Mathematics 307—November 27, 1995

Hints for the last home work

1. Use the fundamental principle of the course.

2. If A is any matrix then $B = A\,{}^tA$ is a symmetric matrix, and is positive definite. Its eigenvalues are real and positive. Therefore $B = XD^2X^{-1}$ where X is orthogonal and D is diagonal — its entries are positive square roots of the eigenvalues of B. We can rewrite this equation as

$$D^{-1}X^{-1}A\,{}^tAXD^{-1} = I$$

or

$$Y^tY = I$$

where

$$Y = D^{-1}X^{-1}A, \quad A = XDY$$

so that Y is also orthogonal. This is called the **singular value decomposition** of A. This work is much simpler, incidentally, if A itself is symmetric.

3. and 4. If you cannot finish the job, at least do a few steps carefully. I expect many of you to write a simple program to help you do these, but it is not necessary if you think carefully about how to save work.

5. Be a bit careful about exactly when you can stop if you want 8 decimals.

6. No problem.

7. I want 24 terms with a $\pm$ attached.

8. Do the job in your mind, and count each time you multiply two numbers. Count a division also as a multiplication.

9. This is not hard since M is already diagonal.

10. Positive definite means that the eigenvalues are are positive, or also that when you factor the matrix as LD^tL the entries of D are all positive, since this means you can find a coordinate system in which the quadratic function determined by A is a sum of squares. But it will also be positive definite if you can write the quadratic function in any way as a sum of squars, and this is the simplest thing to do here. So start by writing down the quadratic function associated to A.

For example, the matrix

$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

determines the quadratic function

$$2x^2 - 2xy + 2y^2 = x^2 + y^2 + (x-y)^2$$

is positive definite on these grounds.

If you want your last assignment graded, you must hand it in on Thursday and on time! Hand in what you can on time and hand in the rest later, if necessary. I will count late work as done but will probably not be able to grade it numerically.