 |
| 0.9062500000 | 0.5289187067 | 0.1830077508 |
| 0.9375000000 | 0.5346376989 | 0.1607600491 |
| 0.9687500000 | 0.5396614505 | 0.1390288694 |
| 1.0000000000 | 0.5440061026 | 0.1178197969 |

| | | |
|---|---|---|
| 0.0000000000 | 0.0000000000 | 1.0000000000 |
| 0.0156250000 | 0.0156250000 | 0.9843750000 |
| 0.0312500000 | 0.0310058594 | 0.9687500000 |
| 0.0468750000 | 0.0461425781 | 0.9531288147 |
| 0.0625000000 | 0.0610352159 | 0.9375151992 |
| ... | | |
| 0.9375000000 | 0.5294287593 | 0.1646150967 |
| 0.9531250000 | 0.5320008702 | 0.1537706615 |
| 0.9687500000 | 0.5344035368 | 0.1430554813 |
| 0.9843750000 | 0.5366387787 | 0.1324701842 |
| 1.0000000000 | 0.5387086253 | 0.1220153566 |

The graph should look like this: