Mathematics 307—September 25, 1995

Second homework — due Tuesday, October 10

Exercise 1. If a particle with position vector $\mathbf{i} - \mathbf{j} + 2\,\mathbf{k}$ is rotating clockwise around the axis $x = y = z$ (clockwise as seen looking from this vector towards the origin) with a speed of 1^r per second, what is its linear velocity?

Exercise 2. Let $u = (1, 1, 0)$, $v = (1, 2, 1)$. What is the projection of u onto the line along v? The projection of u onto the plane perpendicular to v? The vector you get by rotating u by $30°$ around the axis along v?

Exercise 3. Suppose that

$$T = I + \Omega\,\Delta t + \text{ terms of order } \Delta t^2$$

is an orthogonal transformation for all t. What condition must Ω satisfy? (Hint. Write out ${}^tT\,T$).

Exercise 4. (a) Find the centre of gravity of a tennis racket. Assume it is constructed by adding a circle of radius 10 cm to a thin handle of length 20 cm, and that the linear density is 1 gm/cm around the circle, 2 gm/cm in the handle. This calculation will use the sum of two integrals, one over each component.

(b) Find its moment of inertia matrix $\mathcal{I}$ with respect to its centre of gravity—its principal axes (clear) and eigenvalues.

Exercise 5. Do the same for a system made up of three objects: (i) mass 3, location $-\mathbf{i}$; (ii) mass 1, location $\mathbf{i} + \mathbf{j}$; (iii) mass 2, location $\mathbf{i} - \mathbf{j}$.

Exercise 6. How does differentiation act on the space of functions of the form

$$a \cos \omega x + b \sin \omega x?$$

Choose a basis and write down the matrix.

Exercise 7. How does differentiation act on the space of polynomials of degree at most n? Choose a basis and write down the matrix.

Exercise 8. Find a formula for

$$\int x^n e^{cx}\, dx$$

by this method.

Exercise 9. Let T be the linear operator

$$Tf = f'' + f$$

acing on the space of functions $P(x)e^{-x}$ where $P(x)$ has degree at most 4. What is its matrix?

Exercise 10. There exists a unique solution of the form $P(x)e^{-x}$ of the differential equation

$$y'' + y = x^4 e^{-x}$$

Find it by this method, considering the operator $y \mapsto y'' + y$ as a linear operator.

Exercise 11. How does $T : f \mapsto f' + f$ act on the space $P(x)e^{-x}$ with P of degree at most 3? Choose a basis and write down the matrix.

Exercise 12. Suppose that $u = (a, b, c)$ is a $3D$ vector. The map taking v to the cross-product $u \times v$ is a $3D$ linear transformation. What is its matrix? What are its eigenvectors and values?