Mathematics 266 — Spring 1999 — Part II

Chapter 5. Formalities of the Fourier transform

The Fourier transform of a function $f(t)$ defined for all real numbers allows one to resolve $f(t)$ as a 'sum' of simply periodic components. This means we can write

$$f(t) = \int_{-\infty}^{\infty} \widehat{f}(\omega) e^{2\pi i\omega t}\, d\omega$$

where ω represents the frequency in cycles per unit of t. The function $\widehat{f}$ is called the **Fourier transform** of $f(t)$, and $f(t)$ is called the **inverse Fourier transform** of $\widehat{f}(\omega)$.

The formula for $\widehat{f}$ is this:

$$\widehat{f}(\omega) = \int_{-\infty}^{\infty} f(t) e^{-2\pi\omega t}\, dt\ .$$

Complex calculus can be used to evaluate such integrals. In fact, we have made similar calculations before; all that is new here is the terminology.

Exercise 0.1. *Find the inverse Fourier transform of* $1/(1+\omega^2)$*; of* $1/(1+\omega+\omega^2)$.

Exercise 0.2. *Find the Fourier transform of* $f(t)$ *where*

$$f(t) = \begin{cases} t & \text{if } |t| \le R \\ 0 & \text{otherwise} \end{cases}$$

Exercise 0.3. *Find the Fourier transform of* $f(t)$ *where*

$$f(t) = \begin{cases} e^{ct} & \text{if } t \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

and c *is a complex number with real part negative.*

Exercise 0.4. *Find the Fourier transform of* $f(t)$ *where*

$$f(t) = \begin{cases} te^{ct} & \text{if } t \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

and c *is a complex number with real part negative.*

Exercise 0.5. *Find the Fourier transform of* $f(t) = e^{-\pi t^2}$*. This will require a trick we will discuss in class. Recall that*

$$\int_{-\infty}^{\infty} e^{-\pi t^2}\, dt = 1$$