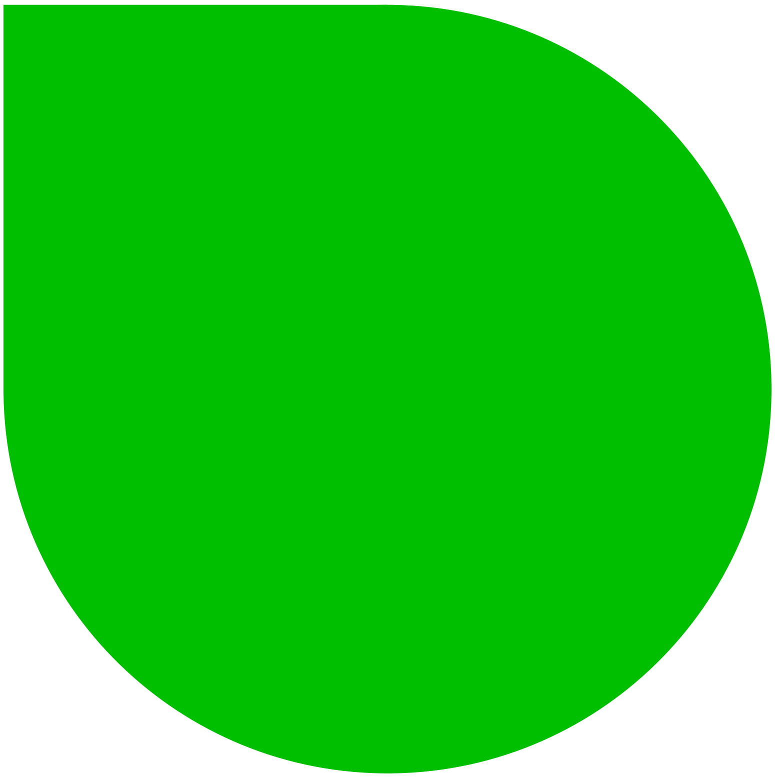
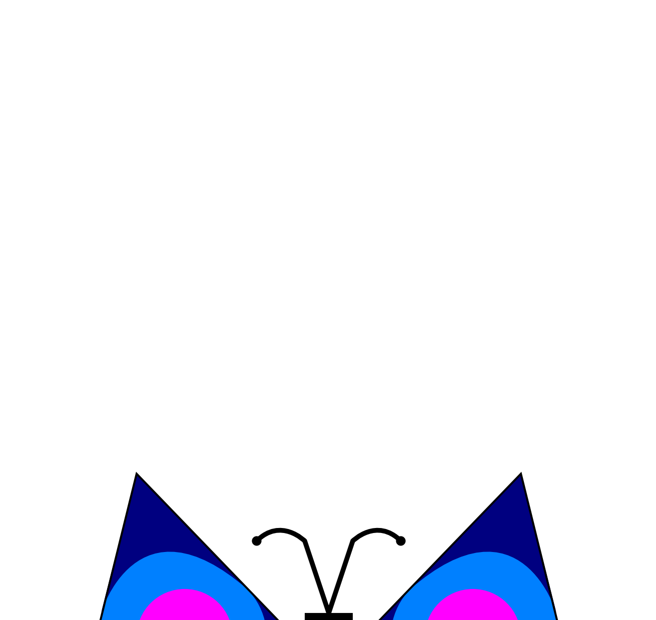
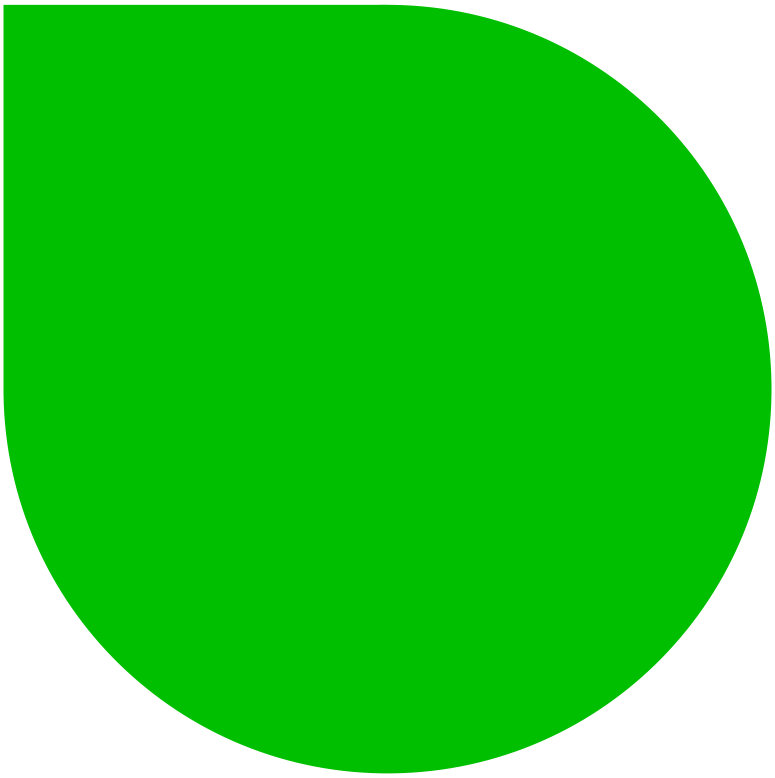
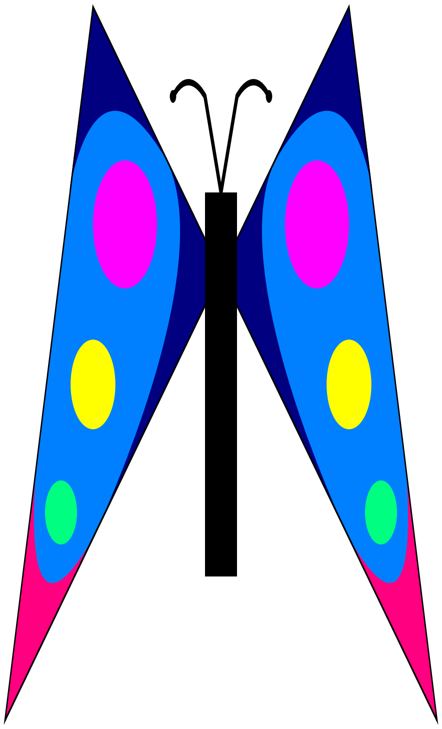
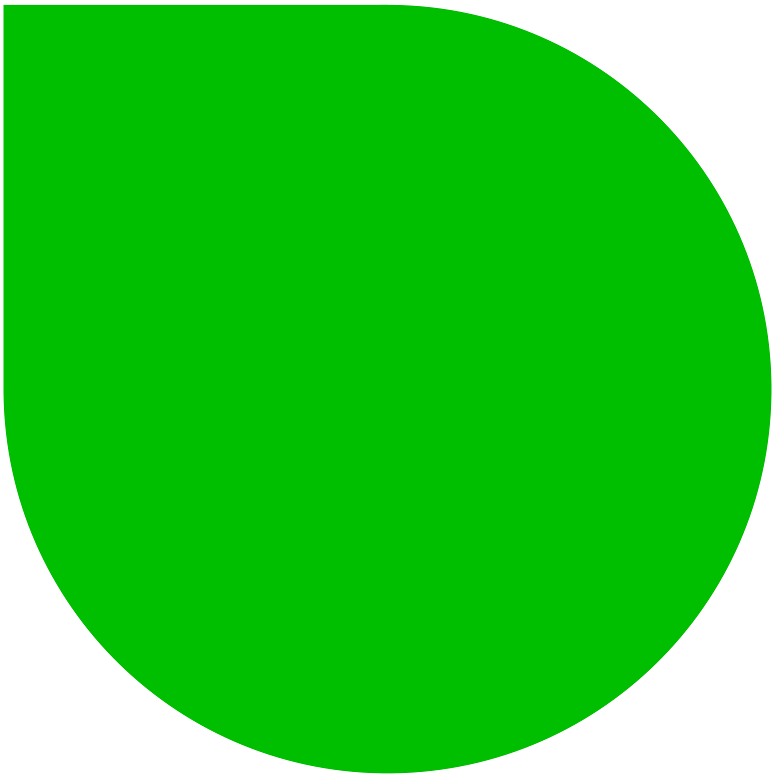
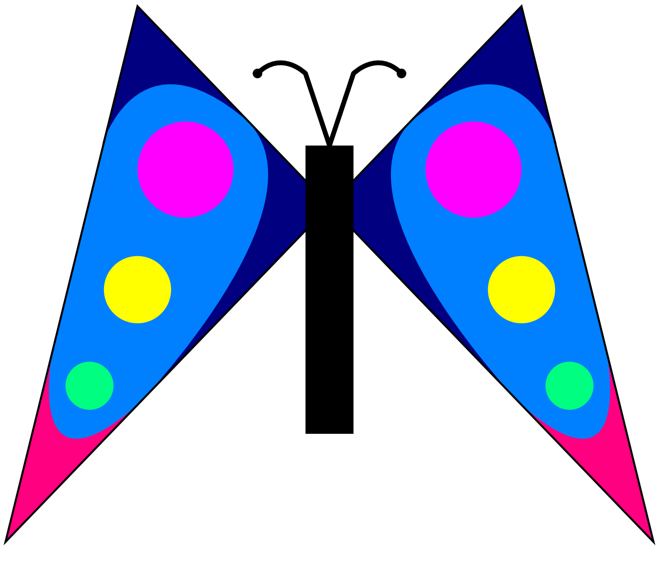
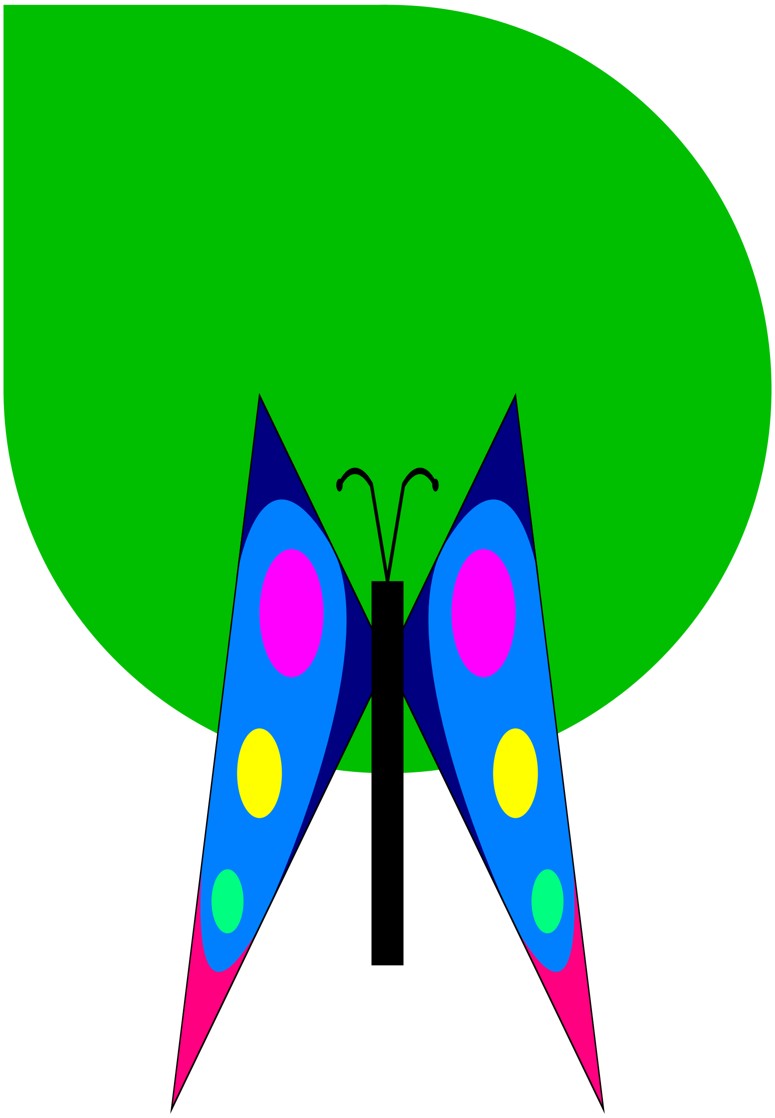


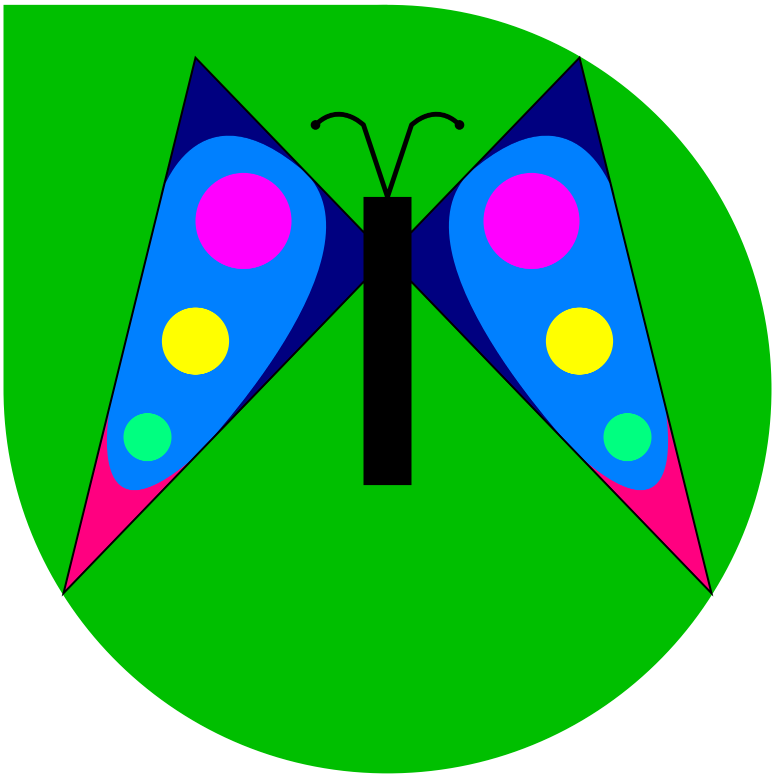


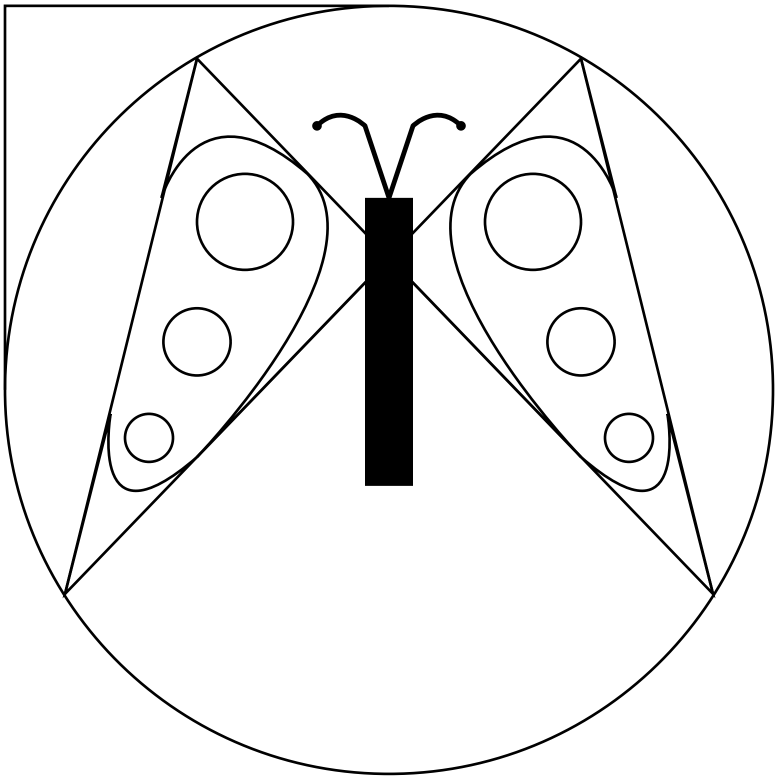


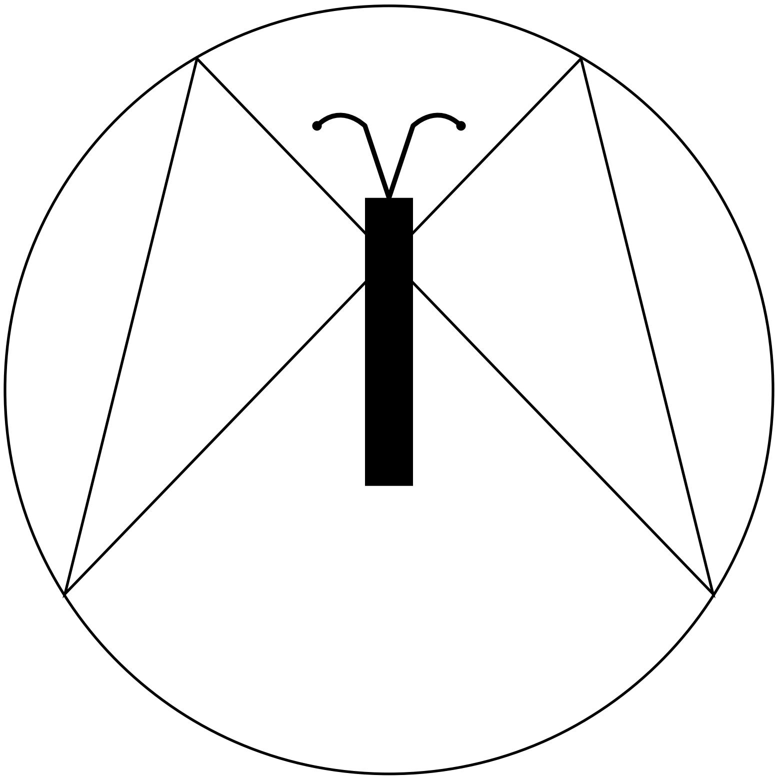


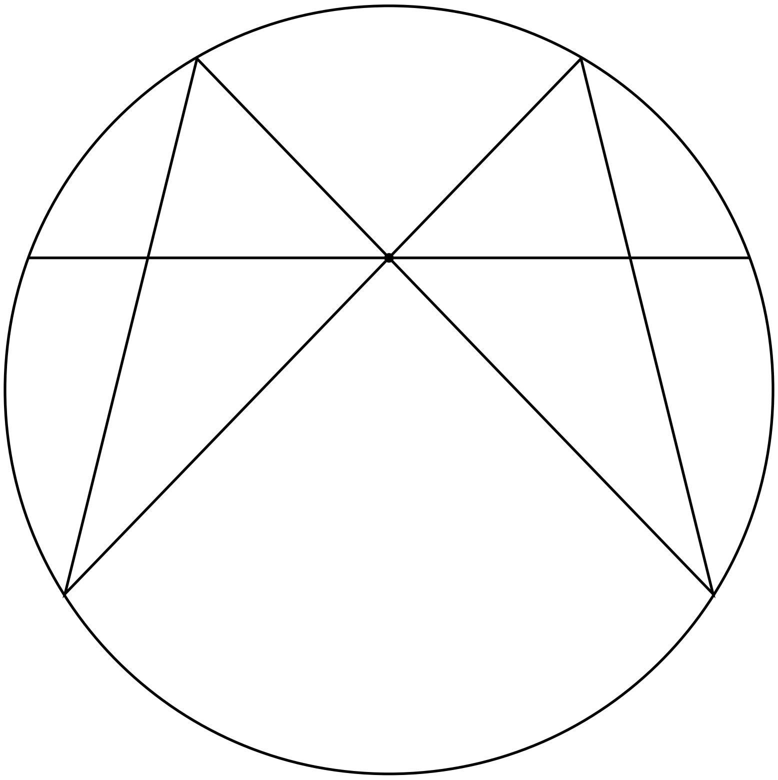



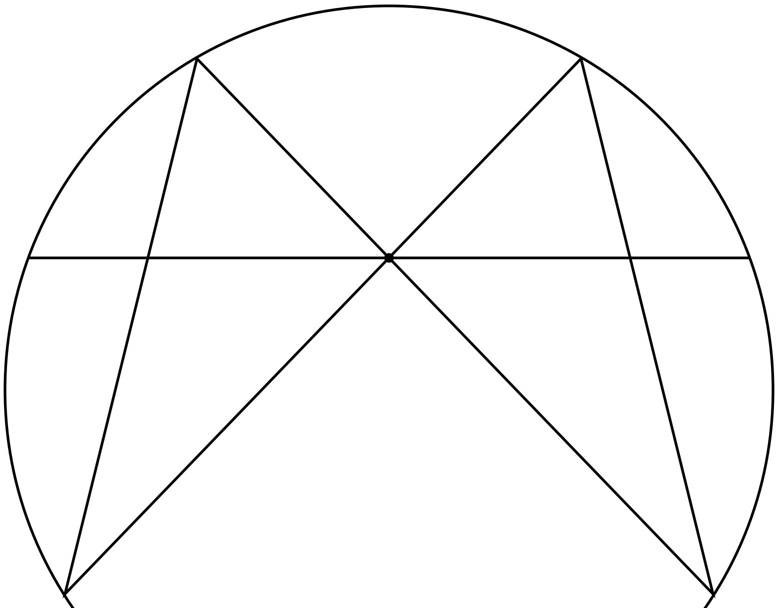






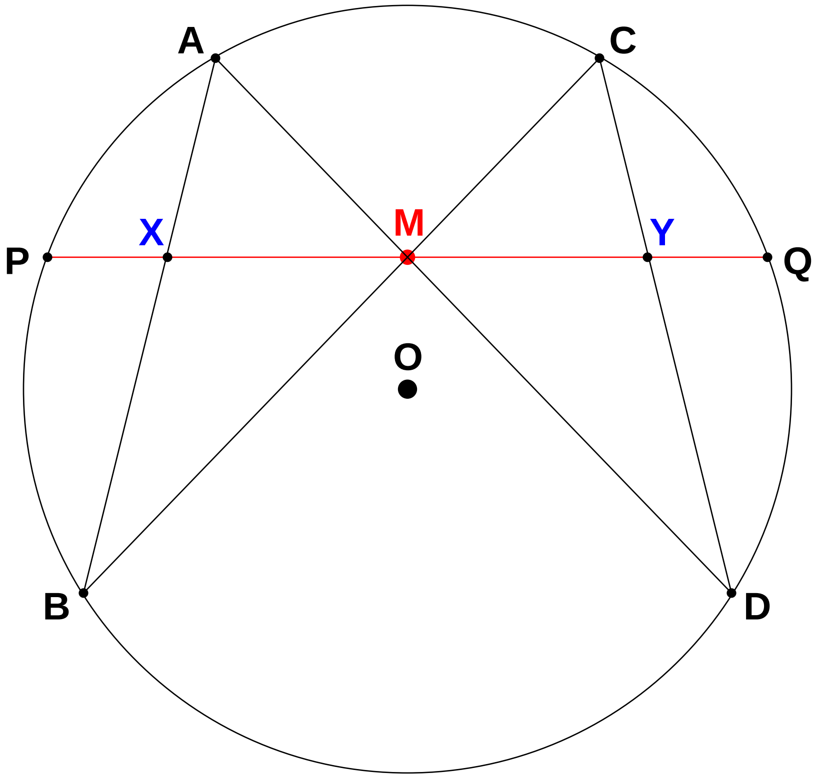






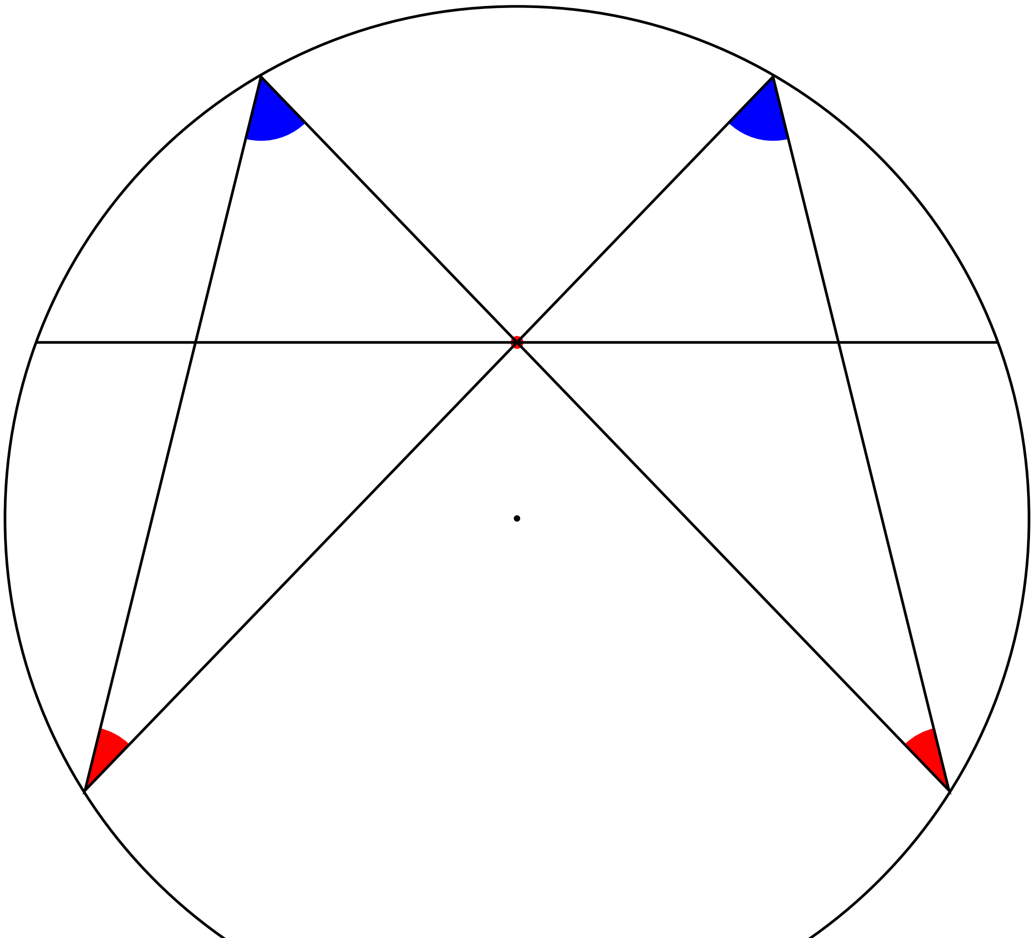
Math 307 Project

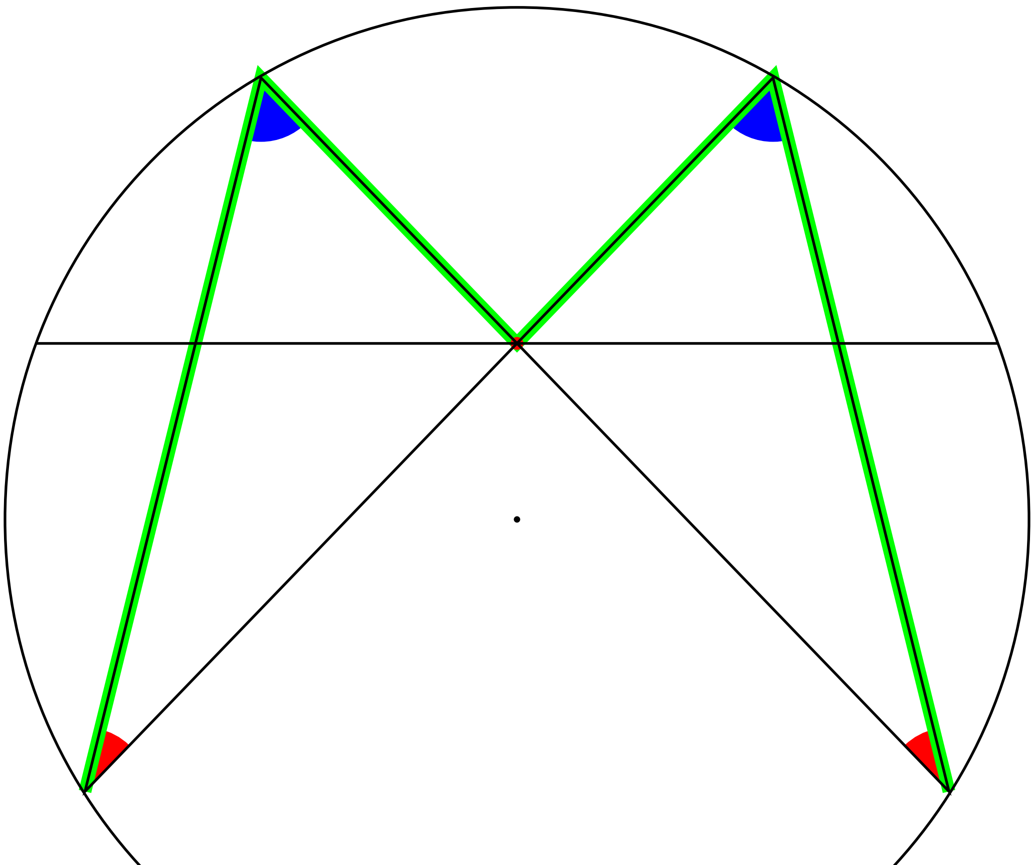
The Butterfly Theorem

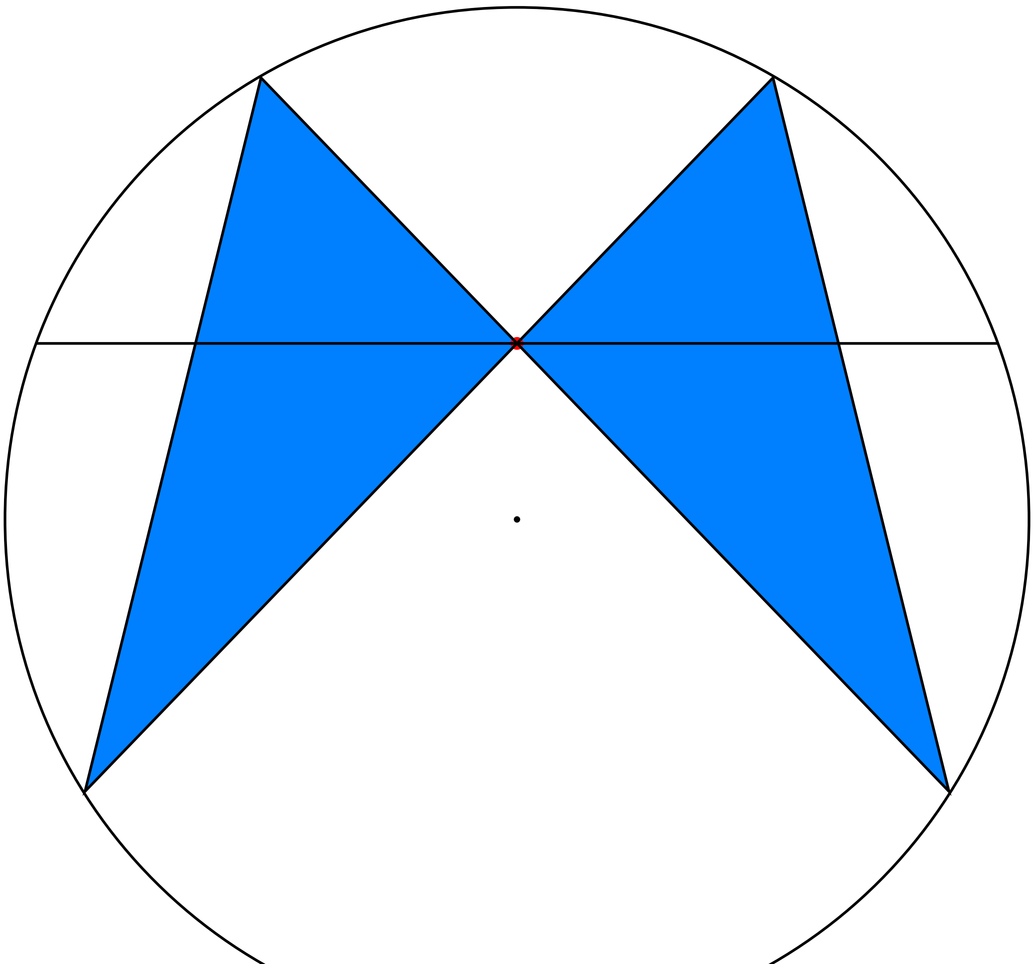


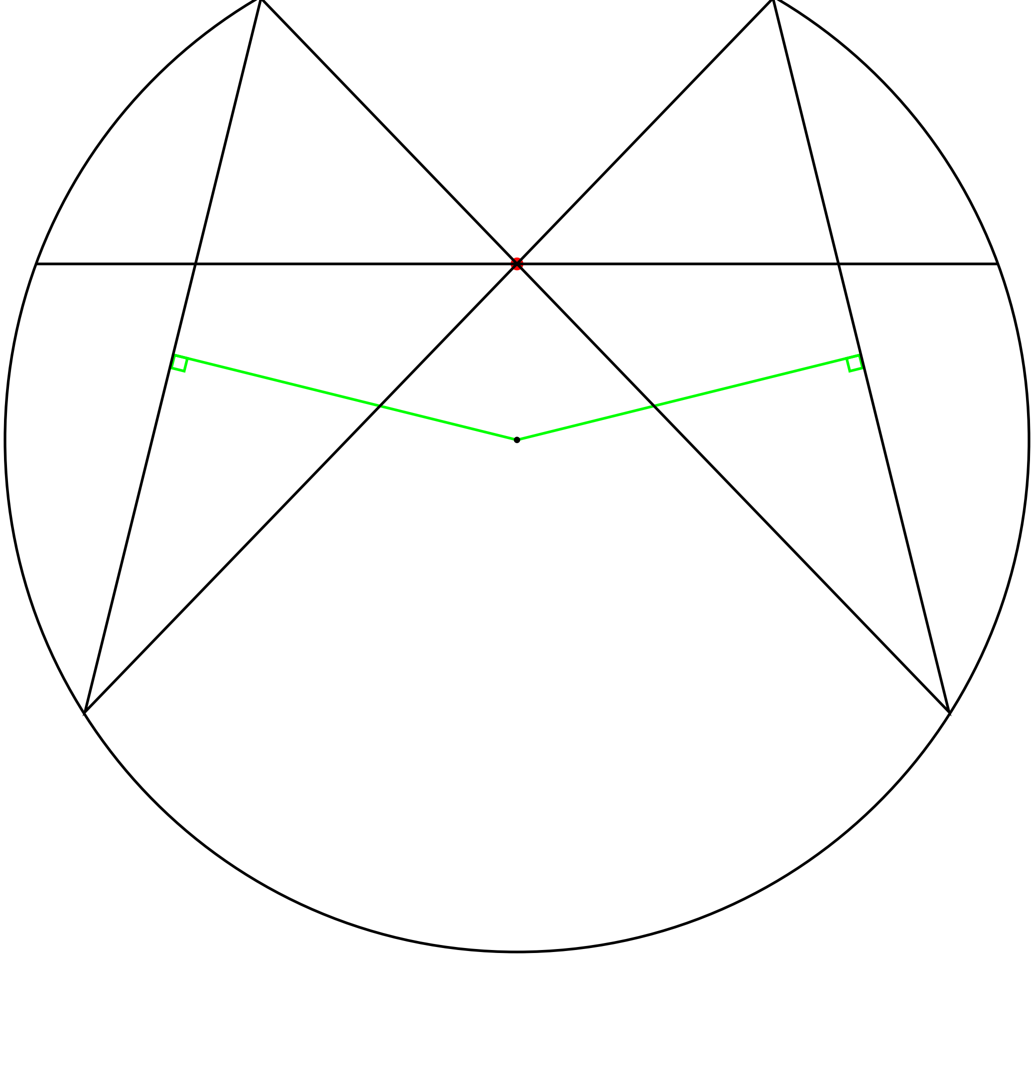
Theorem: Let M be the midpoint of a chord PQ of circle, through which two chords AB and CD are drawn; AB cuts PQ at X and CD cuts PQ at Y.

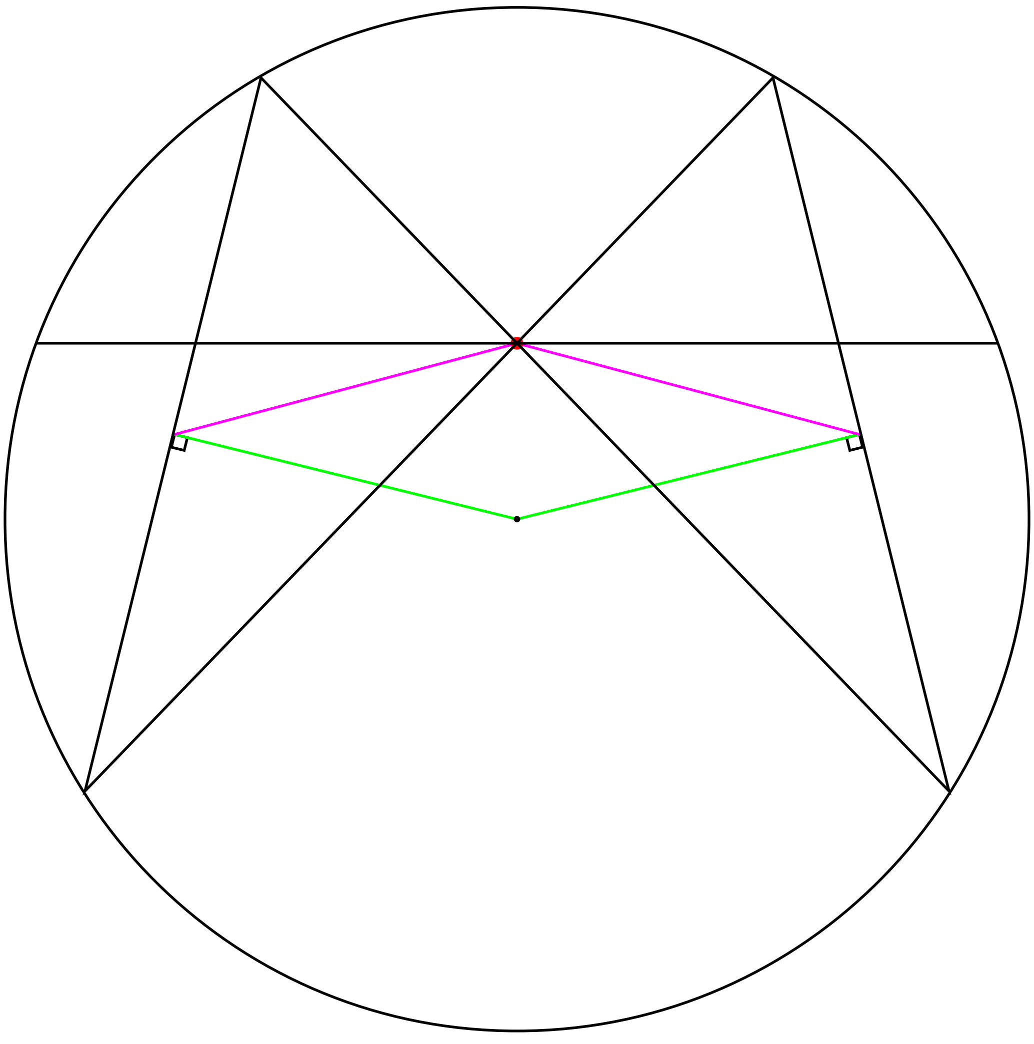
Prove that M is the midpoint of XY

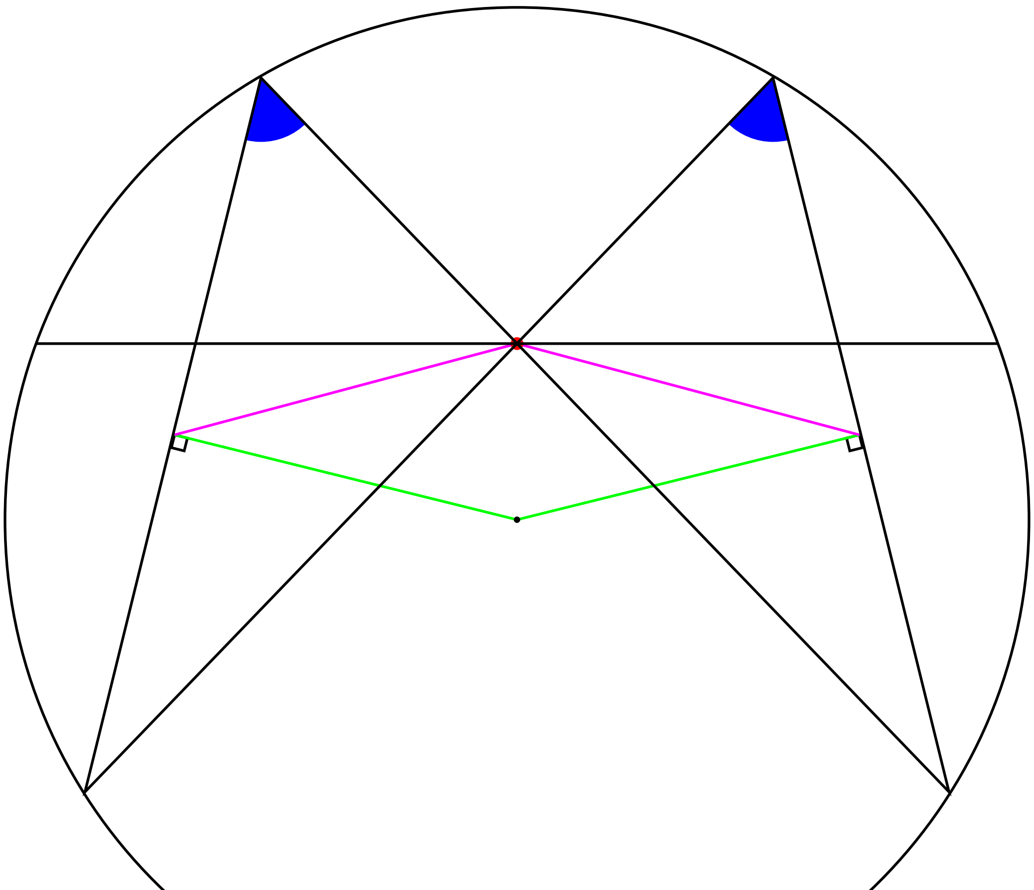


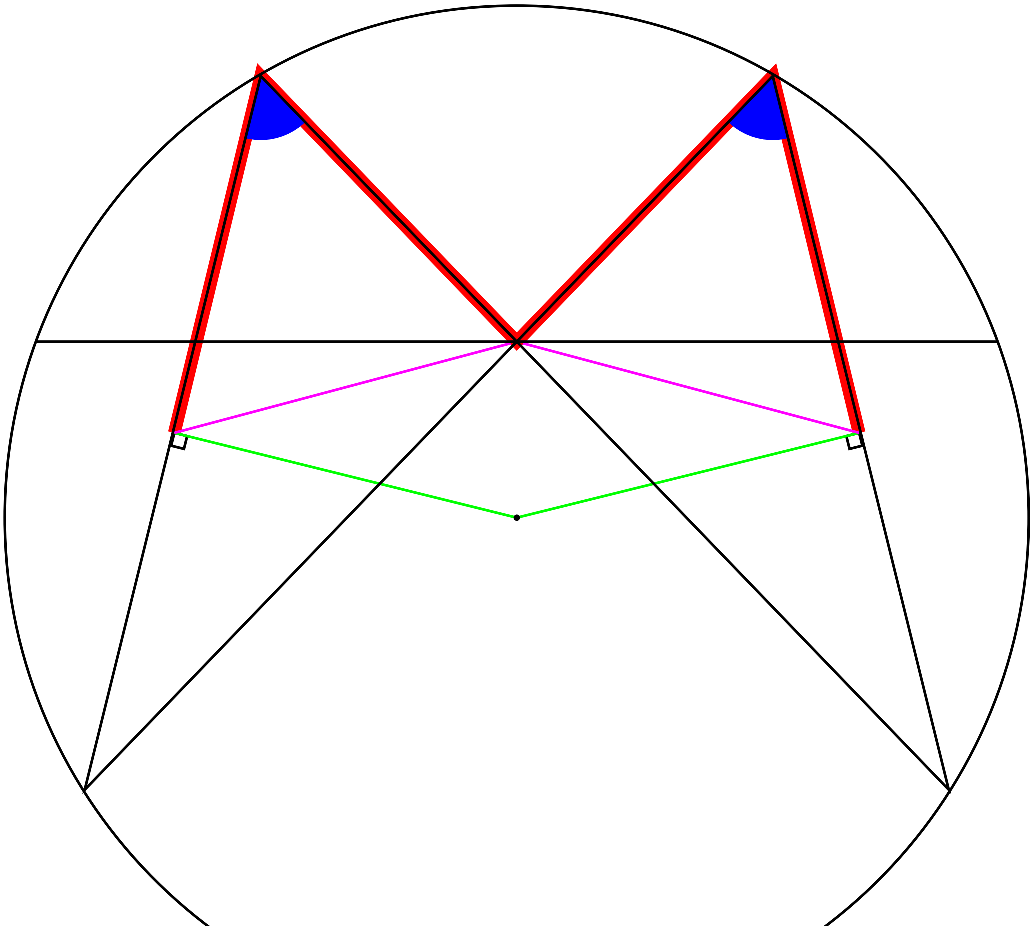


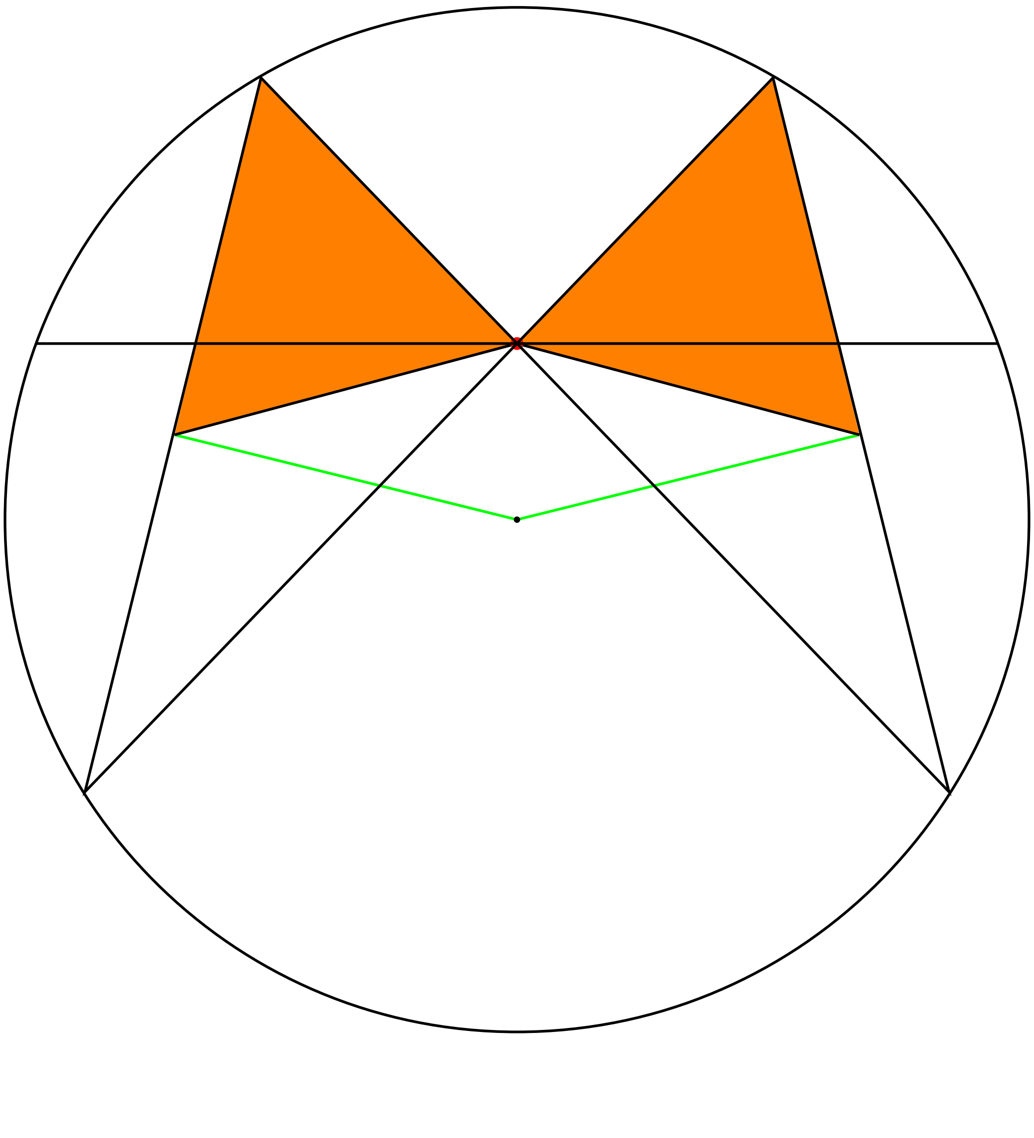


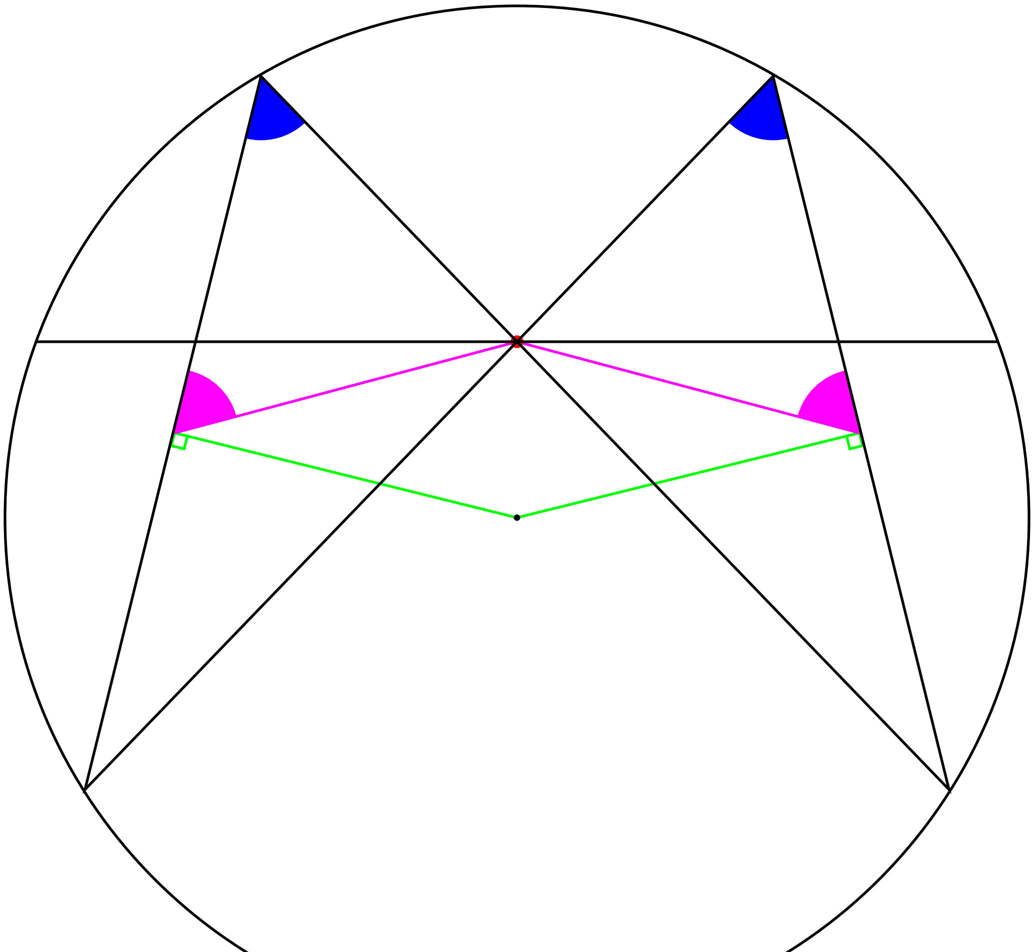


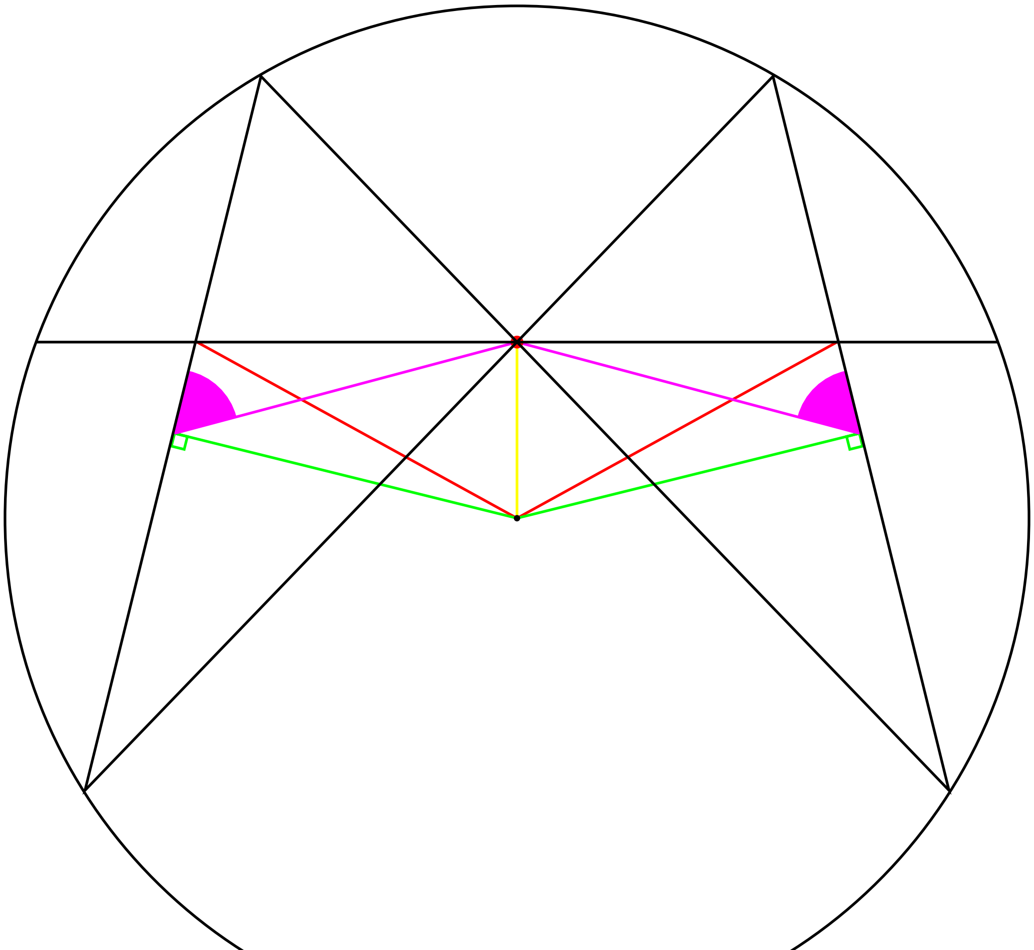


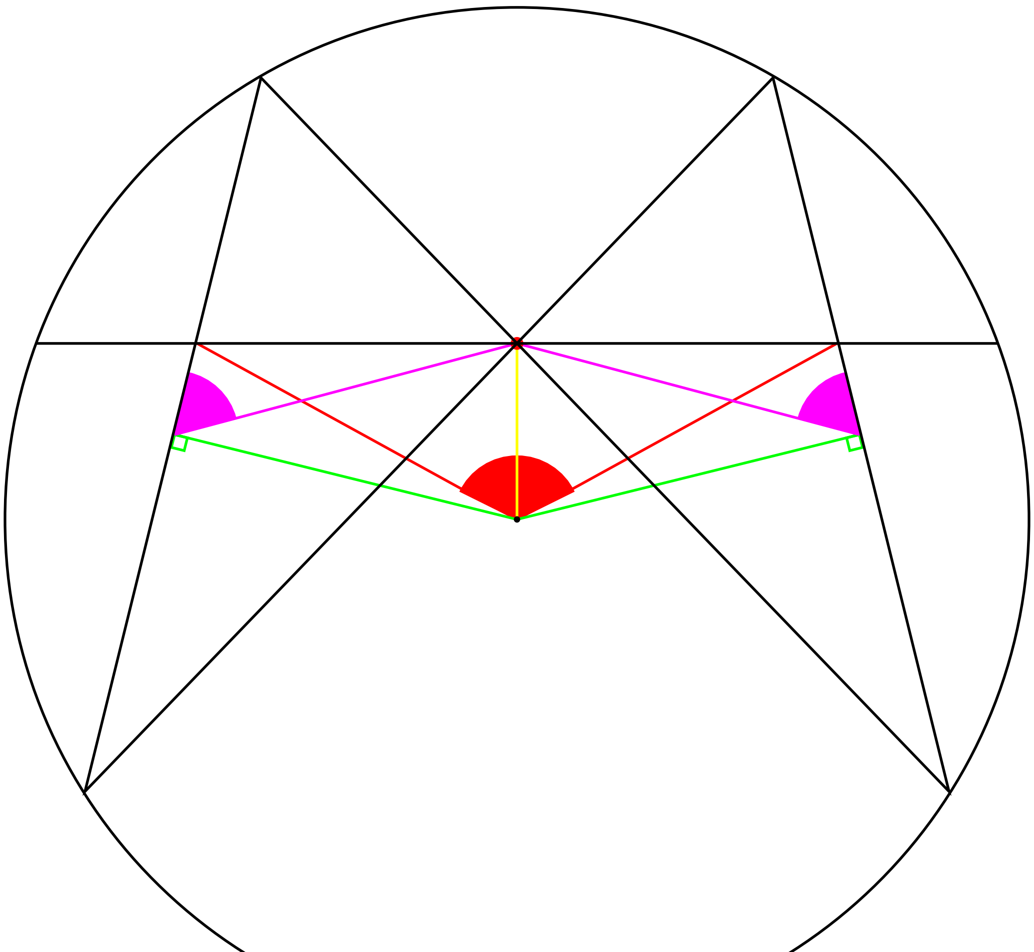


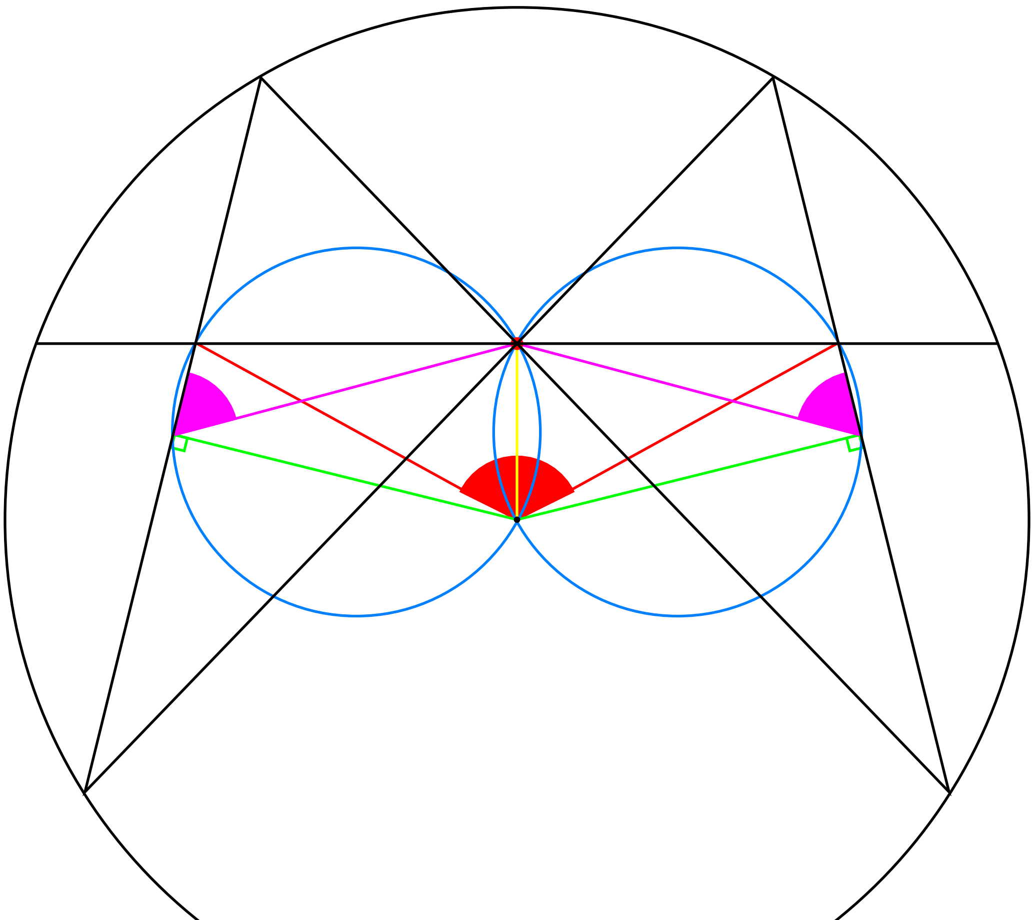


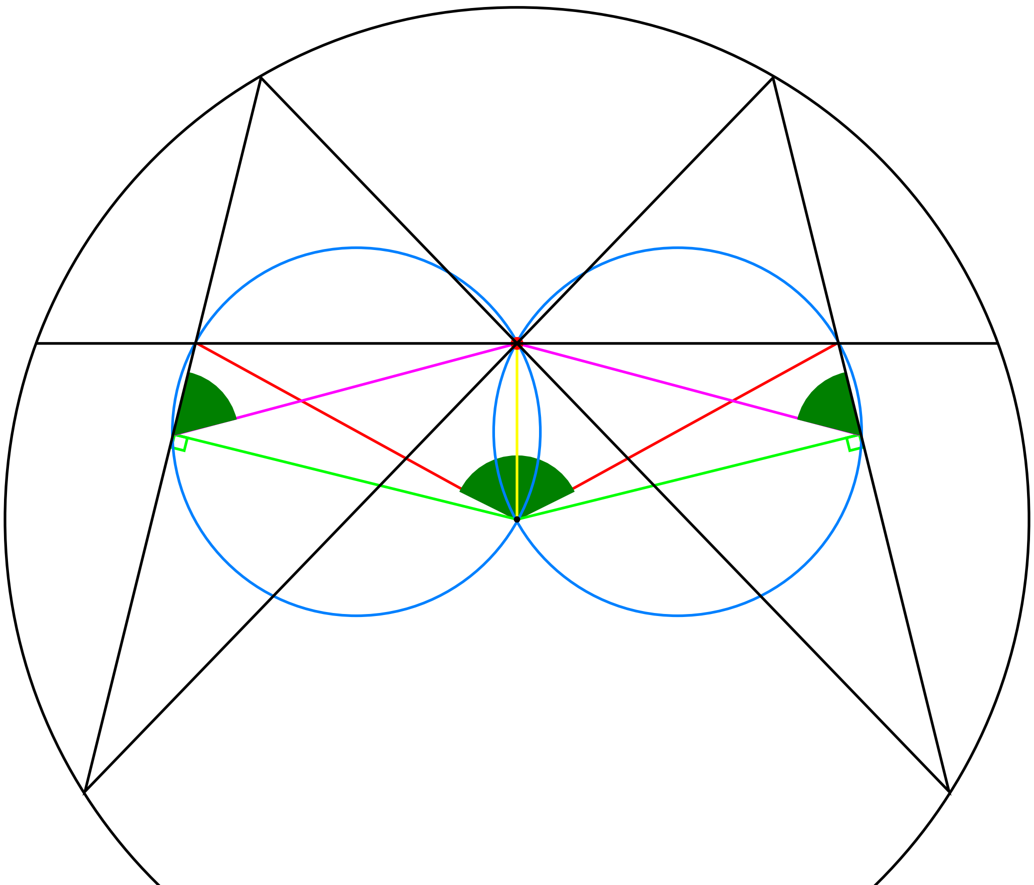


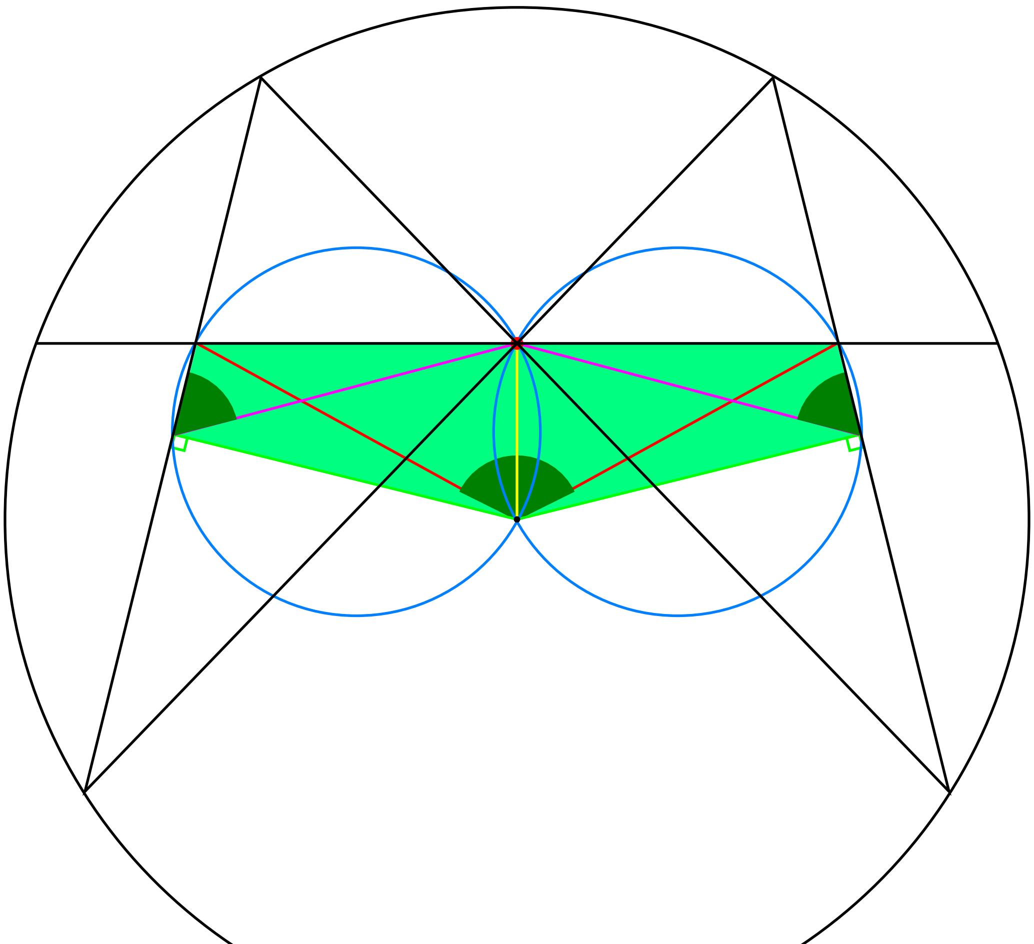


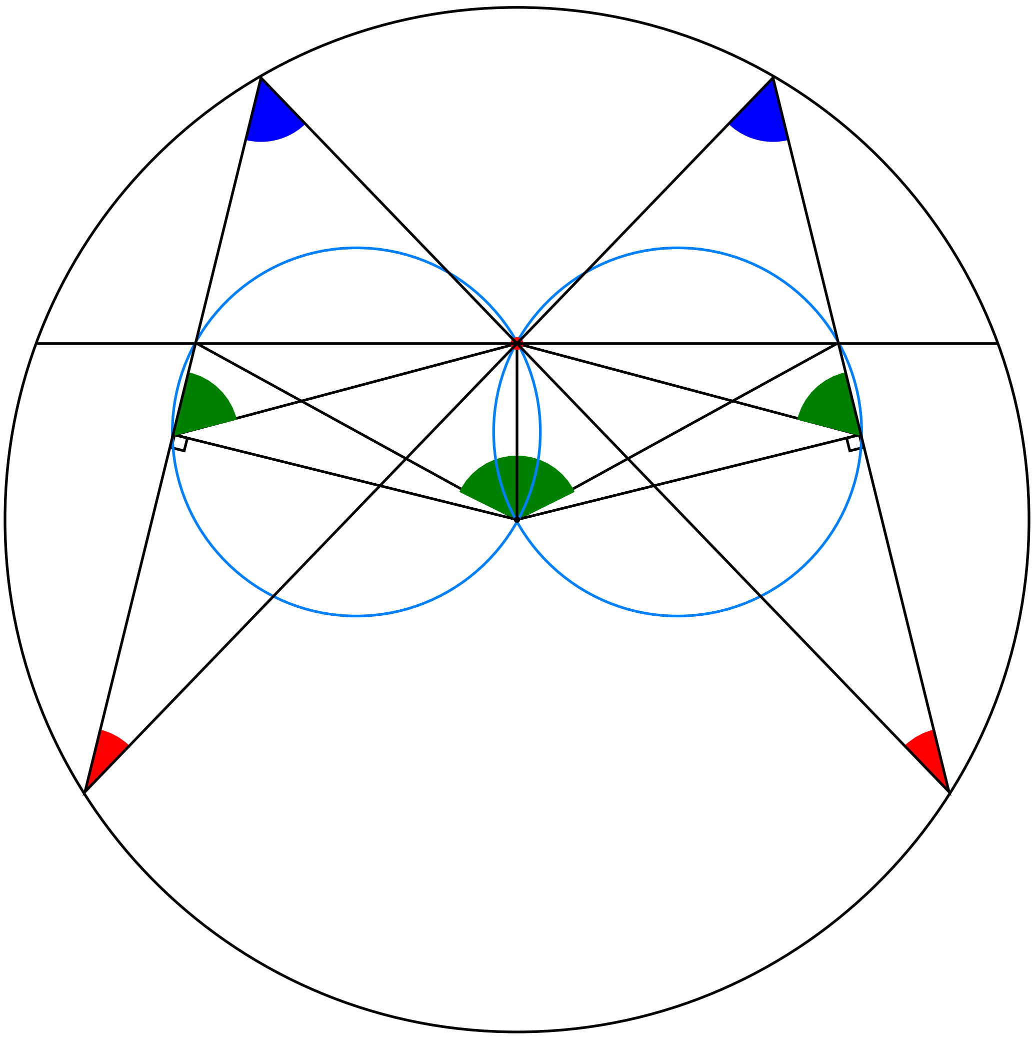


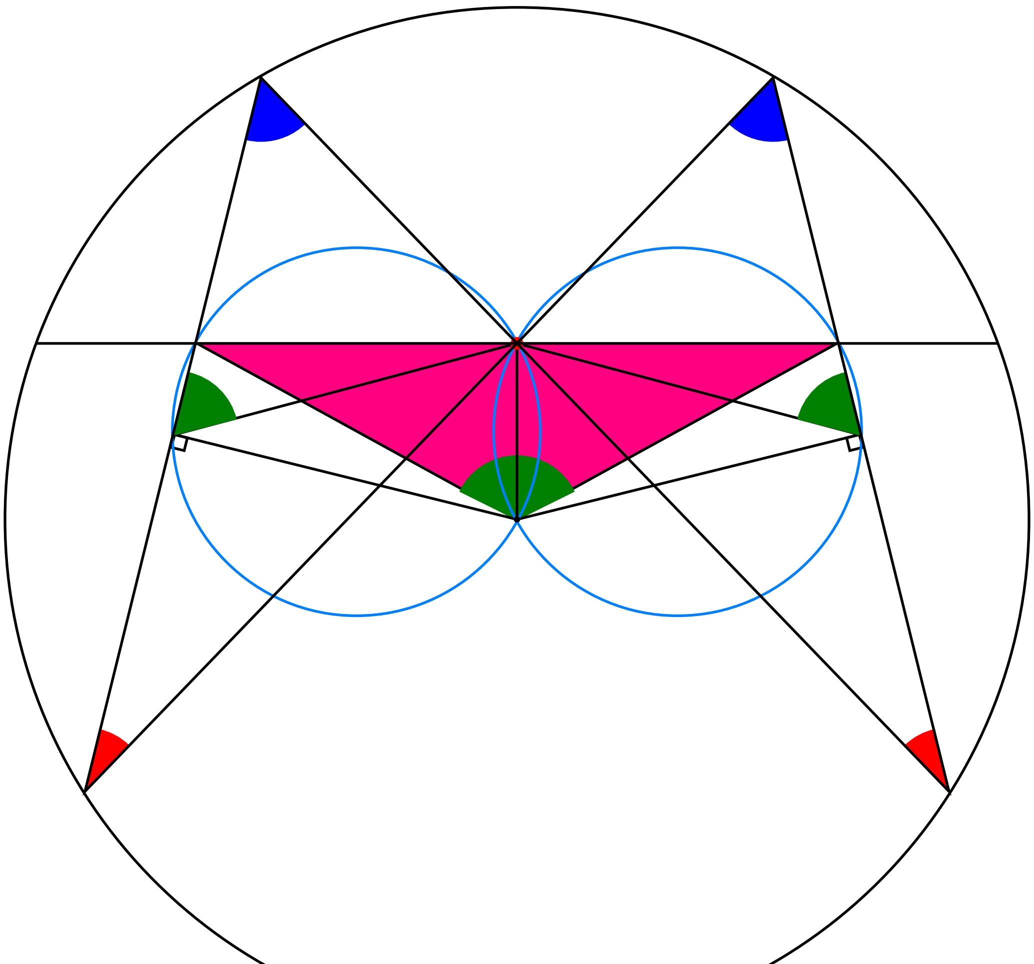


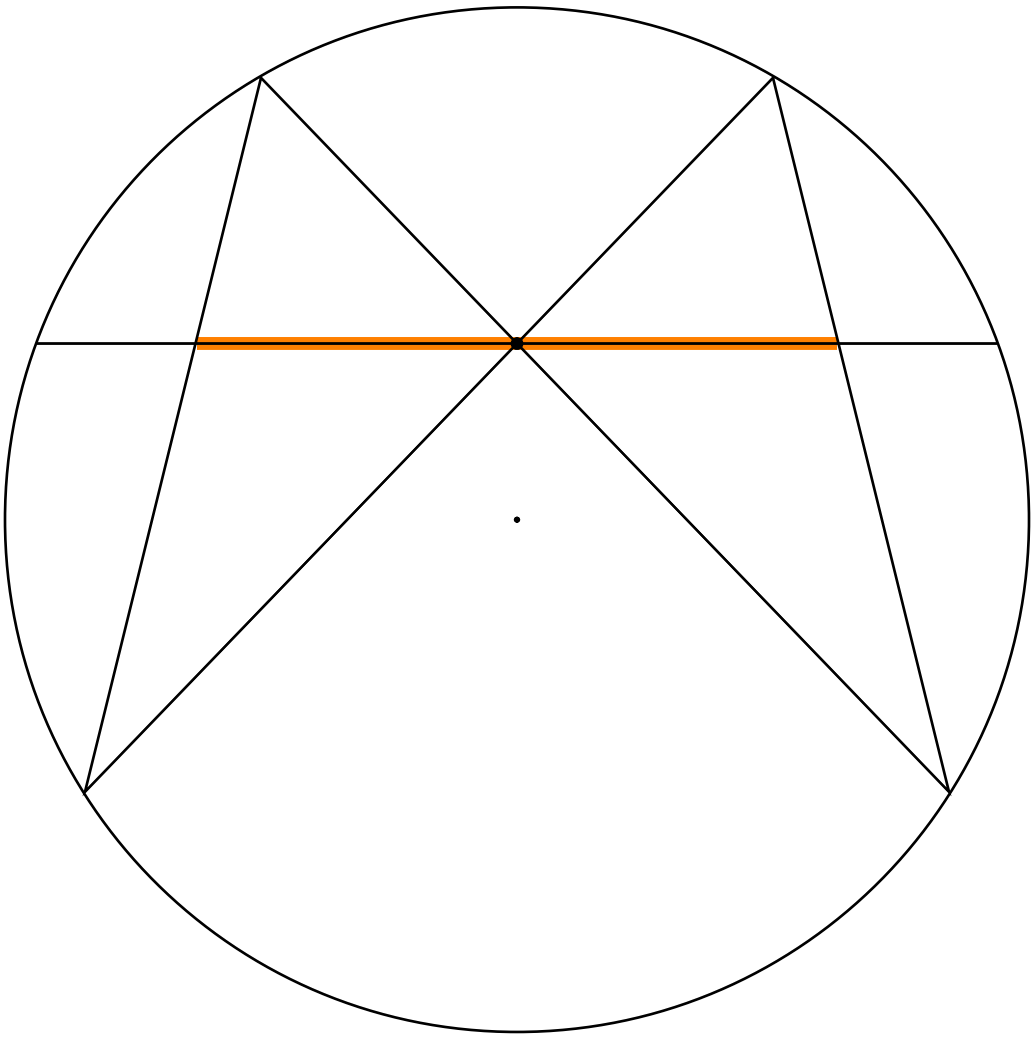






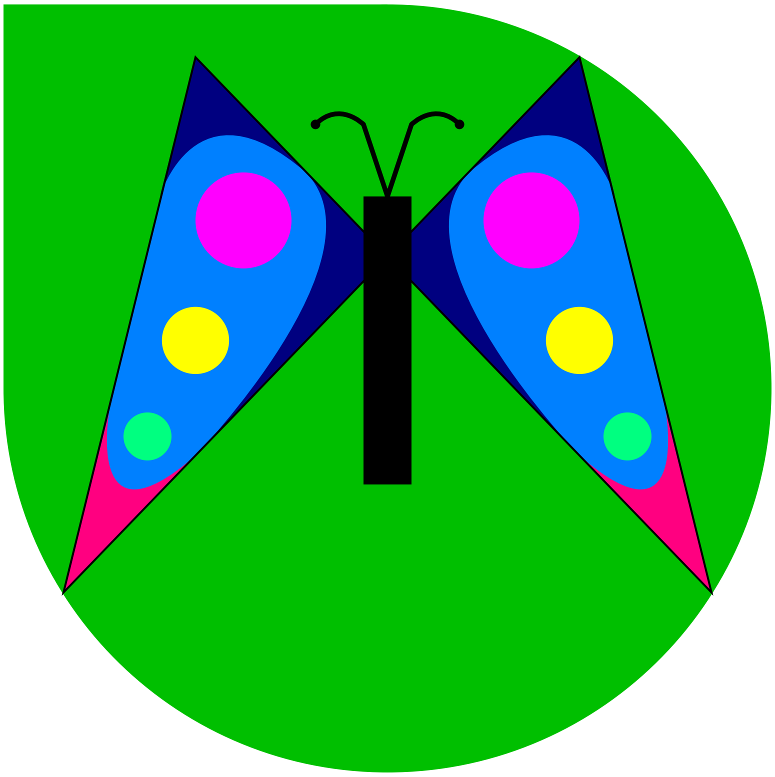


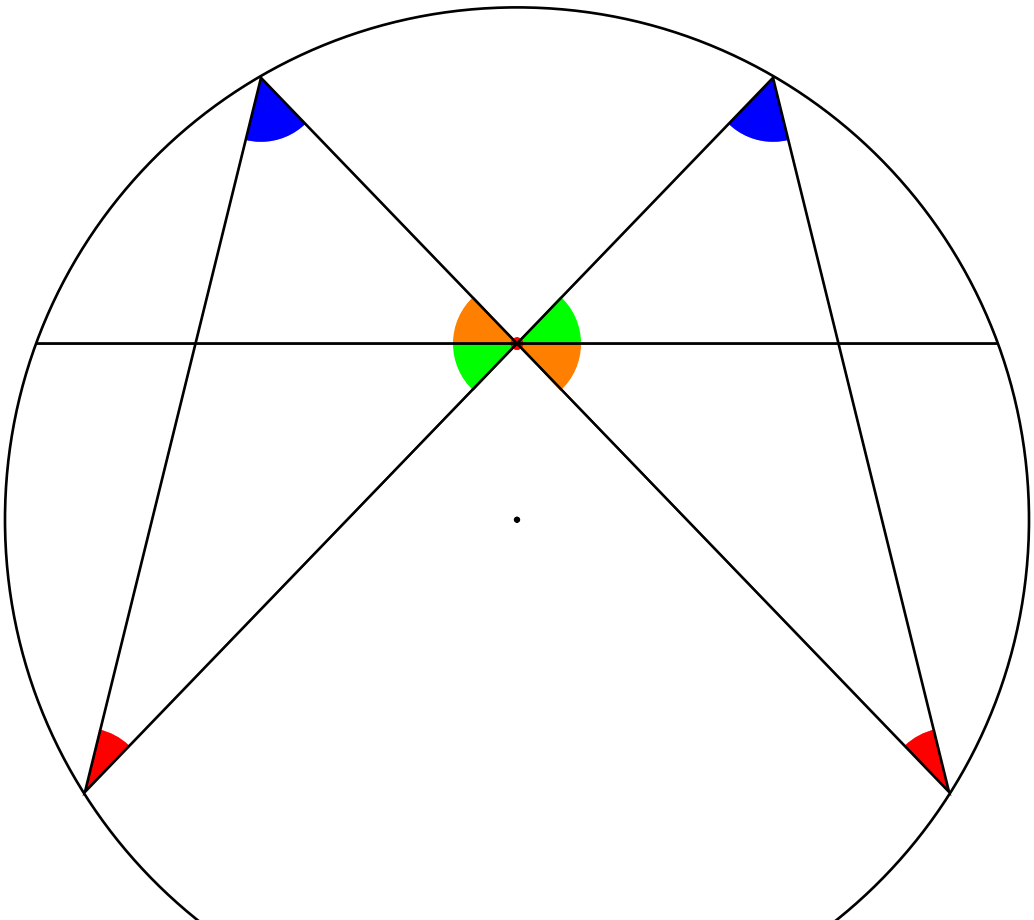


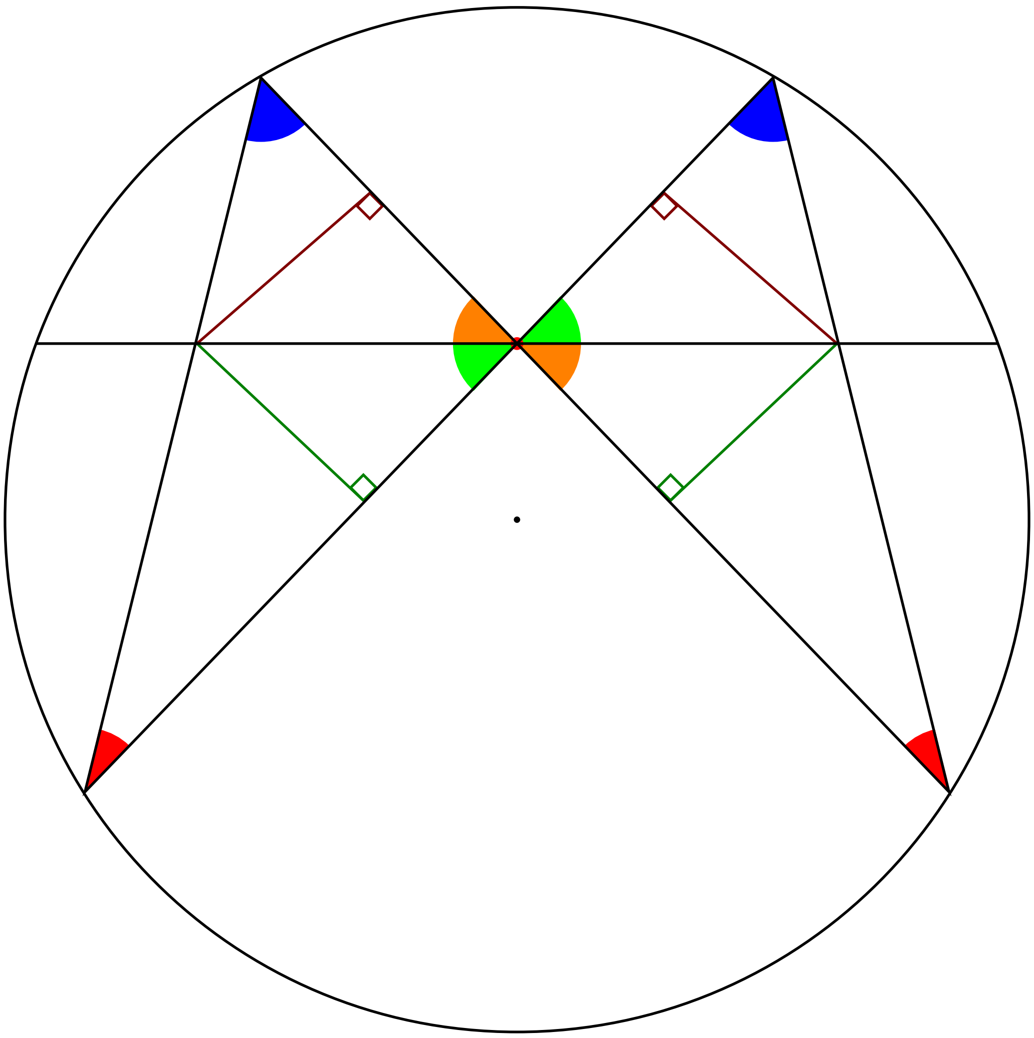


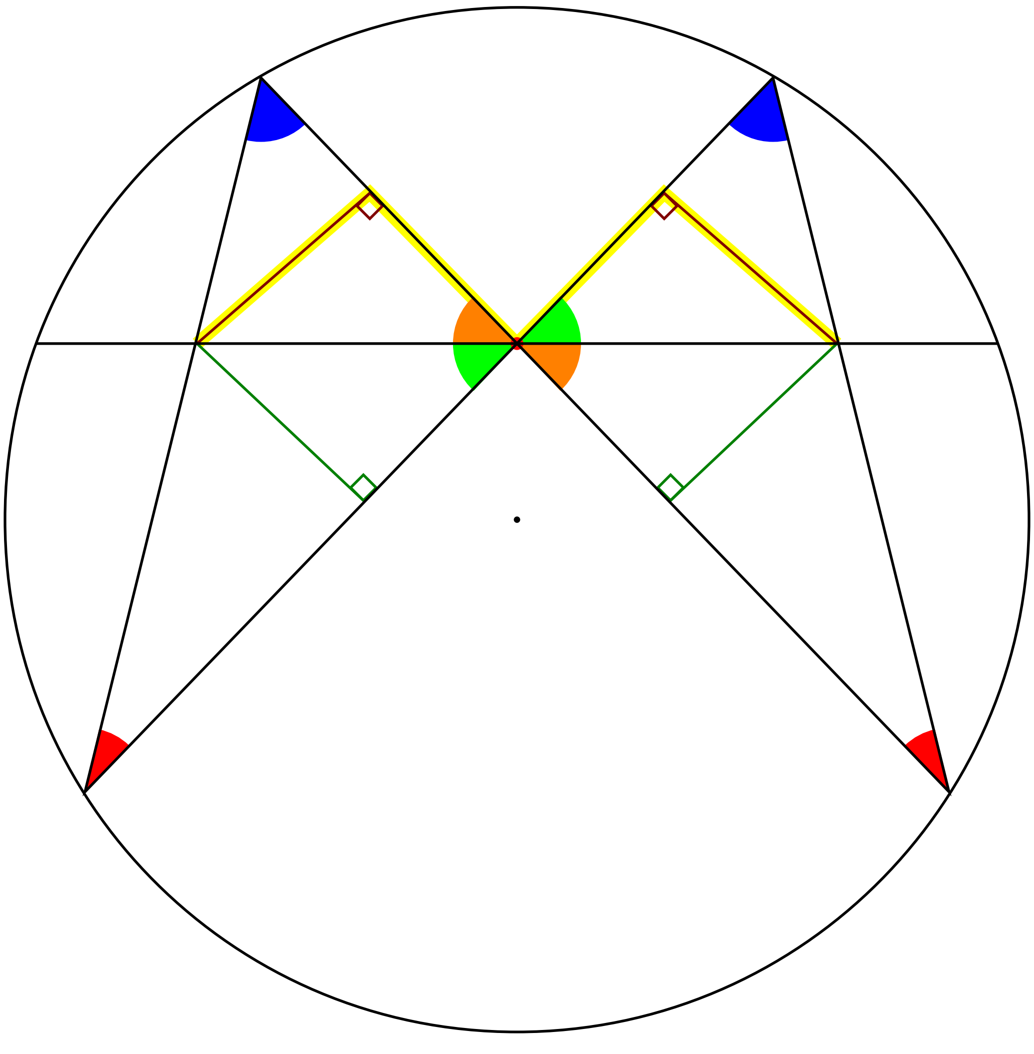
The Butterfly Theorem

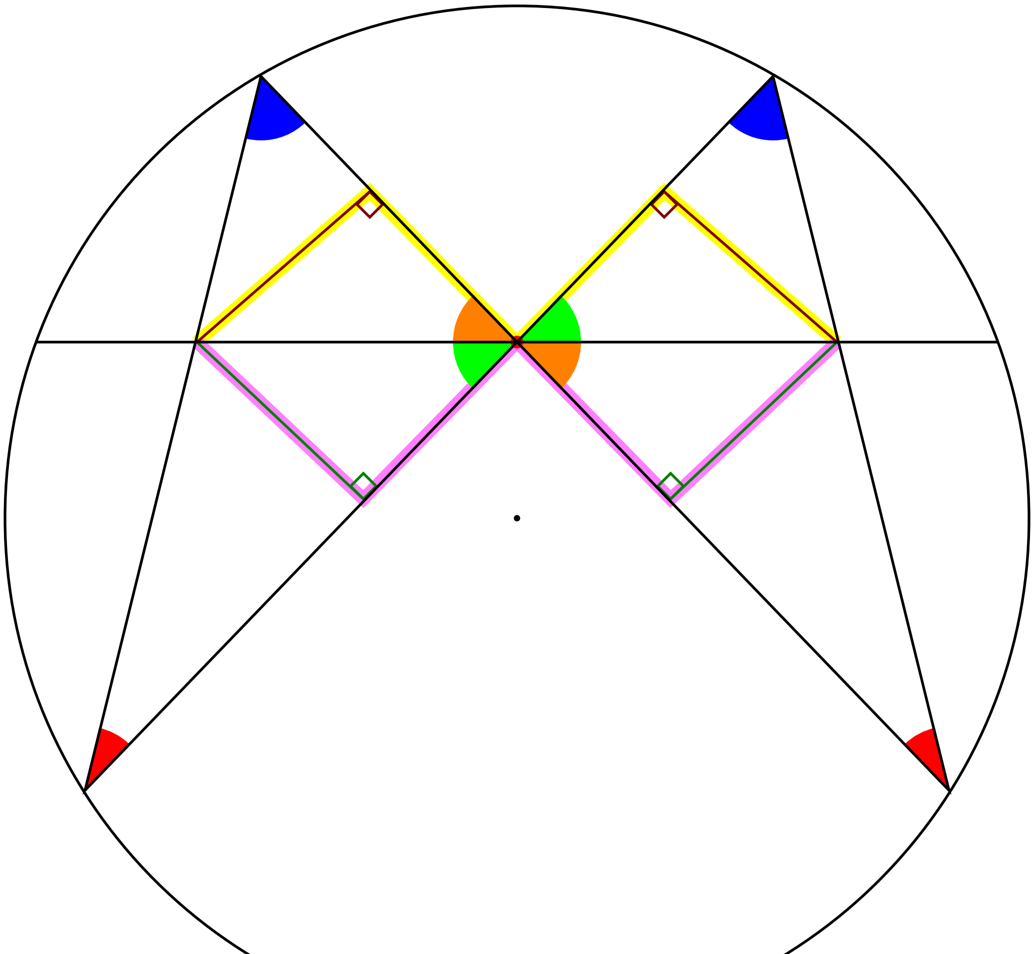
Second Proof

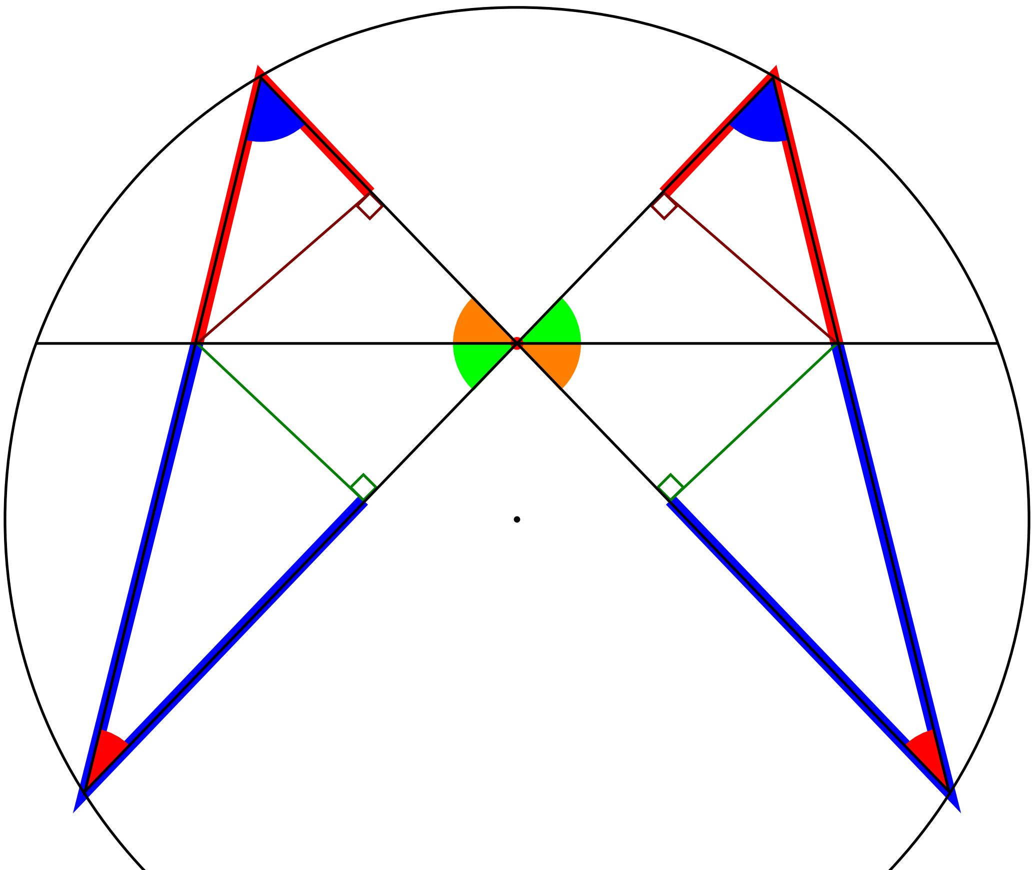


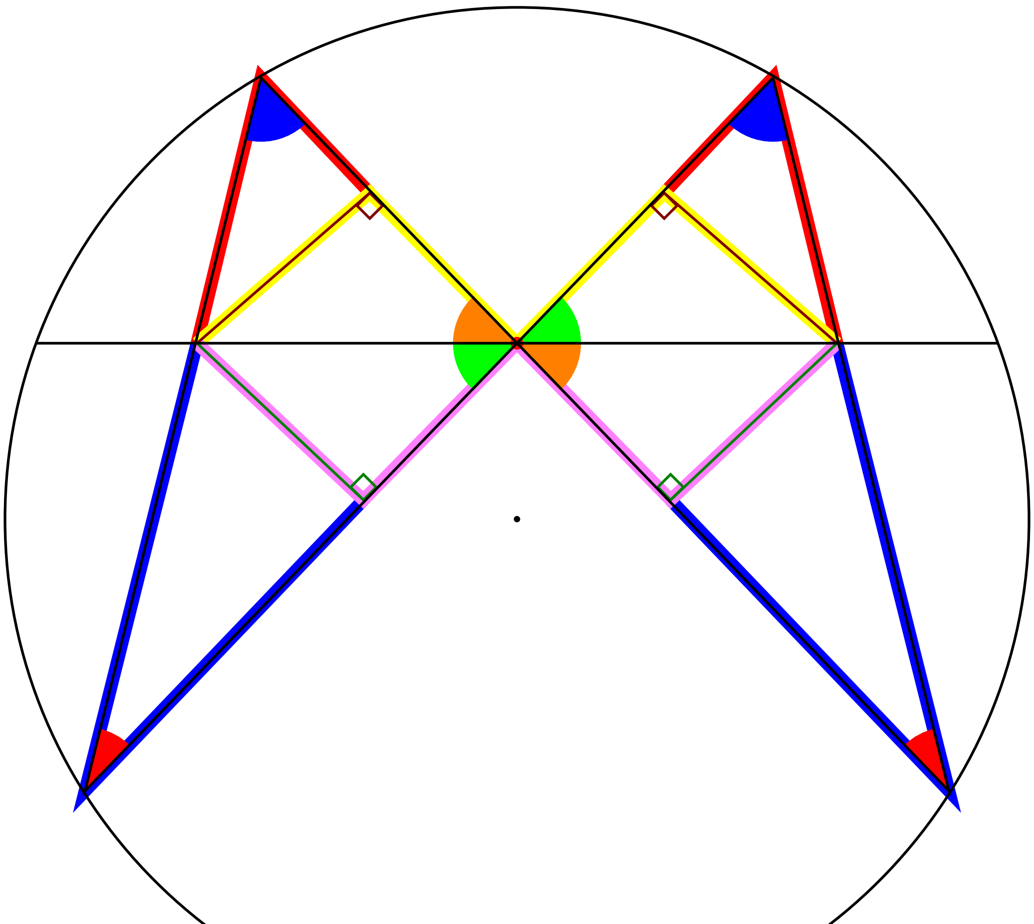


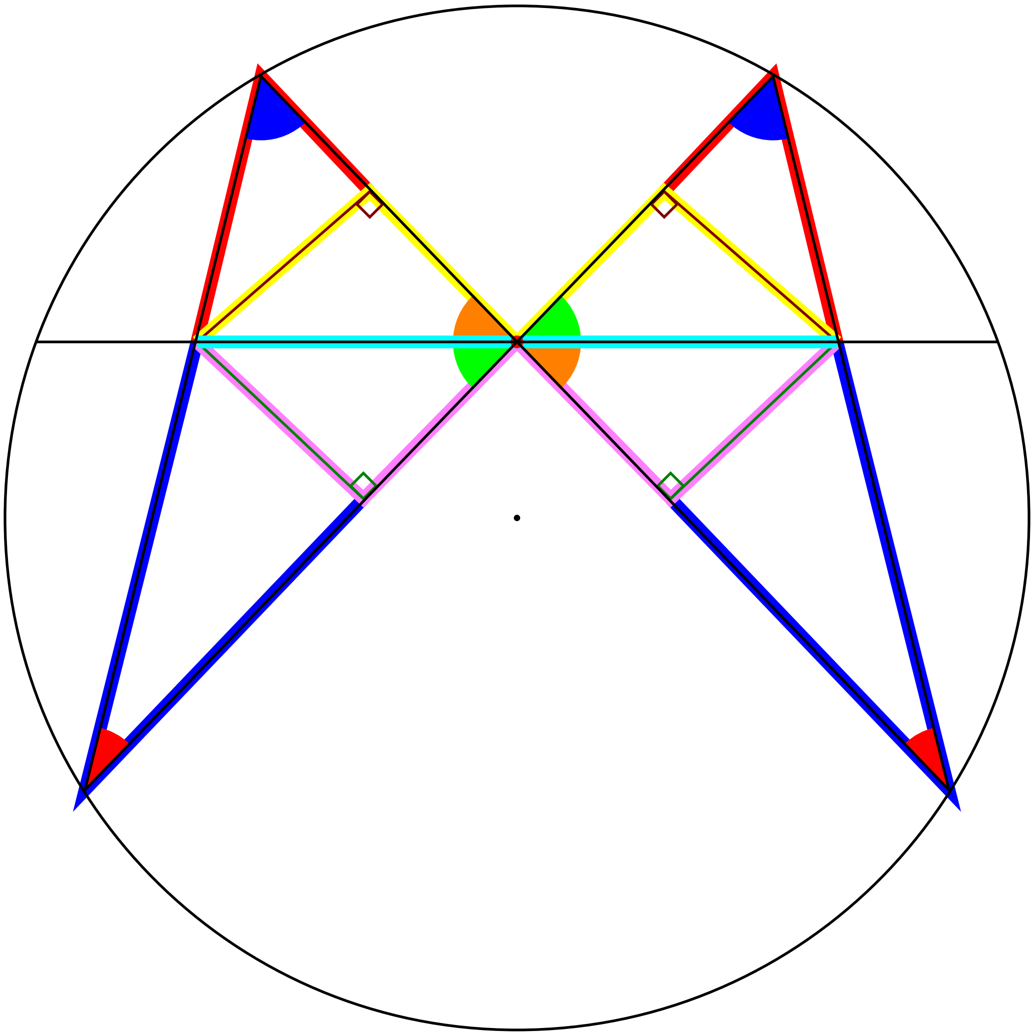


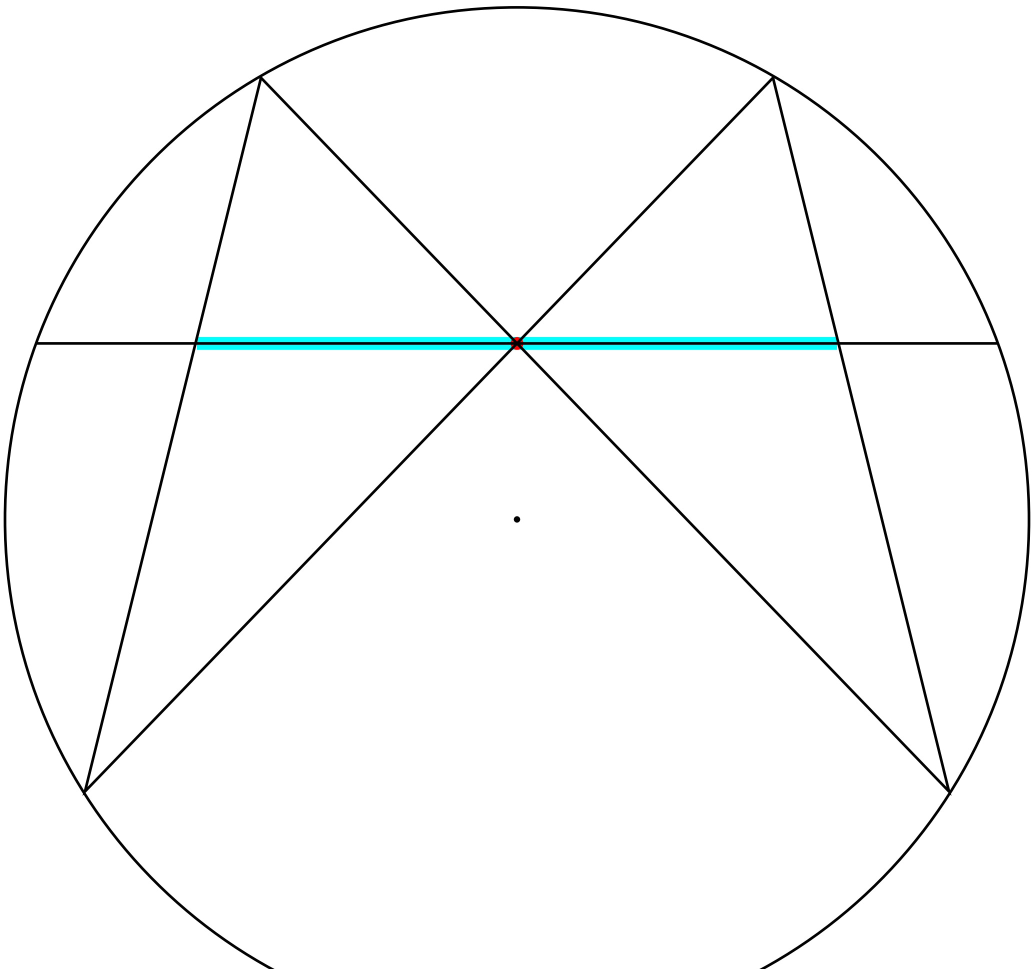






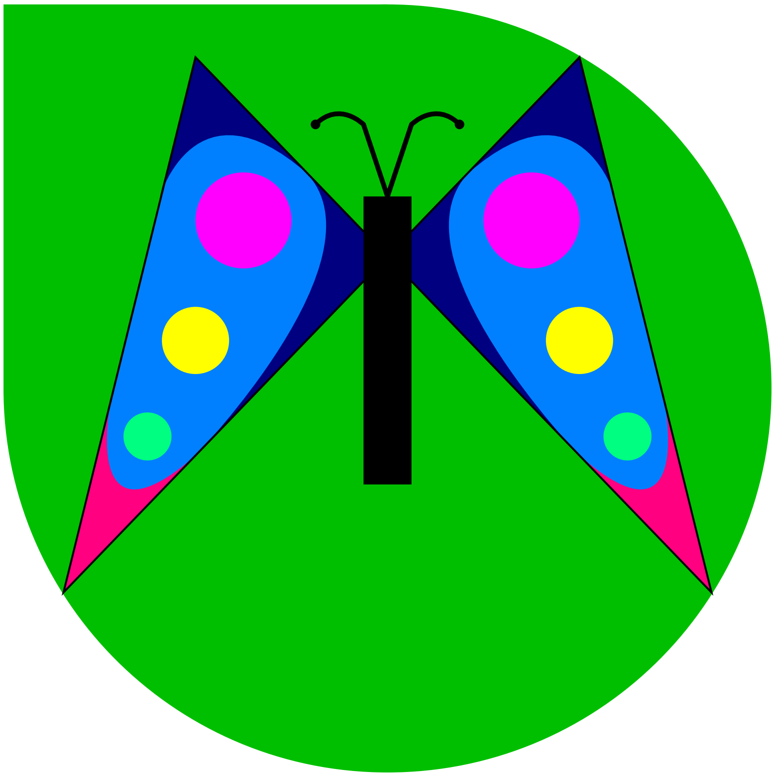


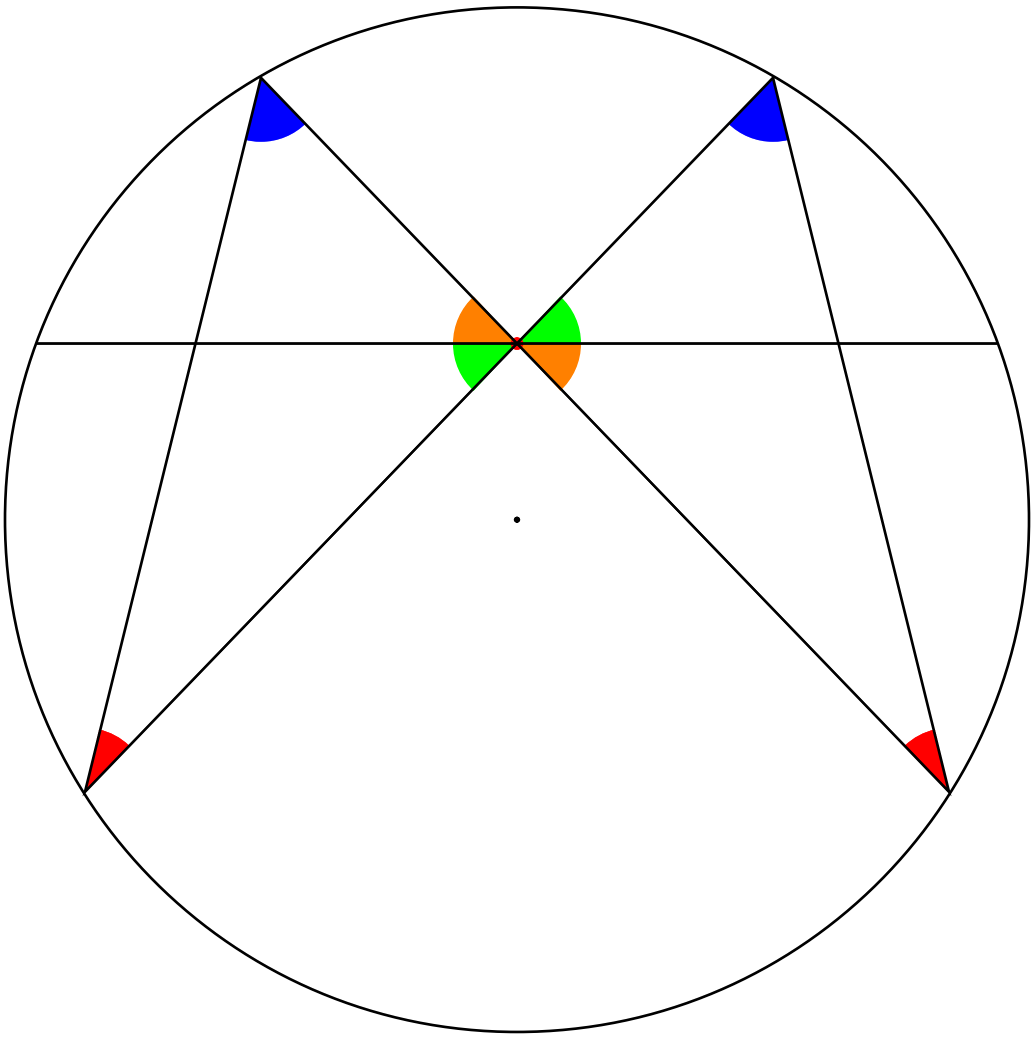


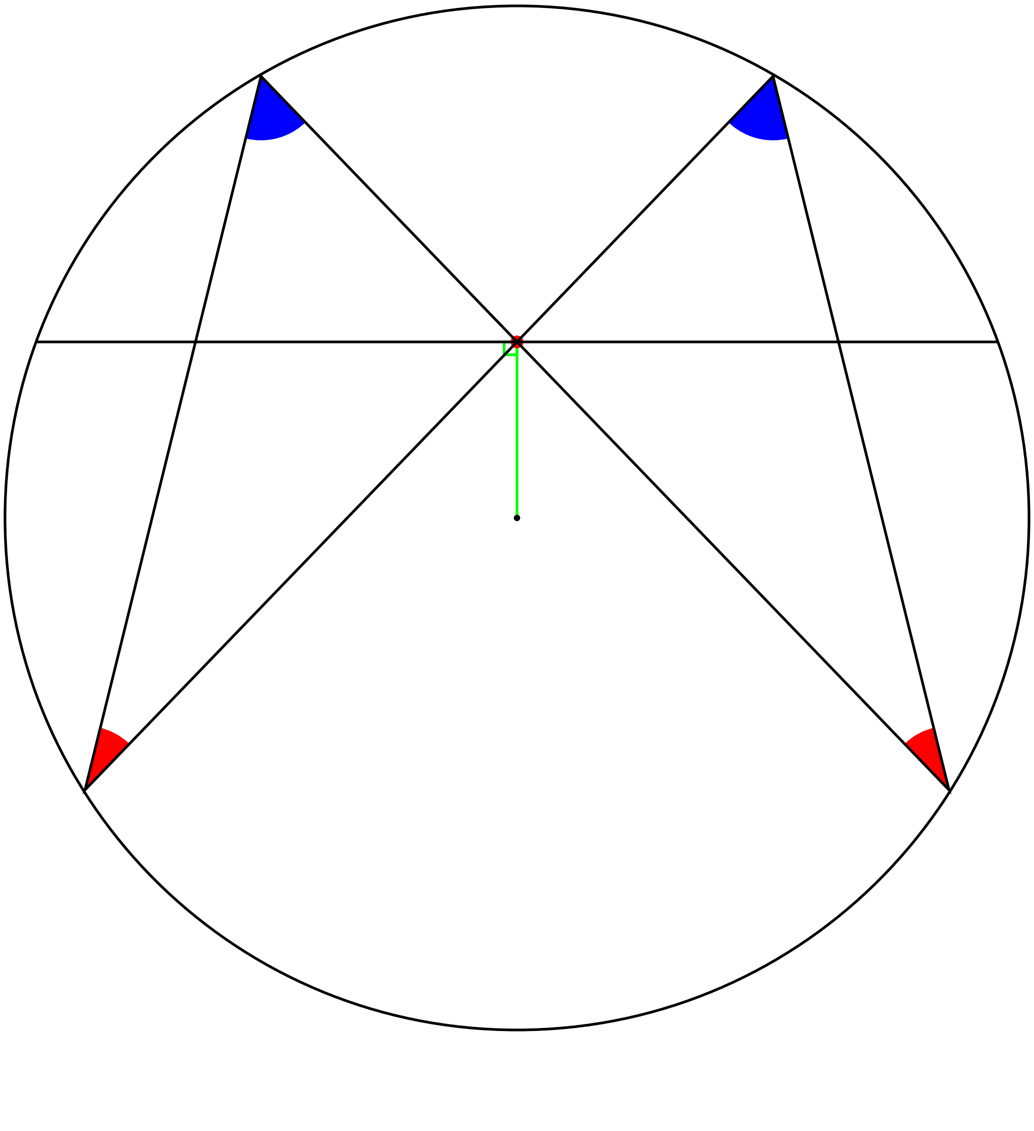


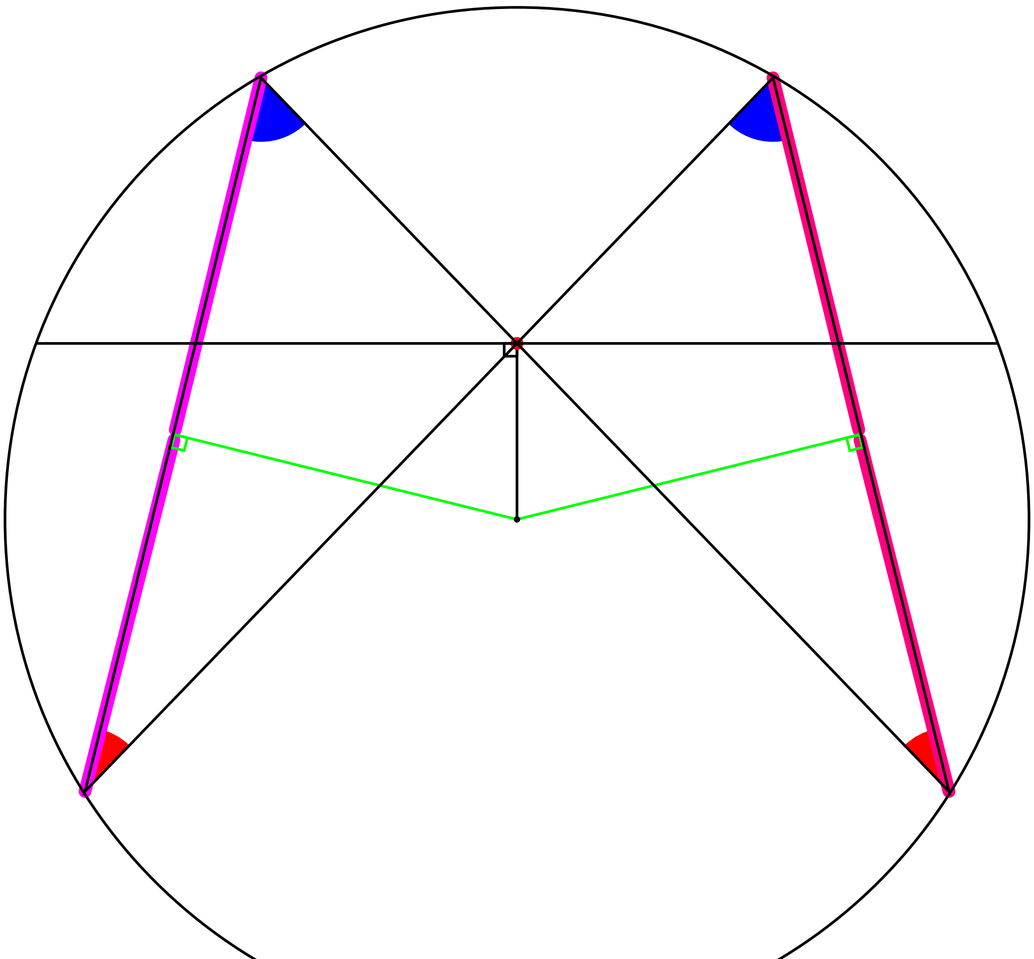
The Butterfly Theorem

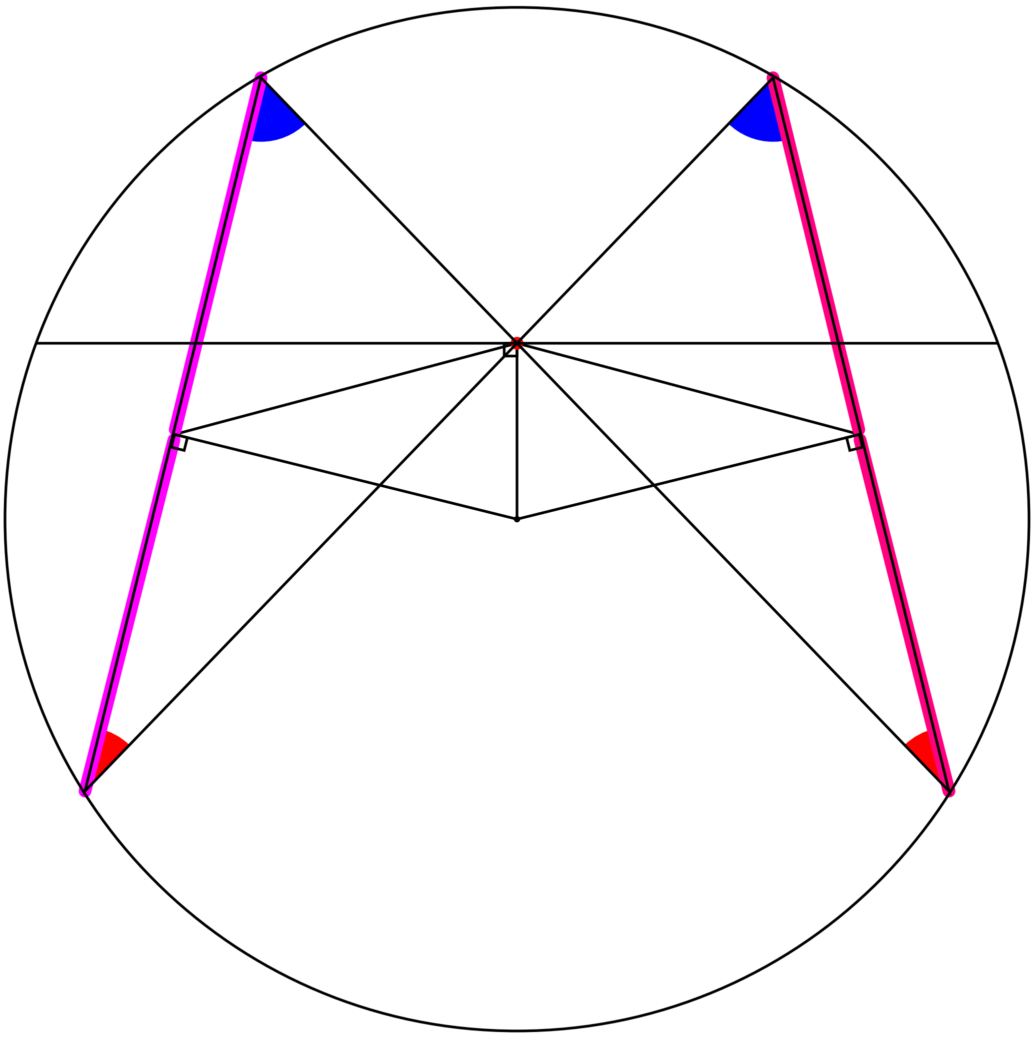
Third Proof

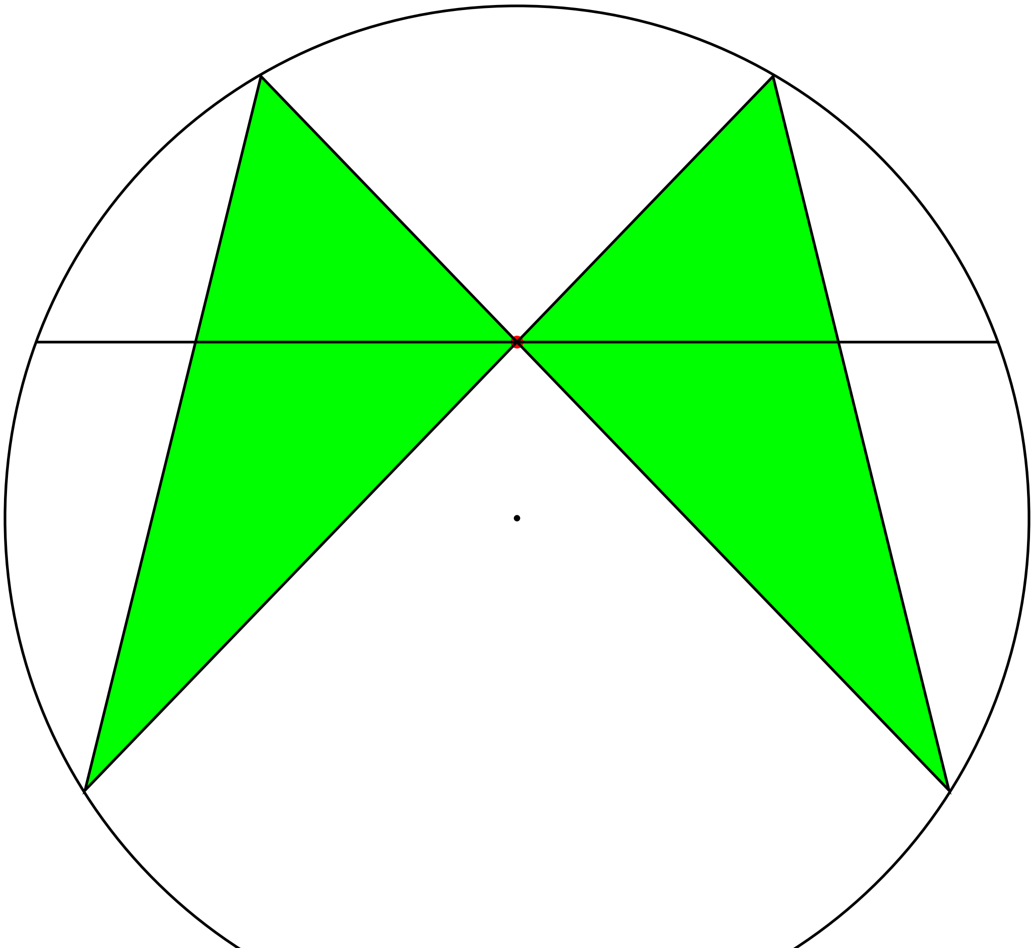


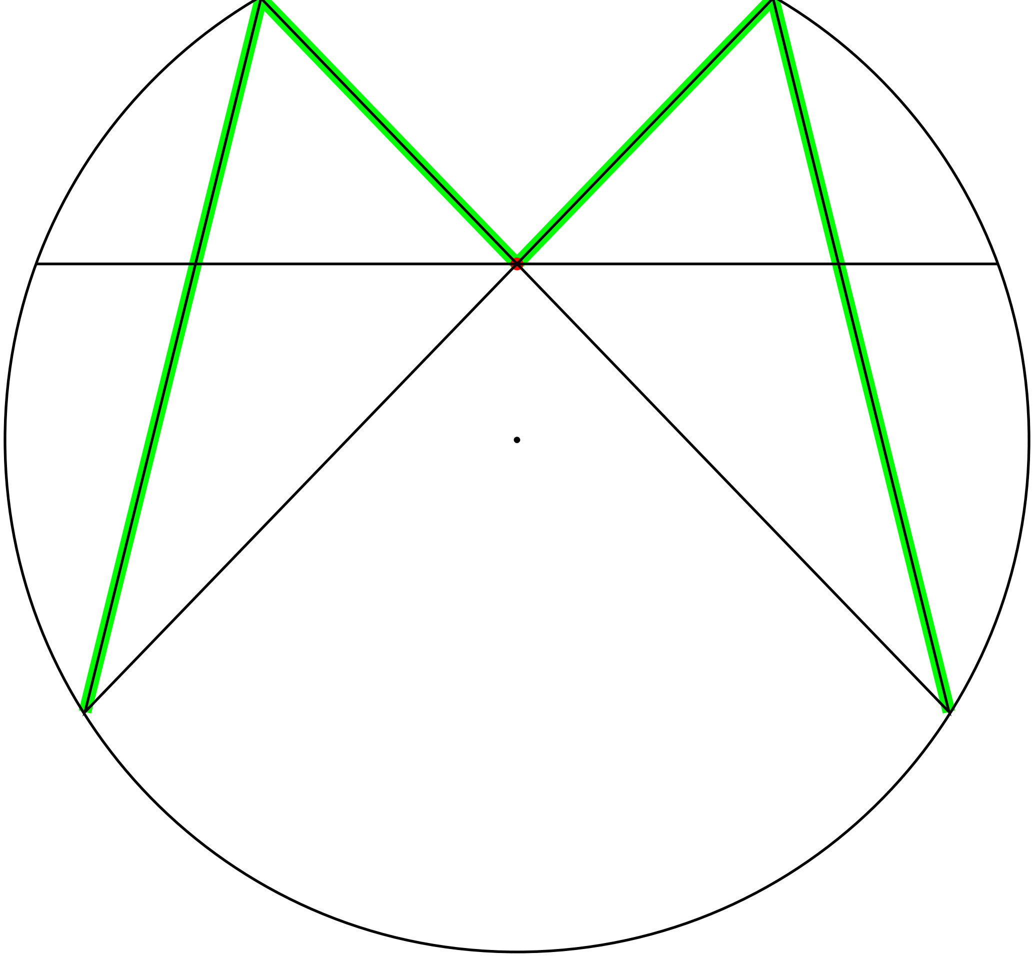


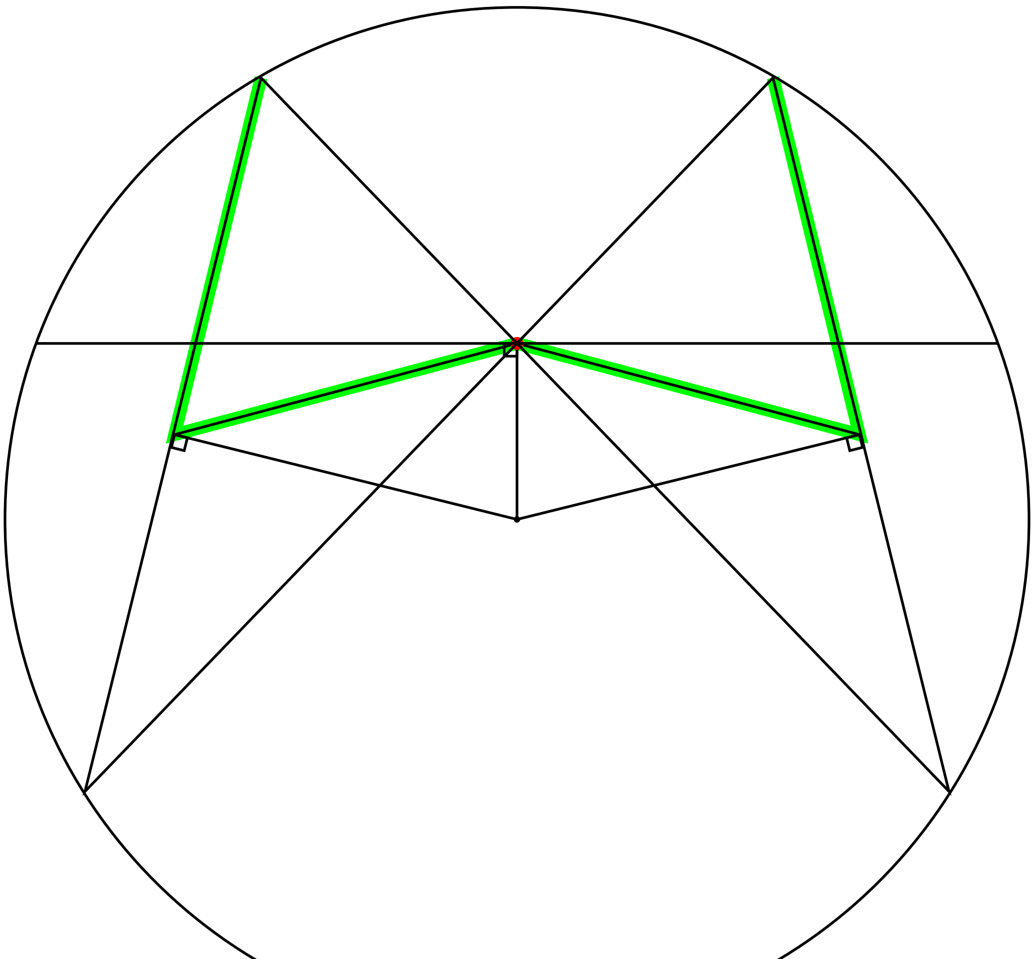


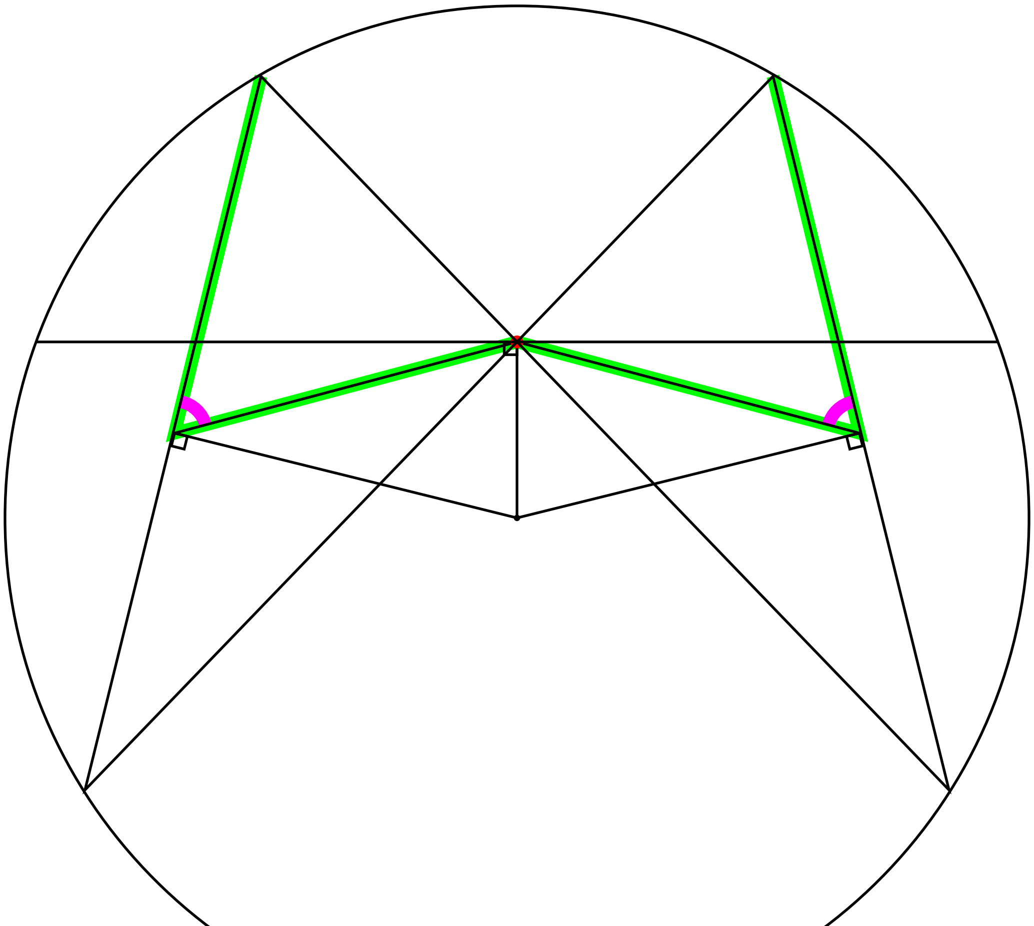


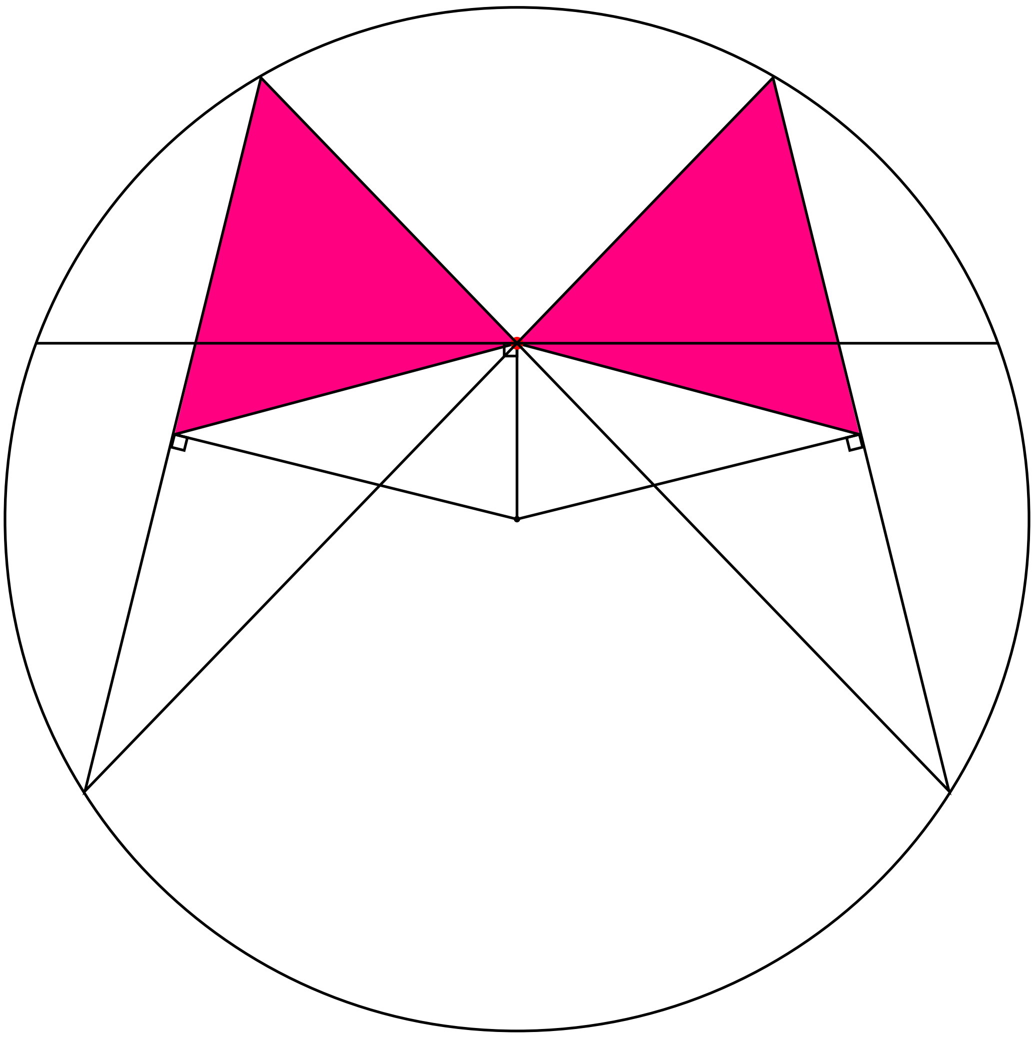


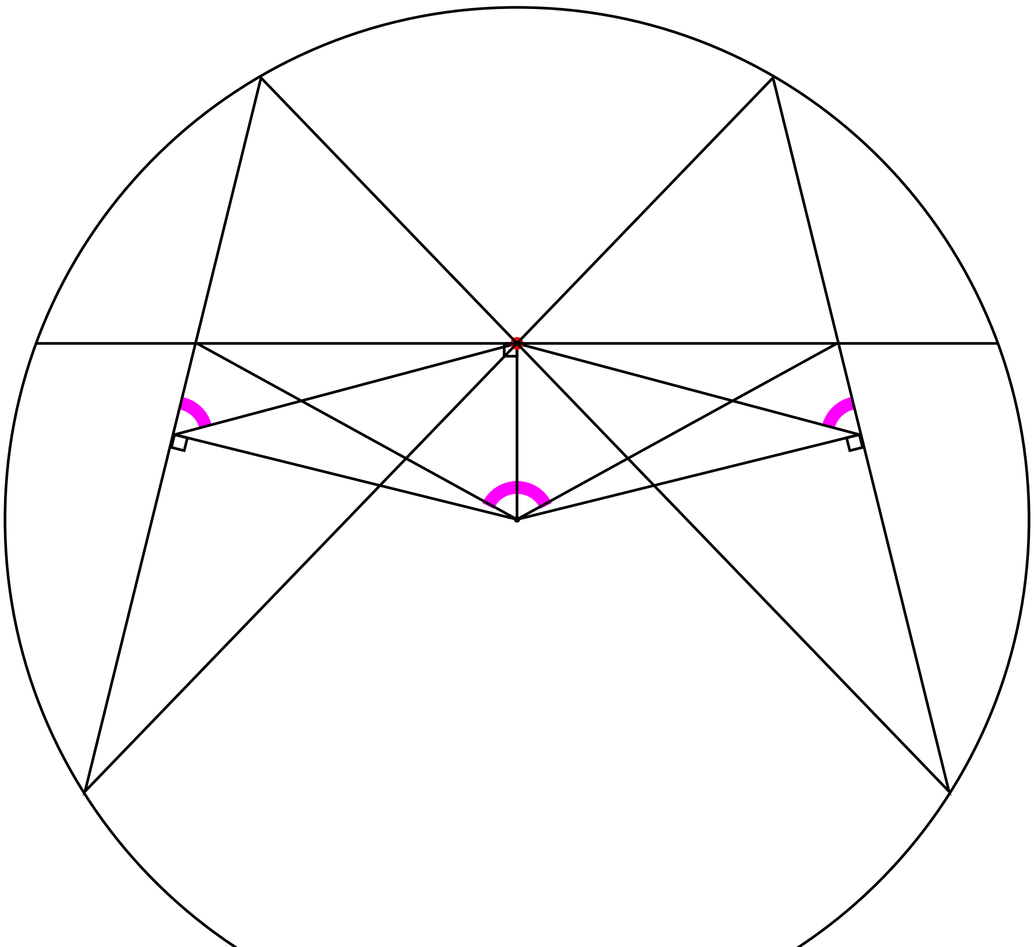


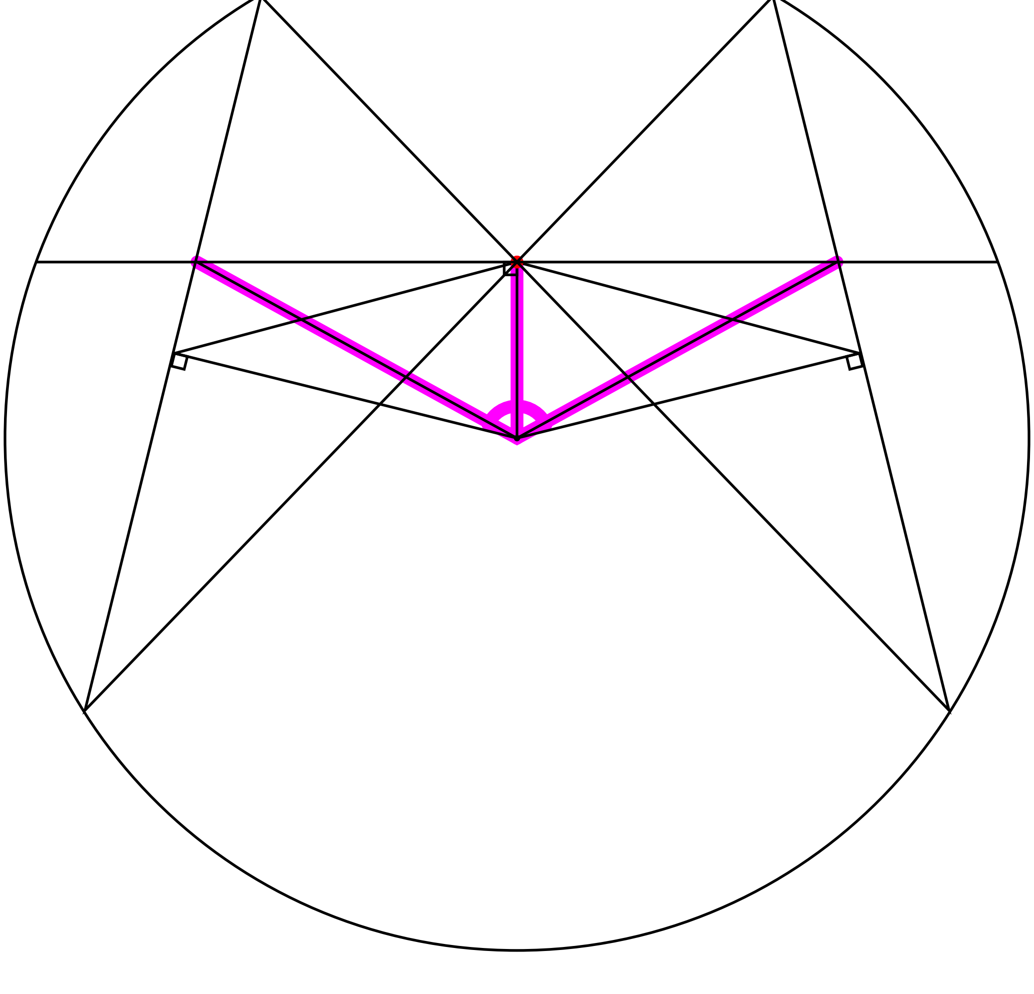


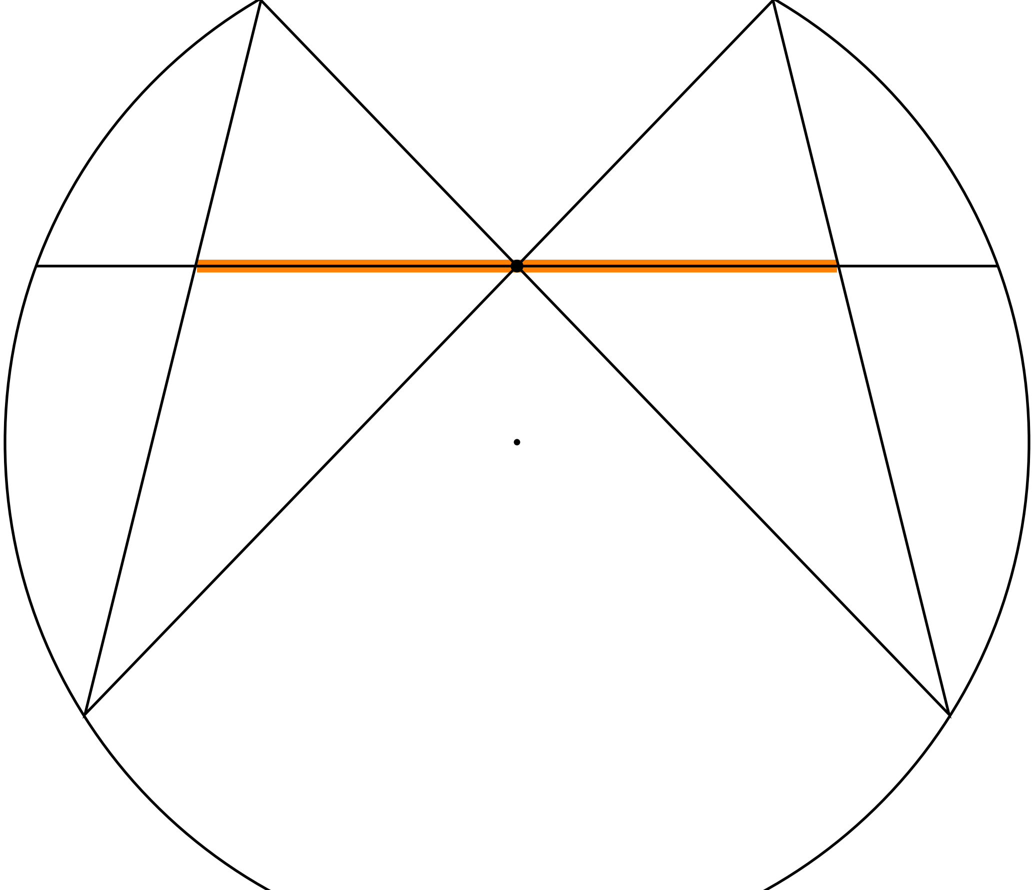


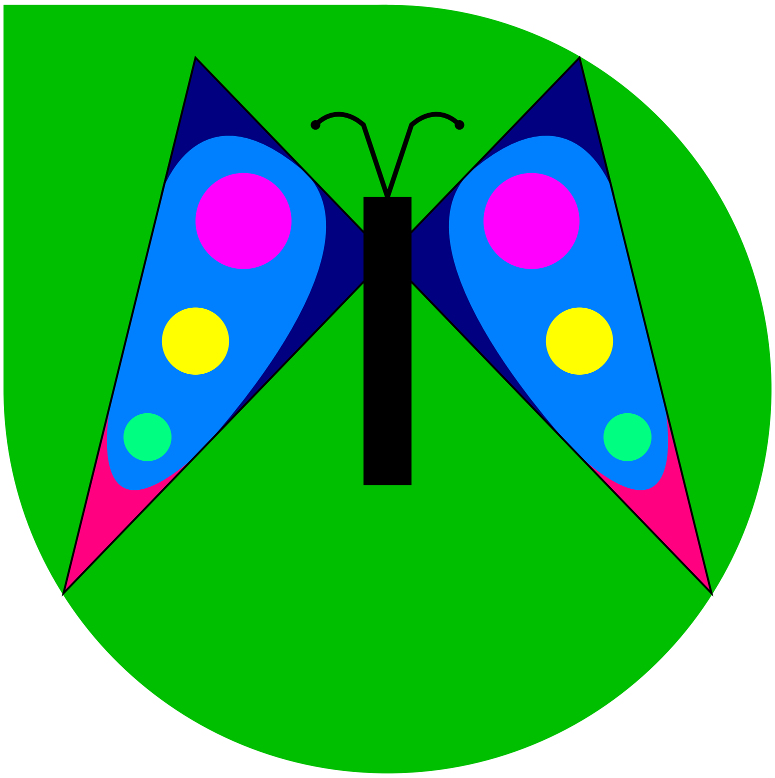


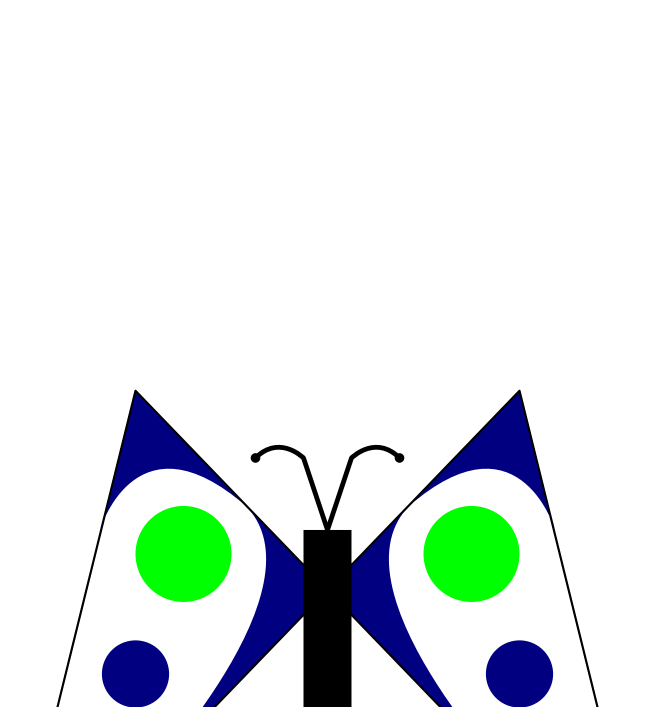


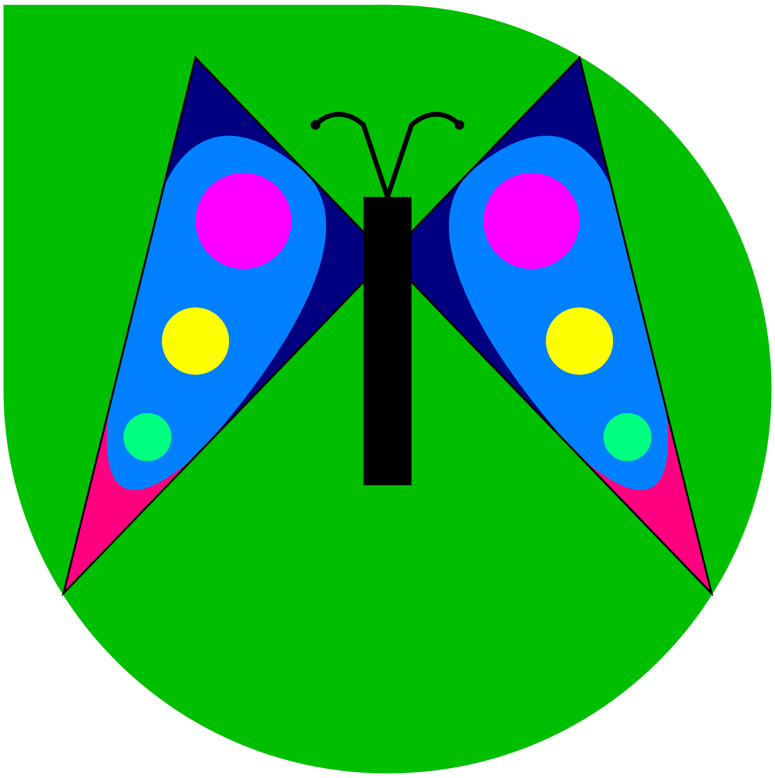


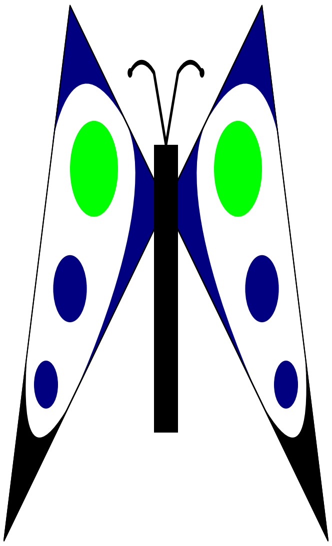


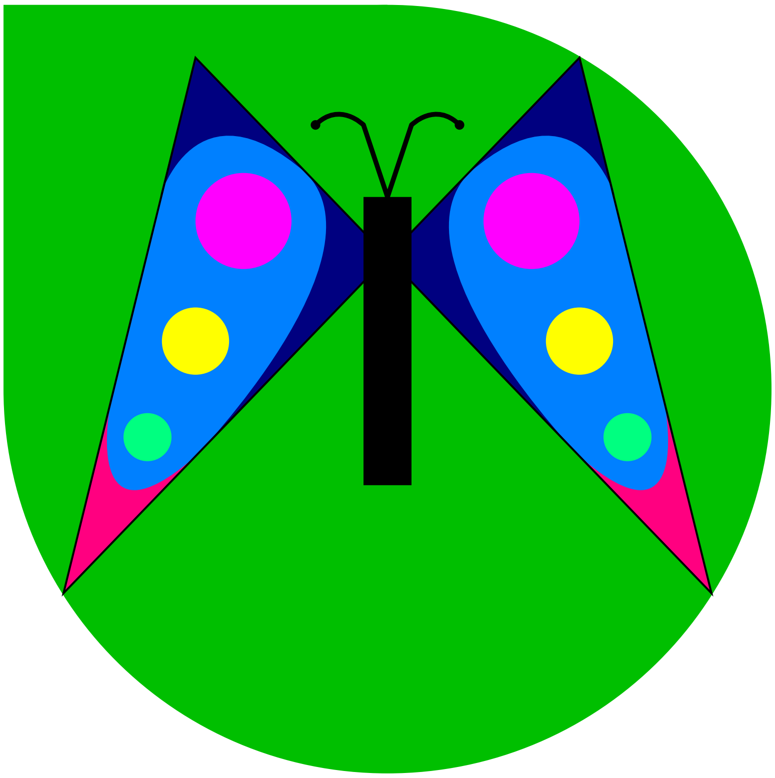


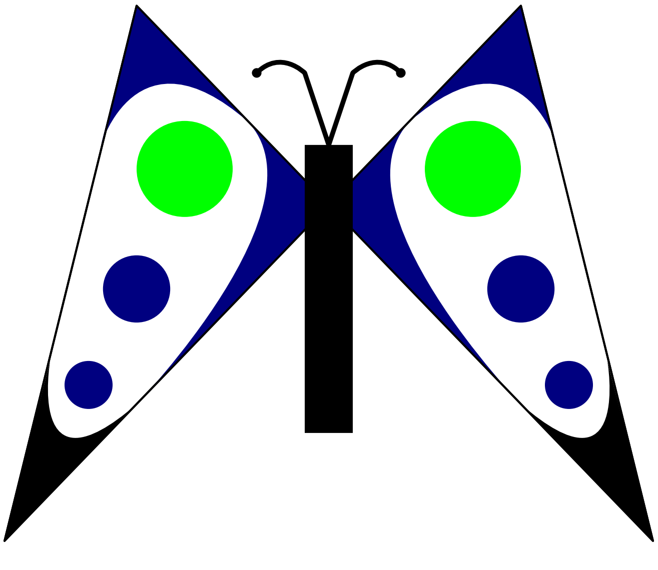


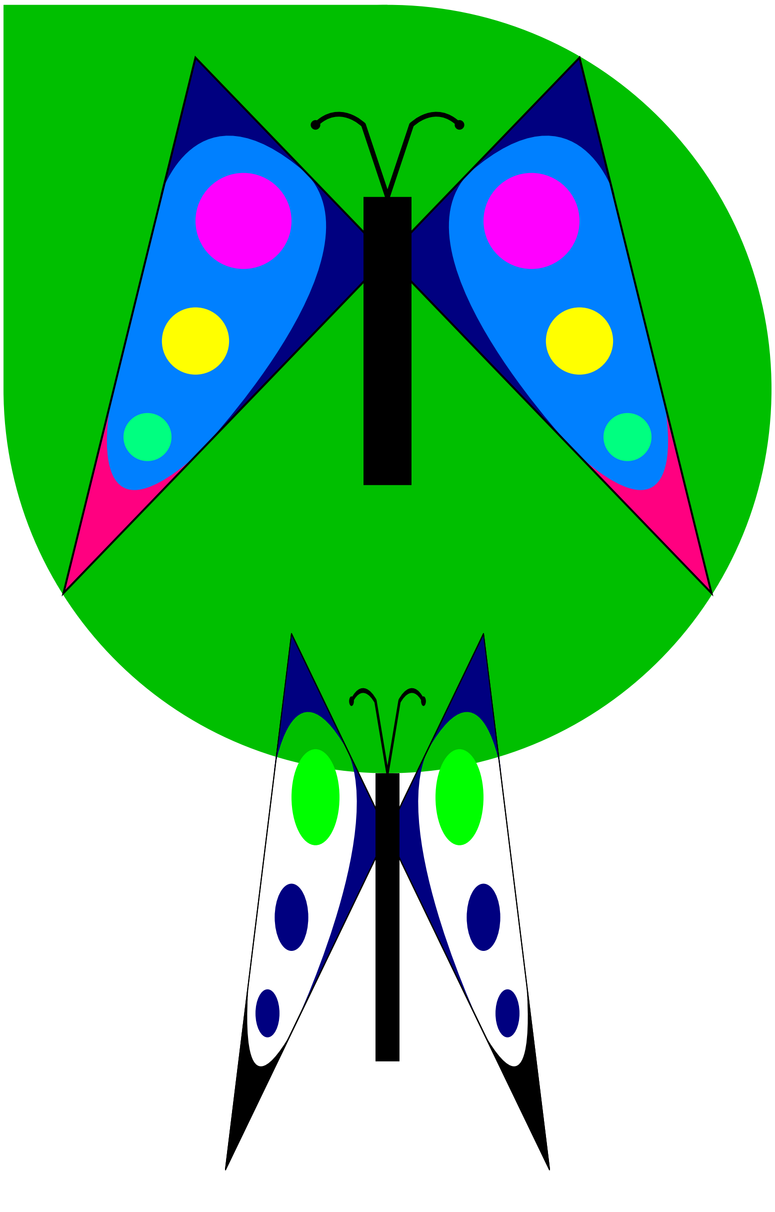


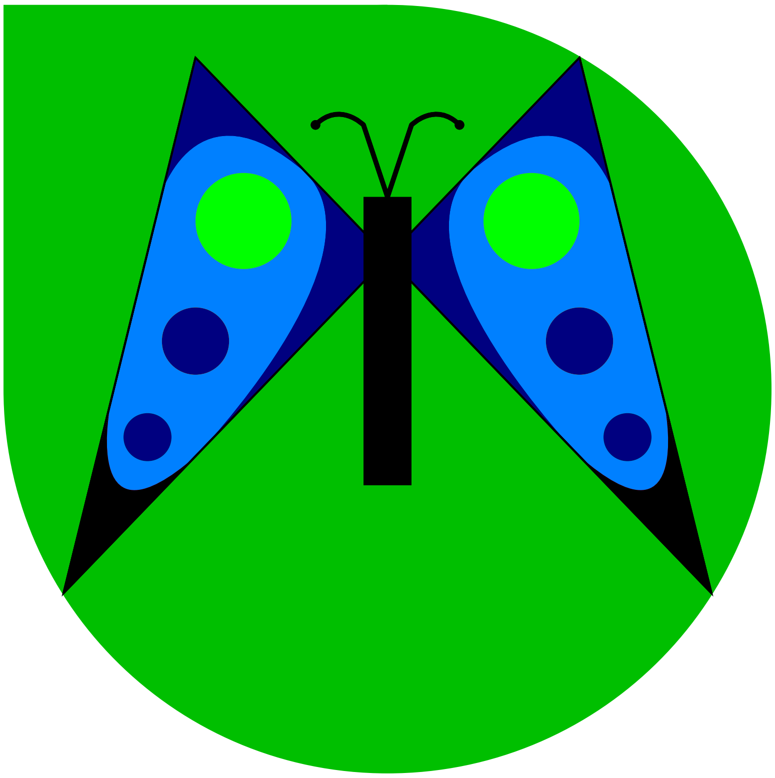




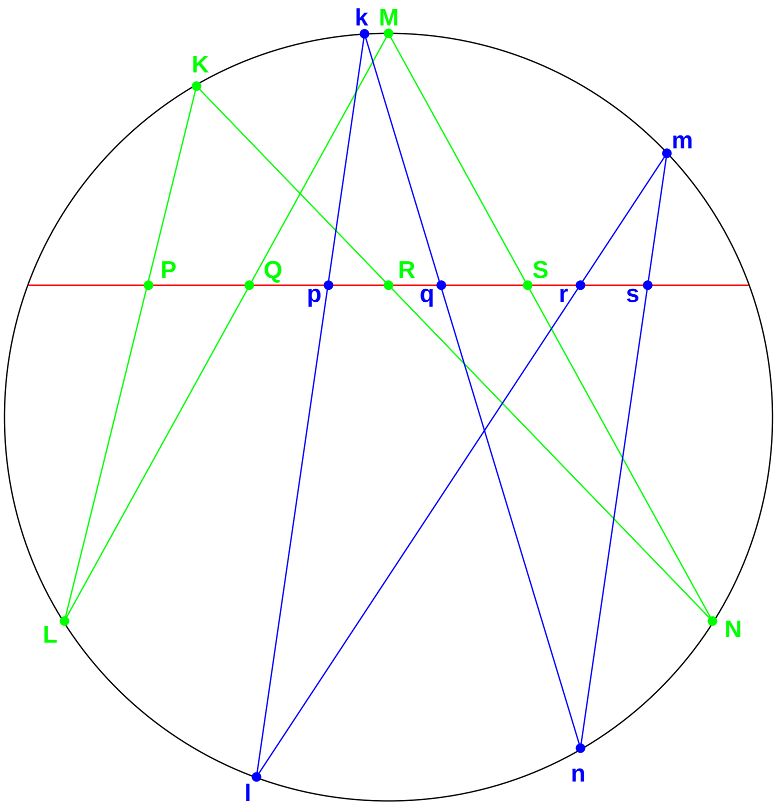








The Two Butterflies Theorem



Theorem: Let two self-intersecting quadrilaterals KLMN and klmn be inscribed in the same circle. Assume the two intersect cord AB at points PQRS and pqrs (P lies on KL, etc) Assume that none of the vertices of the quadrilaterals coincide with either A or B.

Prove that the same three of the points P,Q,R,S coincide with three points p,q,r,s then the remaining two points also coincide

Using the Butterfly Lemma...

It states that P,Q,R,and S lie on a conic given by a second degree f=0, then for some constant X and Y,

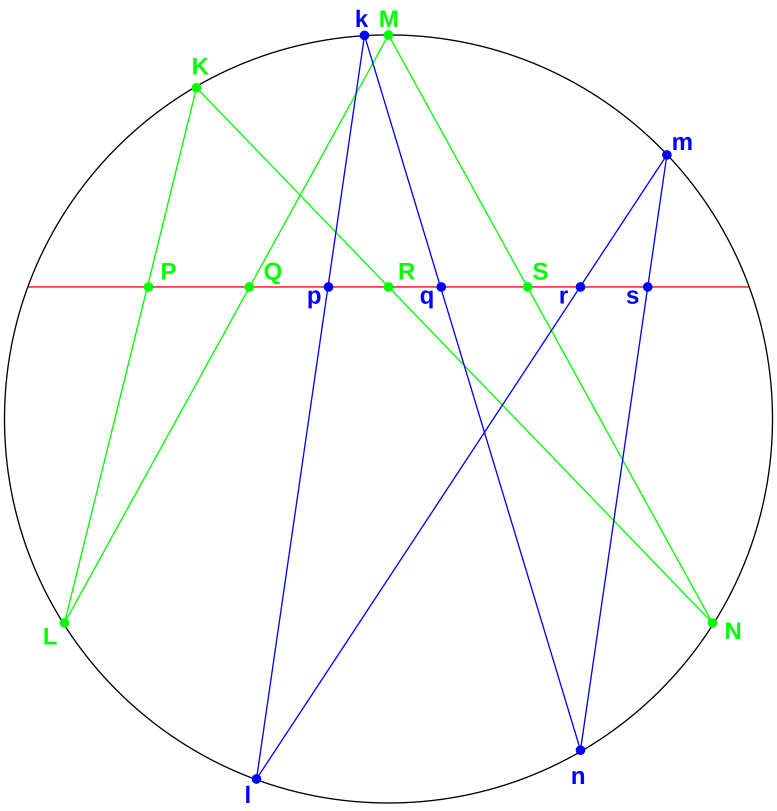
f=X1(PQ)1(RX) + Y1(PS)1(QR)

where 1(PQ)=0, 1(RS)=0, 1(PS)=0, and 1(QR)=0 are the linear equations of PQ, RS, PS, QR.

In our case, the equations are:

f=A1(KL)1(MN) + Y1(KN)1(ML) and

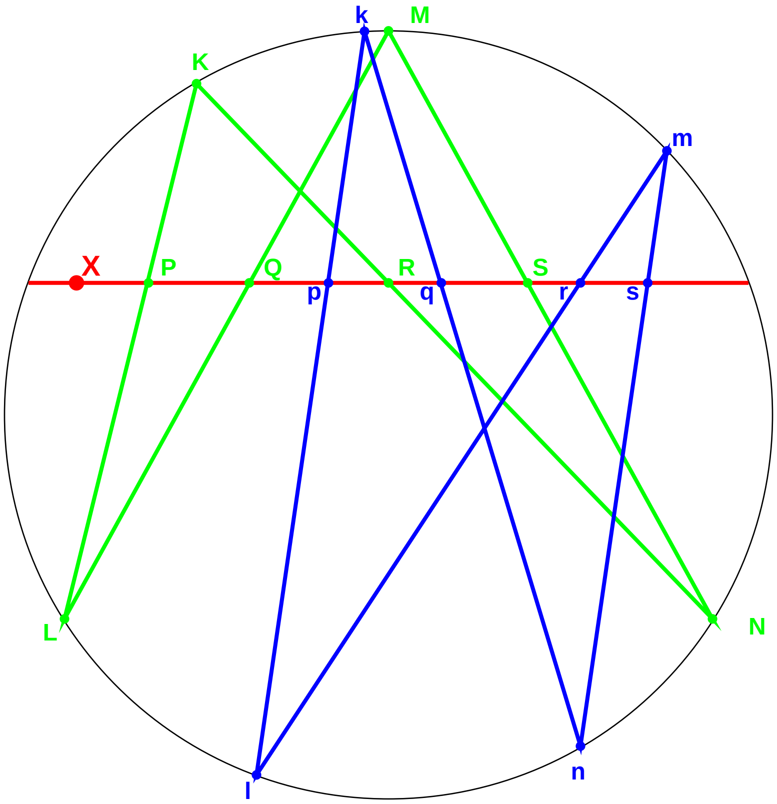
f=A1(kl)1(mn) + Y1(kn)1(ml)



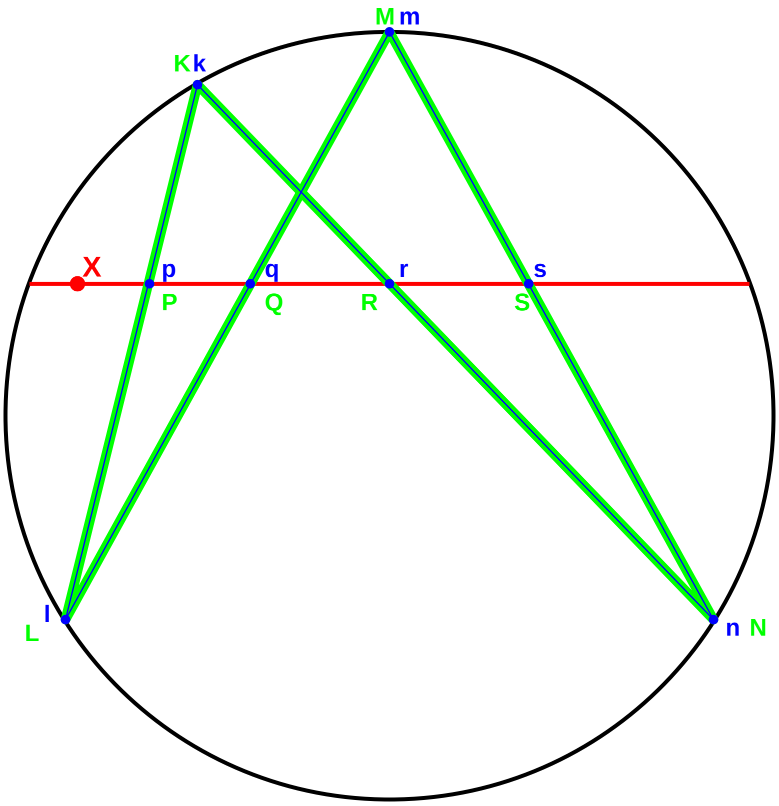
Consider AB as a number line...(with coordinate x)

Assume P=p,Q=q,S=s, the we get the equation(from the lemma):

$$A(x-P)(x-R)+B(x-Q)(x-S) = a(x-p)(x-r)+b(x-q)(x-s)$$



We are going to show that S=s



If A=a, then B=b and S=s but Q many not be equal to P or R therefore (x-P)(x-R) is not divisible by (x-Q) therefore A=a thus the theorem holds true.