**Mathematics 266 — Spring 1999**

**Review**

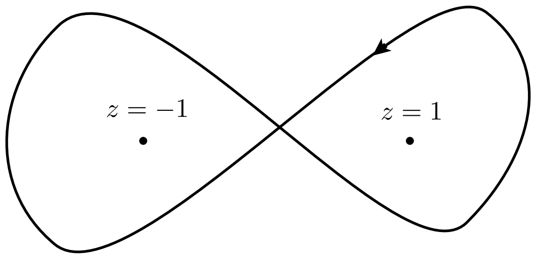
We are making up this list of questions to help you focus your efforts in preparing for the final examination. Many of the questions on the examination will be *closely related* to them, and you should be able to answer most. The instructors will offer no advice concerning these questions, except to resolve ambiguities.

In addition, you should review all homework problems done during the term.

1. Find

$$\int_C \frac{z^3+1}{z^2-1}\,dz$$

where $C$ is the folowing curve:




2. Suppose that $z^n = 1$ but $z \neq 1$. What is

$$z^{n-1} + z^{n-2} + \cdots + z + 1?$$

Why?

3. If $P(z)$ is a polynomial

$$P(z) = p_0 + p_1 z + p_2 z^2 + \cdots + p_{n-1} z^{n-1}$$

of degree $n-1$ then its **finite Fourier transform** of order $n$ is the function assigning to $k$ in the range $[0, n-1]$ the value of $P(z)$ at $z = e^{2\pi ik/n}$:

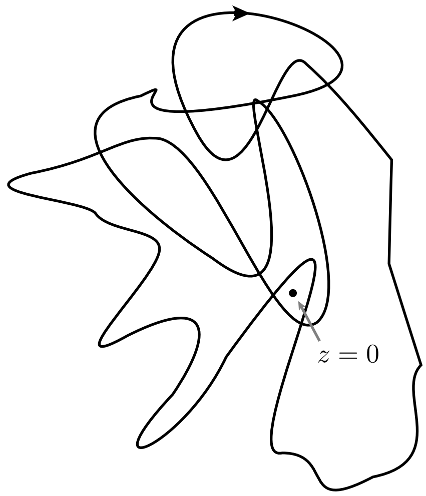
$$\widehat{p}(k) = P(e^{2\pi ik/n}) \ .$$

Find a formula for $p_0$ in terms of the numbers $\widehat{p}_k$. For the $\ell$-th coefficient $p_\ell$. (Hint: if you add all the $\widehat{p}(k)$ together, what do you get? What happens if you multiply $P(z)$ by $z$? $z^2$? If nothing else works, try a few small values of $n$ such as $n = 3$.)

4. Find

$$\int_C \frac{\cos z}{z}\,dz$$

if $C$ is this curve:




5. What is the circulation of the vector field

$$[-y, x]/r^2$$

around the square with corners $(\pm 1, \pm 1)$?

6. What is the flux of the vector field

$$[x, y, z]/r^3$$

through the surface of the cube with corners at $(\pm 1, \pm 1, \pm 1)$?

7. What is the flux of the vector field $[x, y, z]$ through that same cube?

8. Through that same cube with its bottom face removed?

9. Suppose

$$P(z) = p_0 + p_1 z + \cdots + p_{n-1} z^{n-1} + p_n z^n \ .$$

Under what circumstances is the sum of the roots of $P$ equal to $0$?

10. What is the curl of $[x, y, z]/r^n$? The divergence? For what values of $n$ is the divergence equal to $0$?

11. What is the flux of $[-y, x]/r^2$ through the square with corners at $(\pm 1, \pm 1)$? If a small square of area $A$ starts out at $(1, 0)$ in the flow for this vector field, what is its approximate area a small time interval $\Delta t$ later? What can you say about a formula for the area at any time $t$?

12. Let $f(z) = z^3 - z + 1$. Draw an approximation to the image of the square with corners at $(1 \pm 0.1, 1 \pm 0.1)$ under $f(z)$.

13. Find the Fourier transform of

$$f(t) = \begin{cases} 0 & t < 0 \\ e^{-t} \sin t & t \geq 0 \end{cases}$$

14. Find the inverse Fourier transform of

$$\frac{z}{z^2 + 2}$$

15. Find the circulation of $[y^2, x^2]$ around the square with corners at $(0, 0)$, $(1, 0)$, $(0, 1)$, $(1, 1)$.

16. Find

$$\int_C \frac{z}{z^2 + 2} \, dz$$

where $C$ is the circle of radius $3$ centered at $0$.

17. Find the first three non-zero terms of the Laurent series of

$$\frac{1}{(e^z - 1)^2}$$

at $z = 0$.

18. Explain in your own words what the relationship between Cauchy's First Integral Theorem and Stokes' Theorem is.

19. If $f(x + iy) = U(x, y) + iV(x, y)$ is complex differentiable, what is

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} \quad ?$$

20. What is

$$\int \frac{1}{z^2 - 1}\, dz$$

where $C$ is the path starting at $z = 2$ and then going by straight line segments to $2 + i$, $-2 + i$, $-2$.