Mathematics 255—January 10, 1996

First homework—due Friday, January 12

Exercise 1. Use a spread sheet and the numerical method suggested in the notes to calculate and graph the solution of the differential equation plus initial conditions in the range indicated:

$$y' = xy + 1, \quad y(0) = 1, \quad 0 \le x \le 1$$

Use intervals $\Delta x = 0.5, 0.25, 0.125, 0.1$.

Exercise 2. The same for the equation

$$y' = x^2 + y^2, \quad y(0) = 1$$

in as much of the range $[0, 1]$ as you can.

Exercise 3. Find the general solutions of the equations

$$y' = y + x$$
$$y' = y + e^x$$
$$y' = 2y + \sin x$$
$$y' = y^2$$

Exercise 4. (a) Write down the differential equation for a tank of water, draining at the bottom through a hole of effective area 1cm and of varying of cross-sectional area $h^2\text{cm}^2$ at height h (in other words, a tank the shape of a coffee filter). Assume other data as in the example discussed in class.

(b) Find the equilibrium height. if it exists, or if not tell why it doesn't exist.

(c) Write down the linear equation approximating yours near equilibrium height.

(d) Plot the slope field for heights from 0 to equilibrium, $t = 0$ to 20 seconds.

(e) Tell whether it is a separable equation or not. If it is, find an implicit equation for $h(t)$.