Mathematics 257—January 12, 1996

Second homework — due Wednesday, January 17

Read:

§10.2	Exercise #29
§§10.3–4	(again, perhaps)

Exercises:

§10.3	6, 9, 10, 11, 15
§10.4	13, 15, 17, 21, 22, 25, 37

Also:

Exercise 1. (a) Use the identities

$$\sin(at + bt) = \sin at \cos bt + \cos at \sin bt$$
$$\sin(at - bt) = \sin at \cos bt - \cos at \sin bt$$

to find a formula for

$$\int \cos at \sin bt \, dt$$

(b) Show by calculation that

$$\int_0^T \cos 2\pi m \frac{t}{T} \sin 2\pi n \frac{t}{T} \, dt = 0$$

for all positive integers m, n.

Exercise 2. Suppose that f and g are functions of period T. Recall that the dot product of f and g is defined to be

$$f \bullet g = \int_0^T f(t)g(t) \, dt \ .$$

Let A be the linear operator taking f to f''.

(a) Use integration by parts to show that

$$Af \bullet g = f \bullet Ag$$

(b) Show that each of the functions 1, $\cos 2\pi m(t/T)$, $\sin 2\pi m(t/T)$ (where m is a positive integer) is an eigenvector for A. What are the eigenvalues?