Mathematics 255—January 14, 1996

Second homework—due Monday, January 22

Exercise 1. (a) Use a spread sheet and the improved Euler's method to calculate and graph the solution of the differential equation plus initial conditions in the range indicated:

$$y' = xy + 1, \quad y(0) = 1, \quad 0 \le x \le 1$$

Use intervals $\Delta x = 0.5, 0.25, 0.125, 0.0625, .03125$.

(b) Estimate the error in each of the values you get for $y(1)$.

Exercise 2. The same for the equation

$$y' = x^2 + y^2, \quad y(0) = 1$$

in as much of the range $[0, 1]$ as you can.

Exercise 3. The solution of the equation in #2 goes off to ∞ at some finite time t. Estimate to 3 decimals what that time is, by any method you can think of. Explain your method.

Exercise 4. Use Newton's method and the implicit equation for the height $h(t)$ in the solution of the problem

$$h' = 2 - \sqrt{0.196\, h}, \quad h(0) = 10$$

to calculate $h(t)$ exactly for $t = 1, 2, 10, 20$.

Exercise 5. A drop of water evaporates at a rate proportional to the surface area of the drop. If the drop is spherical, and in the time span $t = 0$ to $t = 30$ minutes goes in size from a 3mm radius to a 2mm radius, find a formula for the radius of the drop at all times $t > 0$. Graph it as a function of t.

Exercise 6. Newton's law of cooling says that the rate of change of a cooling object is proportional to the difference between the temperature of the object and the surrounding temperature.

(a) Write down the corresponding differential equation, if $\theta(t)$ is the temperature, T is the surrounding temperature, and k is the constant of proportion.

(b) If a cup of coffee has initial temperature 96° and at time $t = 1$ minute has cooled to 92° in a room at temperature 25°, find a formula for $\theta(t)$. Graph $\theta(t)$ as a function of t.

Exercise 7. An object of mass m dropped from a height in free air falls under the acceleration of gravity, and also feels resistance $k|v|$ proportional to the magnitude of its velocity v.

(a) Write down a first order differential equation for v.

(b) Write down a formula for its solution, assuming $v = 0$ at $t = 0$.

(c) Graph the solution.

(d) Find a formula for the length of time it takes to achieve 90% of its limiting velocity.