Mathematics 257—February 5, 1996

Fourth homework — due Wednesday, January 31

Read:

§10.5

Exercises:

§10.5 1, 2, 3, 4, 7, 10, 12, 13, 14, 15

Also:

Exercise 1. Find the solution of the heat equation in $2D$

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$$

on the unit square $0 \leq x, y \leq 1$ with initial distribution

$$u(0, x, y) = f(x, y) = x$$

and boundary conditions

$$u(t, x, y) = 0 \quad (t > 0)$$

for all (x, y) on the edge of the square.

Exercise 2. (a) Using separation of variables, describe all the fundamental solutions (eigenvectors) of the same heat equation on the unit square subject to the homogeneous boundary value conditions

$$u(0, y) = 0, \quad \frac{\partial u}{\partial x}(x, 0) = 0, \quad \frac{\partial u}{\partial x}(x, 1) = 0, \quad \frac{\partial u}{\partial y}(1, y) = 0$$

corresponding to a box insulated on three sides and open on the fourth.

(b) Find an estimate for the solution of this system if the initial temperature distribution is a constant 1.

(c) At what time t does the centre of the square reach temperature $1/2$? At that time t, can you estimate (with reasons) where the temperature is highest?