Mathematics 307—December 6, 1995

Third homework — due Tuesday, November 7

Exercise 1. Let

$$M = \begin{bmatrix} 1 & 3 & -1 \\ -1 & 2 & 0 \\ -2 & 1 & 1 \end{bmatrix}$$

Apply the Gram-Schmidt process to write $M = KAN$ where K is an orthogonal matrix, A a diagonal one, N an upper triangular one with 1's along the diagonal.

Exercise 2. Make a vector space from the polynomials of degree at most 5. Put a dot product on this vector space according to the formula

$$P \bullet Q = \int_{-1}^{1} P(x)Q(x)\,dx$$

Start with the basis $1, x, \ldots, x^5$ and apply the Gram-Schmidt process to it to find an orthogonal basis. Then find an orthonormal one.

Exercise 3. Find the WLU factorization of the matrices

$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}, \quad \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}, \quad \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}, \quad \begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{bmatrix}$$

Of the similar 10×10 matrix? $N \times N$?

Find the inverse of the 10×10 matrix in this series

Exercise 4. Find the WLU factorization of the matrices

$$\begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}, \quad \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 4 \end{bmatrix}, \quad \begin{bmatrix} 4 & 1 & 0 & 0 \\ 1 & 4 & 1 & 0 \\ 0 & 1 & 4 & 1 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

Of the similar 10×10 matrix? $N \times N$?

Find the inverse of the 10×10 matrix in this series

Exercise 5. Find the WLU factorization of the matrix

$$\begin{bmatrix} 1 & 1/2 & 1/3 \\ 1/2 & 1/3 & 1/4 \\ 1/3 & 1/4 & 1/5 \end{bmatrix}$$

Do this by hand, and then by computer. How different are the answers?

Exercise 6. Find the WLU factorization of

$$\begin{bmatrix} 2 & 3 & 1 \\ 2 & 2 & 2 \\ -6 & -5 & -8 \end{bmatrix}$$

showing essentially all the intermediate steps. Use class rules for finding the pivot.