Mathematics 307—section 103

First homework—due Tuesday September 19, 1995

Exercise 1. Suppose that

$$f_1 = e_1 + e_2$$
$$f_2 = e_1 + 2e_2$$

(a) If x is a point with

$$x_E = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

what is x_F?

(b) If

$$x_F = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

what is x_E?

Exercise 2. Suppose that

$$f_1 = e_1 + e_2 + e_3$$
$$f_2 = e_1 + 2e_2$$
$$f_3 = e_1 - e_3$$

(a) If x is a point with

$$x_E = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

what is x_F?

(b) If

$$x_F = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

what is x_E?

Exercise 3. If T is perpendicular projection onto the line $x = y$ what is its matrix? Perpendicular projection onto the line $y = cx$? Perpendicular projection onto the line through the origin and (a, b)?

Exercise 4. If T is perpendicular reflection through the line $x = y$ what is its matrix? Perpendicular reflection through the line $y = cx$? Perpendicular reflection through the line through the origin and (a, b)?

Exercise 5. Find the matrix of rotation through an angle of $45°$ around the axis through the line $x = y = z$. Of rotation θ around the same axis.

Exercise 6. Suppose that the f's and e's are as in the first exercise. If a linear transformation has matrix

$$M_E = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$$

what is M_F?

Exercise 7. What is the matrix of perpendicular reflection in the plane $x + 2y + z = 0$?

Exercise 8. Classify each of the following matrices A as a (generalized) scaling, rotation, or shear. In each case find a matrix X such that $X^{-1}AX$ has one of the standard forms. In case of a shear, choose the columns of X as orthogonal as possible.

(a)

$$A = \begin{bmatrix} 8 & 12 \\ -3 & -4 \end{bmatrix}$$

(b)

$$A = \begin{bmatrix} 3 & -1 \\ 5 & 1 \end{bmatrix}$$

(c)

$$A = \begin{bmatrix} 3 & 1 \\ 5 & 1 \end{bmatrix}$$

(d)

$$A = \begin{bmatrix} 1 & 3 & -2 \\ -1 & 6 & -3 \\ -1 & 8 & -4 \end{bmatrix}$$

(e)

$$A = \begin{bmatrix} 0 & 2 & -1 \\ -2 & 5 & -2 \\ -3 & 6 & -2 \end{bmatrix}$$