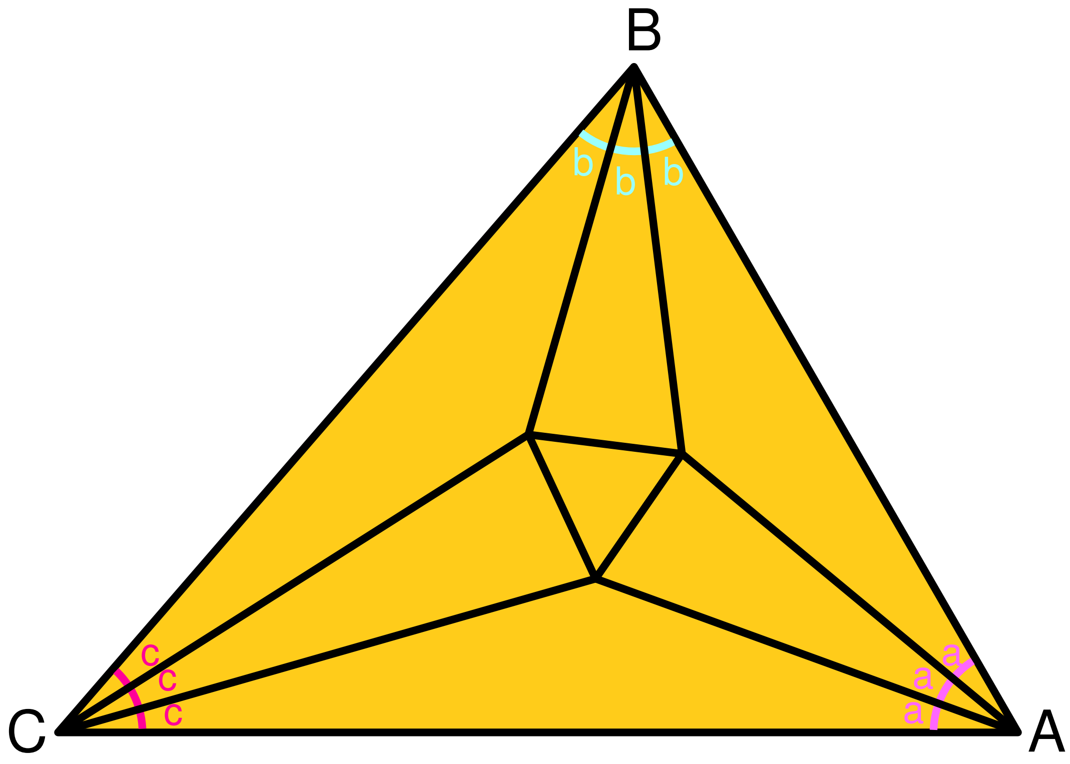
J. Conway's Proof

The proof below is an indirect proof based on the work of Conway dating from around 1995.

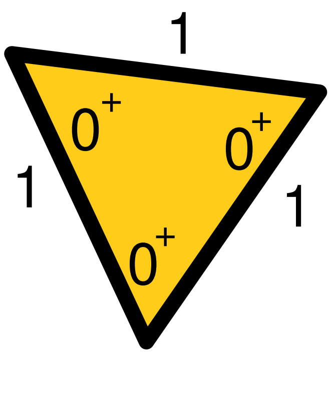


From the above figure we know that $3a+3b+3c = 180$,
which means that $a+b+c= 60$.
Let $x^{+} = x+60$ for any angle x.
since $a+b+c = 60$. Let $0^{+} = a+b+c=60$.
This shows that it is possible to construct an triangle with 3 different types of angle combinations:

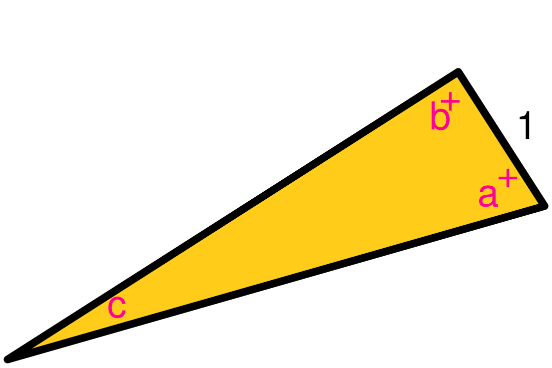
Type 1: $0^{+}, 0^{+}, 0^{+}$;
Type 2: a, b^{+}, c^{+}; a^{+}, b, c^{+}; a^{+}, b^{+}, c
Type 3: a^{++}, b, c; a, b^{++}, c; a, b, c^{++}
since these seven combinations of angles all have a sum of 180.

Instead of working forward, Conway worked backwards. He showed that from an equilateral triangle one can construct a triangle with any angles, i.e. with arbitrary a,b and c that sums to 60. According to Conway, we can make the following constructions:

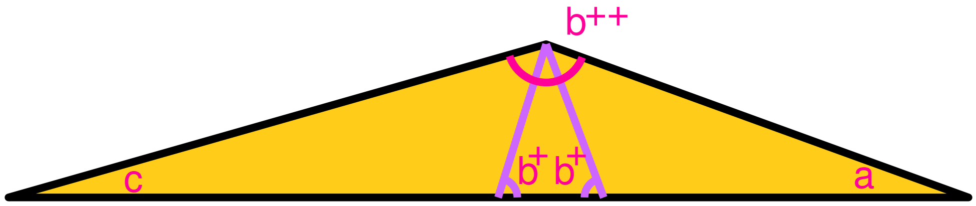
Type 1. Construct an equilateral triangle with length 1.



Type 2. Construct a triangle with the side joining larger angles e.g. a^+ and b^+ to have length 1. for example:



Type 3. Construct two lines that intersects the side opposite to b++ at angle b+, thus forming an isosceles triangle with base angle =b+.



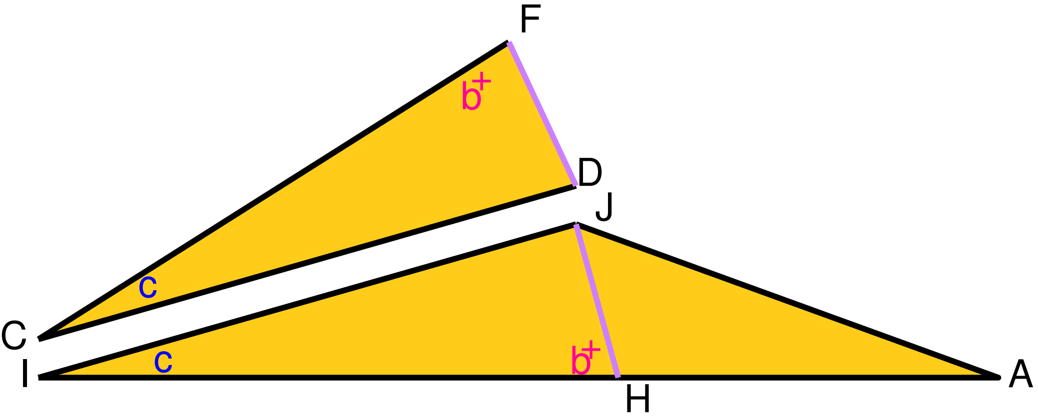
The significance of constructing the isosceles triangle is to prove JHI and DFC are congruent.

1.JHI = DFC = b^+

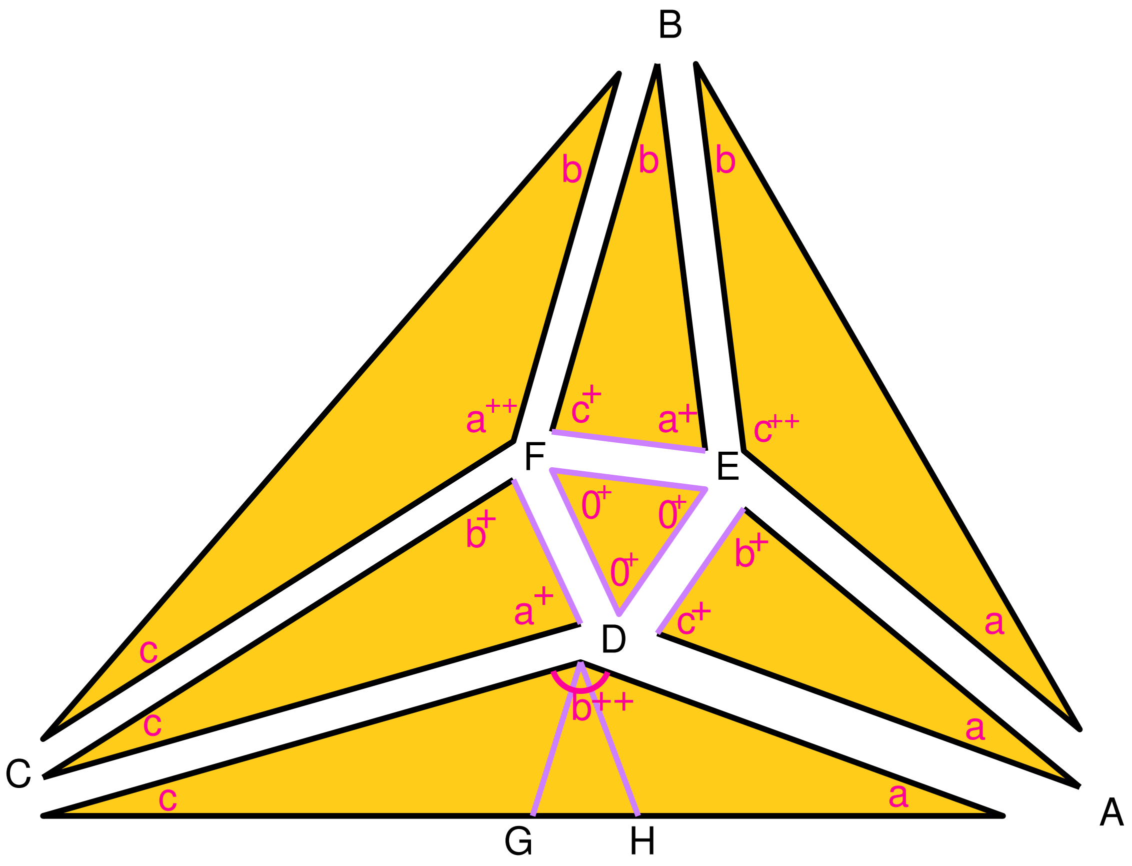
2. JIH = DCF = c

3. By construction, we get JH = DF=1

The above proves that JHI and DFC are congruent. This result is important because it shows that DC = JI are equal, when the two triangles are matched together, they become the common edge, then point J = point D and point I = point C.



Other triangles are constructed in similar way,
and we can get the following:



Note that the angles at the vertex
of the equilateral triangle e.g. D
sums up to 360.
Check: $a^{+}+c^{+}+b^{++}+0^{+}$
$= a+60+c+60+b+120+60$
$= a+b+c+300=360.$

Since the angles at D sums up to 360, and since the sides match due to congruency, we see that the triangles can be assembled perfectly together to form a larger triangle with angles 3a, 3b and 3c, and hence the conclusion.