Mathematics 255—January 28, 1996

Third homework—due Monday, January 29

Exercise 1. (a) Use a spread sheet and the RK4 method to calculate and graph the solution of the differential equation plus initial conditions in the range indicated:

$$y' = xy + 1, \quad y(0) = 1, \quad 0 \le x \le 1$$

Use intervals $\Delta x = 0.25$, 0.125, 0.0625, .03125.

(b) Estimate the error in each of the values you get for $y(1)$.

Exercise 2. (a) For each of the three methods we know, estimate as well as you can without any further calculation what a reasonable choice of step size would be to obtain 16-decimal accuracy in the previous question.

(b) Same differential equation. Assuming an initial trial step size of 0.001, for each of the three methods explain what step size you would use on the first step, and what trial step size for the second step, assuming an overall accuracy of 16 decimals.

Exercise 3. The same as 1.(a) for the equation

$$y' = x^2 + y^2, \quad y(0) = 1$$

in as much of the range $[0, 1]$ as you can. Produce estimates of the error rate as you go with interval values $\Delta = 0.0625$, 0.03125.

Exercise 4. Use Euler's method for second order equations with 4, 8, 16, 32, 64 steps to graph approximate solutions of

$$y'' + y' + y = 0, \quad y(0) = 0, \quad y'(0) = 1$$