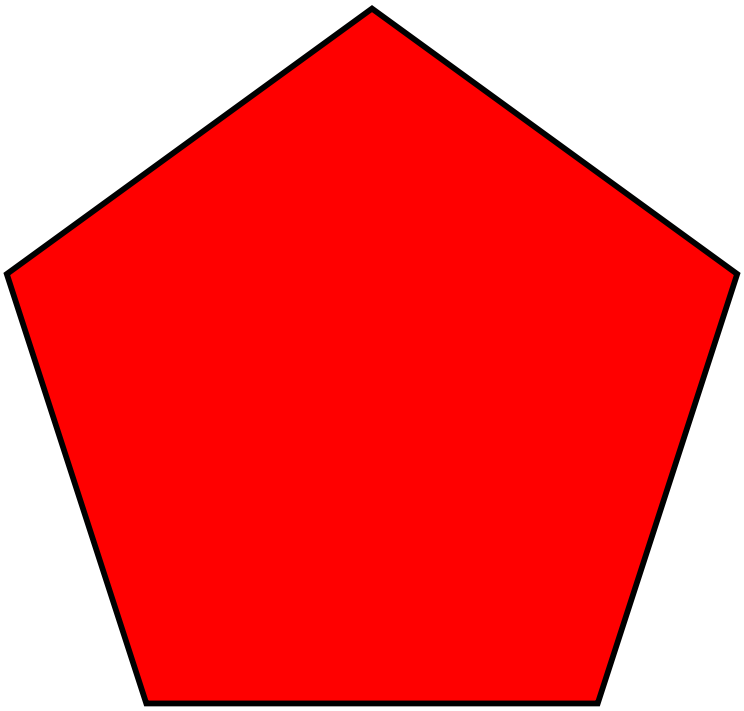
Euclid’s Elements Book 13

Proposition 7

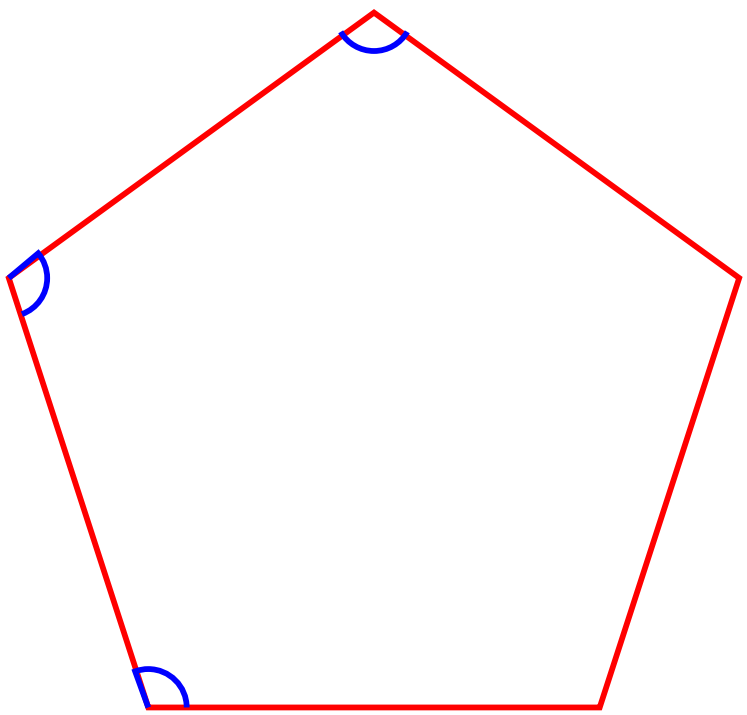
If three angles of an equilateral pentagon, taken in order or not in order, are equal, then the pentagon is equiangular.

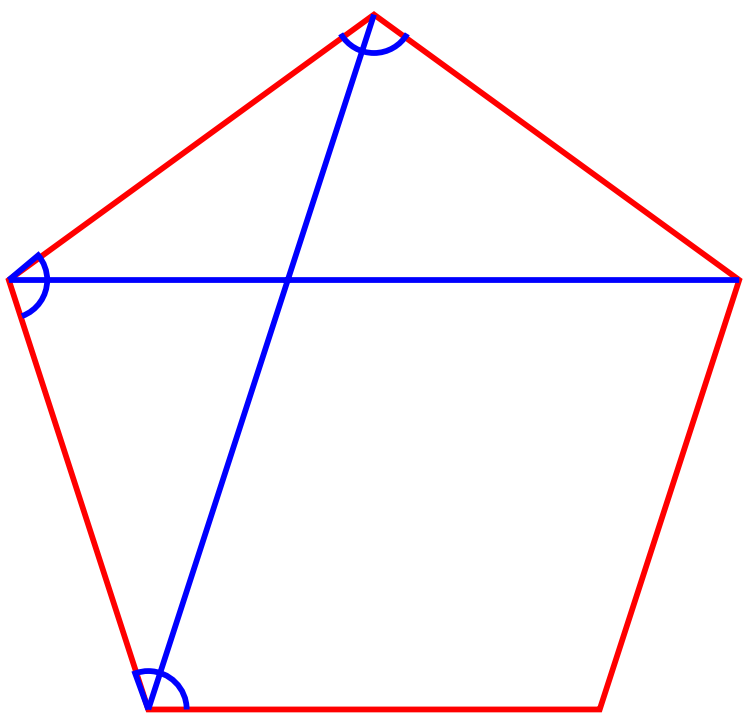
This proposition is used in proposition 17 for the construction of a dodecahedron in a sphere. The faces of a dodecahedron are equiangular and equilateral pentagons.

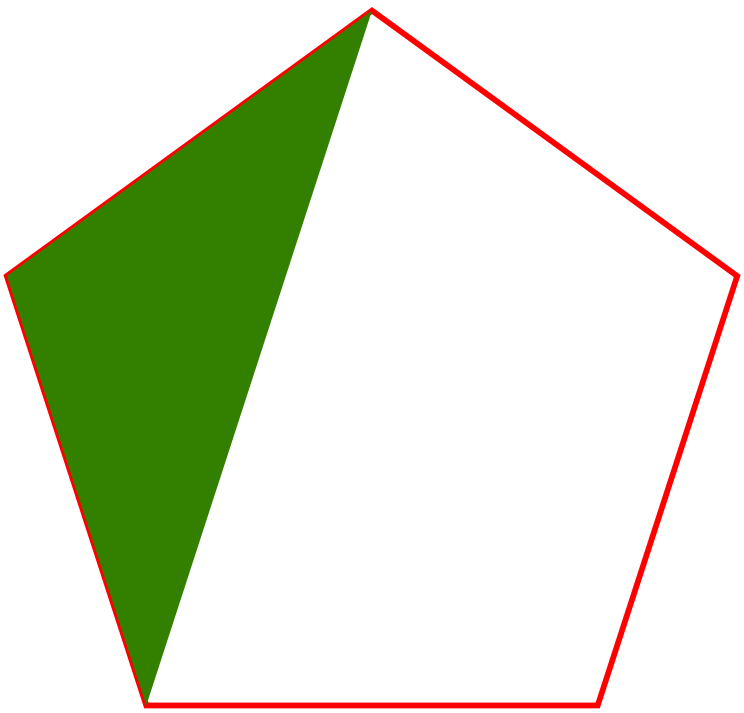
Equilateral pentagon.

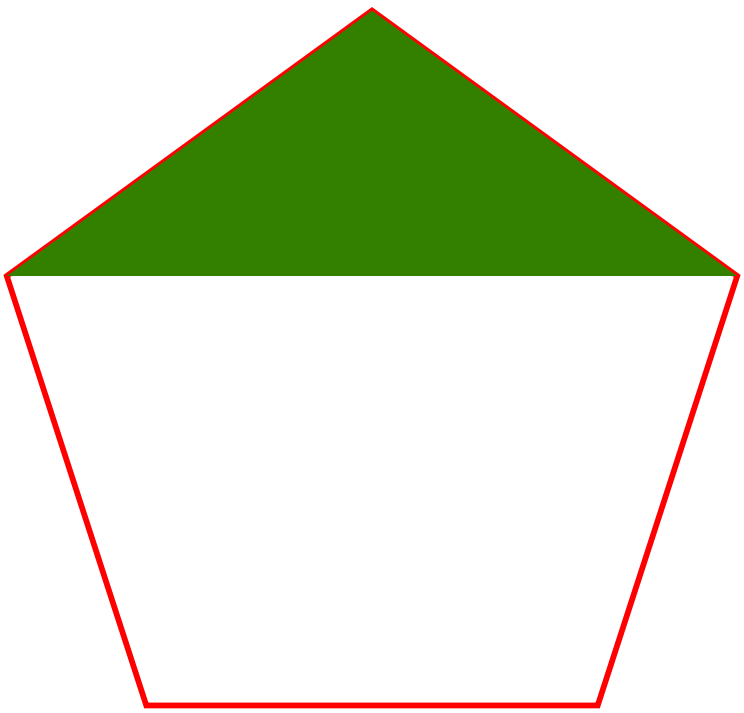


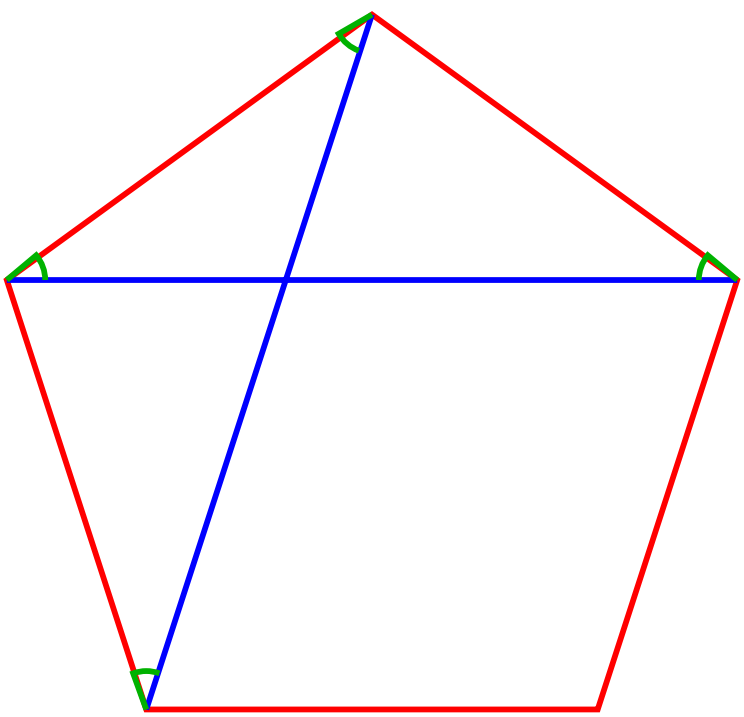
1. Angles are in order.

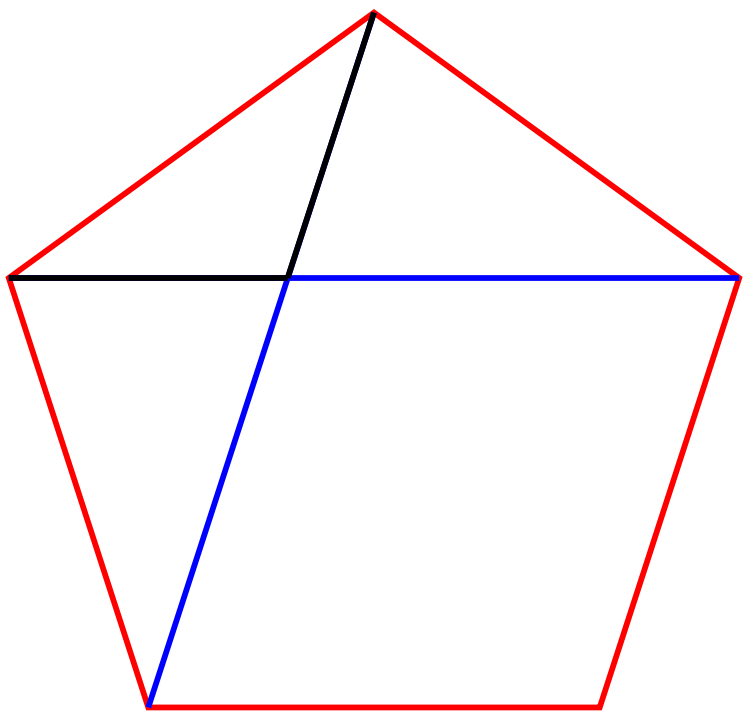


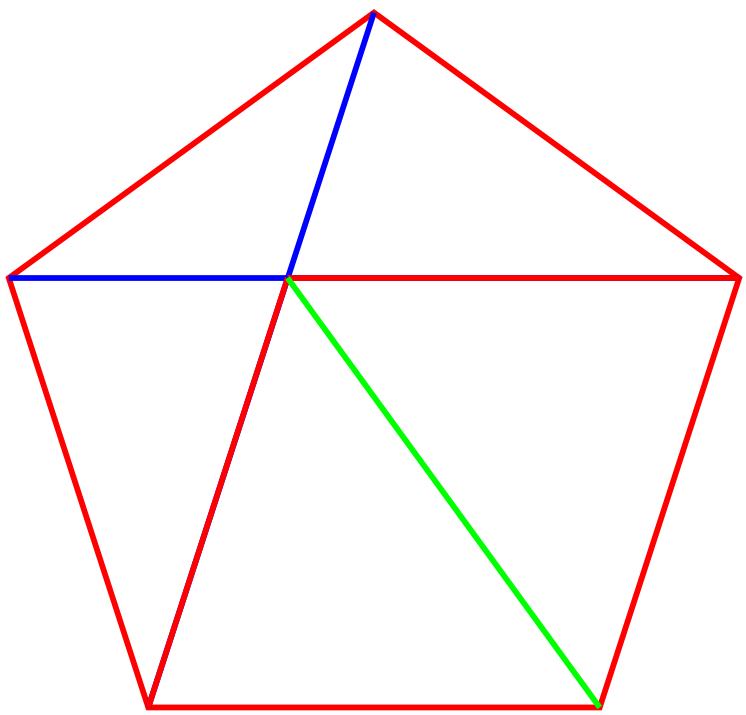


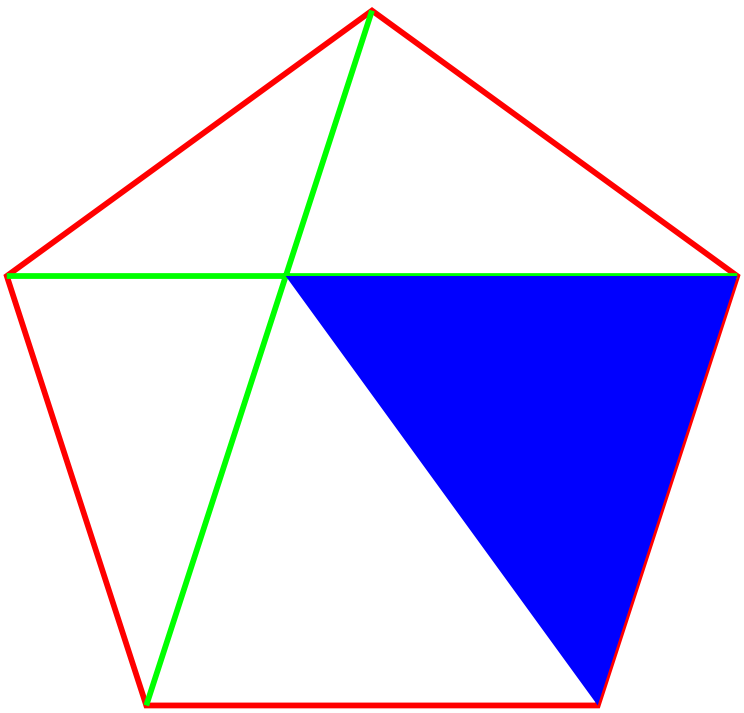


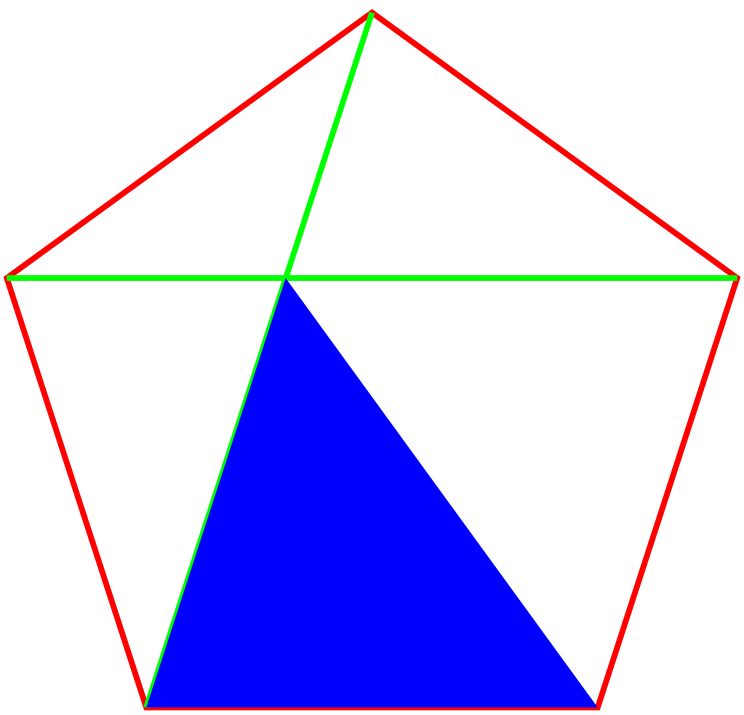


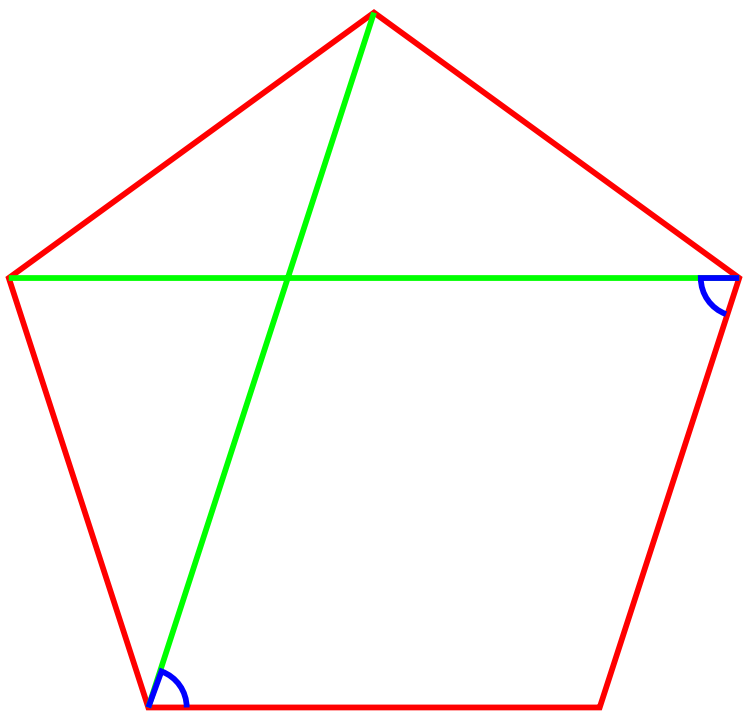


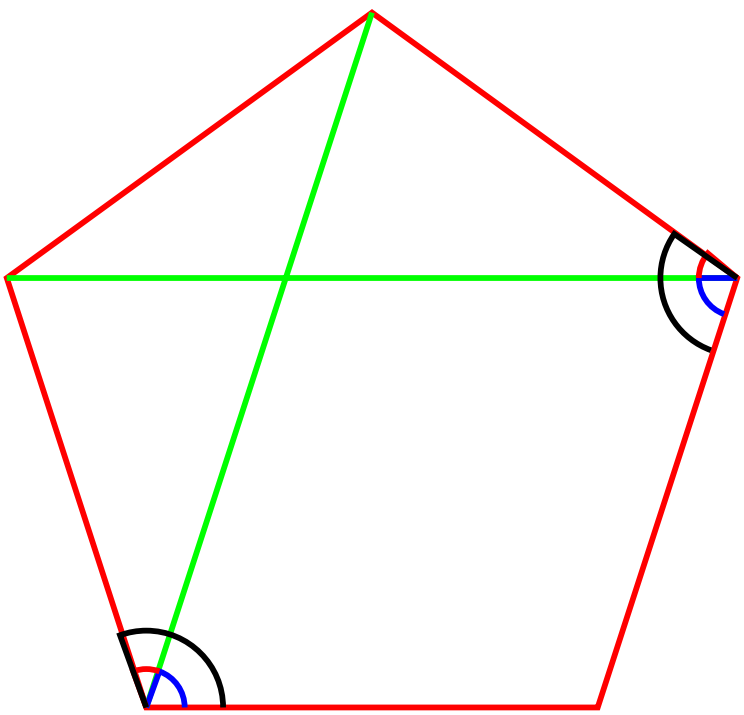


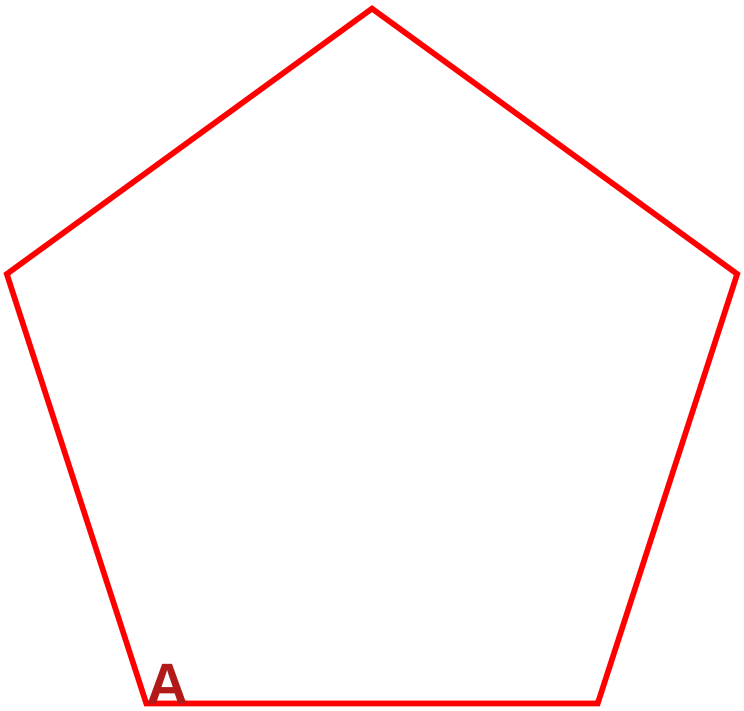






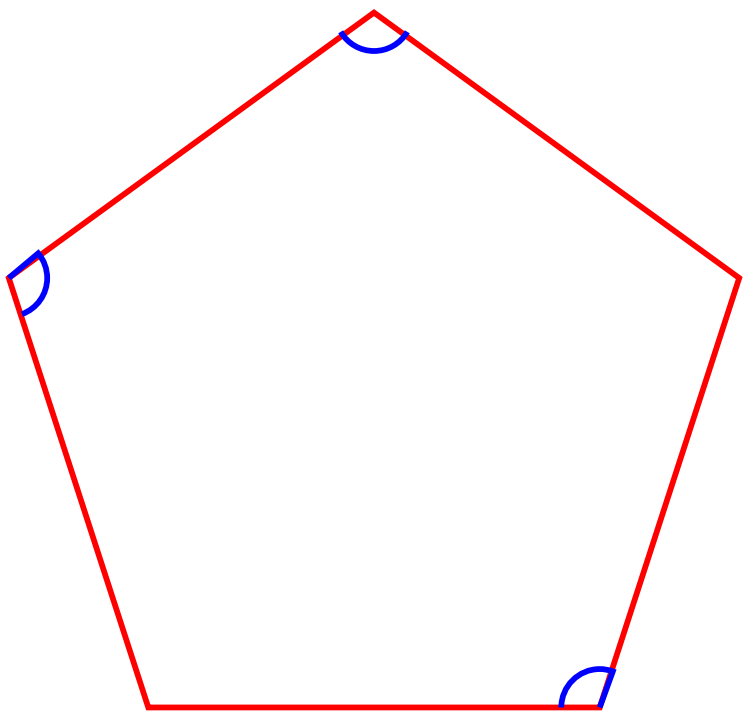


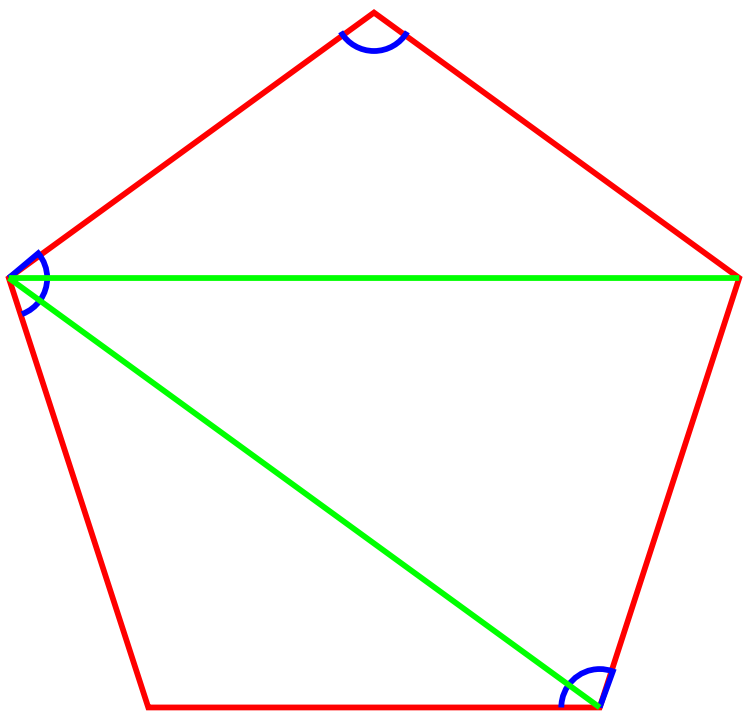


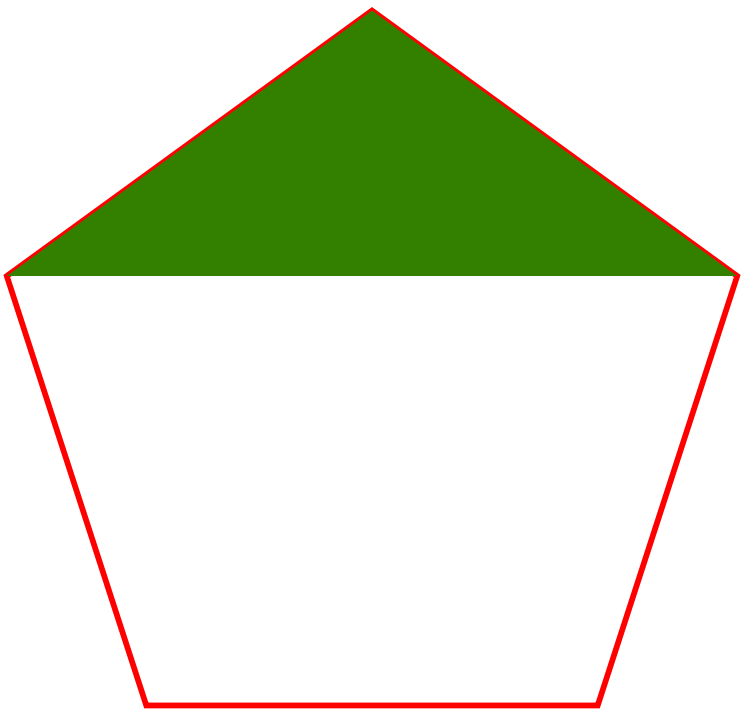


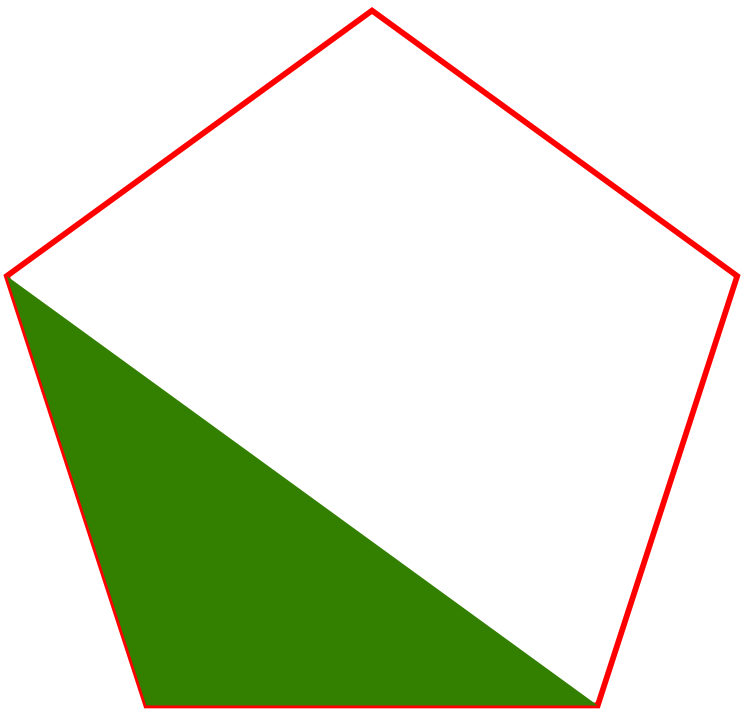
Same proof can show that A is equal to other angles.

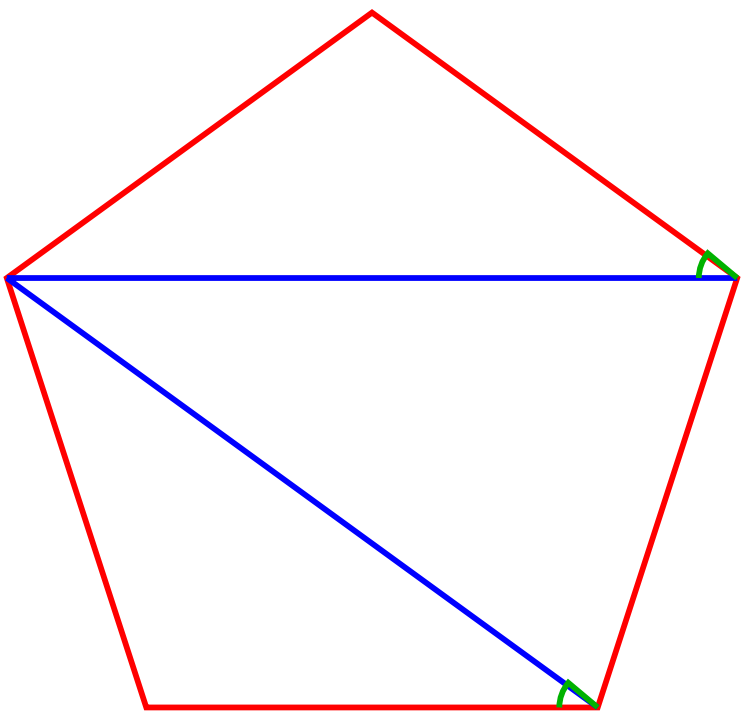
2. Angles are not in order.

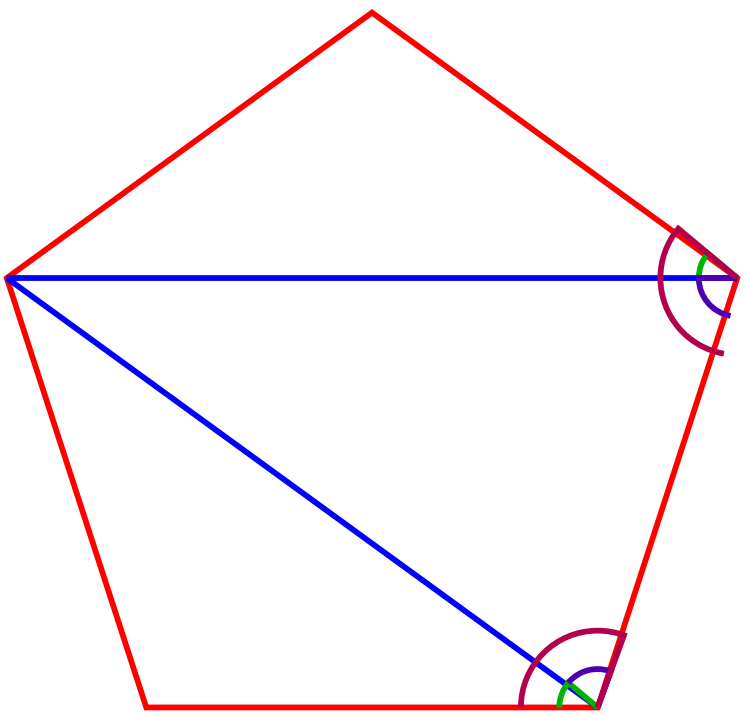


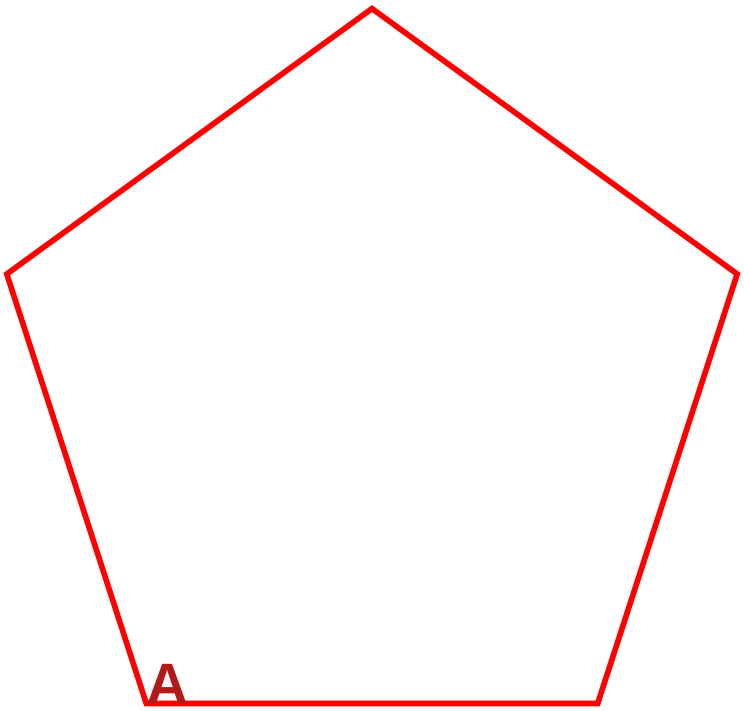






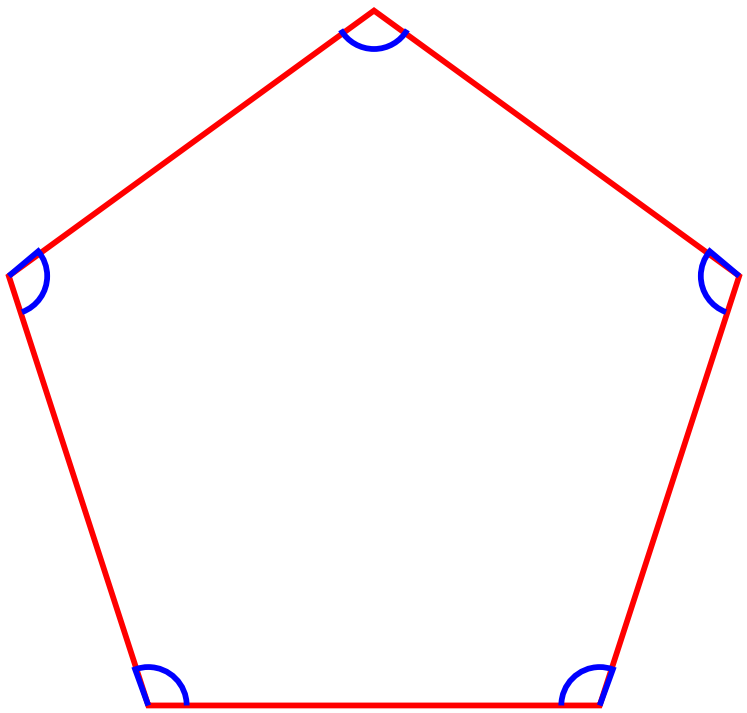






Same proof can show that A is equal to other angles.

All angles are equal.



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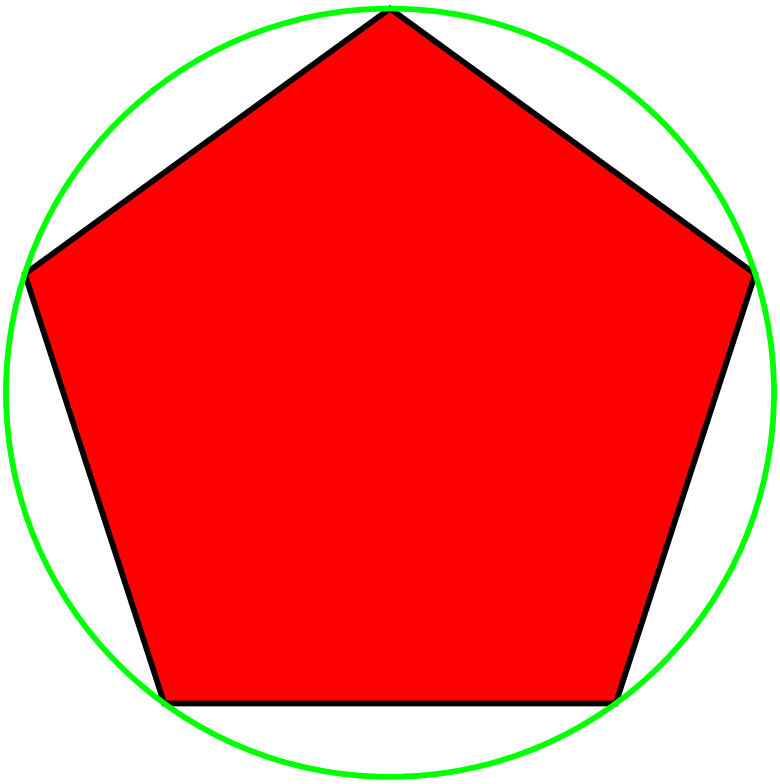
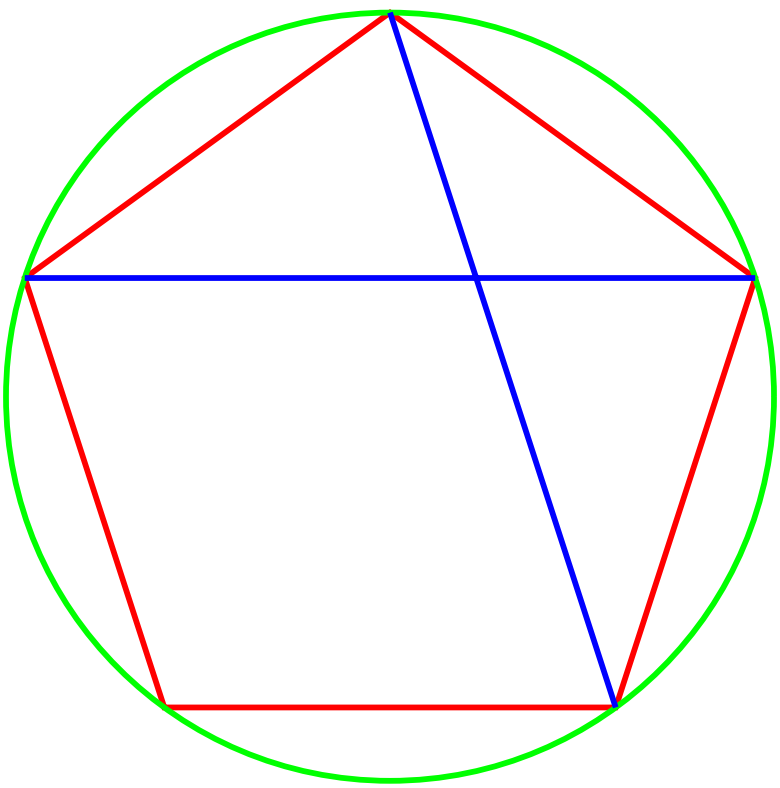
Proposition 8

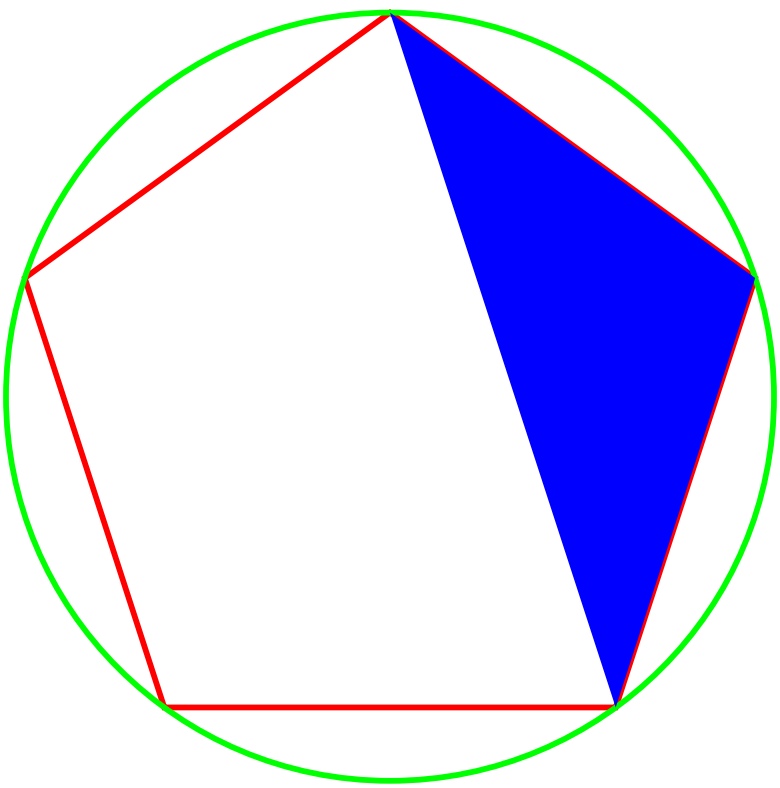
If in an equilateral and equiangular pentagon straight lines subtend two angles are taken in order, then they cut one another in extreme and mean ratio, and their greater segments equal the side of the pentagon.

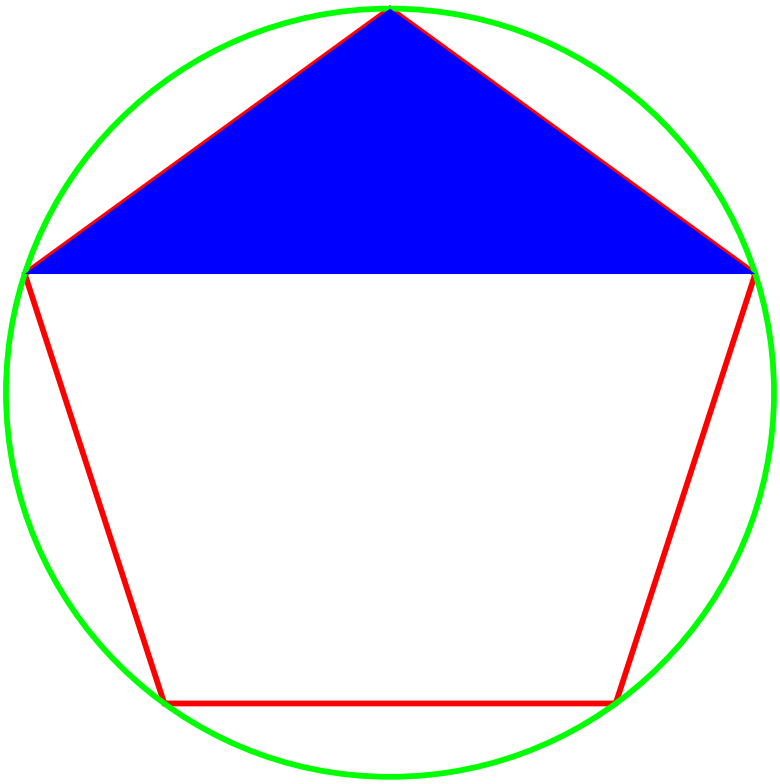
This proposition is used in propostion 11 which says that the equilateral pentagon inscribed in a circle which has its diameter rational, then the side of the pentagon is the irrational straight line called minor. That proposition is later used in proposition 16 that constructs dodecahedron in a sphere. It also proves that the square on the side of the icosahedron is the irrational straight line called minor.

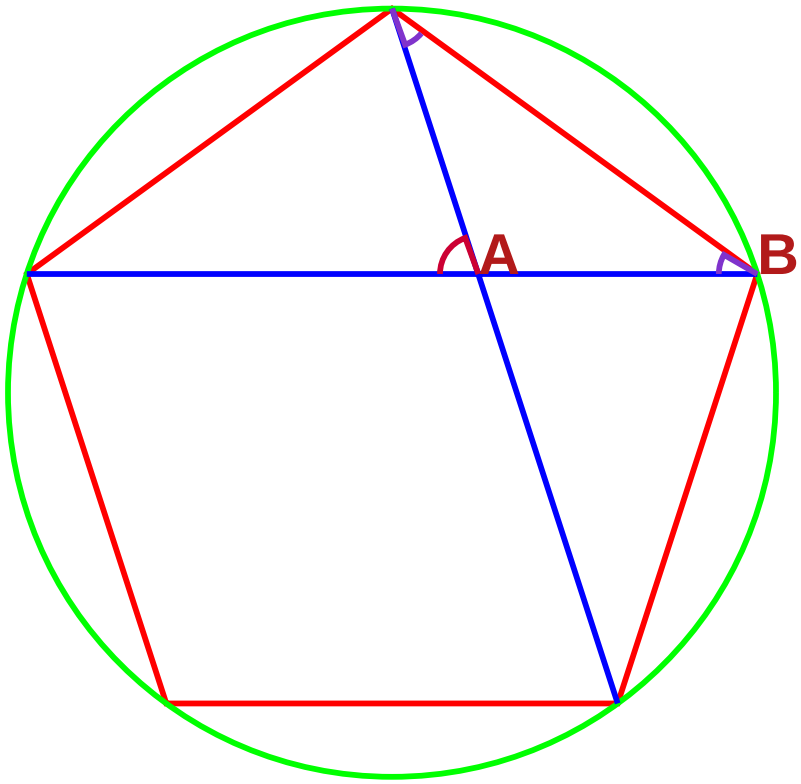
Equilateral and equiangular pentagon.



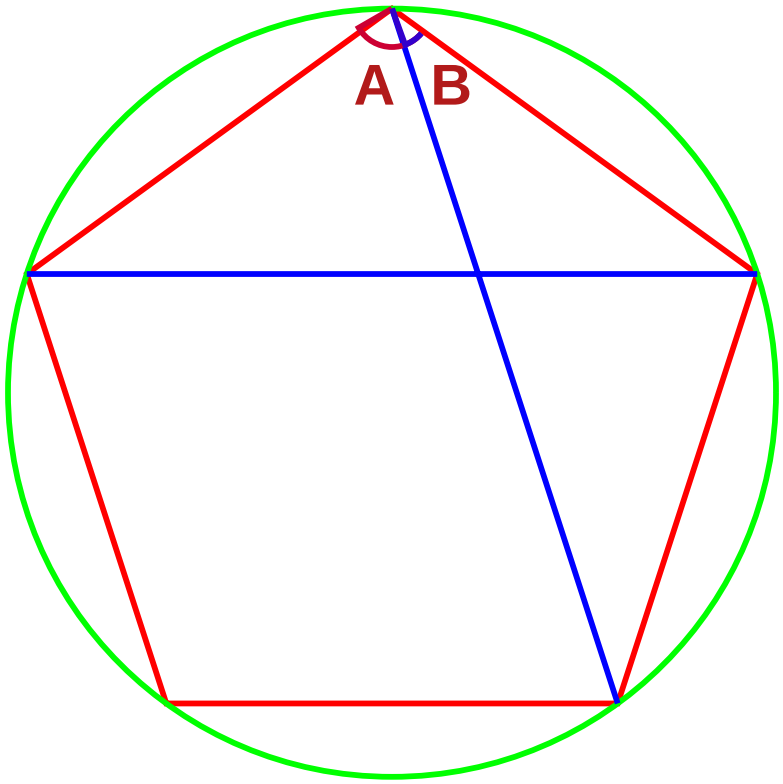




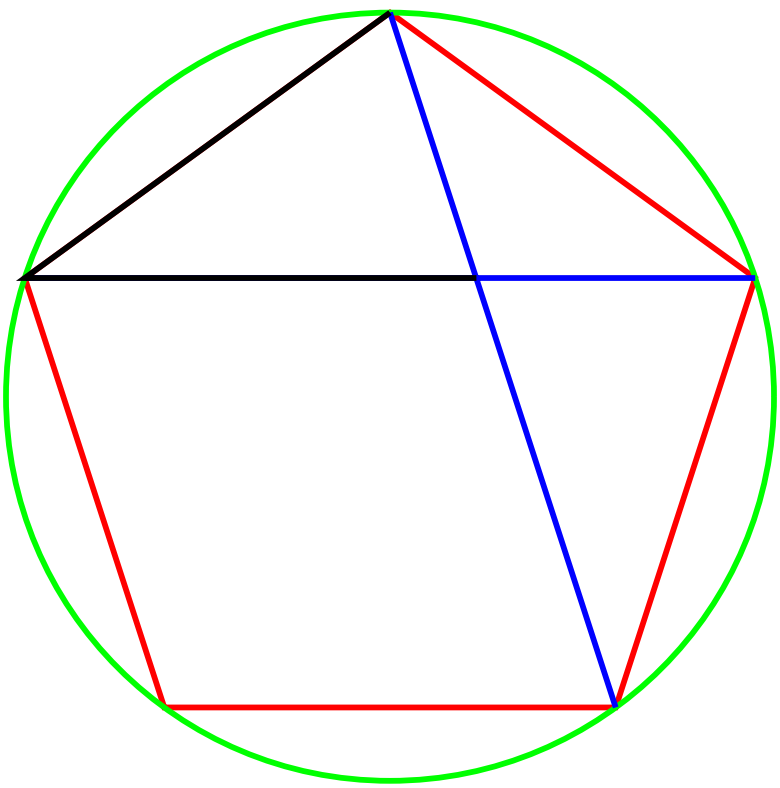


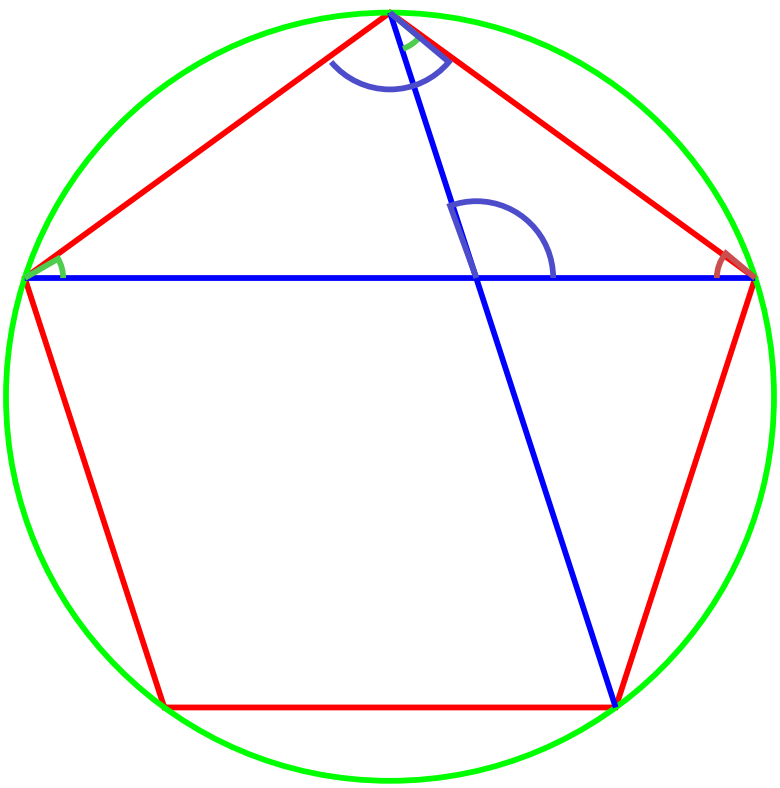


Angle A is double angle B.

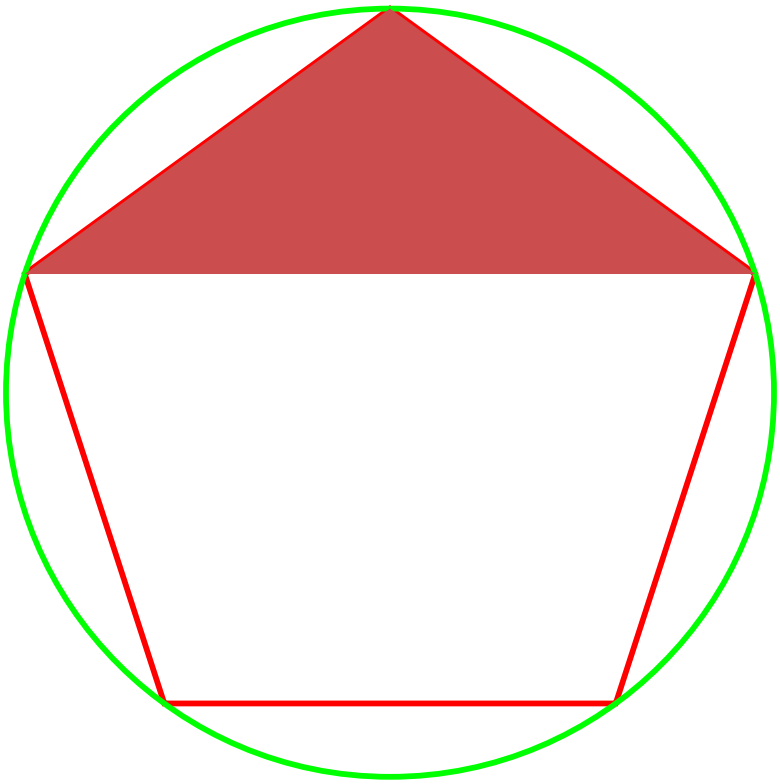


Angle A is double anlge B.

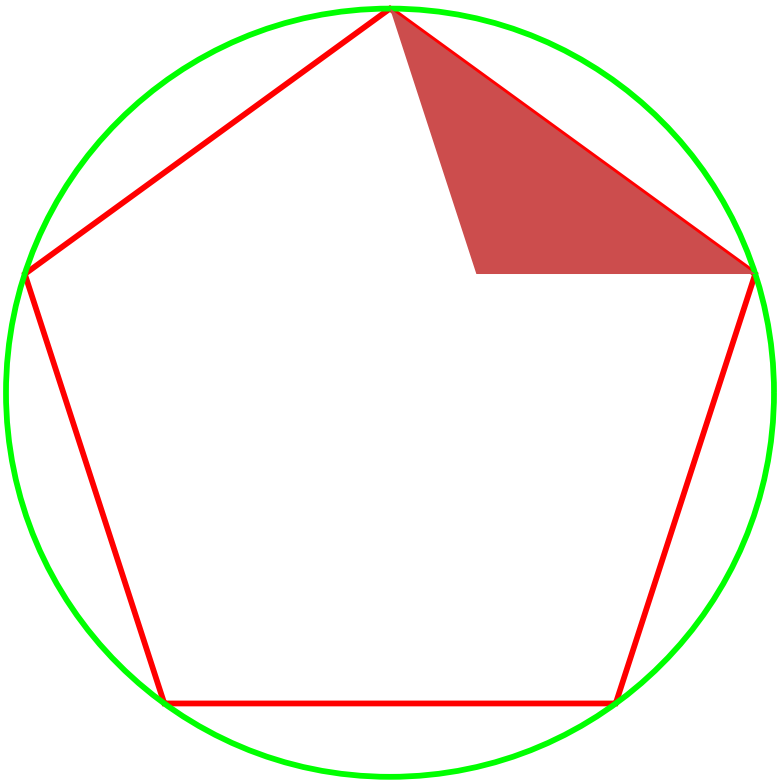




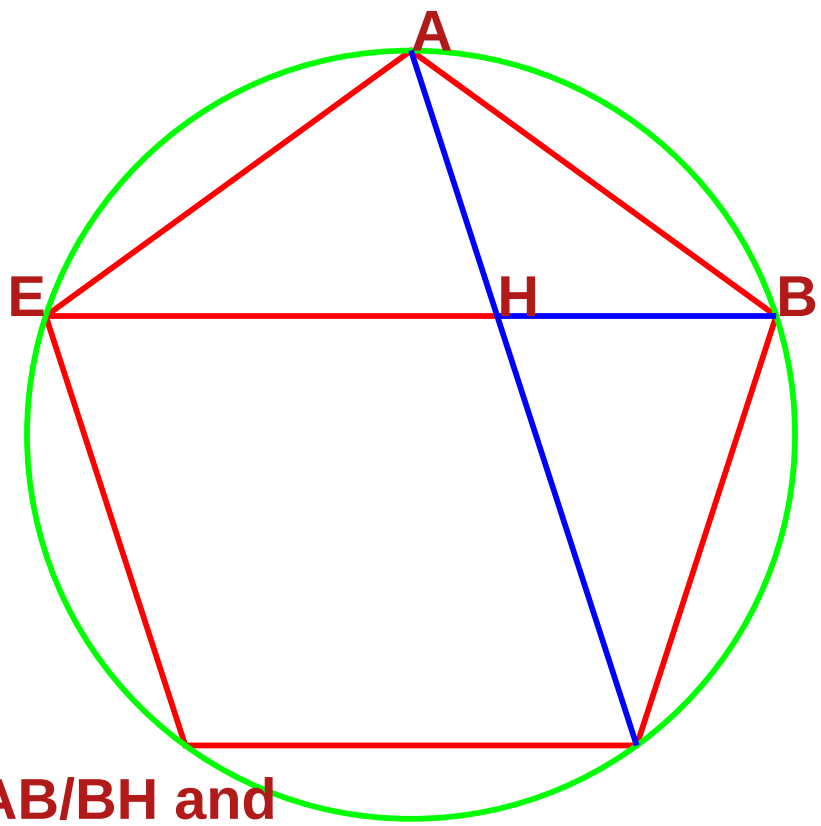
Triangle A.



Triangle B.



Triangles A and B are equiangular.



EB/BA as AB/BH and

since BA=EH,

BE/EH as EH/HB

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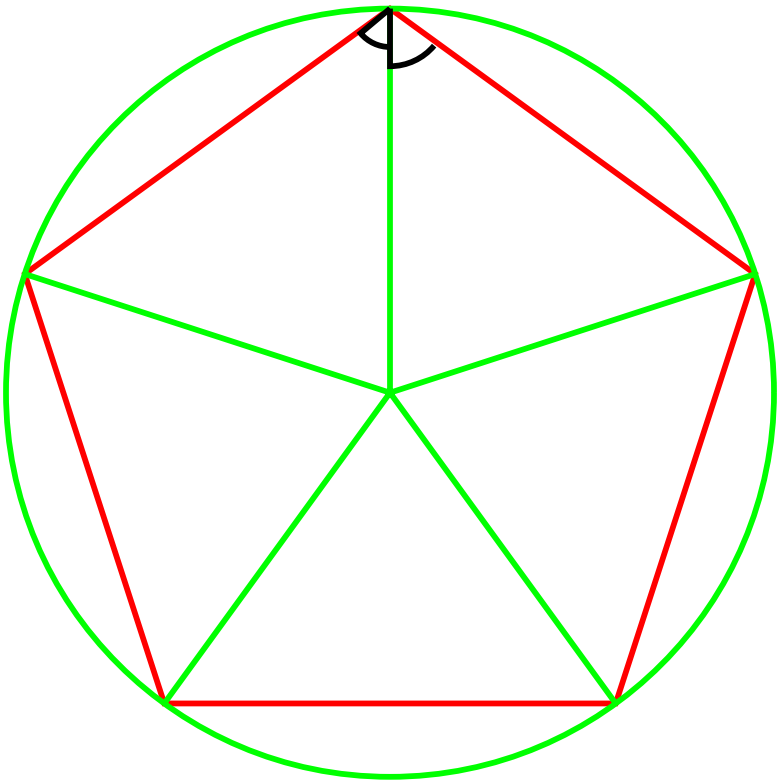
Proposition 18, Lemma

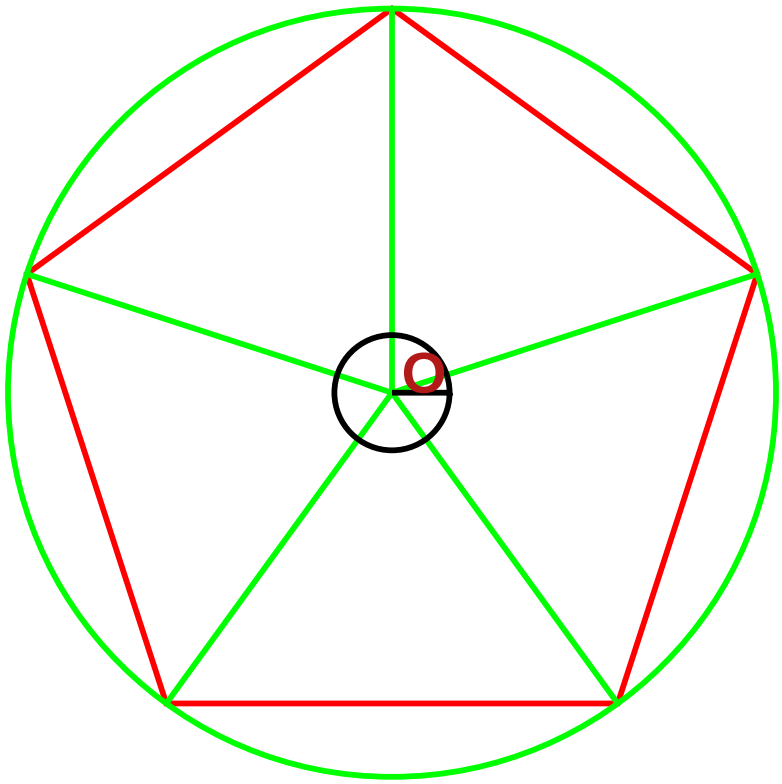
The angle of the equilateral and equiangular pentagon is a right angle and a fifth.

This lemma is proved in proposition 18 which compares the sides of the five regular polyhedra and says that no other figure contained by equilateral and equiangular figures equal to one another can be constructed.

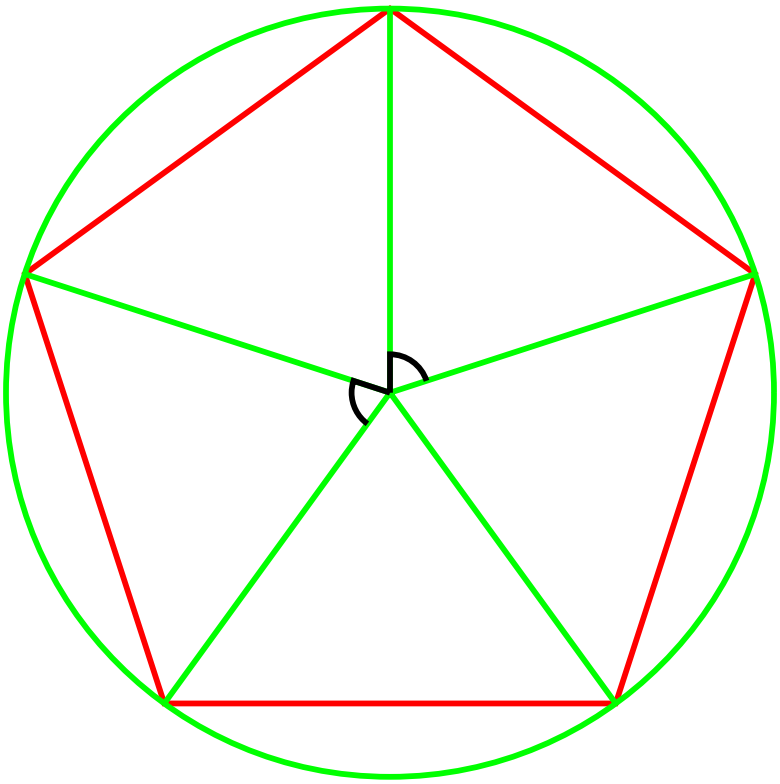
Therefore, tetrahedron, octahedron, cube, icosahedron and dodecahedron are the only such figures.



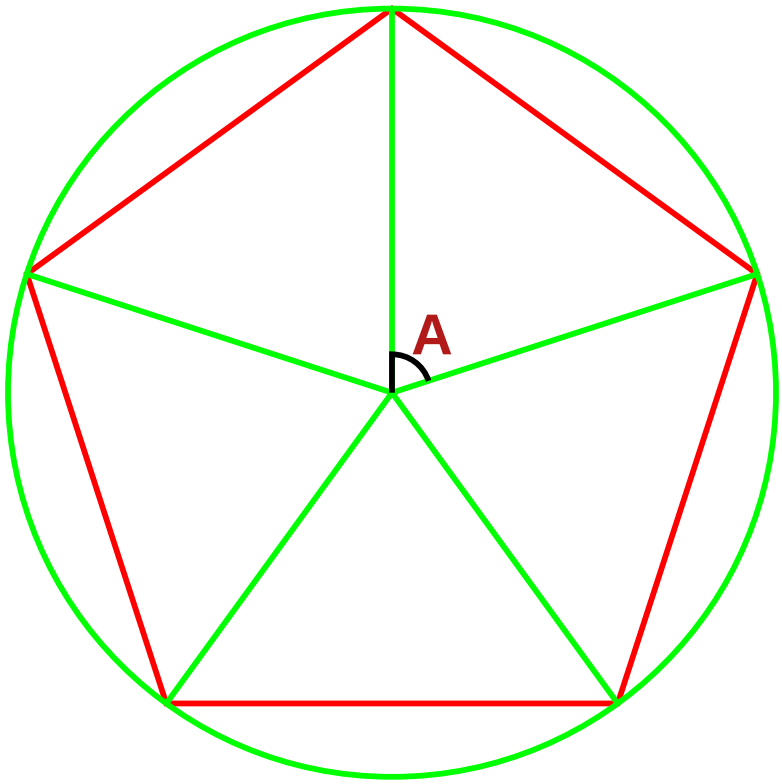
Every anlge of the pentagon is bisected.



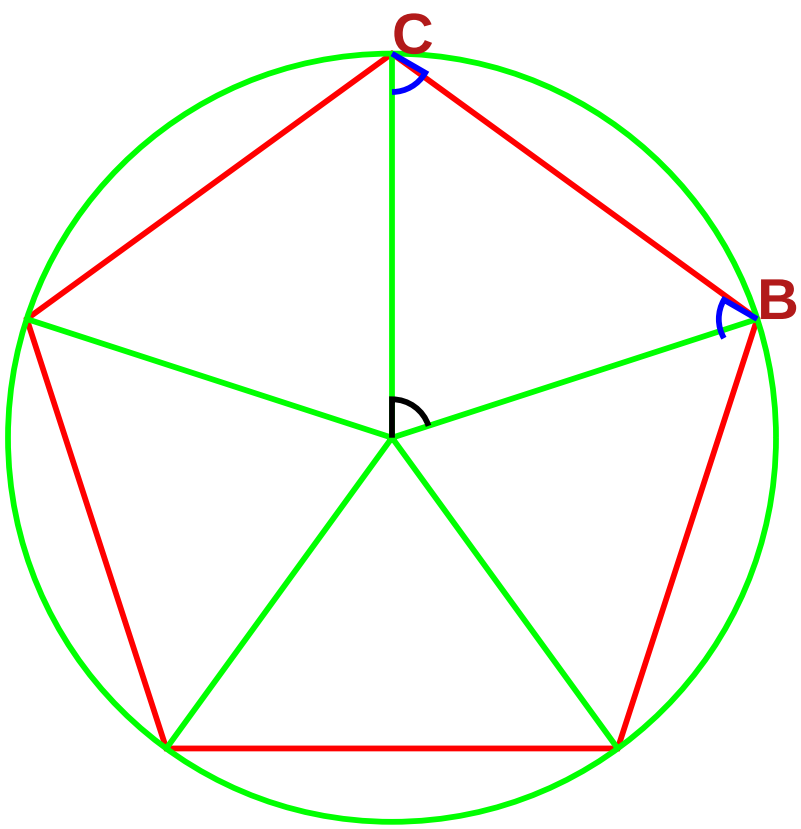
All angles at 0 equal to 360.



For any pair of angles, they are equal one to another.



A is a right angle less a fifth.



Each of B, C is equal to a right angle and a fifth.

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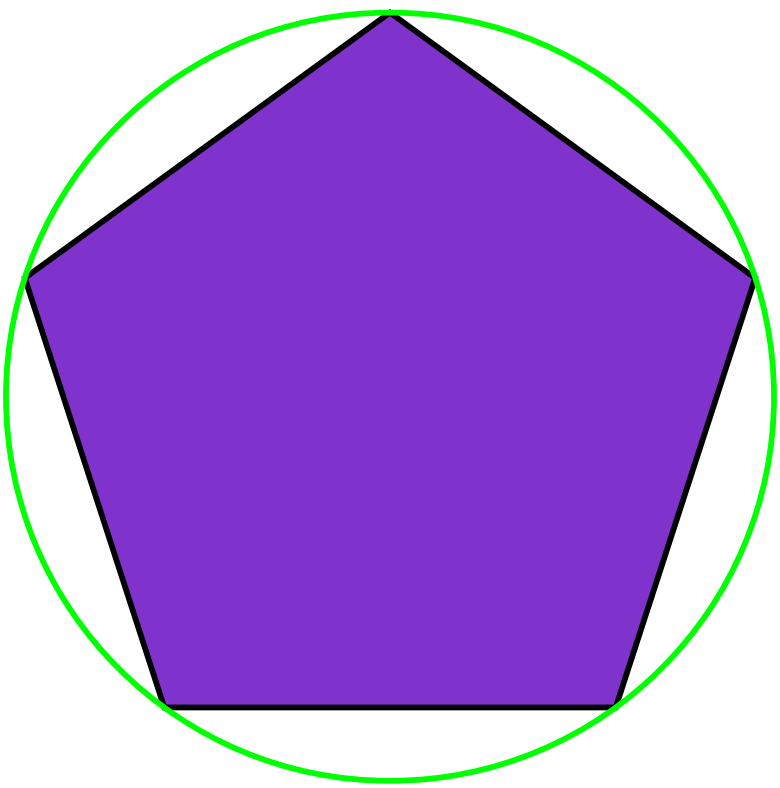
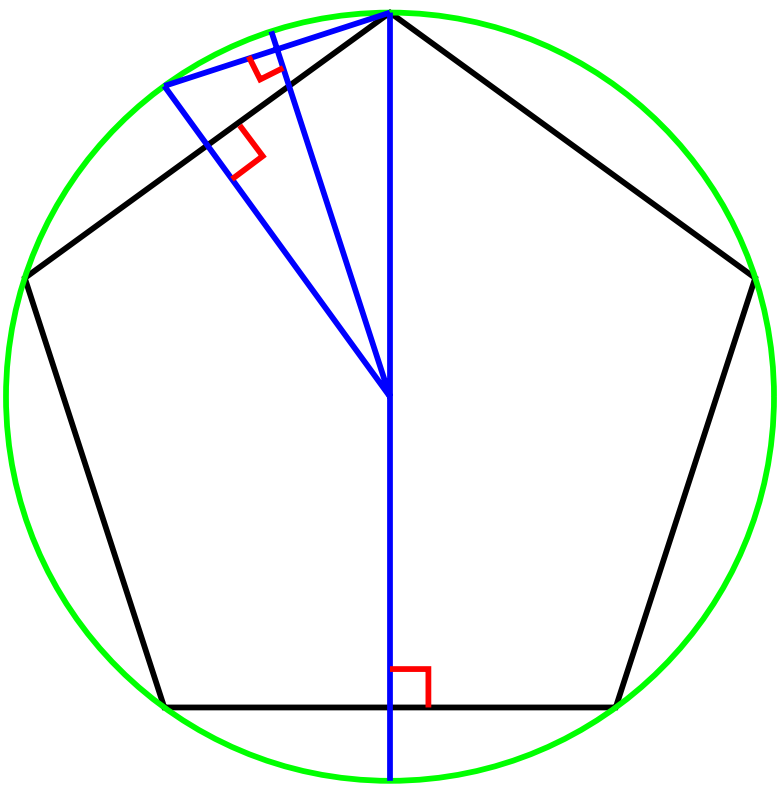
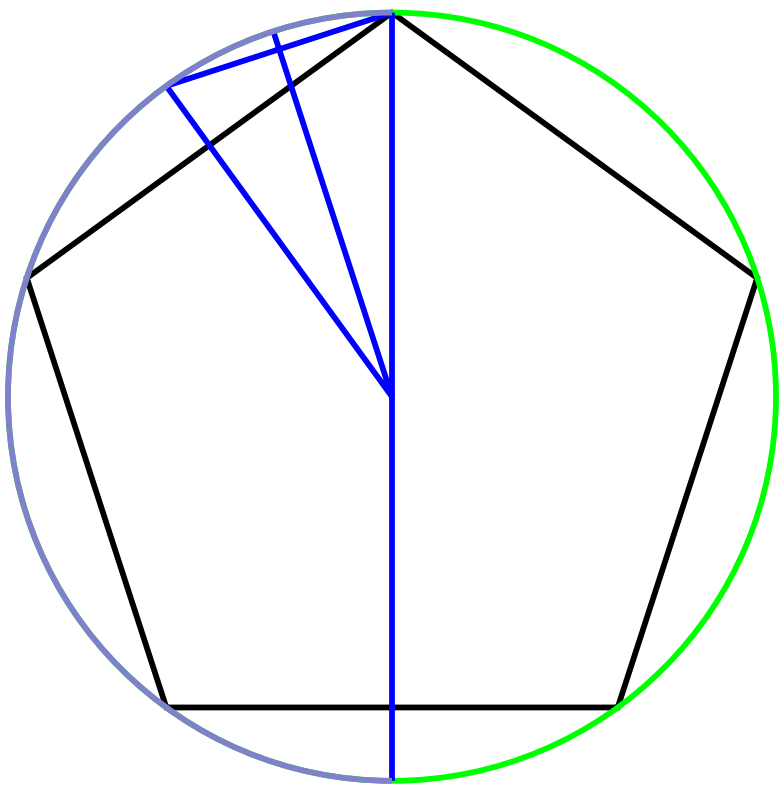
Proposition 10

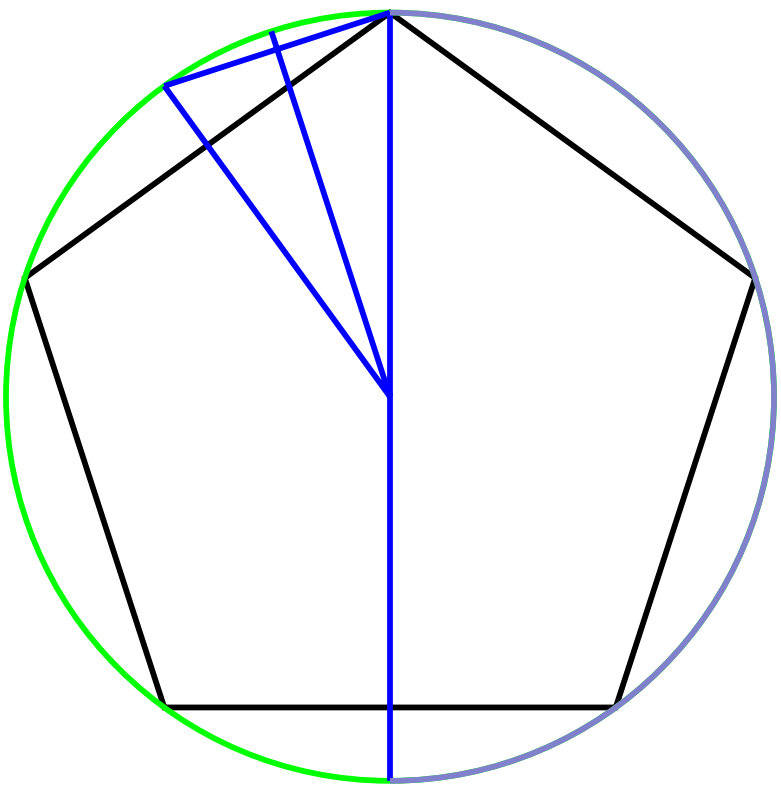
If an equilateral pentagon is inscribed in a circle, then the square on the side of the pentagon equals the sum of the squares on the sides of the hexagon and the decagon inscribed in the same circle.

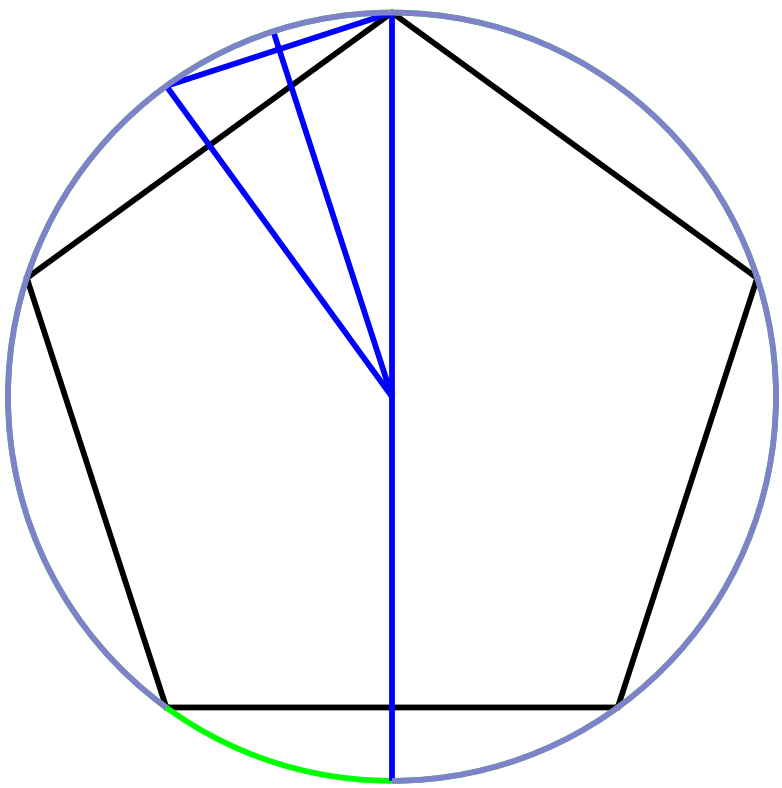
This proposition is used in both propostions 16 and 18. Prop. 16 constructs icosahedron in a sphere, and prop. 18 compares icosahedron to the other 4 regular polyhedra. It is interesting that pentagon is used to construct the polyhedra whose faces are equilateral triangles, but not in prop. 17 which constructs dodecahedron which has equlateral pentagons as its faces.

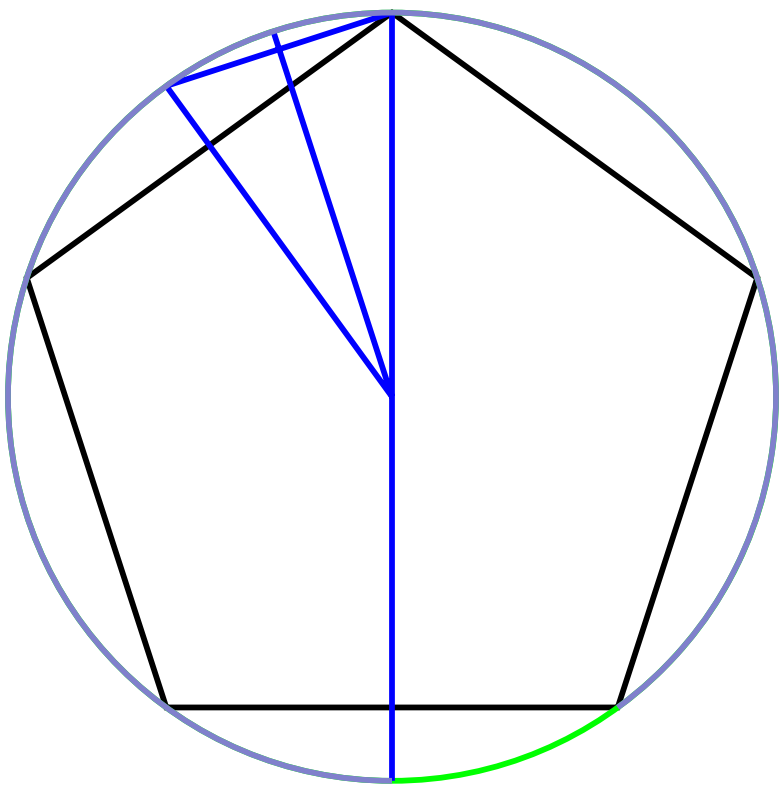


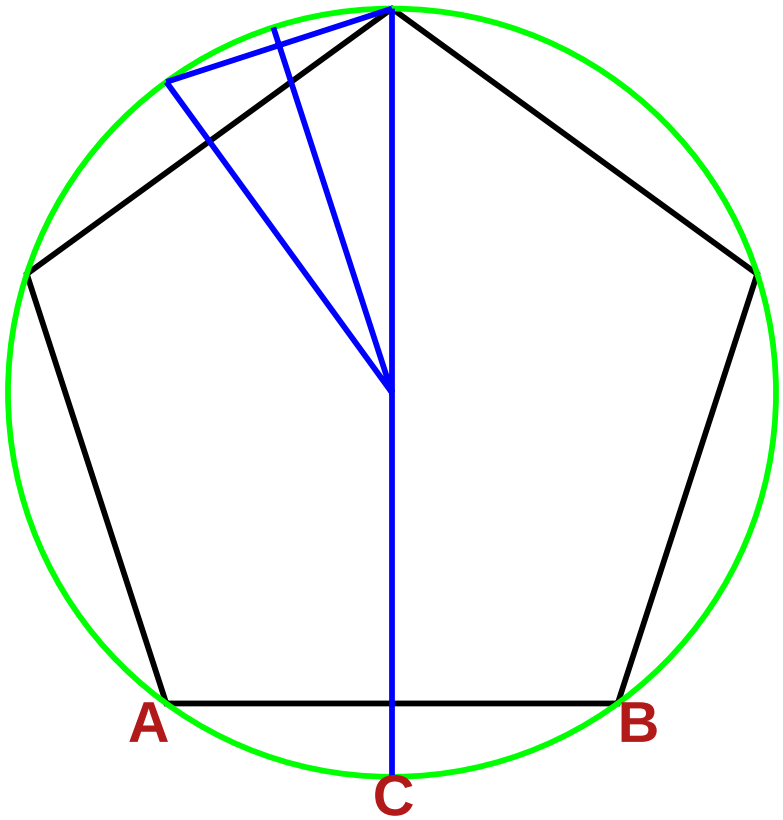




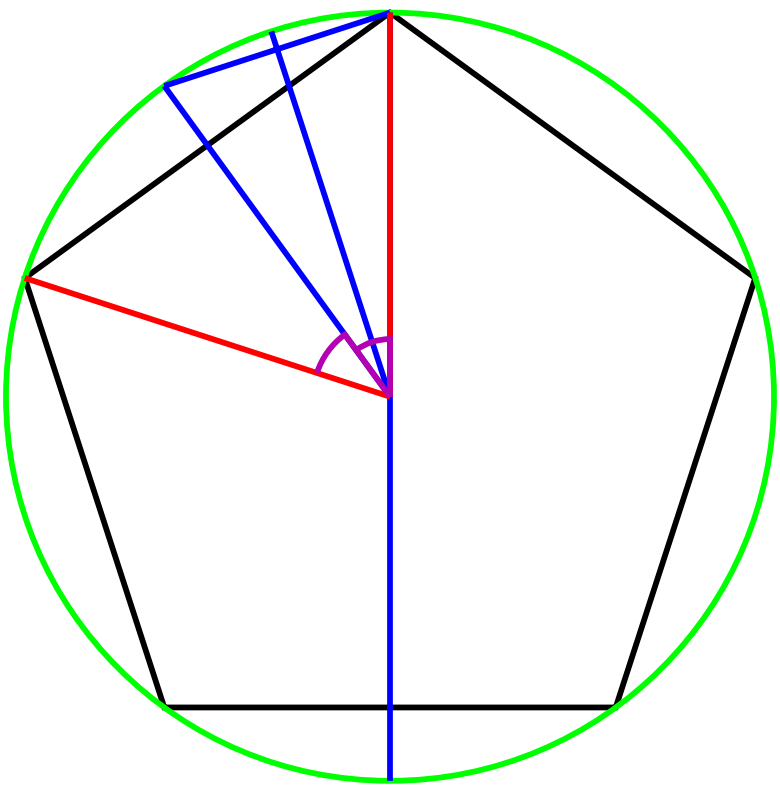


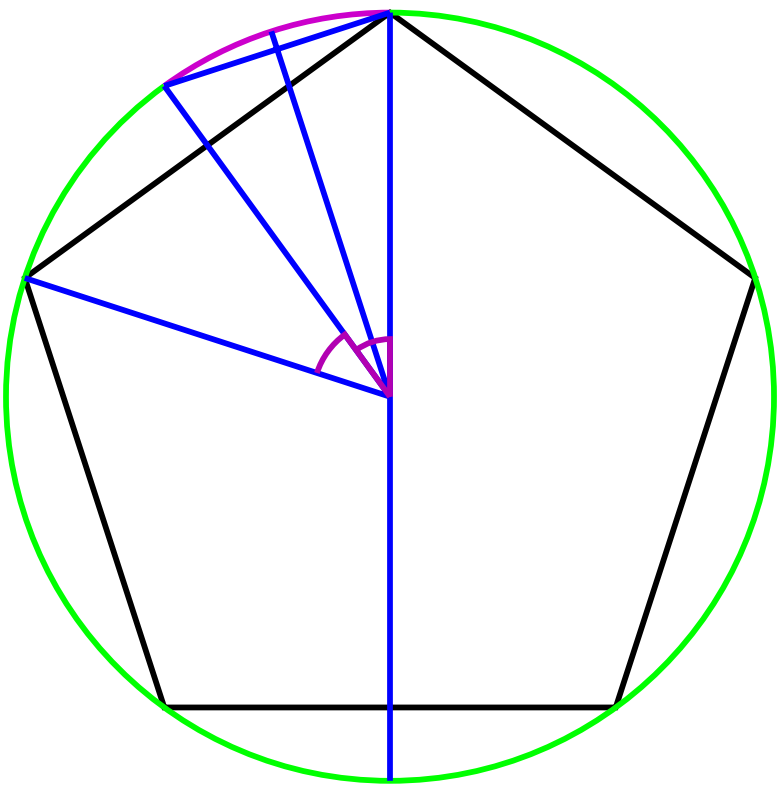


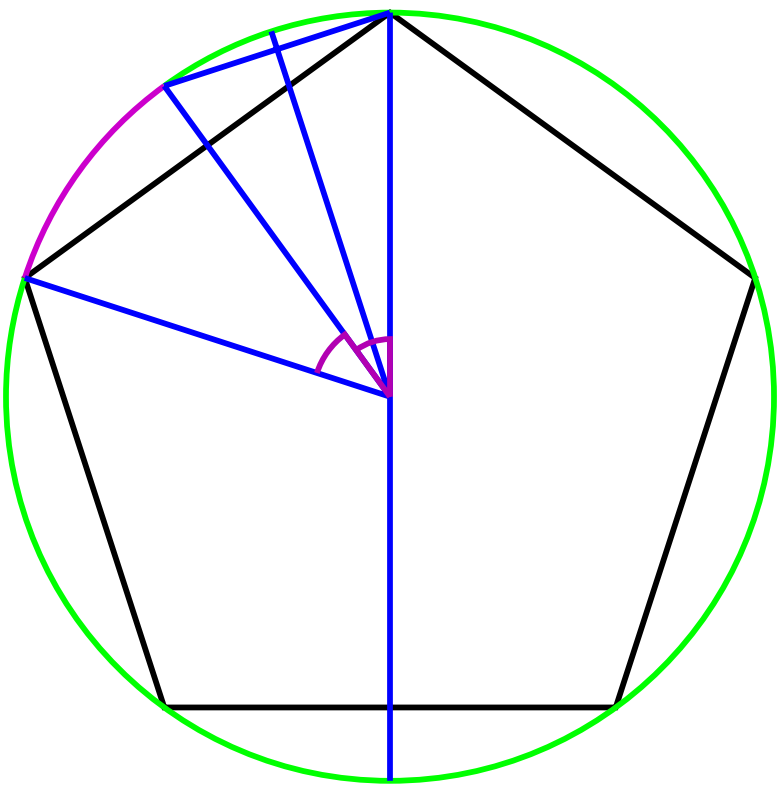


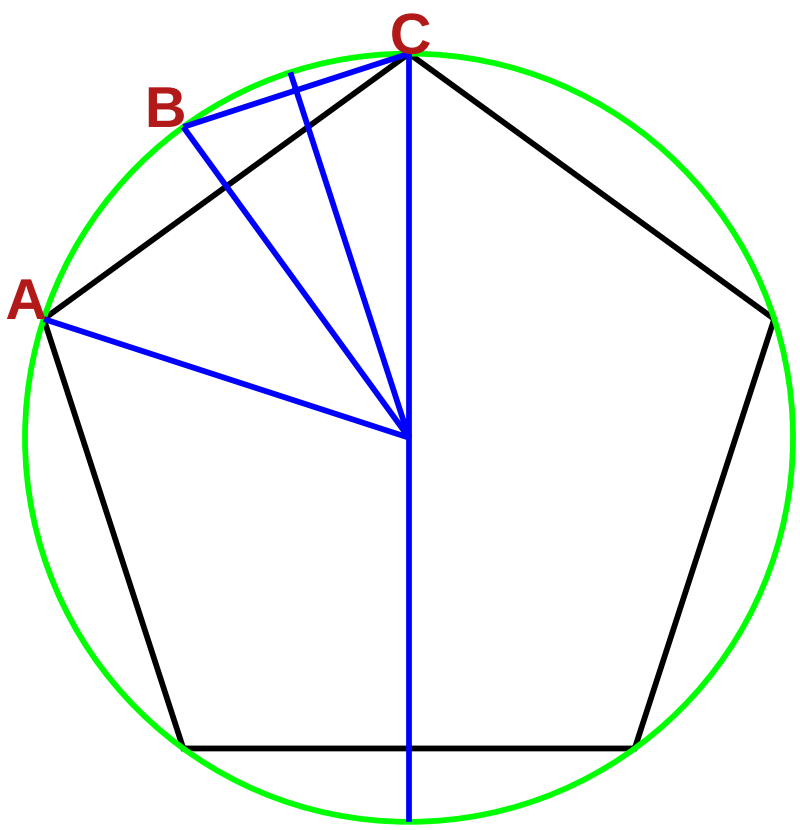


AB belongs to a pentagon; therefore, CB=AB/2 belongs to a decagon.

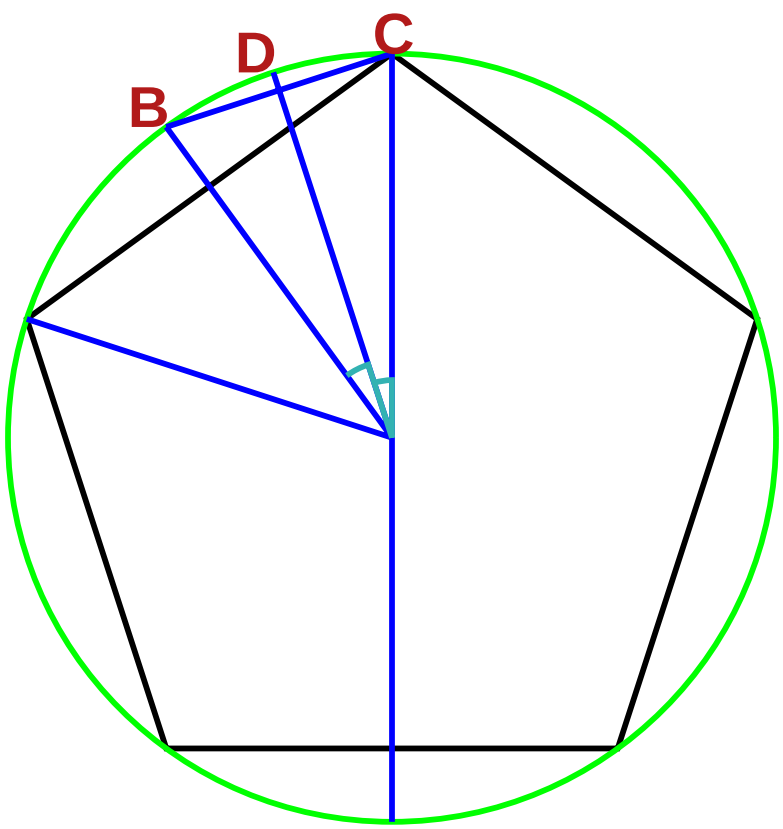






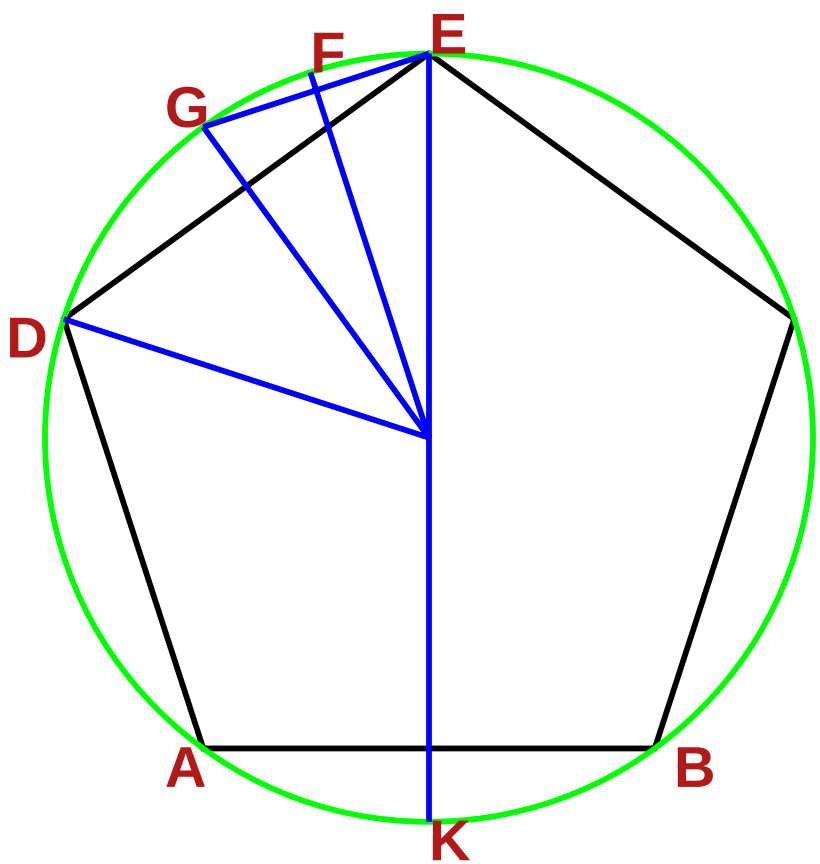


Circumference AC is double AB.



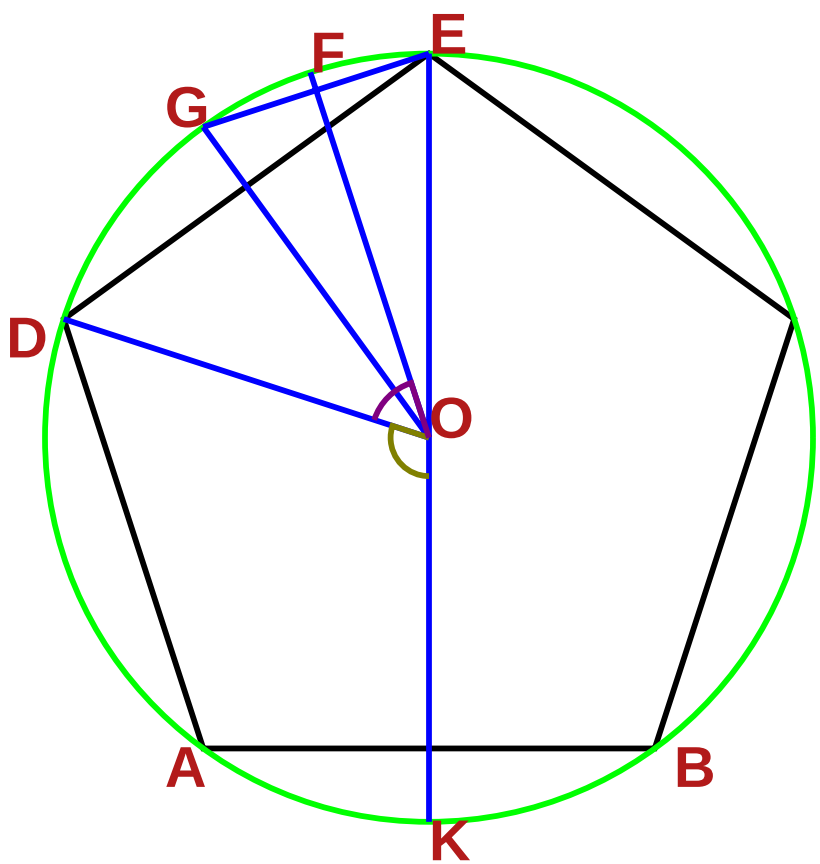
Circumference BD is double BC,

so BC is a side of a decagon.



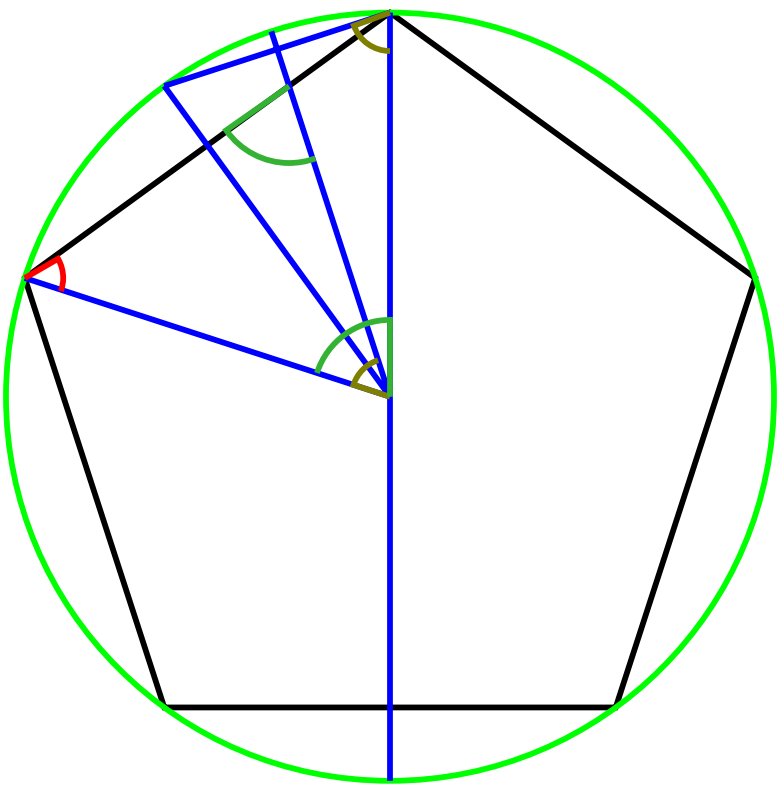
AB=ED, so AB is double GD.

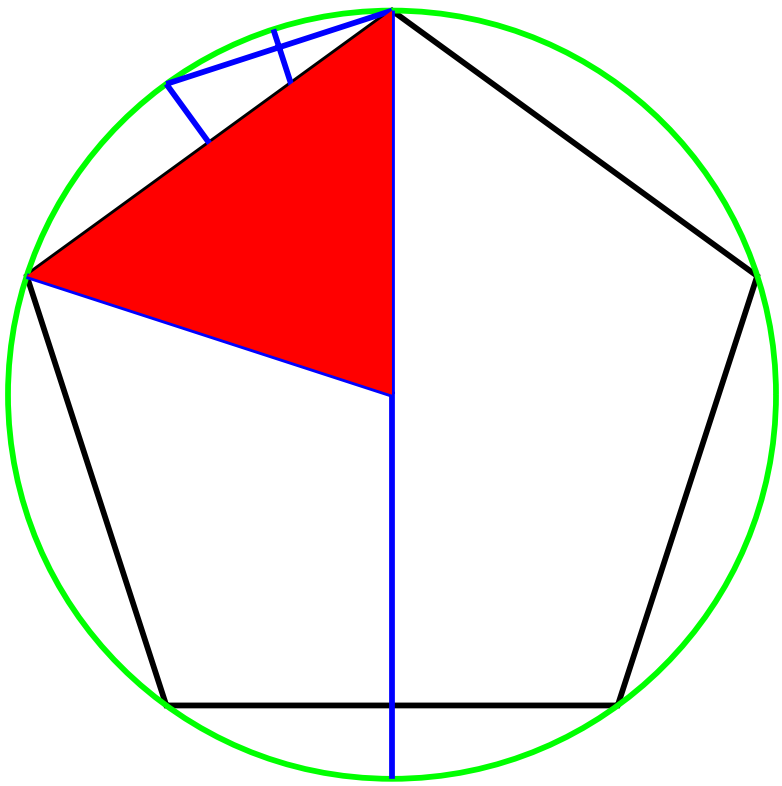
AK=KB, so AK=DG and AK is double FG.



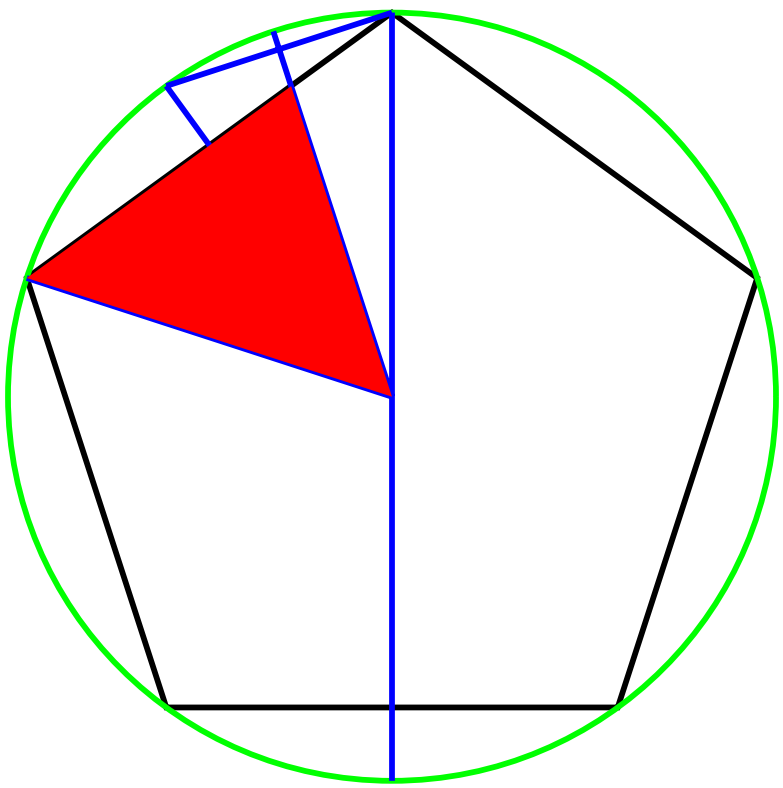
AD=ED, and so DK is double DF.

Angle DOK is double FOD.



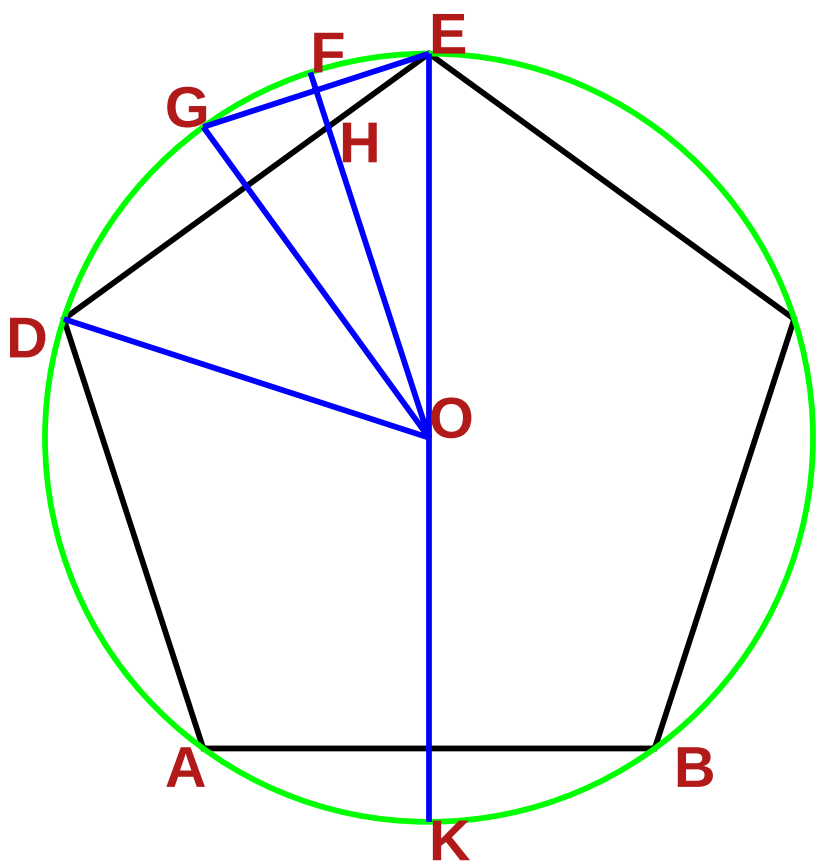


Triangle A.



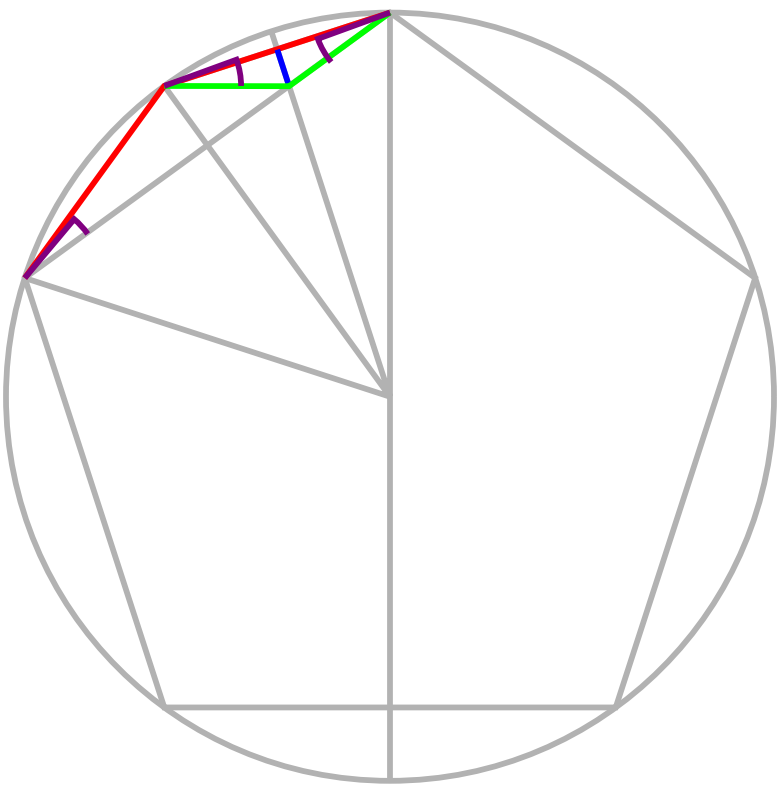
Triangle B.

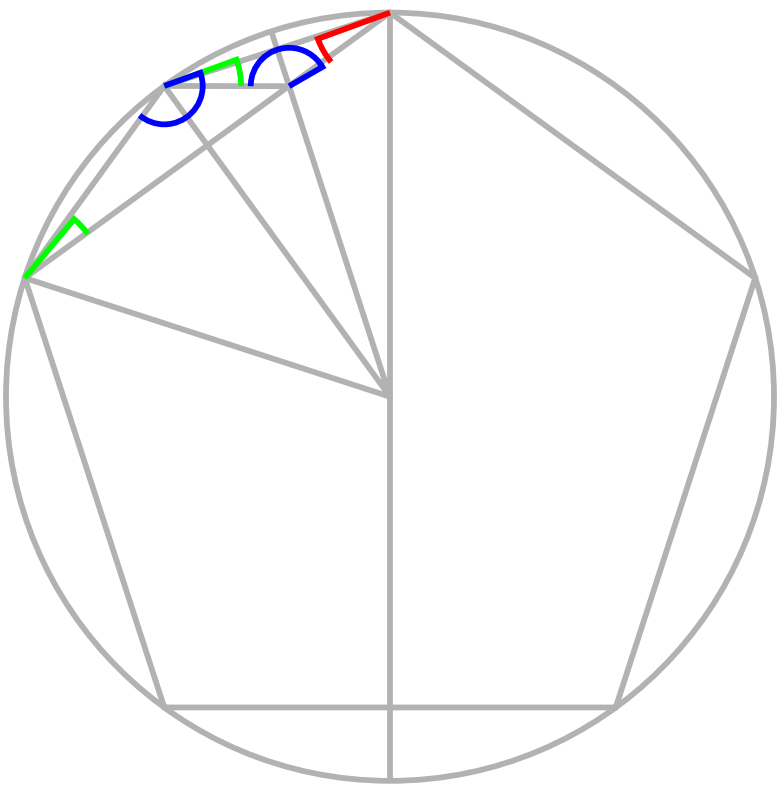
Triangles A and B are equiangular; therefore, their sides are proportional.

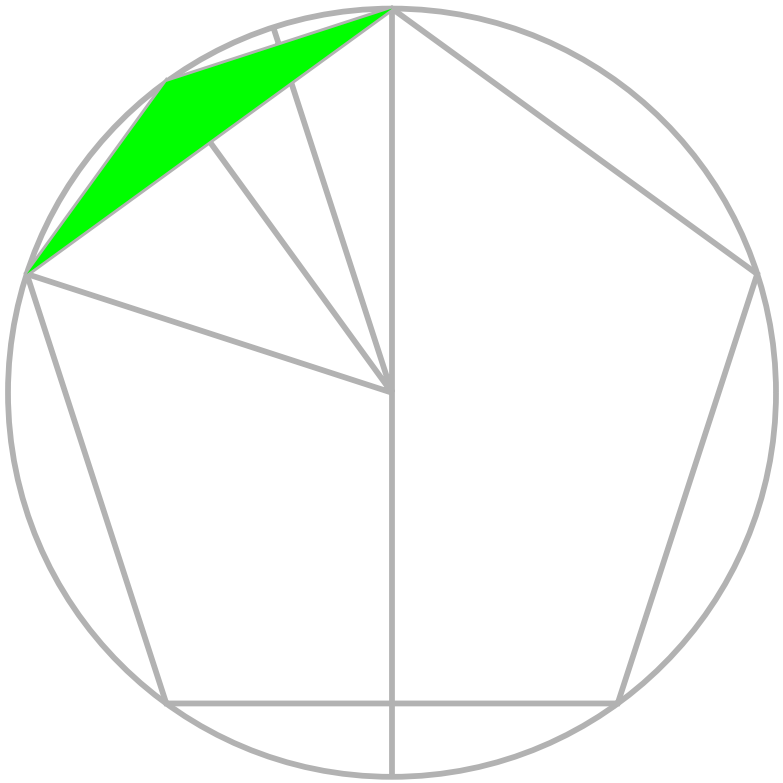


(ED/DO)=(DO/DH)

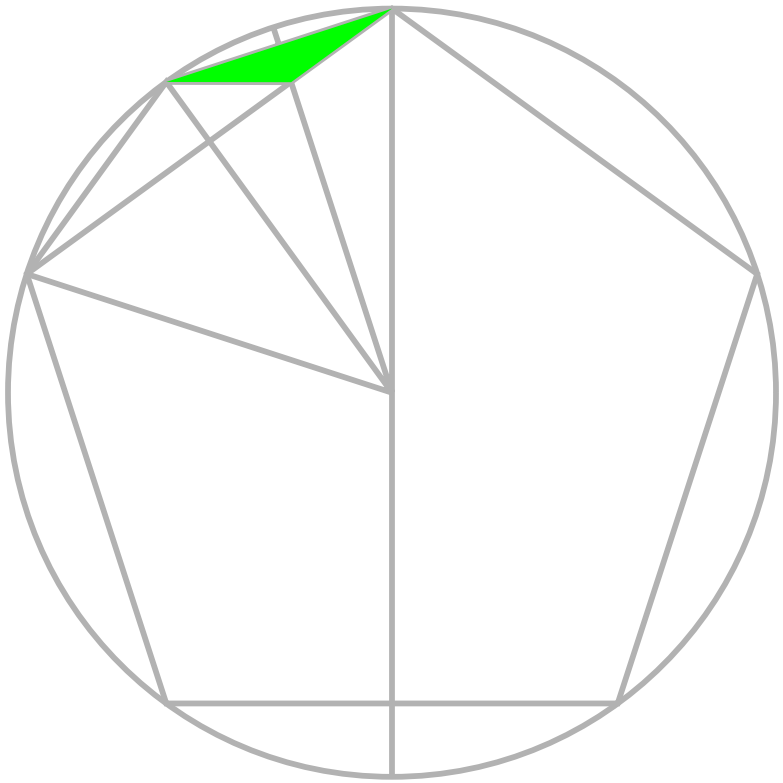
square on DO = rectangle on ED by DH





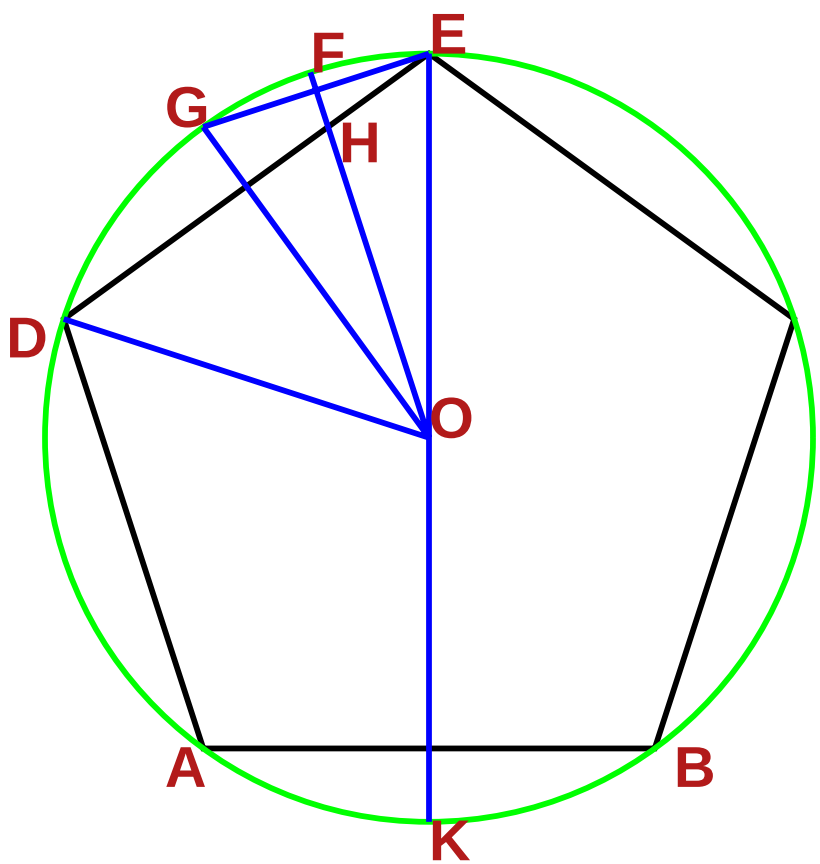


Triangle A.



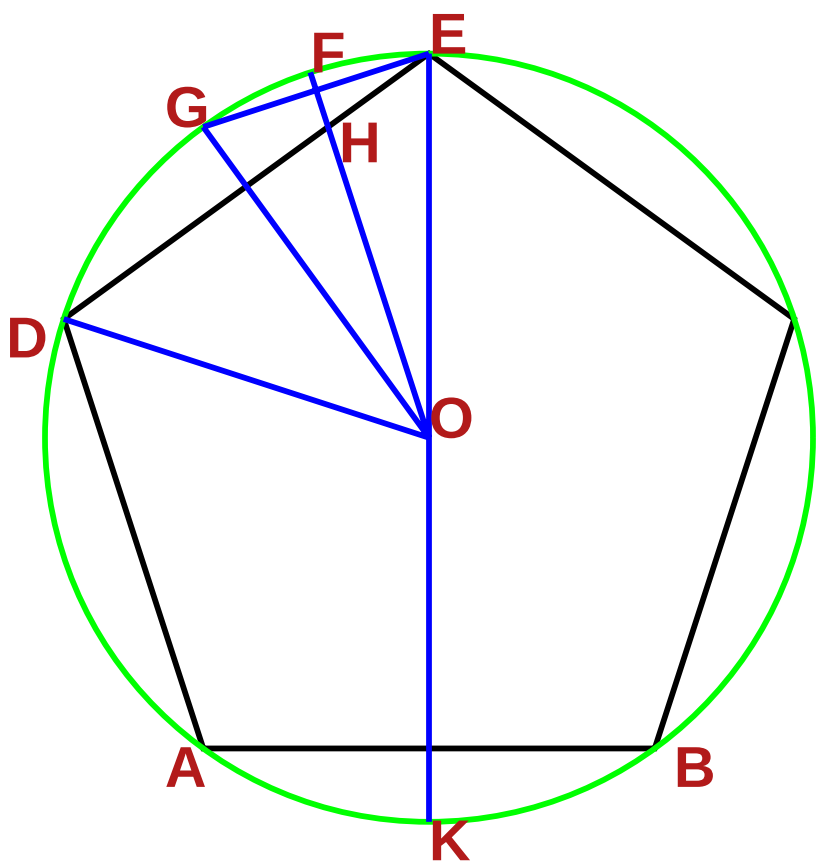
Triangle B.

Triangles A and B are equiangular; therefore, their sides are proportional.



(ED/GE)=(GE/EF)

square on GE = rectangle on ED by EF



ED*EF + ED*DH = GE*GE + DO*DO,

where GE is a side of a decagon,

and DO is a side of a hexagon.