Mathematics 255—February 15, 1996

Fifth homework—due Friday, March 1

Exercise 1. Graph

$$\frac{\cos at - \cos bt}{a^2 - b^2}$$

where $a = 2.1$, $b = 2.0$. Annotate important phenomena on the graph.

Exercise 2. A wooden cube measuring 1 metre on each side is floating in a pool. The density of the wood is 0.65 g/cm^3, a little more than half that of water (which you may assume to be 1 g/cm^3). Recall that the force exerted on an immersed body is equal to the weight of the water displaced.

(a) What proportion of the cube lies above the water line?

(b) The cube is pushed down 5 cm and released. With what frequency does it oscillate?

Exercise 3. A mass weighing 3 lb. stretches a spring 3 in. If it is pulled down an additional 3 in. and released, find a formula for position at time $t > 0$. assume no friction.

Exercise 4. A system satisfies a differential equation of the form

$$y'' + cy' + y = 0$$

(a) Graph the quasi-period as a function of c. (b) For what value of c is it equal to twice that of the free period? (c) For what value of c is the system critically damped?

Exercise 5. A system satisfies a differential equation of the form

$$y'' + cy' + y = \cos \omega t$$

(a) Graph the amplitude of the steady state solution as a function of ω, with $c = 0, 0.5, 1, 2, 4$. For each of these values of c determine that value of ω for which the amplitude is a maximum.

Exercise 6. (a) For which value of ω does the steady state solution of

$$y'' + 4y' + 5y = \cos \omega t$$

have maximum amplitude?

(b) Graph as carefully as you can the steady state solution for this value of ω.

(c) Graph the phase shift as a function of ω also.

Exercise 7. Find and graph the solution of

$$y'' + y = f(t), \quad y(0) = y'(0) = 0$$

where $f(t) = 1$ in $[0, 0.1]$ and 0 elsewhere.