Mathematics 255—February 12, 1996

Fourth homework solutions

Exercise 1. *Find solutions of the following differential equations plus initial conditions:*

	Differential equation	$y(0)$	$y'(0)$
(a)	$y'' + 3y' + 2y = 0$	0	1
(b)	$y'' + 4y' + 4y = 0$	0	1
(c)	$y'' - 3y' + 3y = 0$	0	1
(d)	$y'' + 2y' + 2y = 0$	1	0
(e)	$y'' + 2y' + 2y = 0$	0	1
(f)	$y'' + 2y' + 2y = 0$	1	1
(g)	$y'' + 9y = 0$	0	1

(a)

$$\begin{aligned} y &= Ae^{-t} + Be^{-2t} \\ y' &= -Ae^{-t} - 2Be^{-2t} \\ A + B &= 0 \\ -A - 2B &= 1 \\ B &= -1 \\ A &= 1 \\ y &= e^{-t} - e^{-2t} \end{aligned}$$

(b)

$$\begin{aligned} y &= Ae^{-2t} + Bte^{-2t} \\ y' &= (B - 2A)Ae^{-2t} - 2Bte^{-2t} \\ A &= 0 \\ -2A + B &= 1 \\ A &= 0 \\ B &= 1 \\ y &= te^{-2t} \end{aligned}$$

(c)

$$\begin{aligned} y &= Ae^{3t/2}\cos\left(\sqrt{3}t/2\right) + Be^{3t/2}\sin\left(\sqrt{3}t/2\right) \\ y' &= (3A/2)e^{3t/2}\cos\left(\sqrt{3}t/2\right) + (3B/2)e^{3t/2}\sin\left(\sqrt{3}t/2\right) \\ &\quad - (\sqrt{3}A/2)e^{3t/2}\sin\left(\sqrt{3}t/2\right) + (\sqrt{3}B/2)e^{3t/2}\cos\left(\sqrt{3}t/2\right) \\ A &= 0 \\ (3A/2) + (\sqrt{3}B/2) &= 1 \\ B &= 2/\sqrt{3} \\ y &= (2/\sqrt{3})e^{3t/2}\sin\left(\sqrt{3}t/2\right) \end{aligned}$$

(d)

$$\begin{aligned} y &= Ae^{-t}\cos t + Be^{-t}\sin t \\ y' &= -Ae^{-t}\cos t - Be^{-t}\sin t - Ae^{-t}\sin t + Be^{-t}\cos t \\ y(0) &= A \\ y'(0) &= -A + B \\ A &= 1 \\ -A + B &= 0 \\ B &= 1 \\ y &= e^{-t}\cos t + e^{-t}\sin t \end{aligned}$$

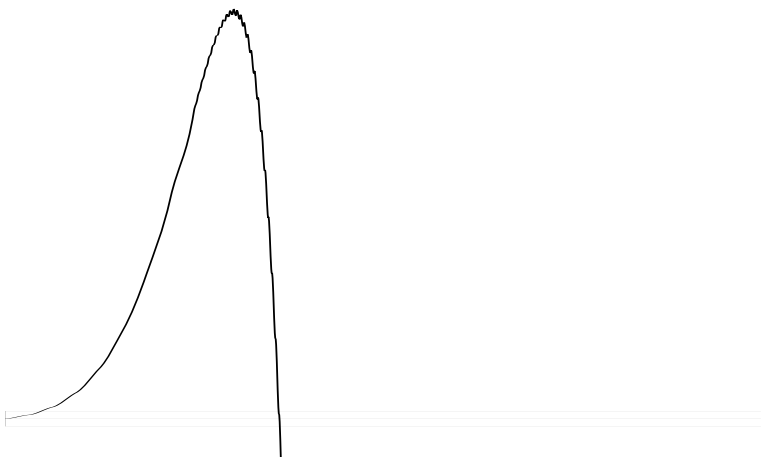
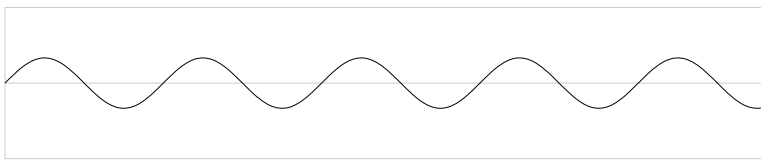
(e)

$$\begin{aligned} A &= 0 \\ -A + B &= 1 \\ B &= 1 \\ y &= e^{-t}\sin t \end{aligned}$$

(f)

$$\begin{aligned} A &= 1 \\ -A + B &= 1 \\ B &= 2 \\ y &= e^{-t}\cos t + 2e^{-t}\sin t \end{aligned}$$

(g)

$$\begin{aligned} y &= A\cos 3t + B\sin 3t \\ y' &= -3A\sin 3t + 3B\cos 3t \\ A &= 0 \\ B &= 1/3 \\ y &= (1/3)\sin 3t \end{aligned}$$

Exercise 2. *Graph each of the solutions you get in #1 in the range* $[0, 10]$. *Use a spread sheet to do a nice job.*