Mathematics 307—December 5, 1995

Fourth homework — due Thursday, November 30

Exercise 1. Find the solutions of

$$\begin{aligned} 0.999x + y &= 1.000 \\ x + 0.999y &= 0.999 \end{aligned}$$

and then

$$\begin{aligned} 0.999x + y &= 0.999 \\ x + 0.999y &= 1.000 \end{aligned}$$

and explain carefully why the answers are so different.

Exercise 2. Find the singular value decomposition of the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{bmatrix}$$

Exercise 3. Find the eigenvalues and eigenvectors of the matrix

$$\begin{bmatrix} 1 & 1/2 & 1/3 & 1/4 \\ 1/2 & 1/3 & 1/4 & 1/5 \\ 1/3 & 1/4 & 1/5 & 1/6 \\ 1/4 & 1/5 & 1/6 & 1/7 \end{bmatrix}$$

by Jacobi's method, showing all intermediate steps.

Exercise 4. Find the eigenvalues and eigenvalues of the matrix

$$\begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{bmatrix}$$

by Jacobi's method.

Exercise 5. Find the highest eigenvalue of the 5×5 Hilbert matrix by the power method, correct to 8 decimals. How many iterations would it take to find it correctly to 12 decimals?

Exercise 6. Draw the curves

$$x^2 + 2xy + 3y^2 = 1, \quad x^2 - 2xy + 3y^2 = 1$$

Exercise 7. Write down the full expression for the determinant of

$$\begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} \\ a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} \\ a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} \\ a_{4,1} & a_{4,2} & a_{4,3} & a_{4,4} \end{bmatrix}$$

Exercise 8. If you apply Gaussian elimination to a tridiagonal $n \times n$ matrix, and you don't have to do any swaps, how many multiplications can you expect to perform? If you apply Gaussian elimination to an arbitrary $n \times n$ matrix?

Exercise 9. Find the generalized eigenvalues and eigenvectors of the problem

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} v = \lambda \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} v$$

Exercise 10. Explain why the matrix

$$\begin{bmatrix} 4 & 1 & 0 & 0 & 0 & 0 \\ 1 & 4 & 1 & 0 & 0 & 0 \\ 0 & 1 & 4 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 & 1 & 0 \\ 0 & 0 & 0 & 1 & 4 & 1 \end{bmatrix}$$

is positive definite.