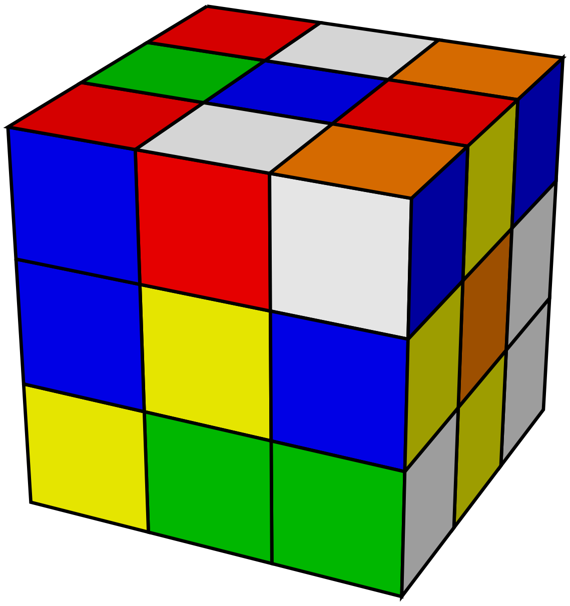
Rubik’s Cube Notation

MATH 308 Project

By Stefanie Krzak

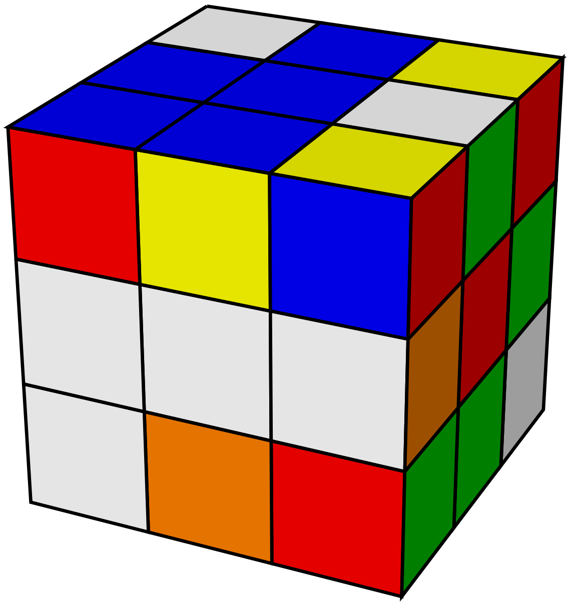
Part what makes the Rubik’s Cube

so neat is that it is a 3d puzzle



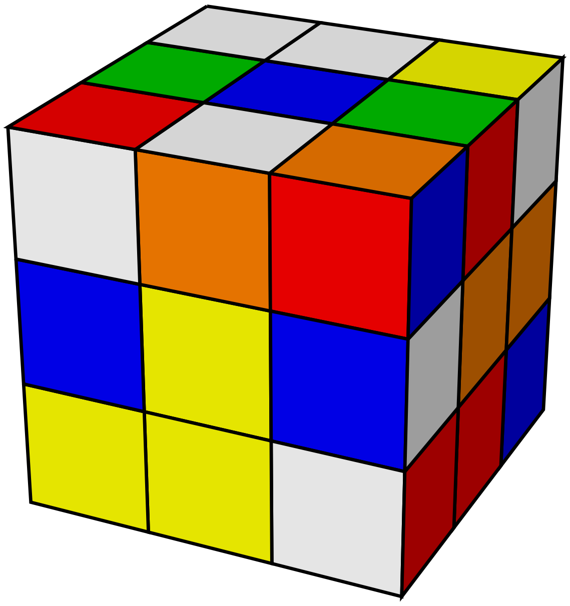
To many it is a fun challenge,

perhaps a way to pass a rainy day

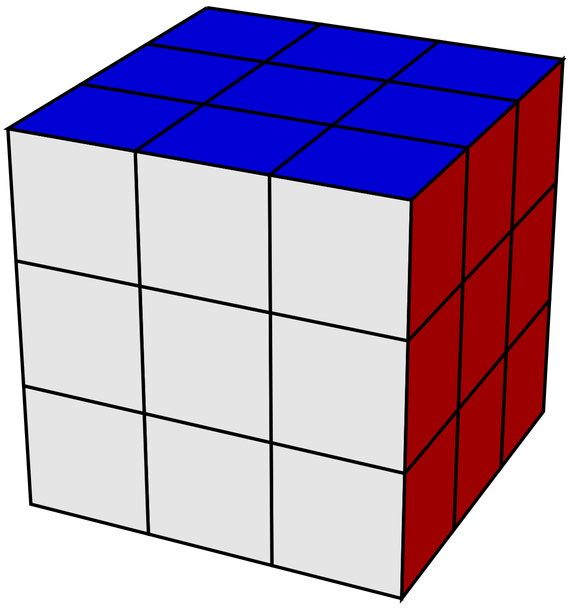


But from a mathermatical point of view

it can be thought of as a permutation puzzle



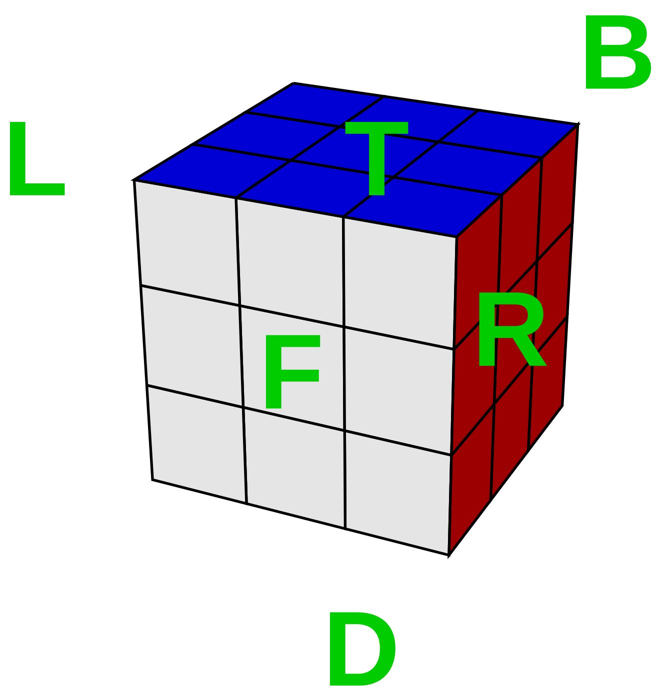
Which can be solved



Returning the cube back to its original one-side / one-colour pattern, or "cubing", is in fact so involved mathematically that there are entire books and even college courses dedicated to explaining it.

With this project I have in no way aimed to explain all that there is to the art of cubing. My goal is to present a consistent notation which may be used to express any of the possible movements you can make with the Rubik’s cube.

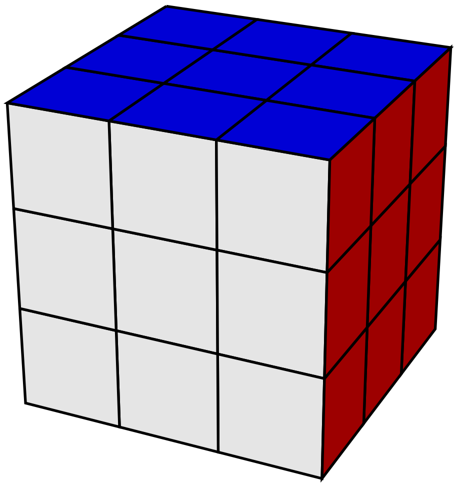
First we give each of the 6 faces a name correspsinging to its location relative to the face we decide is the "front".



F = Front **T = Top** **R = Right**

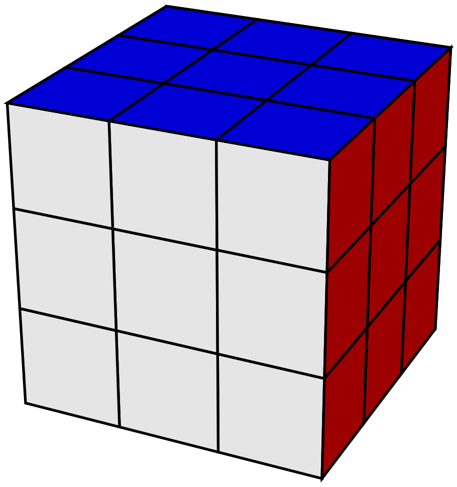
B = Back **D = Down** **L = Left**

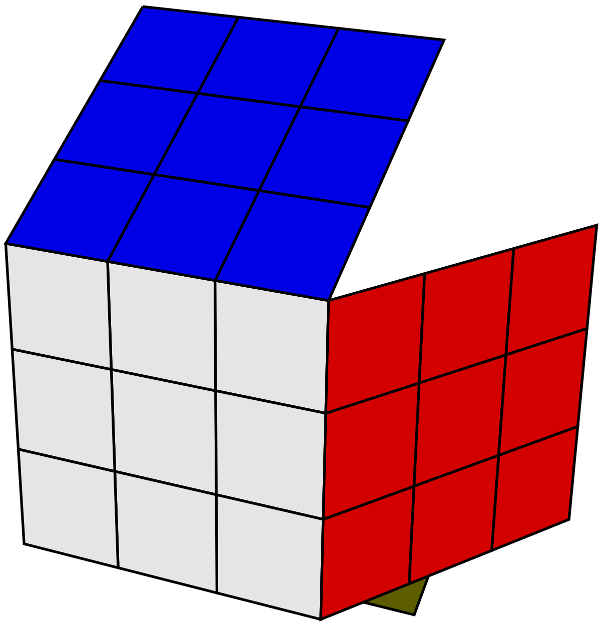
Next we must have a way of identifying each of the 54 facets (6 faces x 9 facets/face) of the cube.

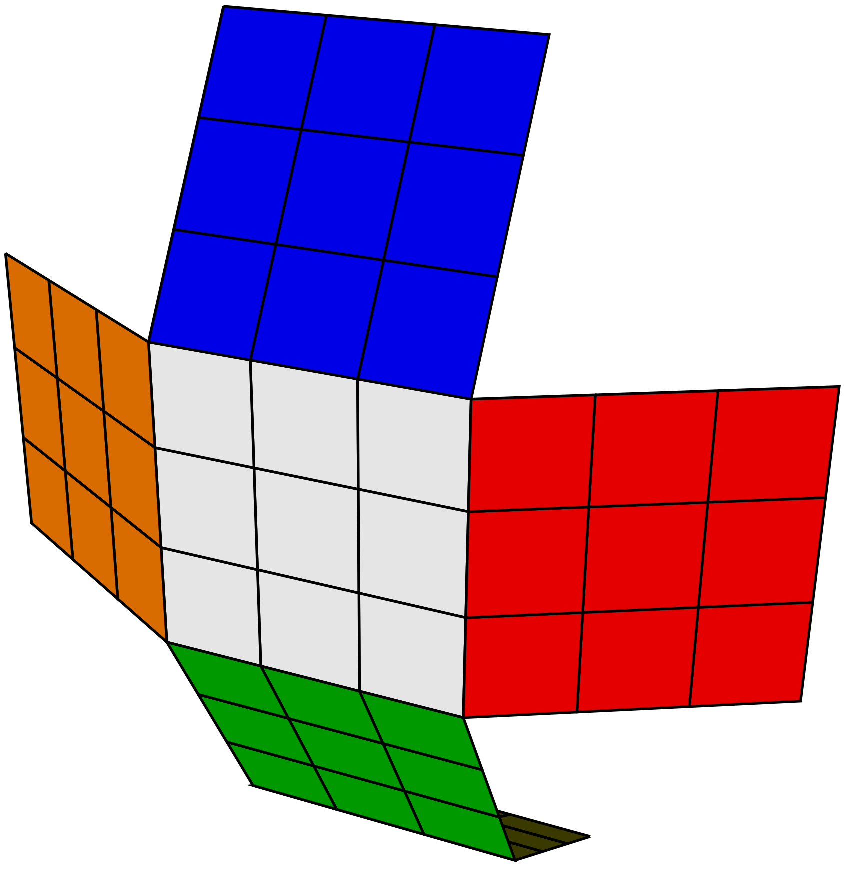


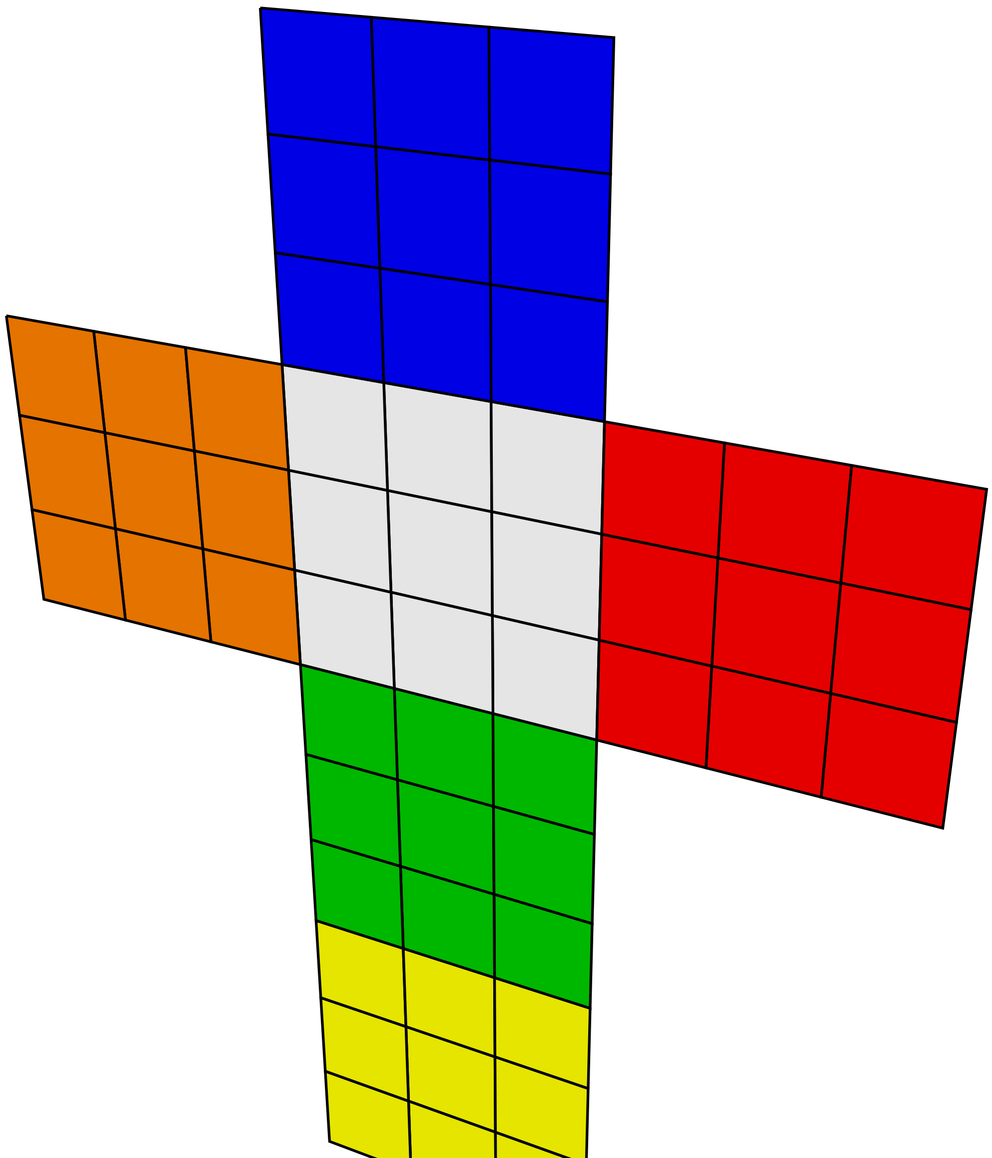
Before we start naming all the facets,

let’s unfold the cube making them all visible:

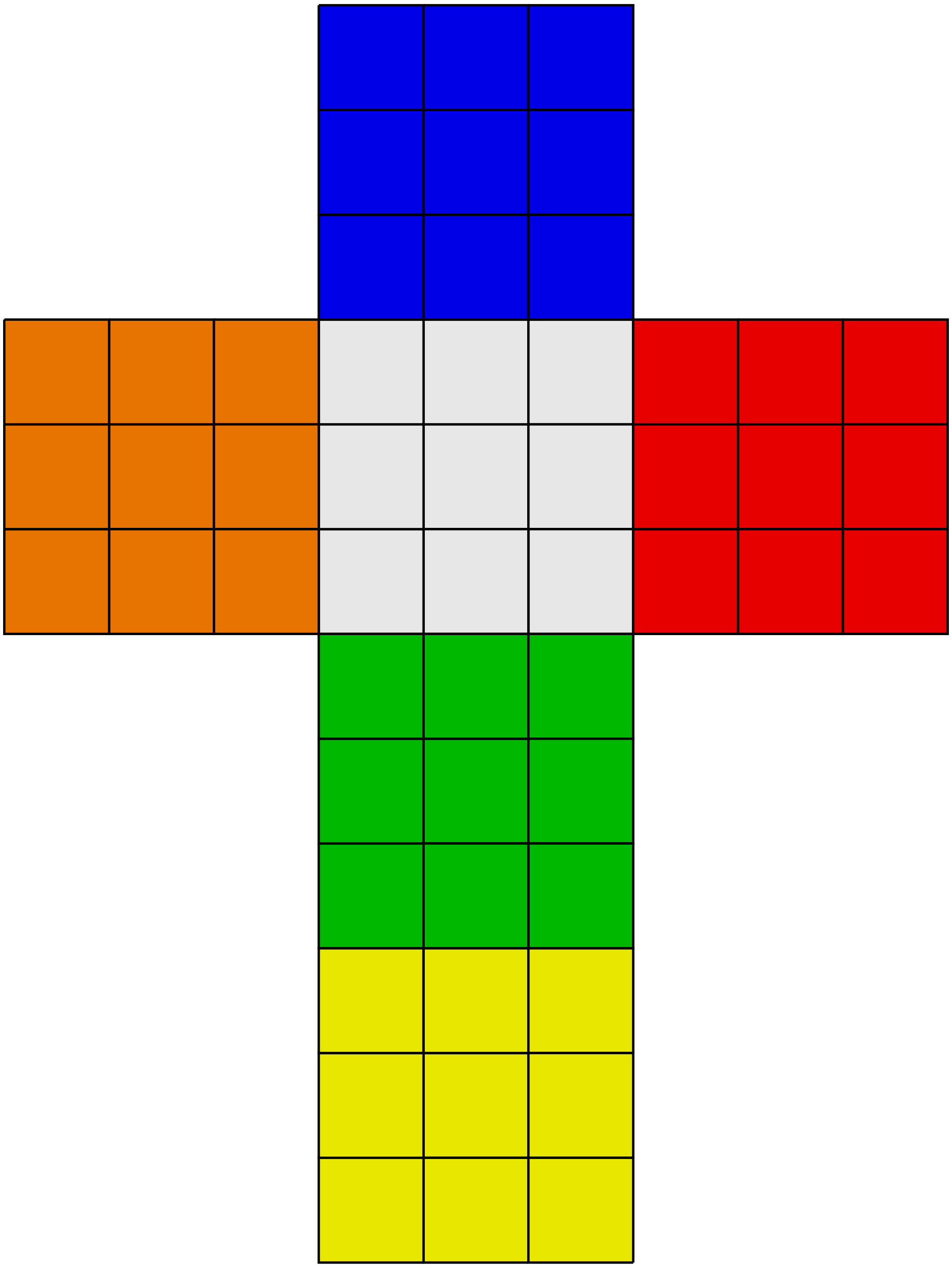




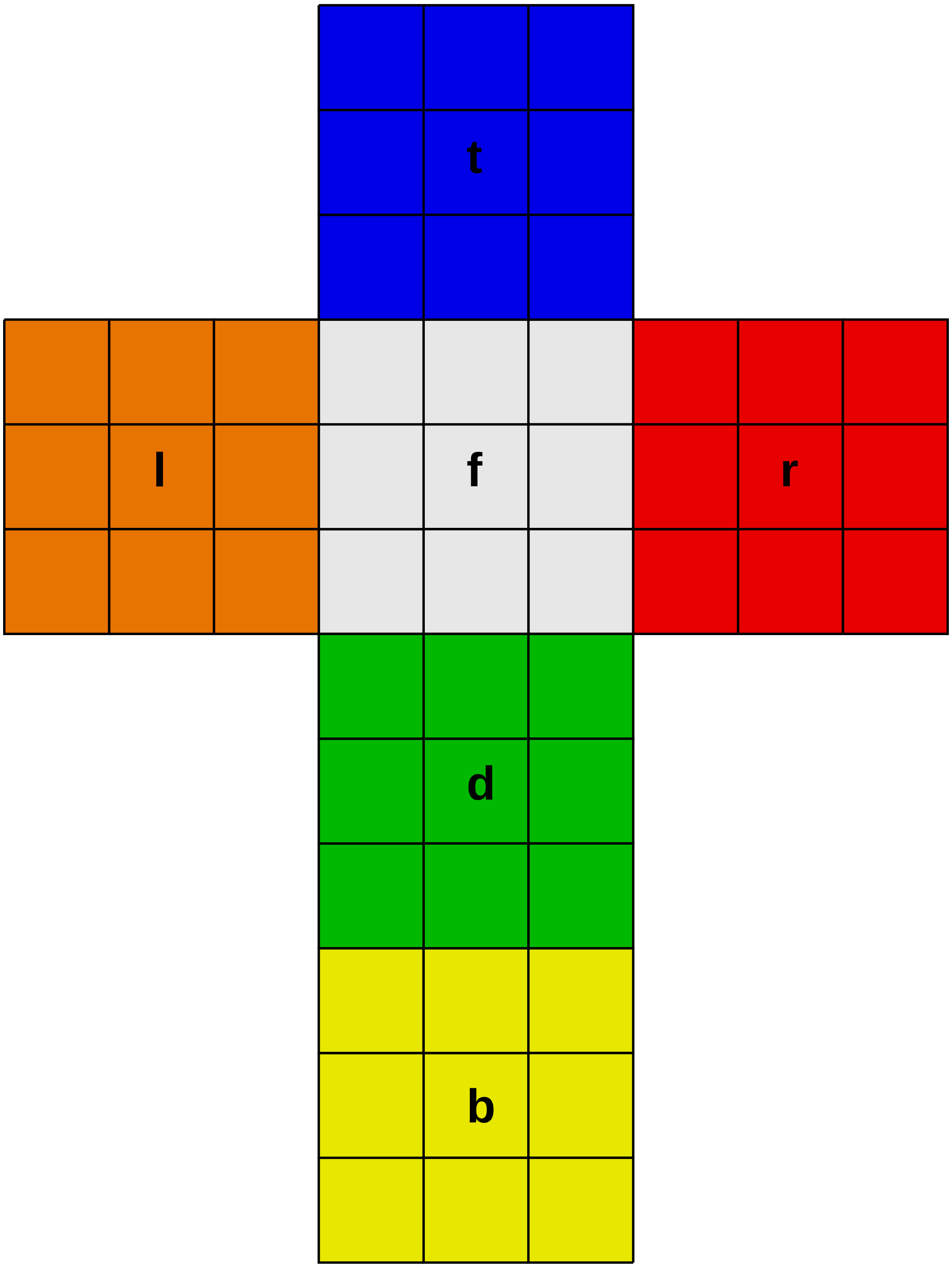




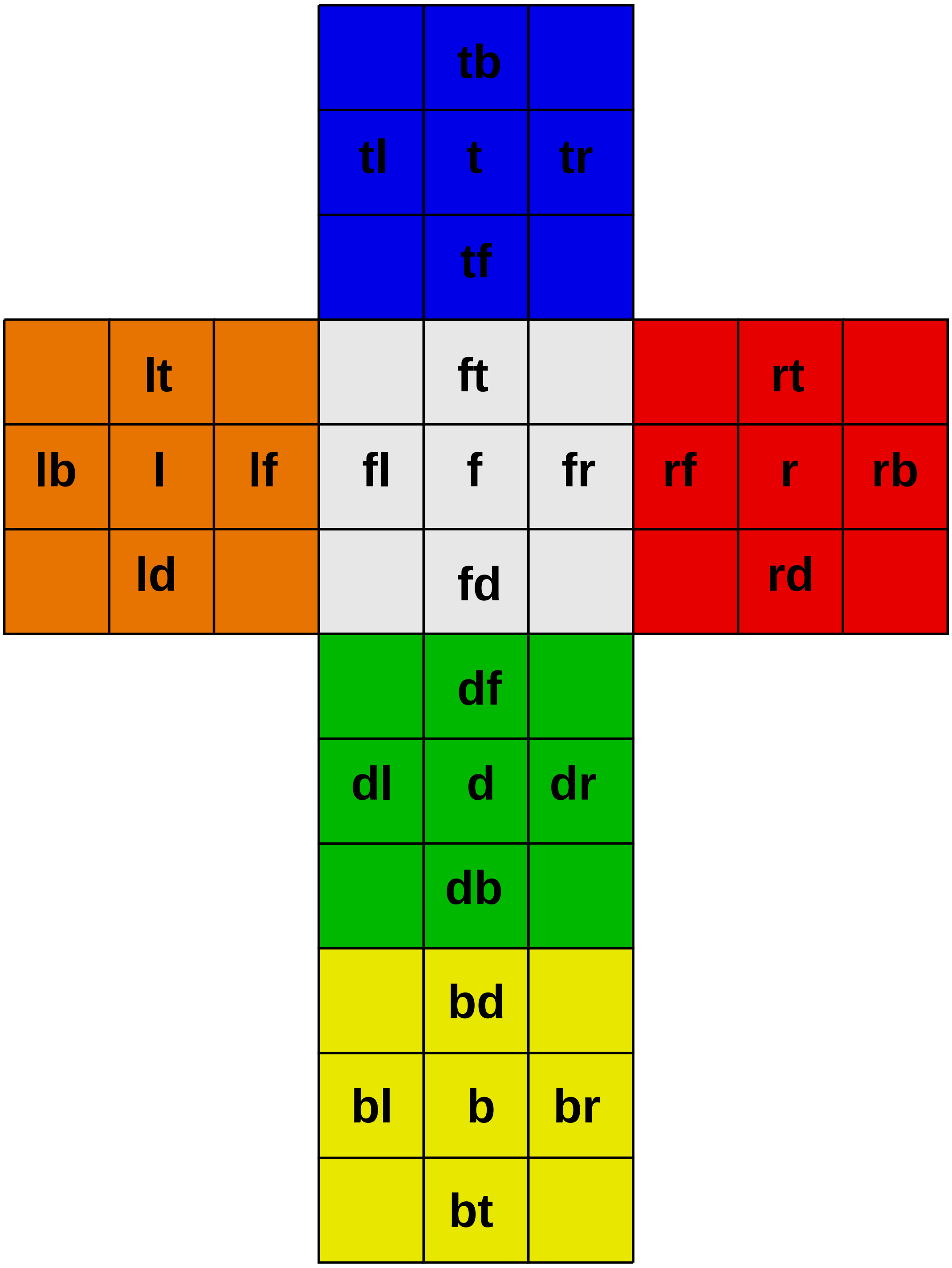
Rather than using numbers to identify each facet, I prefer to use acronyms, which I think are easier to remember. We will use lower case letters that correspond to the way we named the faces.



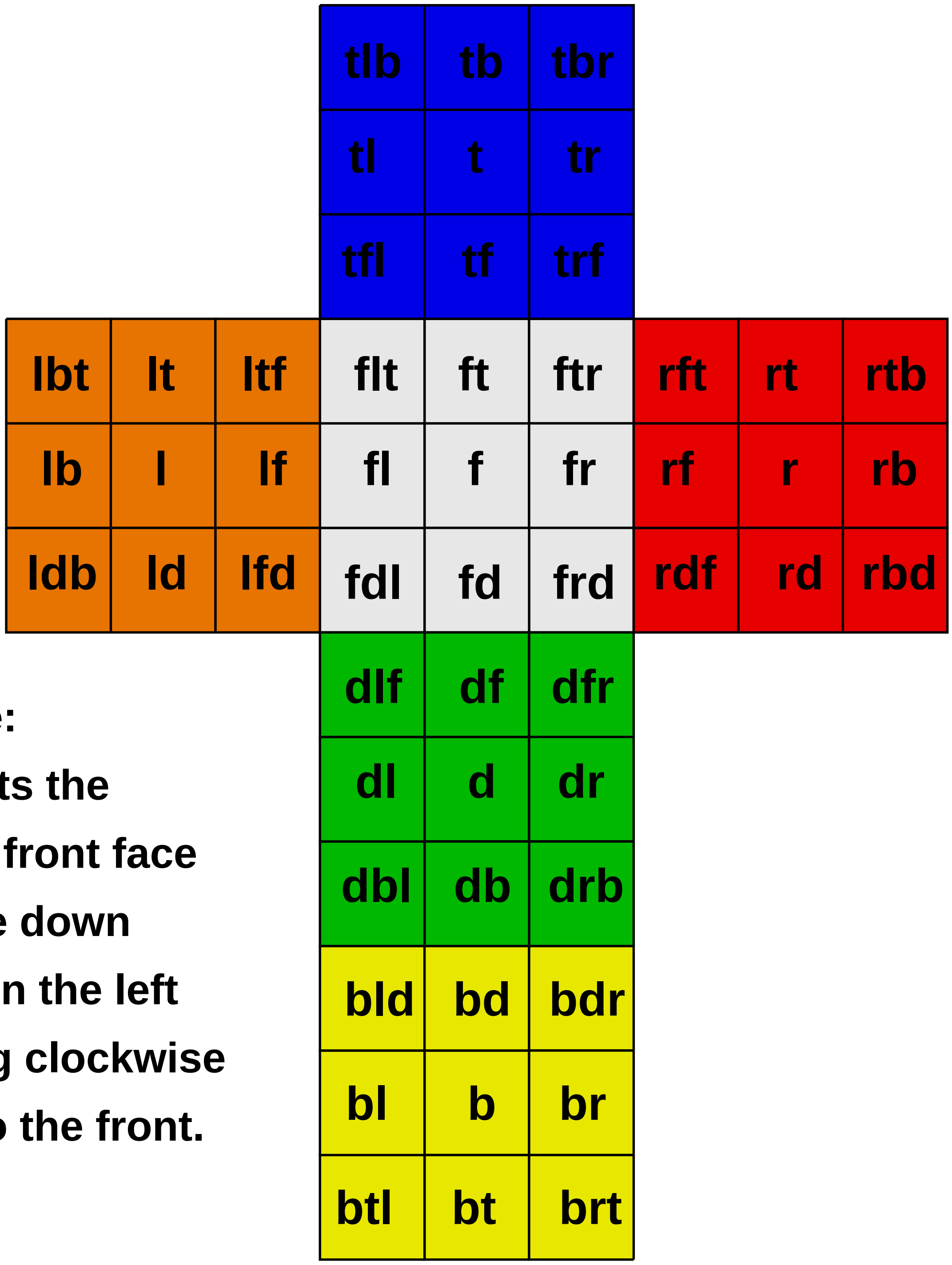
For example, the middle facet of each face will simply be a single letter representing our name for that face.



The middle edge facets that touch only one other face will be named with two letters, the first representing the face it is on, and the second the face that it is touching.

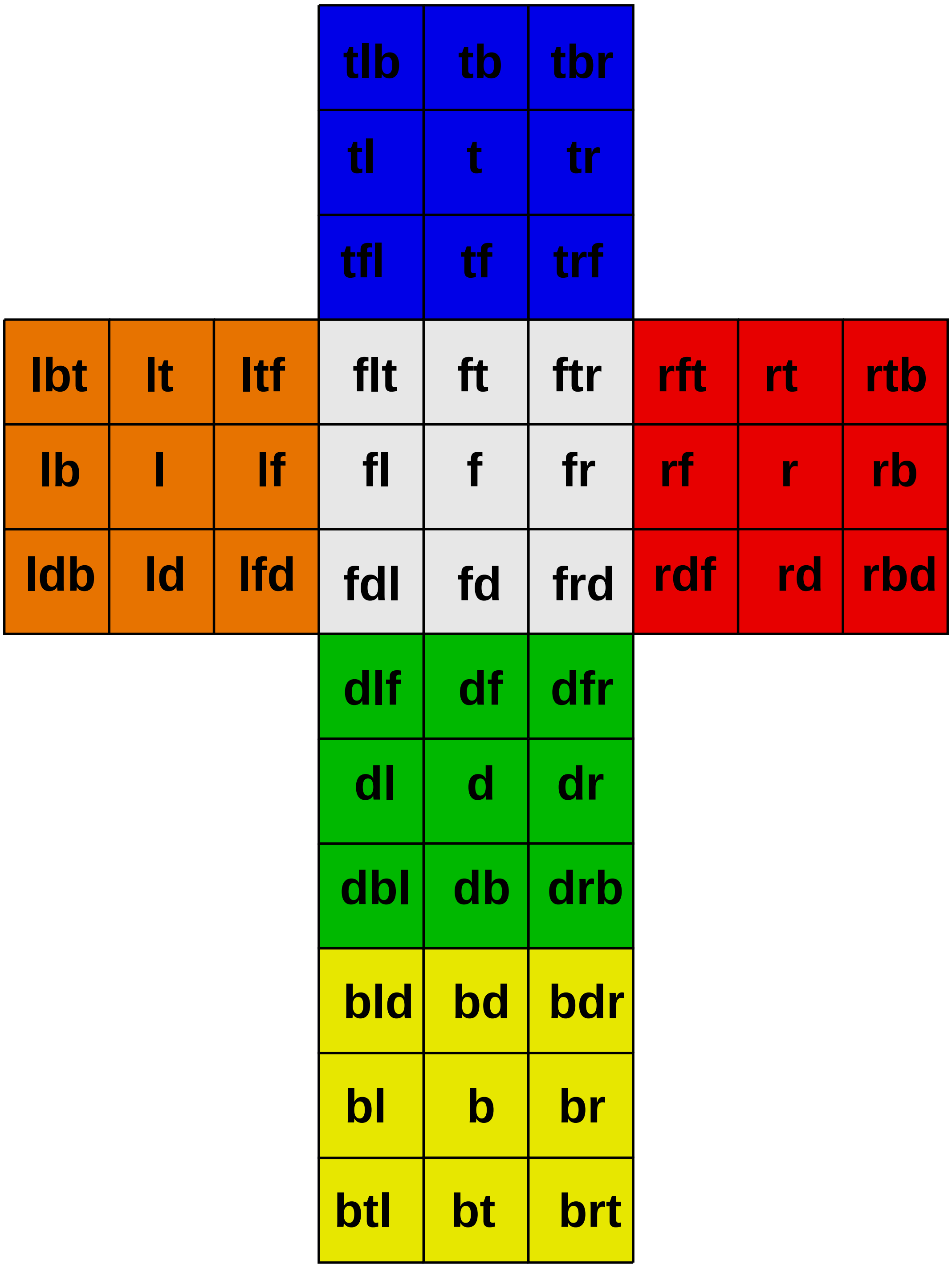


The corner edge facets that touch two other faces are a little more tricky. These facets will be named with three letters. The first is the face it is on, the second and third are the two faces this facet touches in clockwise order according to the face it is on.

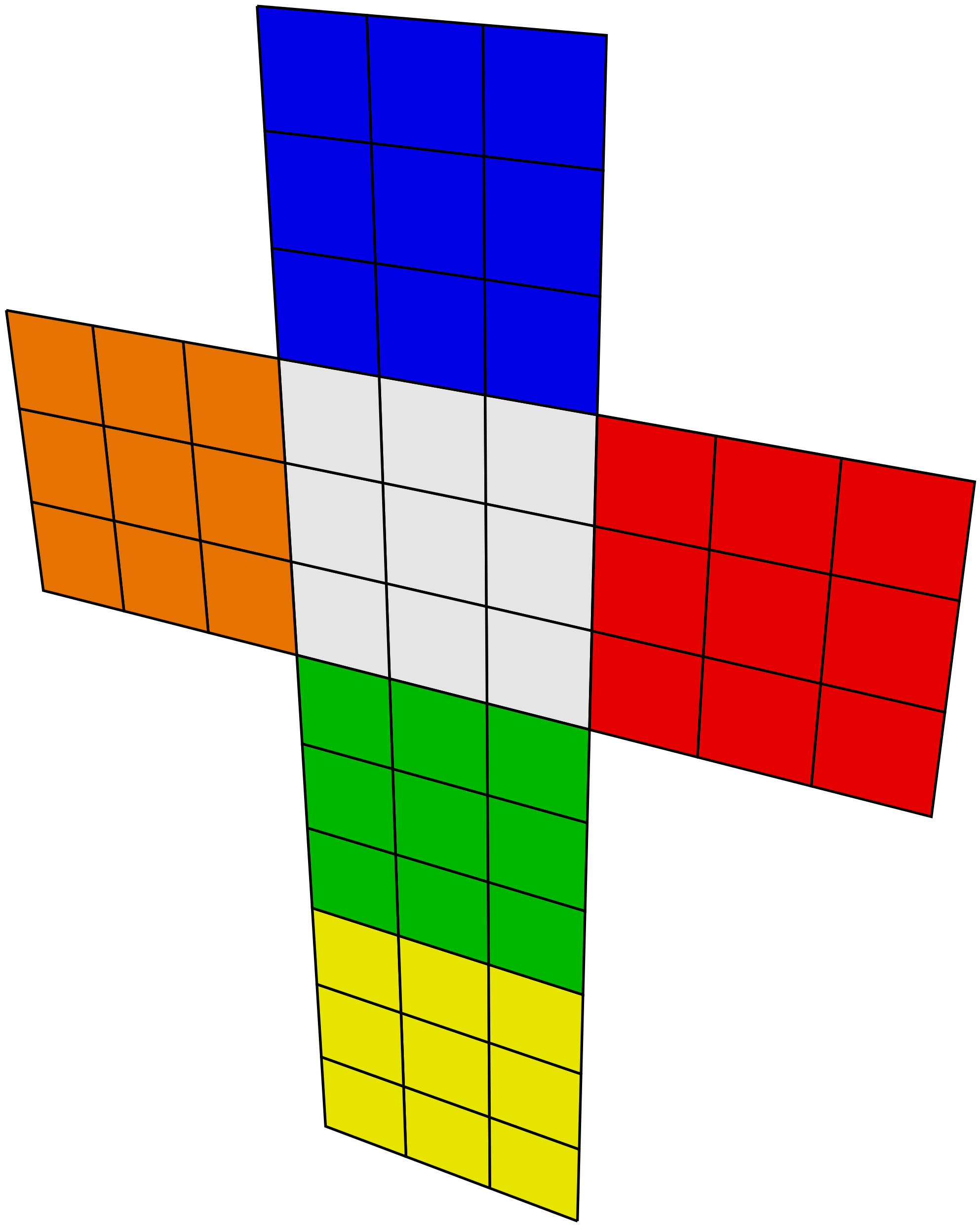


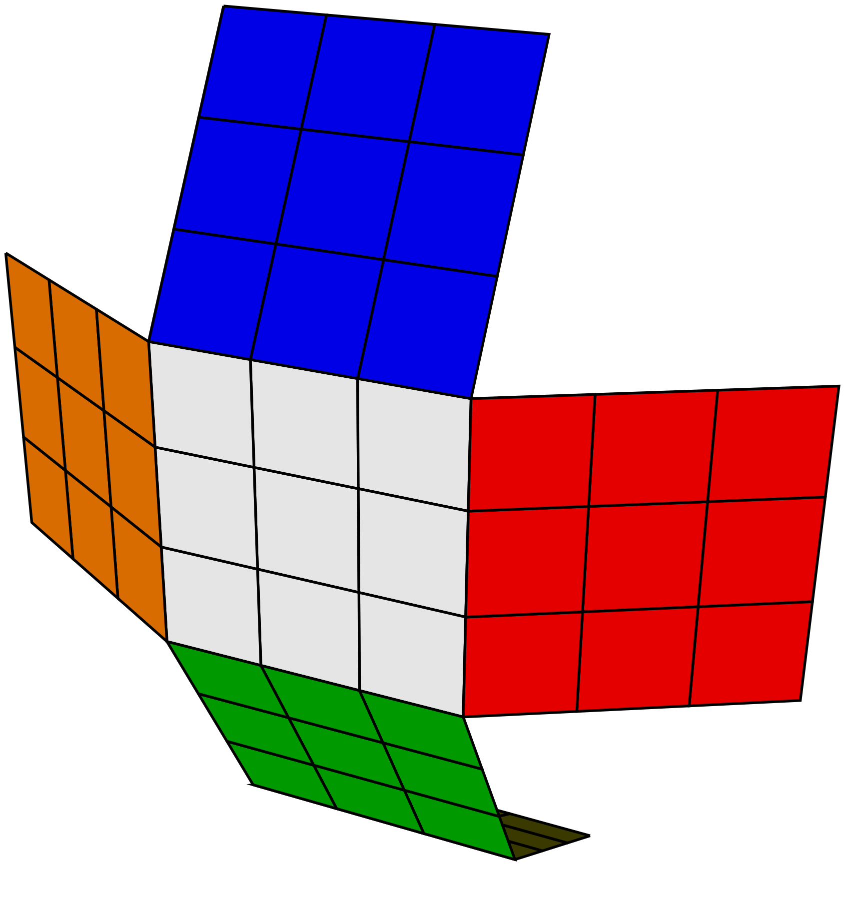
For example:
fdl represents the facet on the front face touching the down face and then the left face, cirlcing clockwise according to the front.

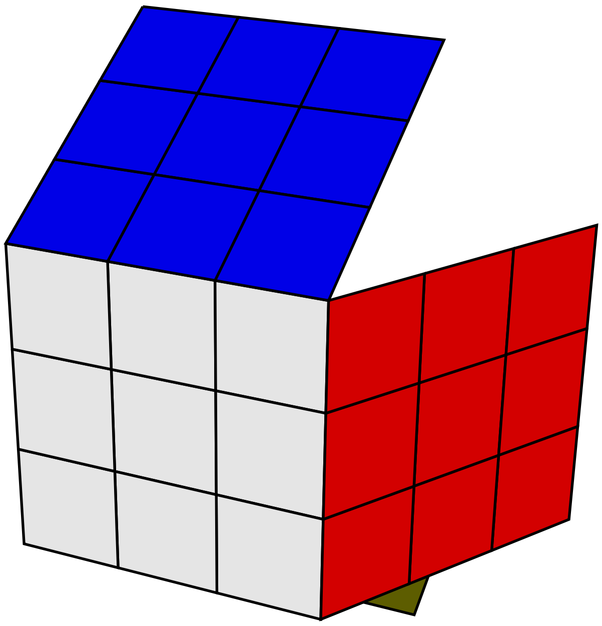
Now that we have named each facet, we can keep track of their colour by tracking how each move changes, or "maps", some of the facets to a new location.

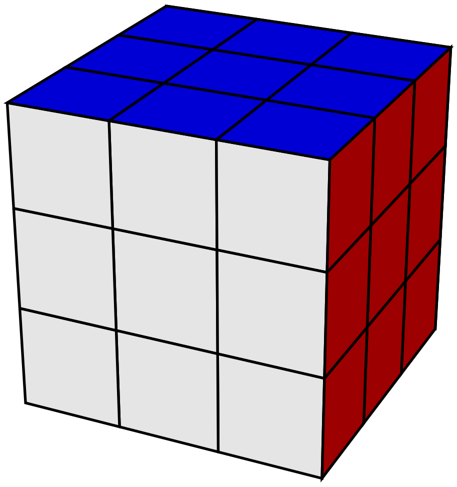


Let’s close the cube back up, and look at how we can move, or twist, the cube.

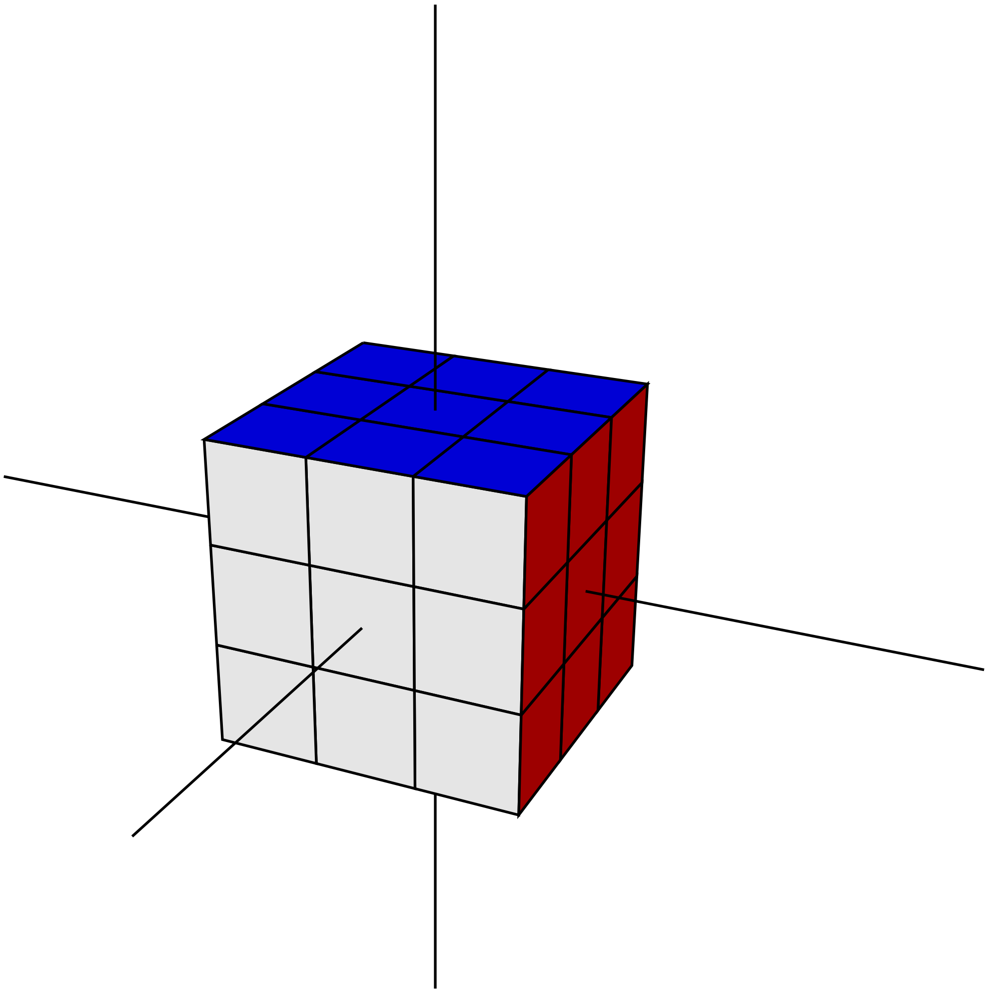




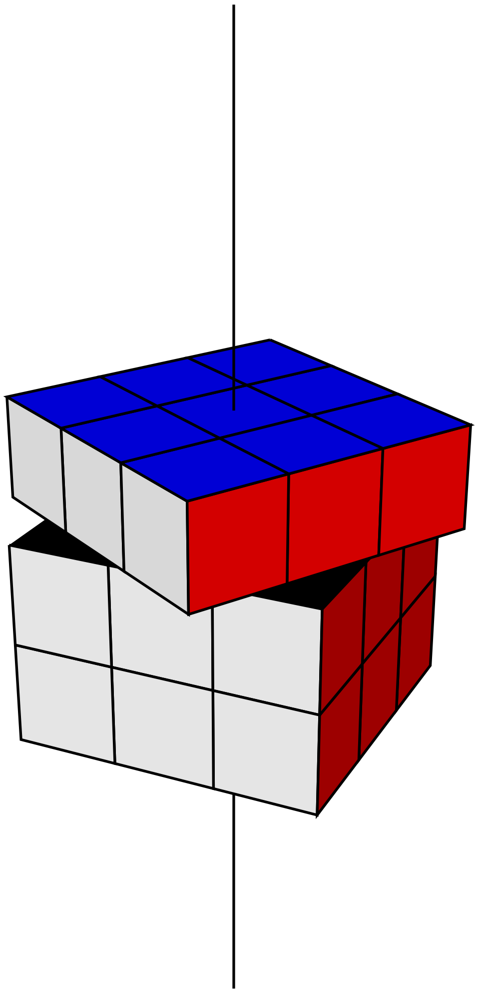




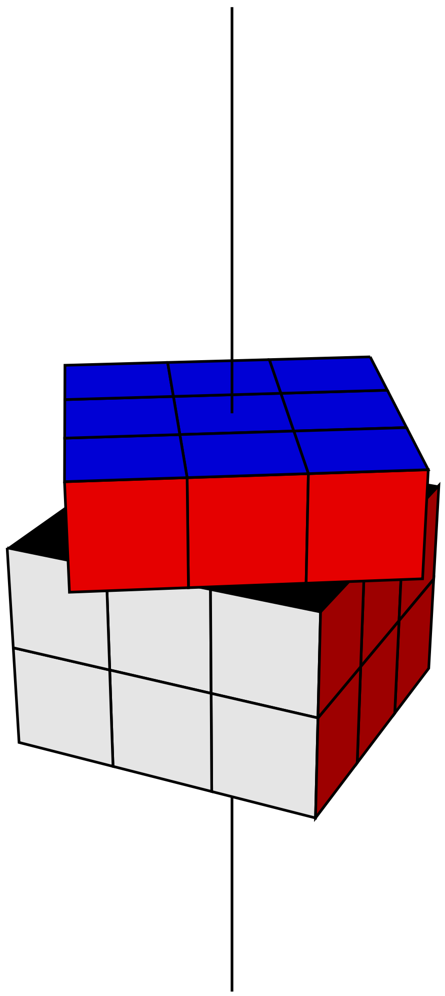
Notice that we can perform twists around three different axes of rotation.



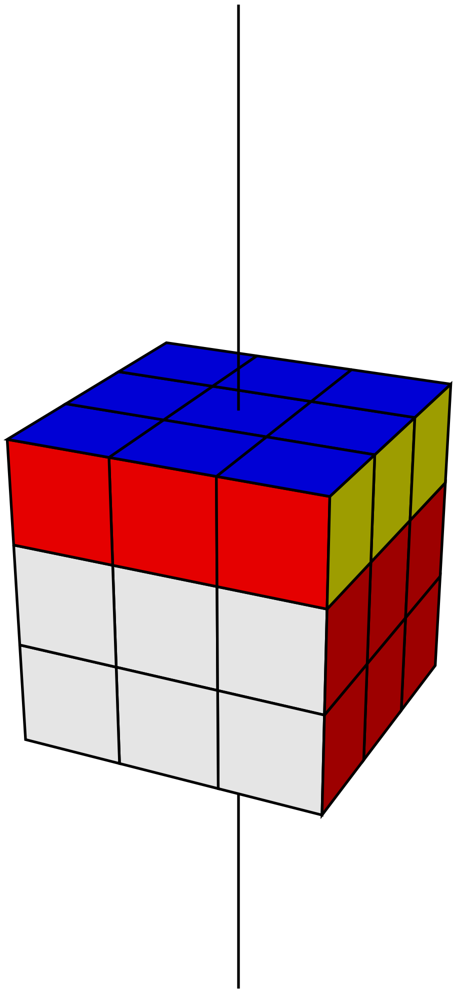
One axis through the top and down faces.



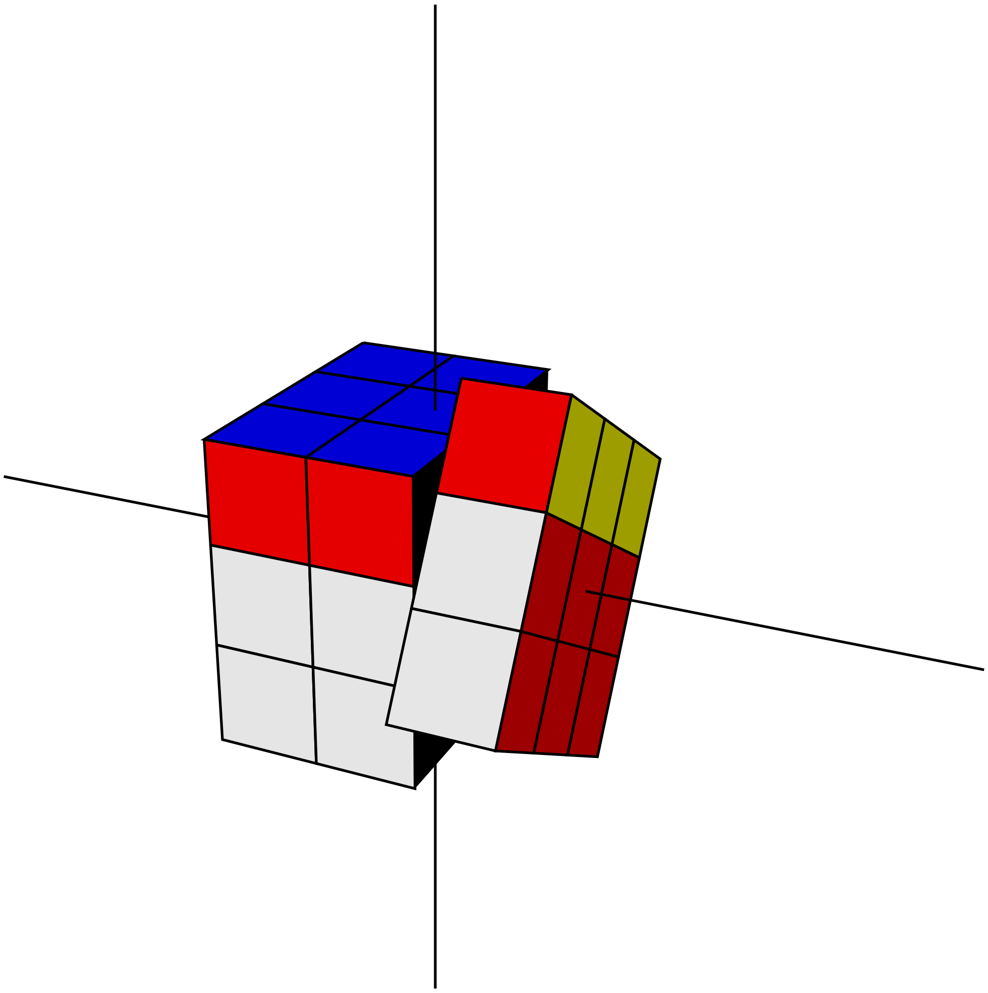
One axis through the top and down faces.



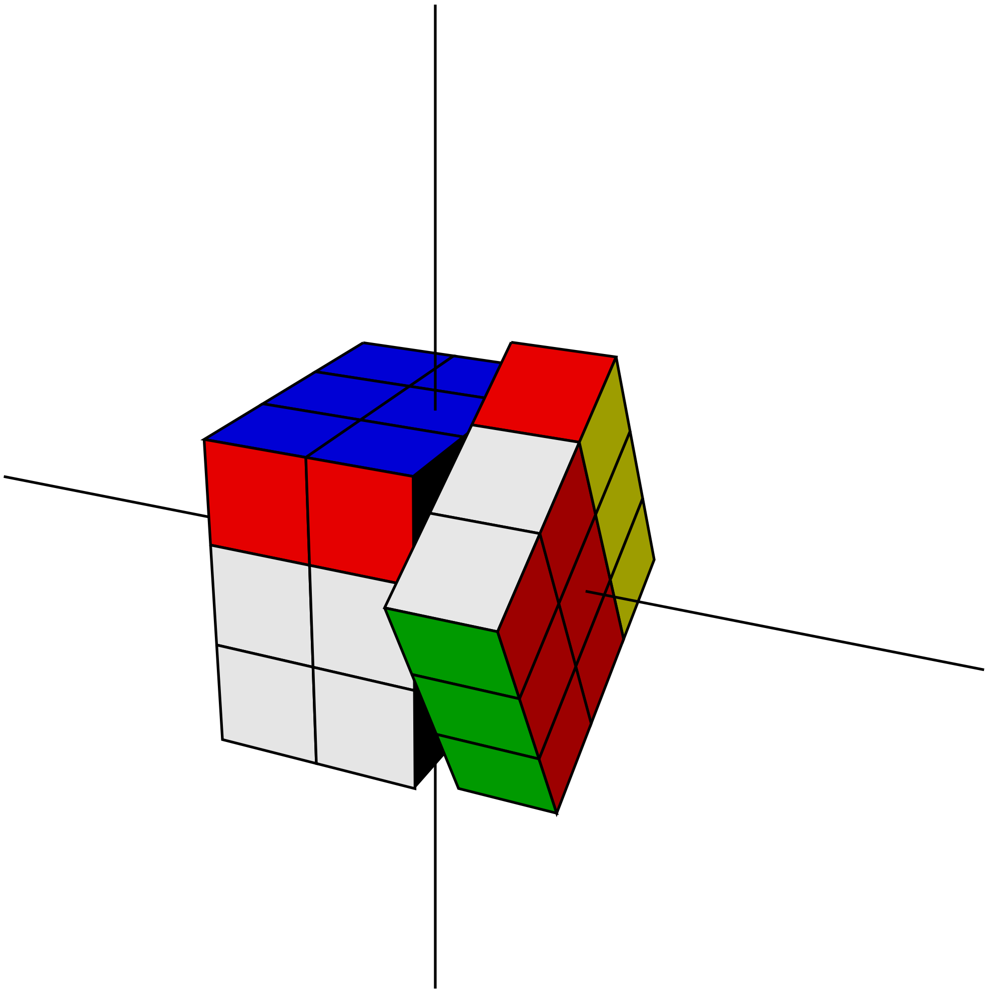
One axis through the top and down faces.



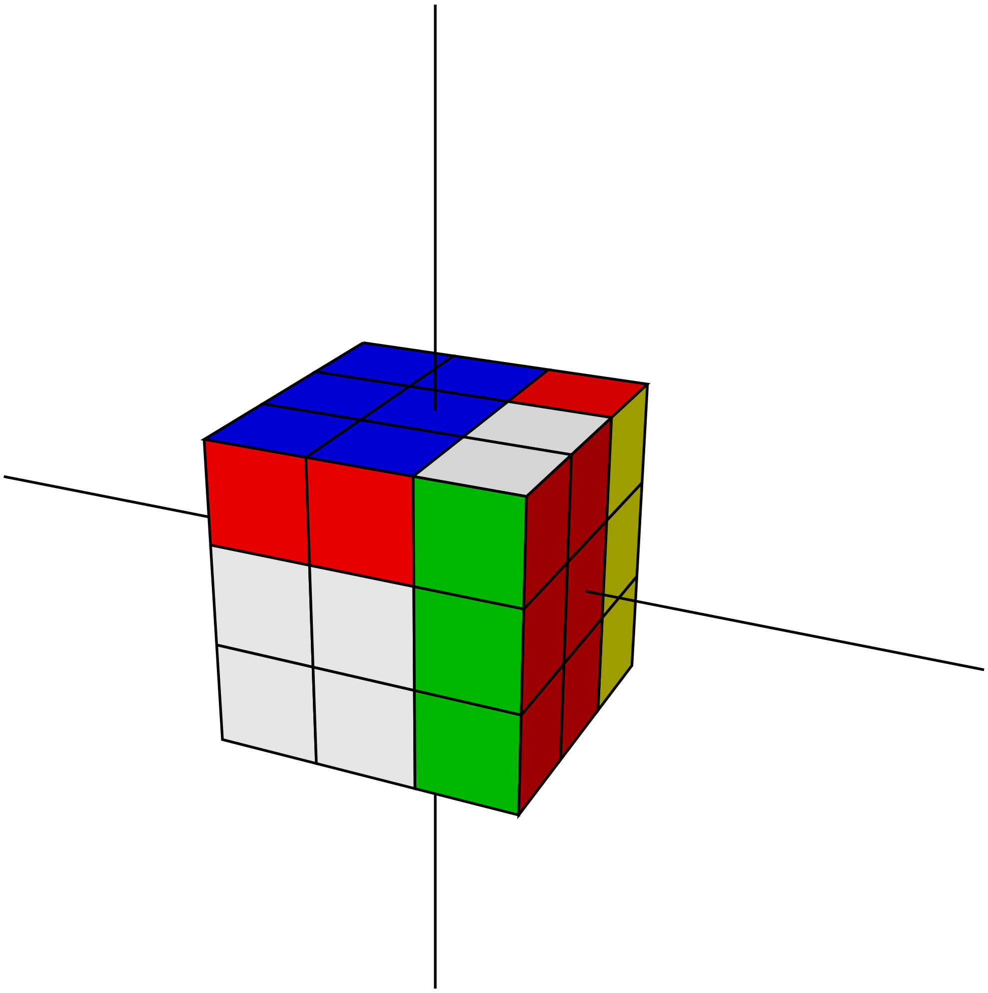
Another axis through the right and left faces.



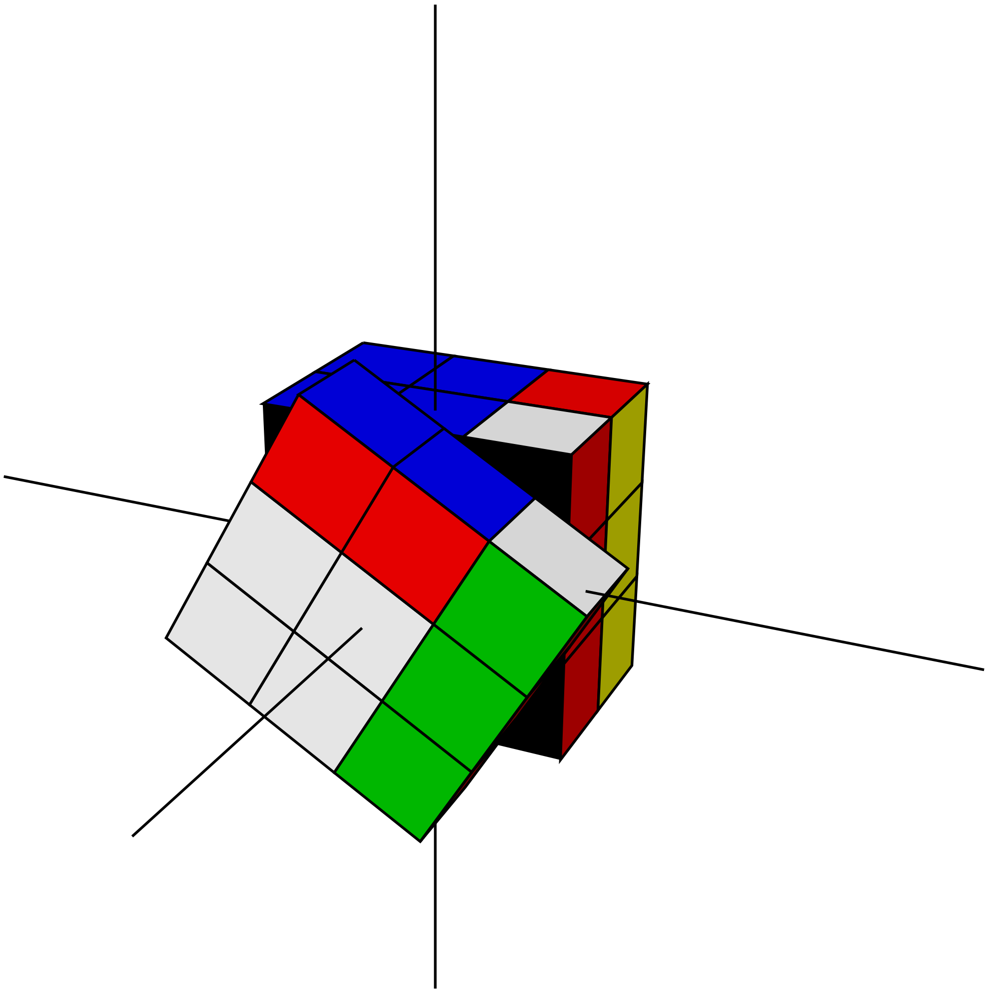
Another axis through the right and left faces.



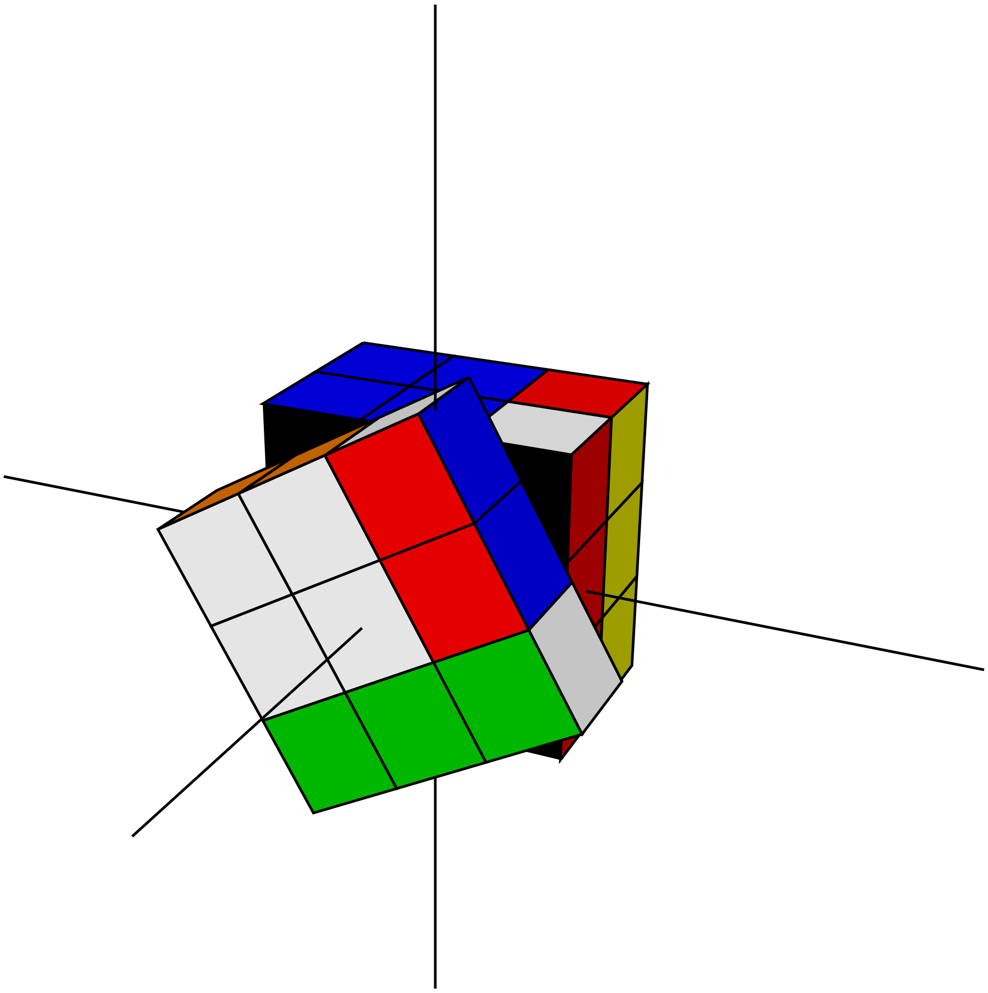
Another axis through the right and left faces.



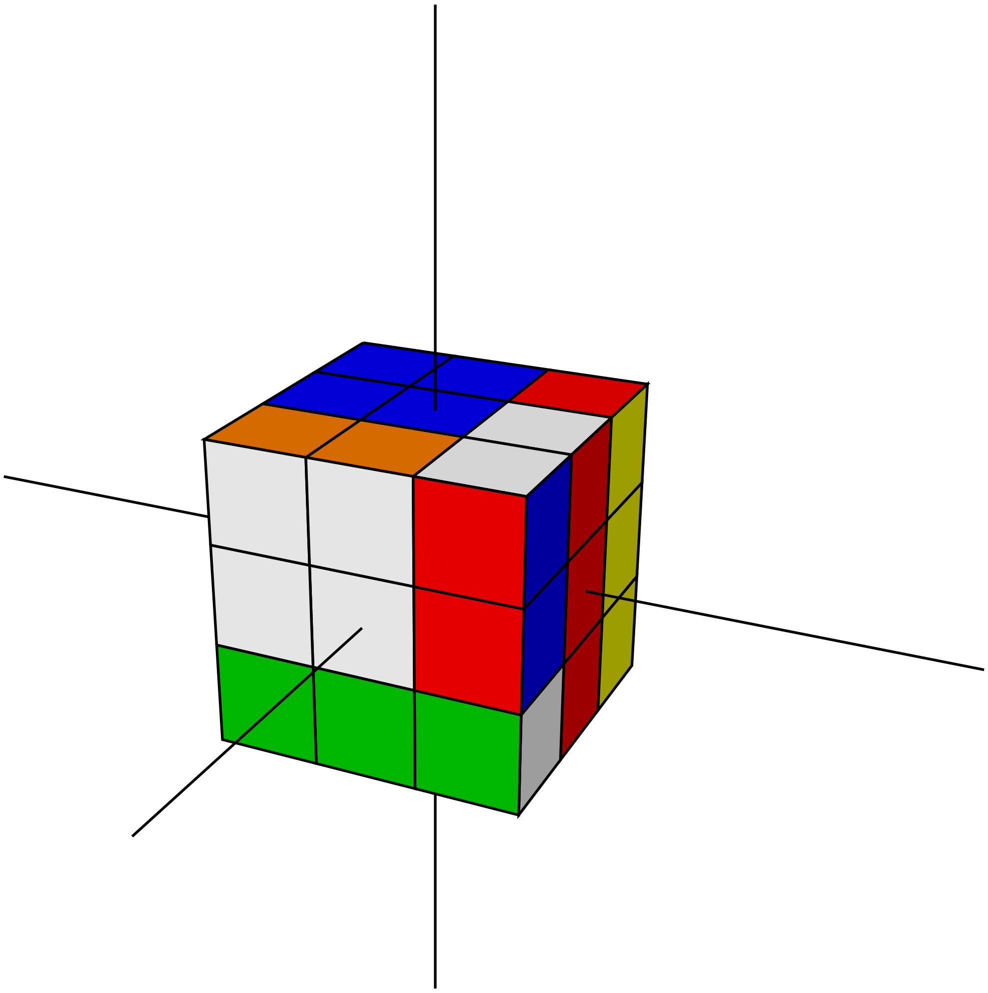
And a third axis through the front and back faces.



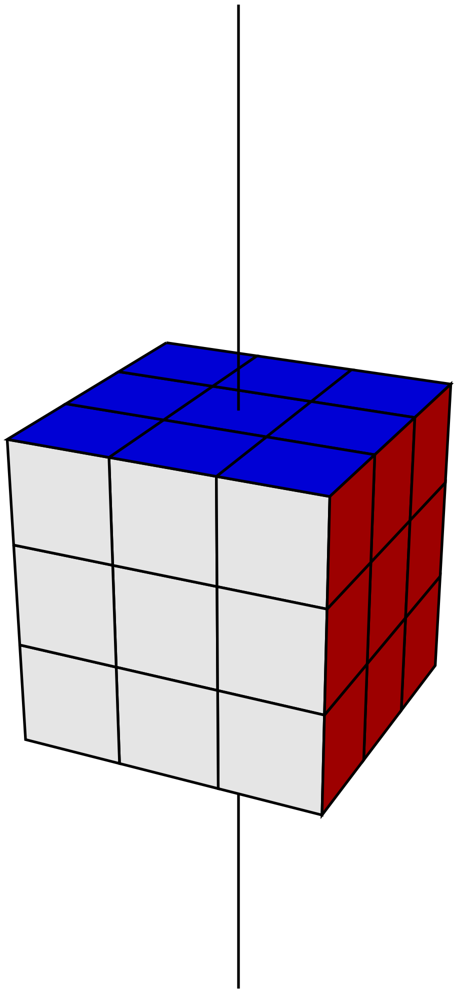
And a third axis through the front and back faces.



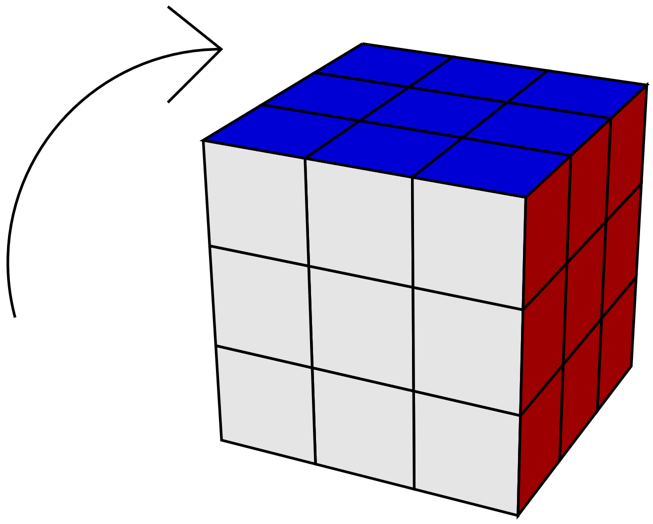
And a third axis through the front and back faces.



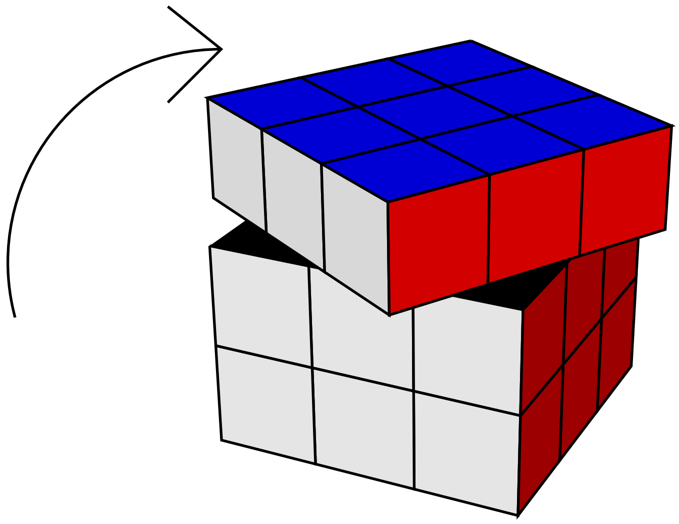
Let's take a closer look at twists around the axis through the top and down faces.



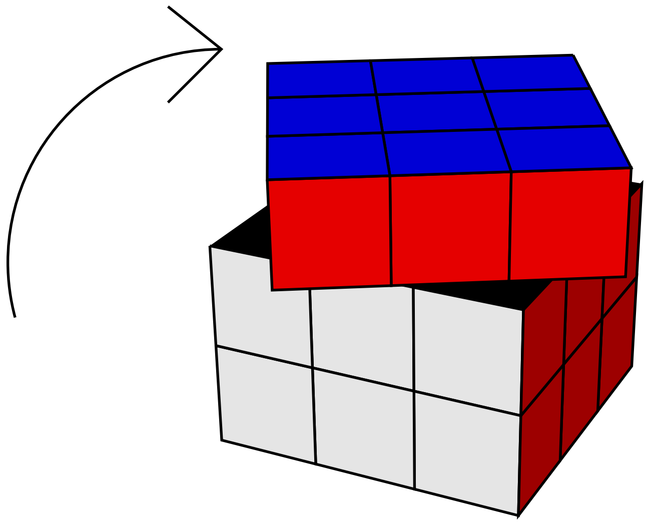
We will use a capital letter to denote a clockwise quarter twist of a face. For example, T represents a clockwise quarter twist of the top face:



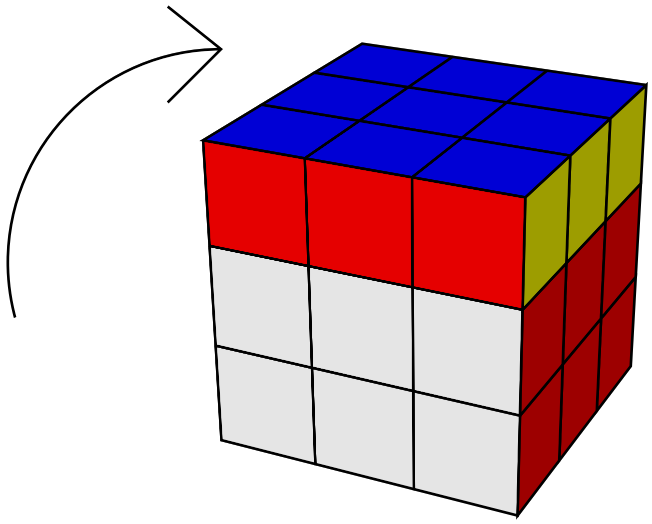
Note that we determine the clockwise direction by looking straight onto the face we are twisting.



The twist T



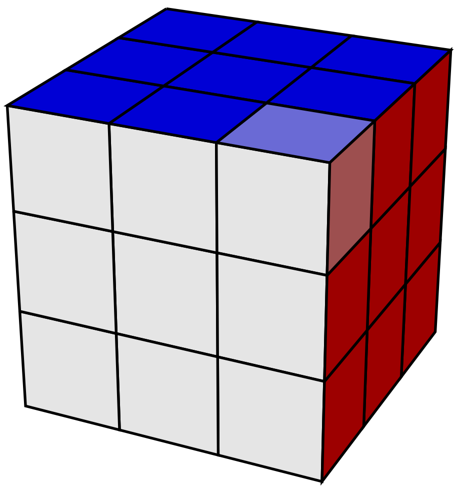
The twist T

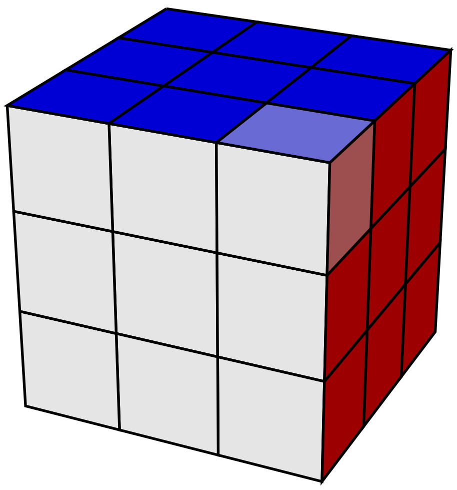


The twist T

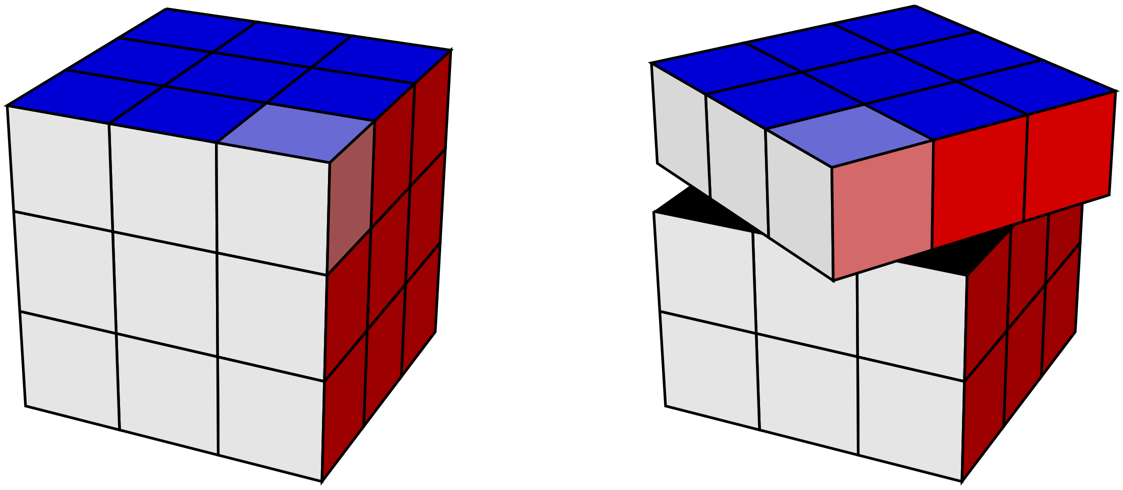
Watch the T twist again, and this time look for which facets move and where they move to:

To help, two facets have been highlighted.

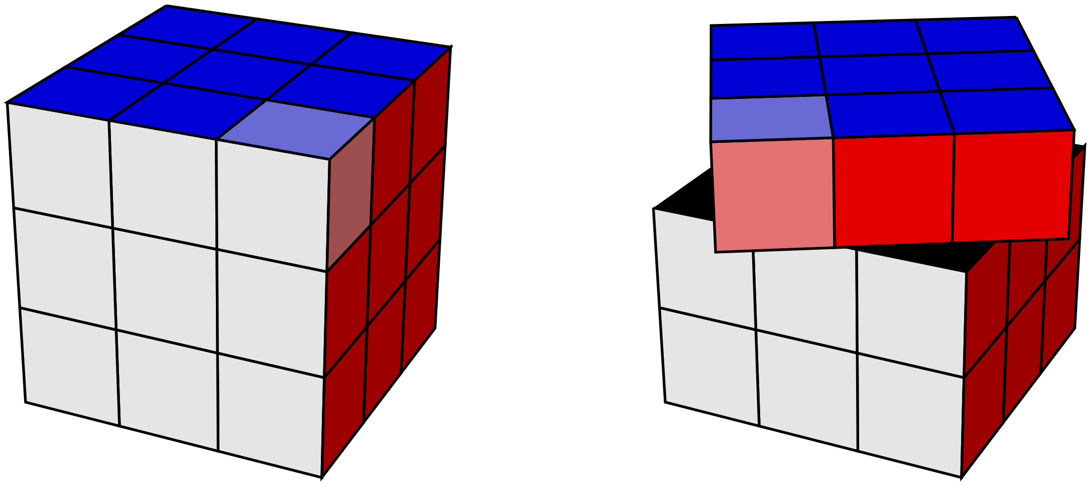




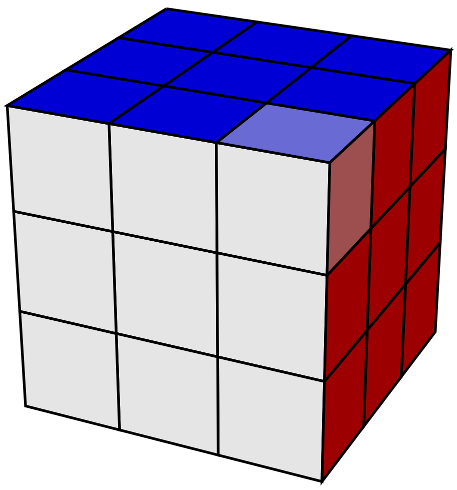
Before T



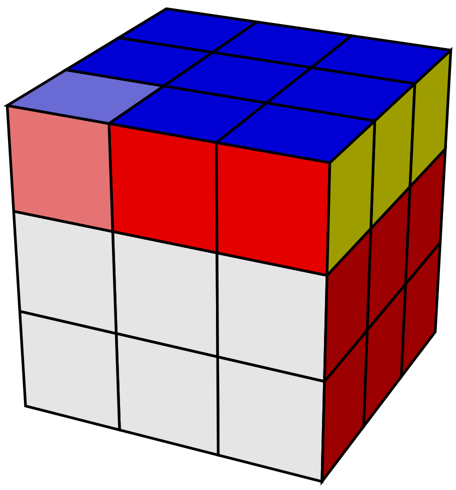
Before T



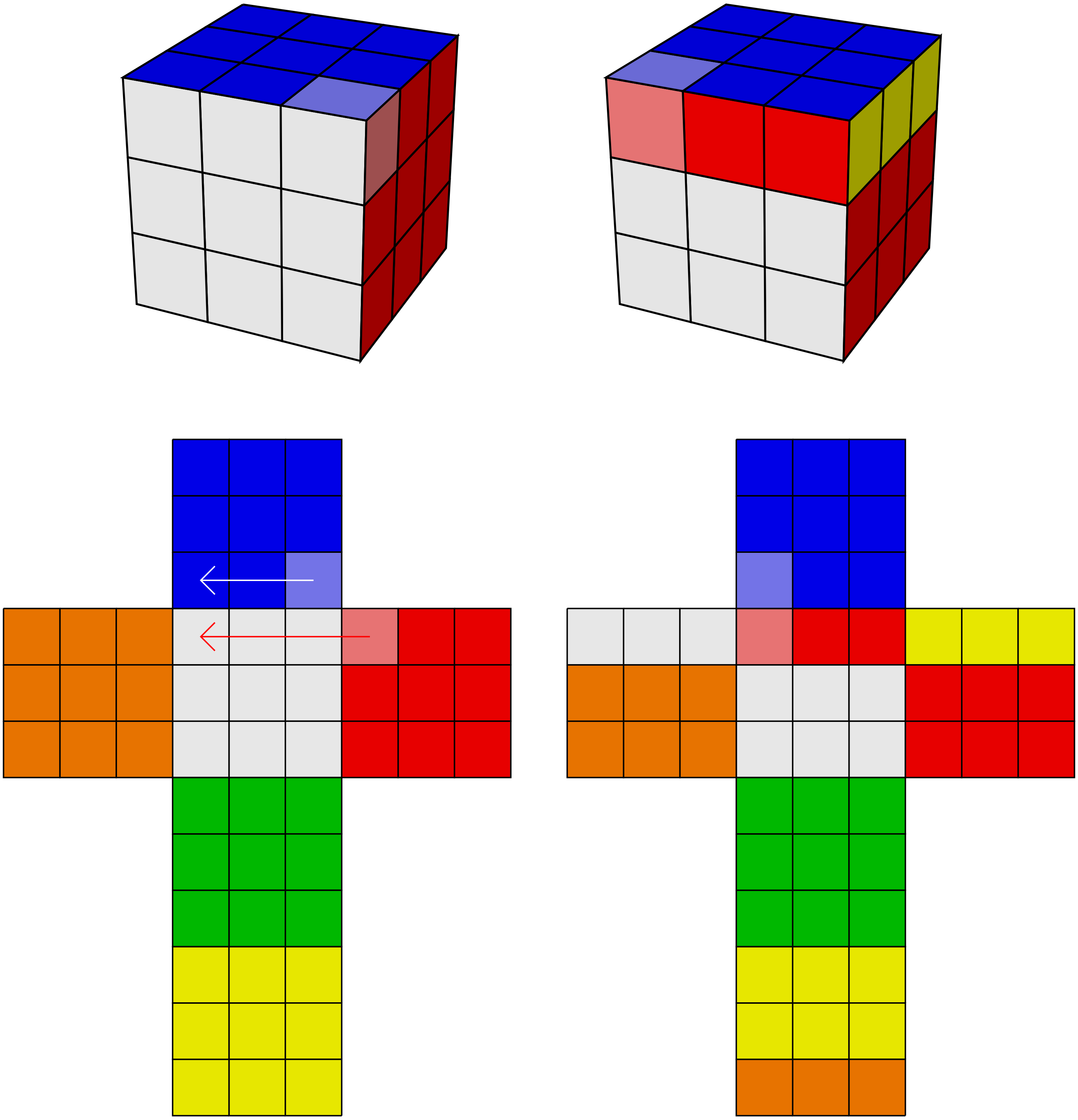
Before T



Before T



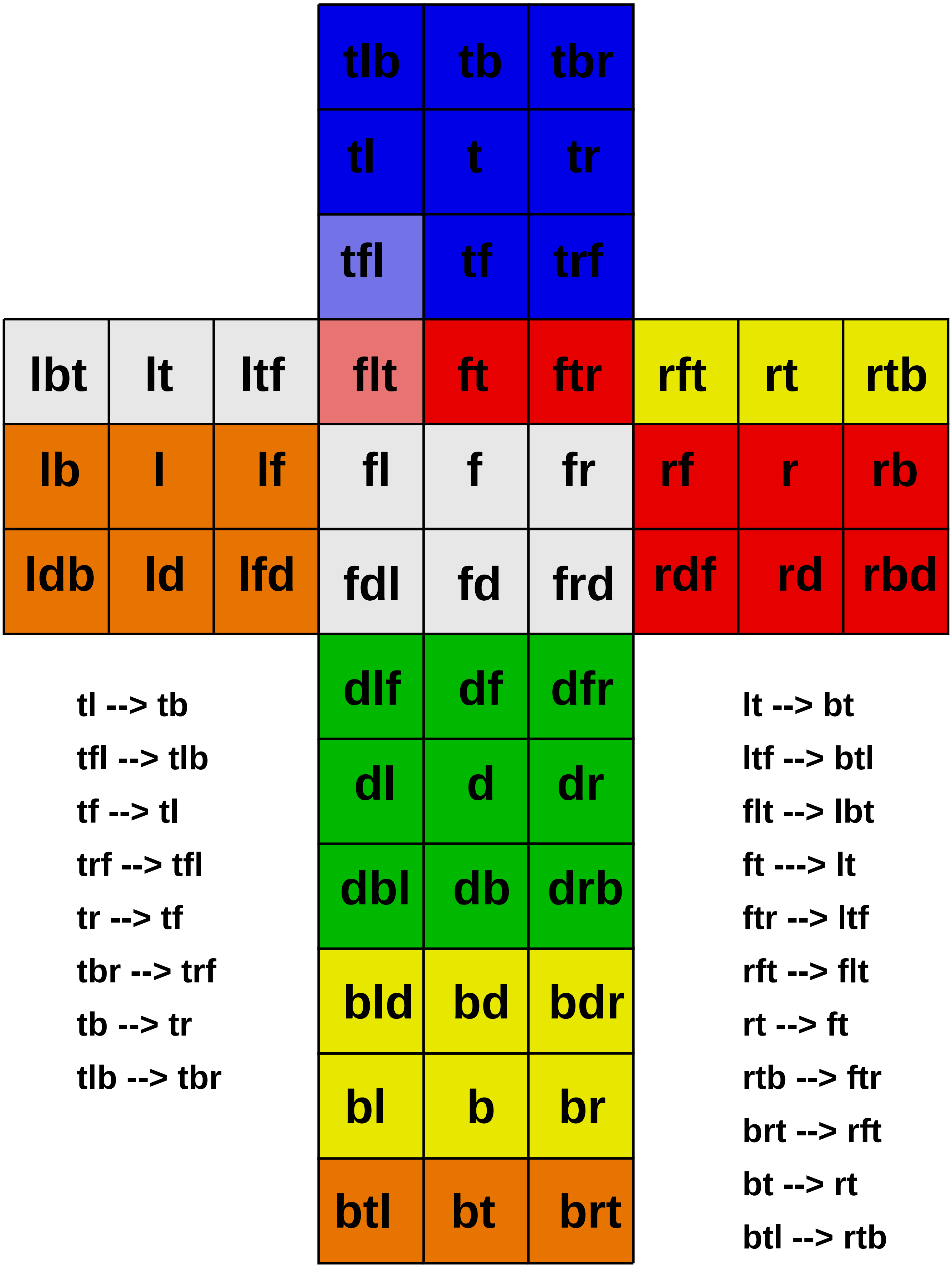
After T



Before T **After T**

The facets on the perimeter of the top face have moved clockwise two spots. The facets on the other faces that are touching the top face have moved around three spots. All the rest of the facets stayed put.

We can use mapping notation to show how the facets changed colour. For example, trf --> tfl means that the colour that was on trf is now on tfl.

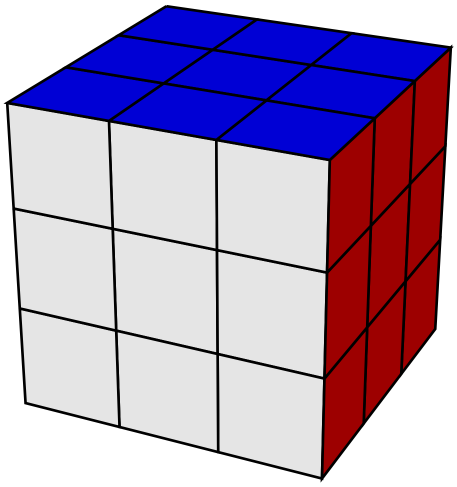


tl --> tb
tfl --> tlb
tf --> tl
trf --> tfl
tr --> tf
tbr --> trf
tb --> tr
tlb --> tbr

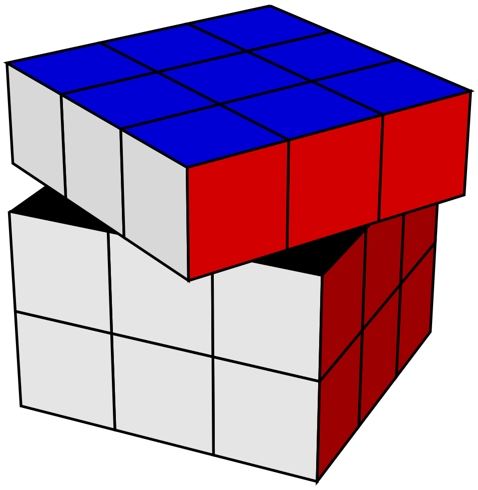
lt --> bt
ltf --> btl
flt --> lbt
ft ---> lt
ftr --> ltf
rft --> flt
rt --> ft
rtb --> ftr
brt --> rft
bt --> rt
btl --> rtb
lbt --> brt

With the twist T we also have the twist TT or T^2

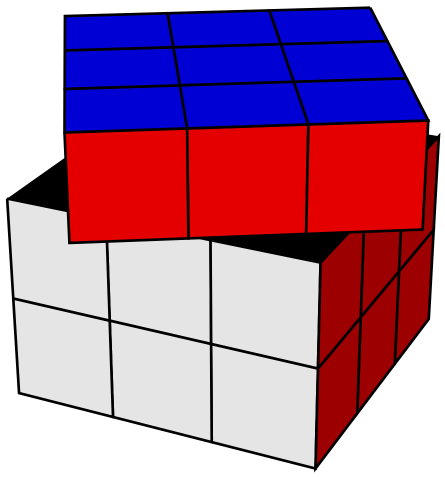
which is two quarter turns of the top face:



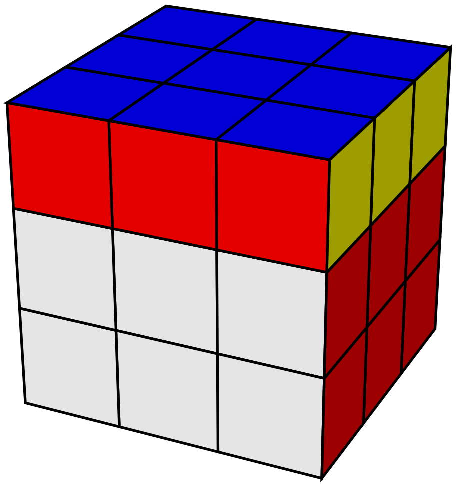
The twist T^2



The twist T^2

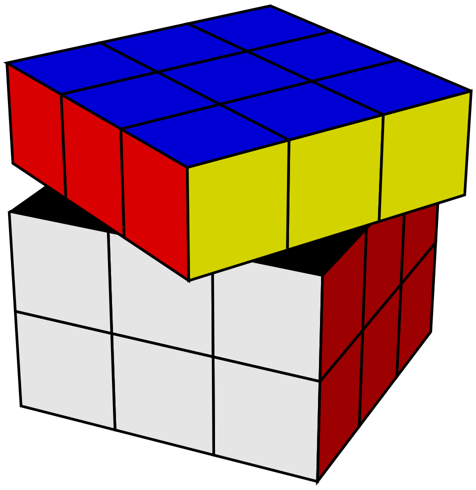


The twist T^2

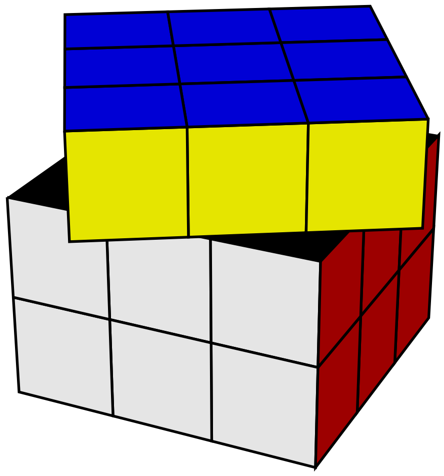


The twist T^2

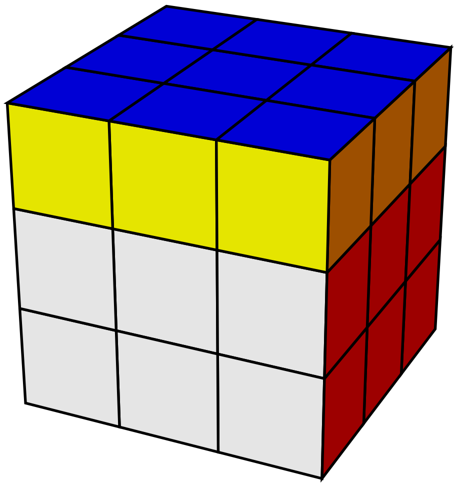
one quarter turn



The twist T^2



The twist T^2

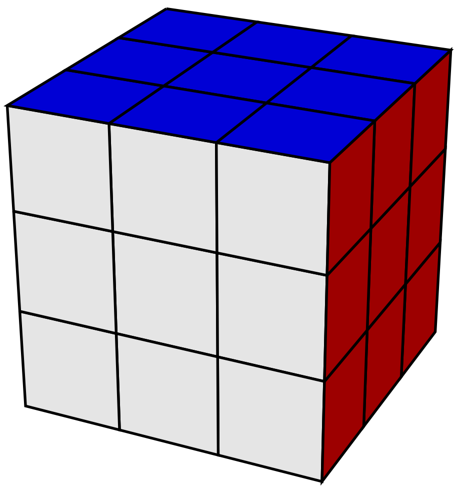


The twist T^2

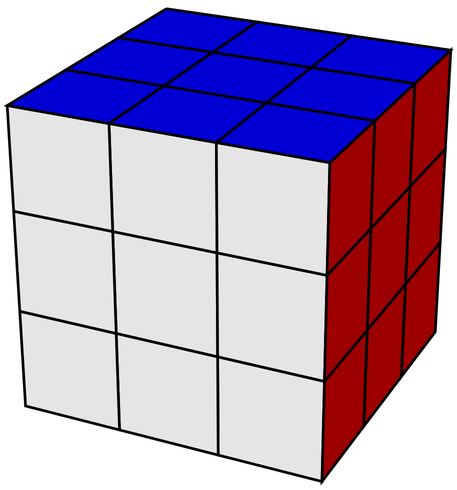
two quarter turns

We also have the twist TTT which is three quarter turns clockwise, and T’ which is one quarter turn counter-clockwise of the top face.

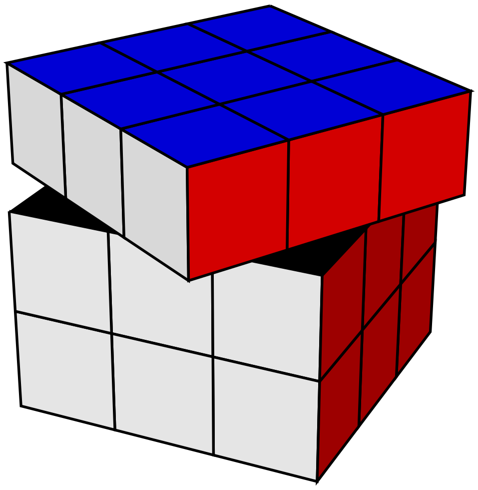
Both TTT and T’ have the same result:



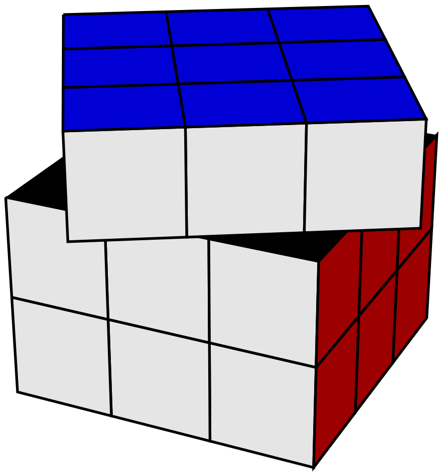
TTT



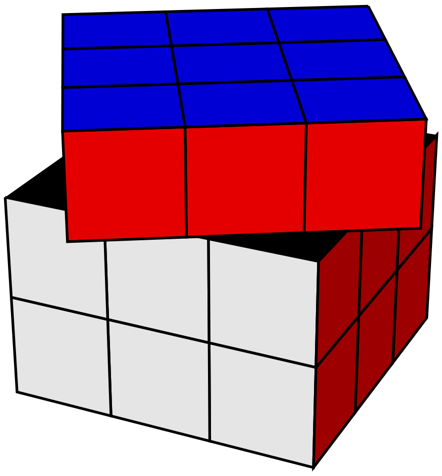
T’

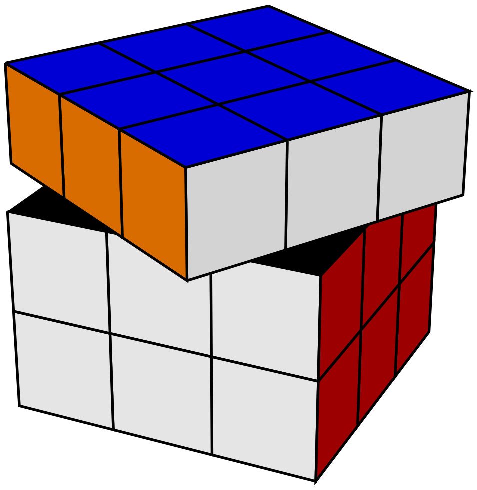


TTT



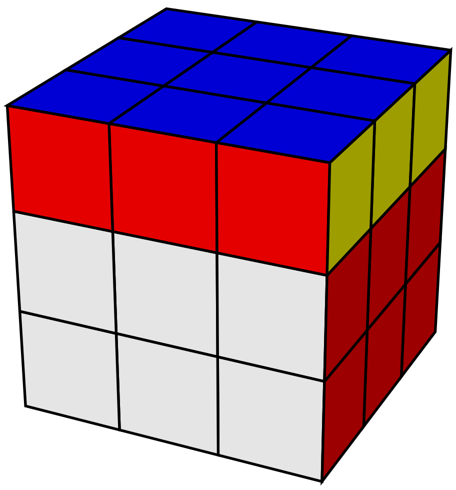
T’

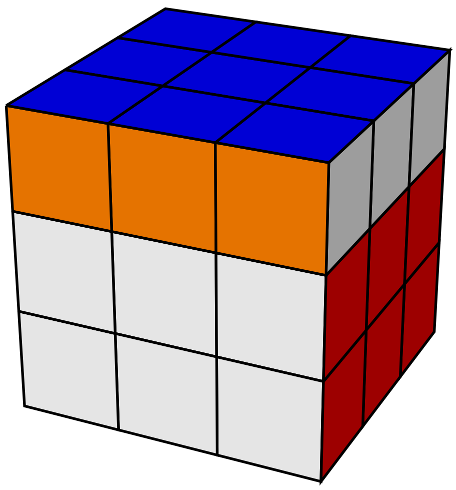




TTT

T'



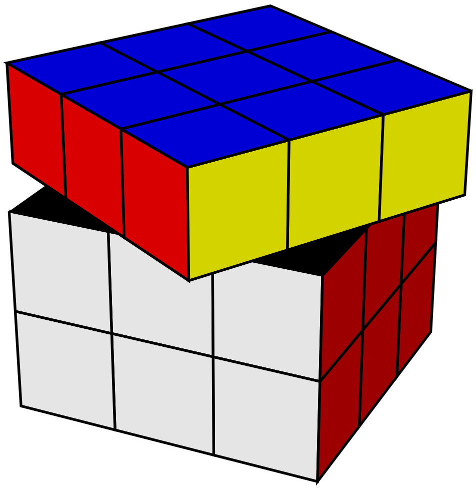


TTT

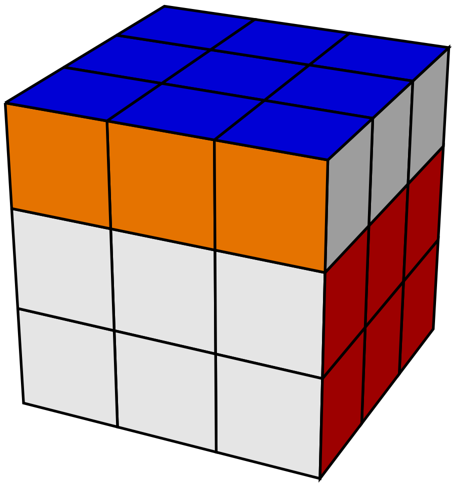
one turn clockwise

T’

one turn counter-clockwise

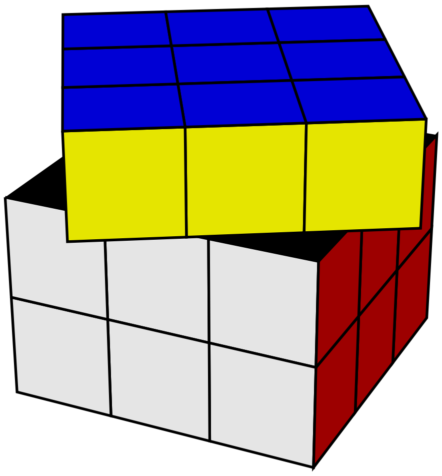


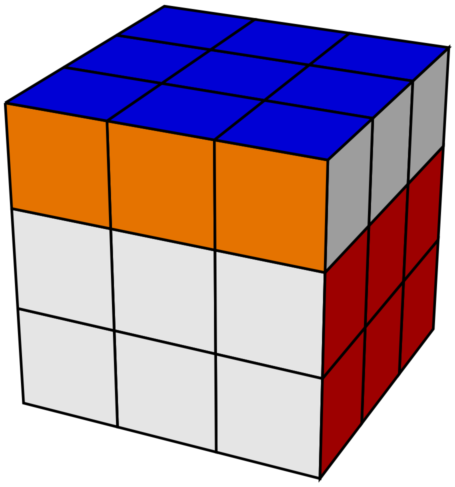
TTT



T'

one turn counter-clockwise

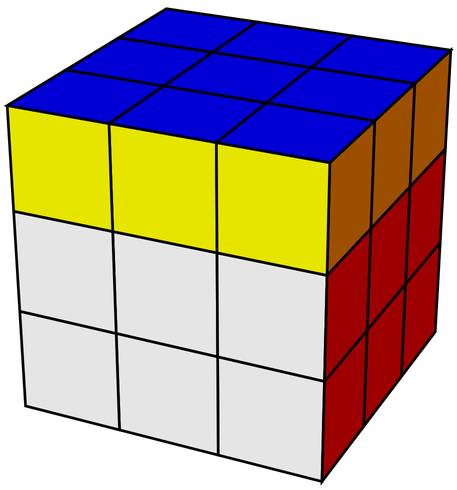




TTT

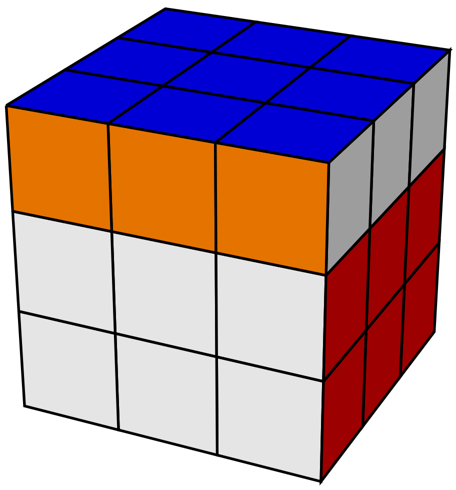
T’

one turn counter-clockwise



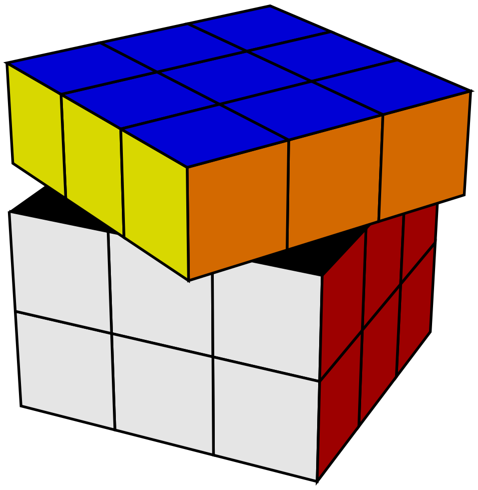
TTT

two turns clockwise

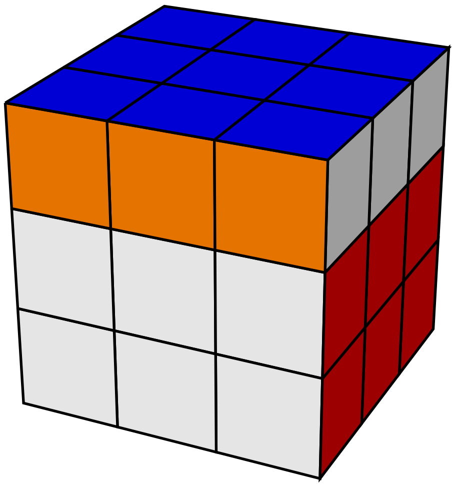


T'

one turn counter-clockwise

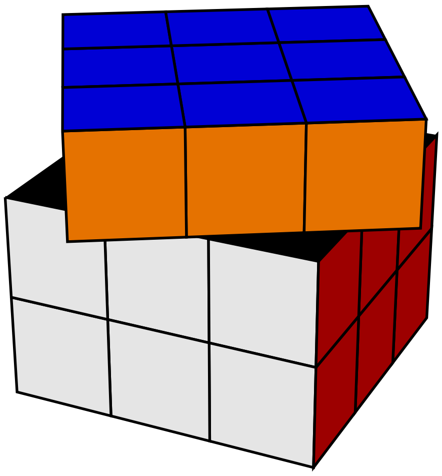


TTT

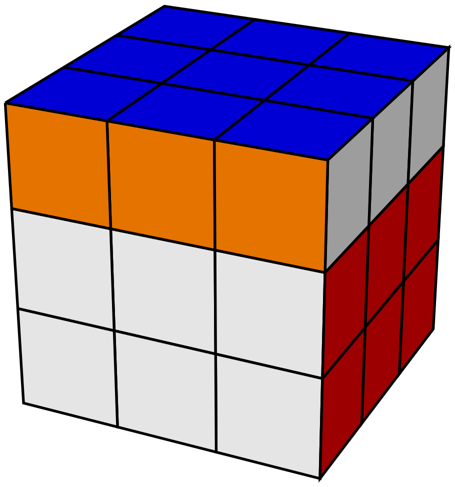


T’

one turn counter-clockwise

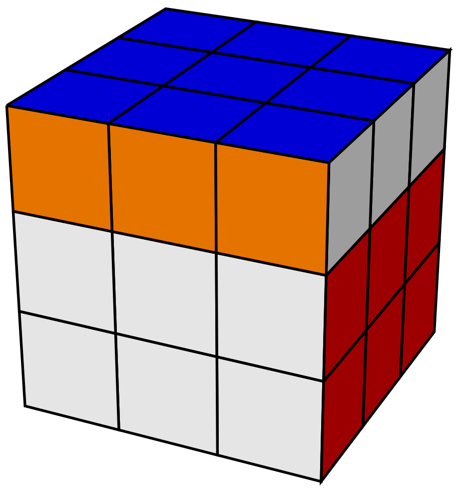


TTT



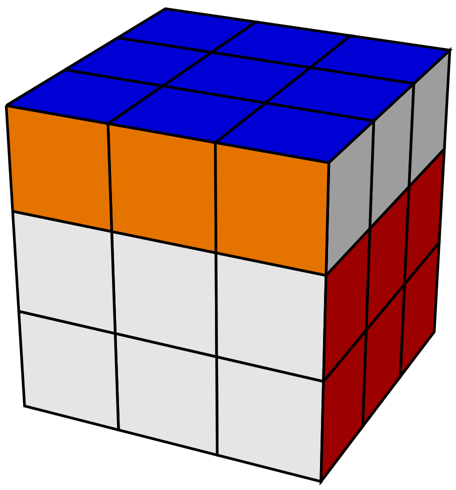
T’

one turn counter-clockwise



TTT

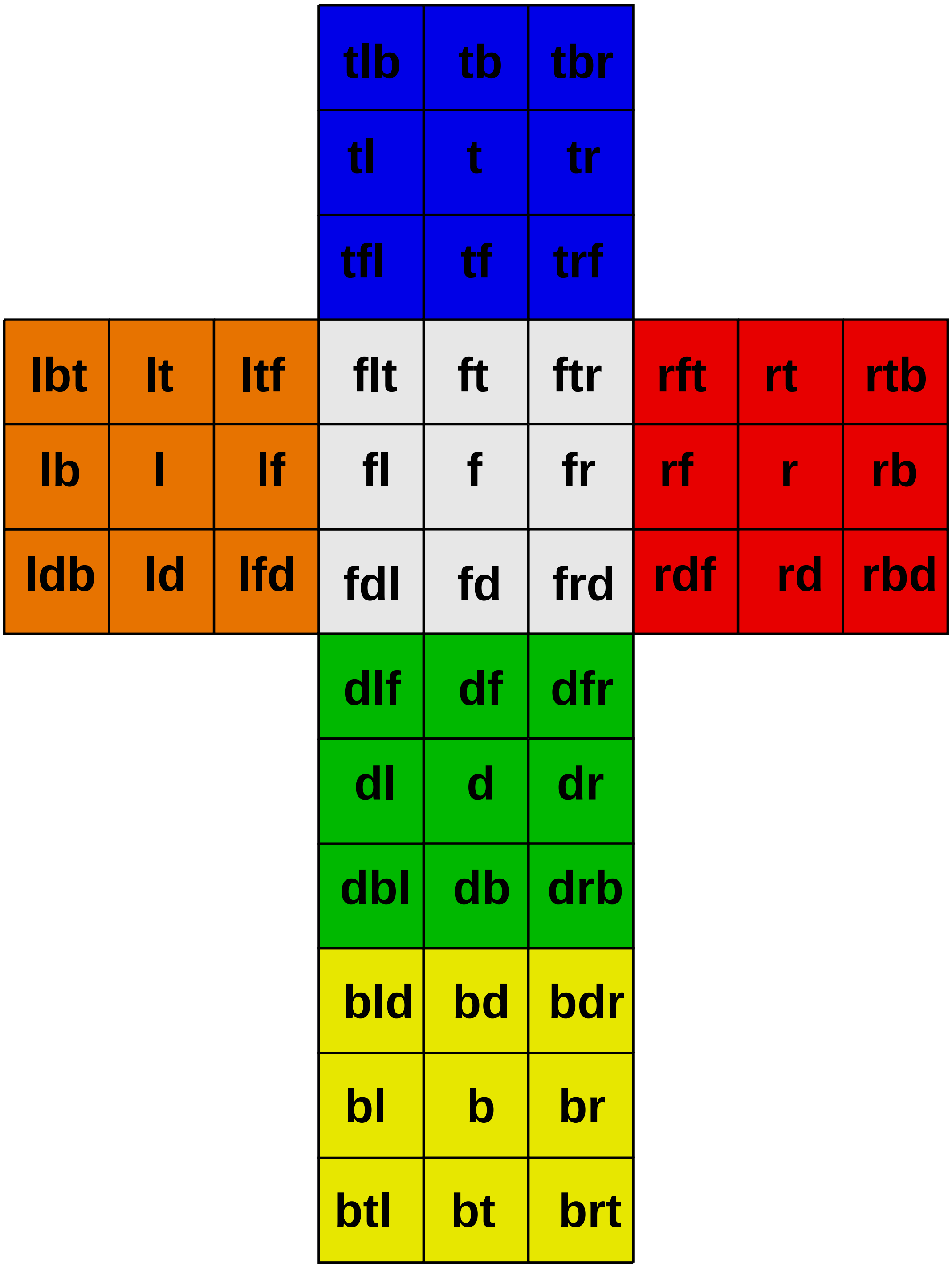
three turns clockwise



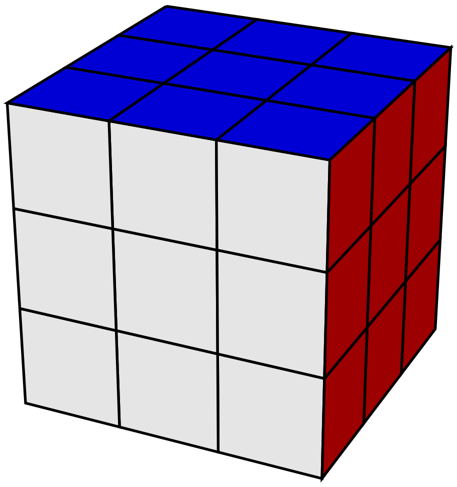
T’

one turn counter-clockwise

The maps corresponding to these twists, as well as the other twists we will look at, can be found in the same way we found the maps for T. That is observing how the twists move the facet colours.

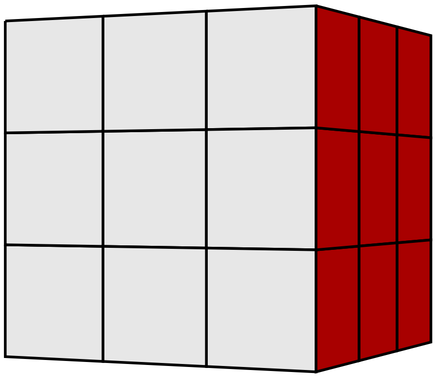


Around this axis we also have the twist D, a a quarter twist clockwise of the down face. Remember that is clockwise looking striaght at the down face.

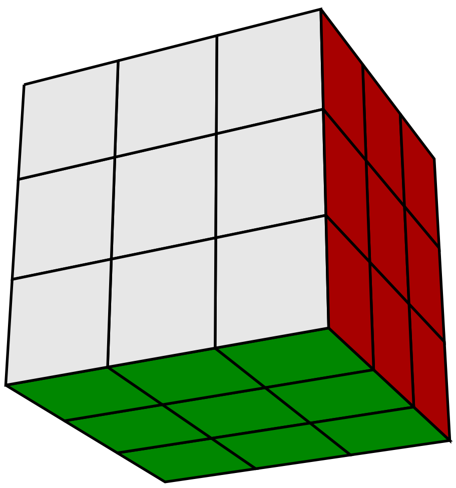


Remember that is clockwise looking striaght

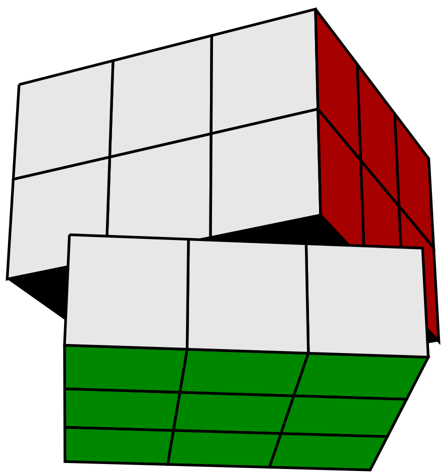
at the down face.



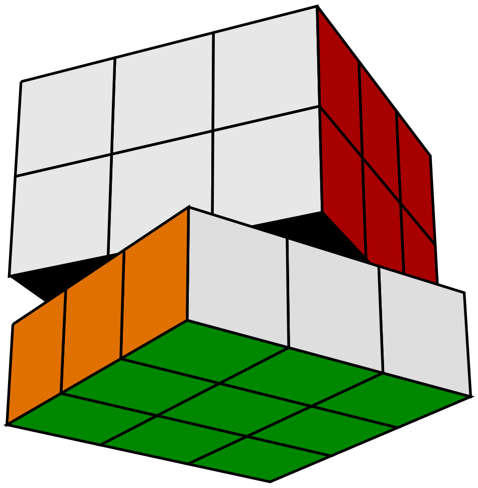
Remember that is clockwise looking striaght at the down face.



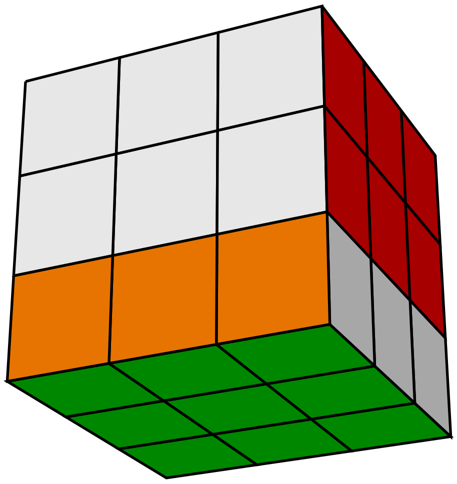
The twist D



The twist D



The twist D



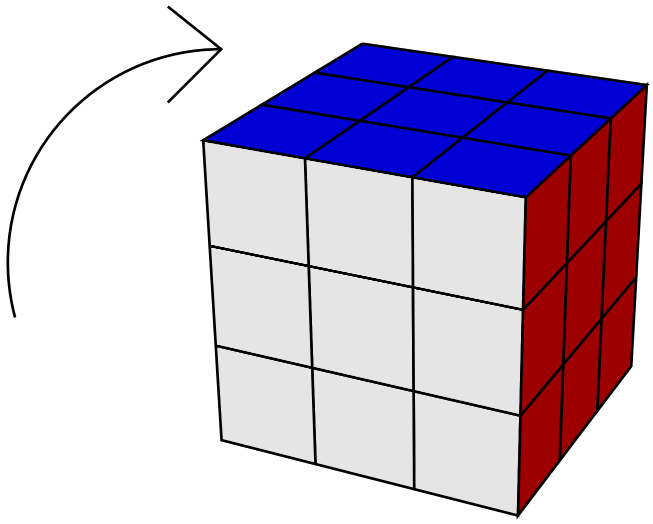
The twist D

And of courses there also exists the twists

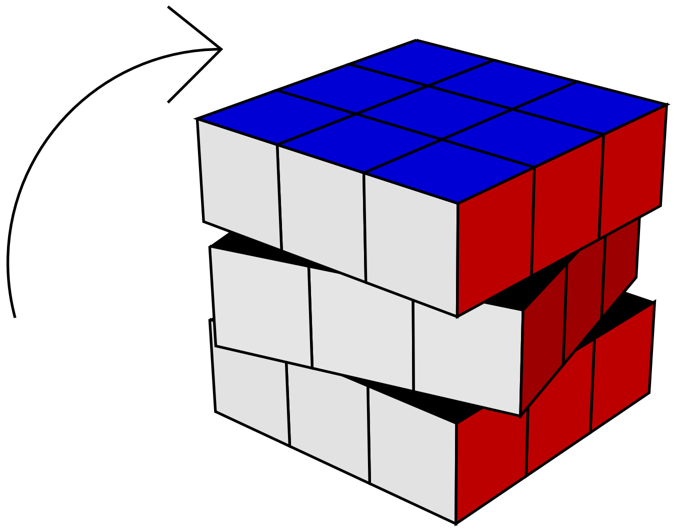
D^2 and D’, as well as corresponding facet maps.

It is also possible to turn two parallel faces in the same direction at the same time. This is called a "slice" and is denoted by the small letter s.

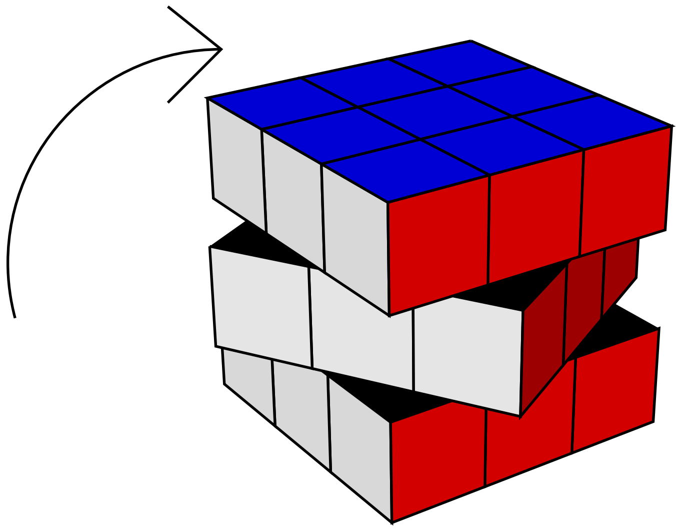
For example, Ts is a slice twist of T and D’, so that both move clockwise according to T.



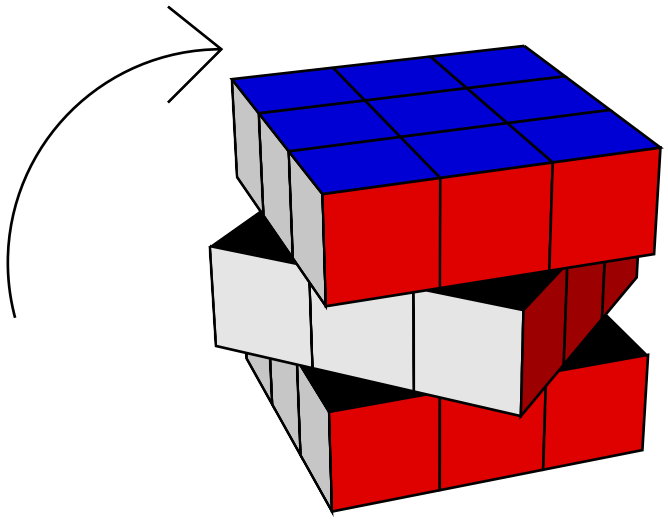
Ts



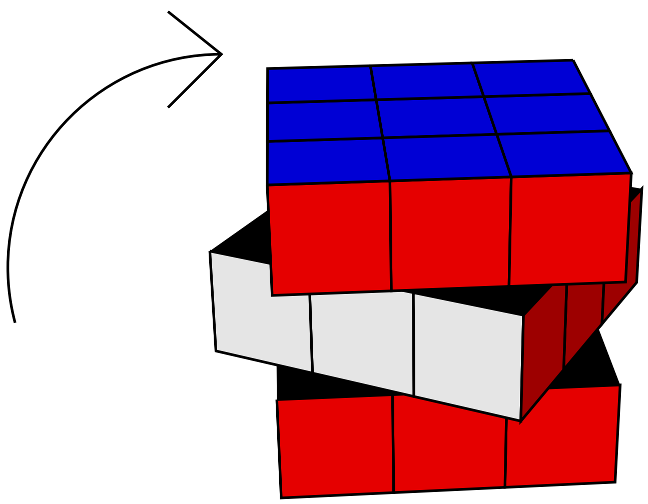
Ts



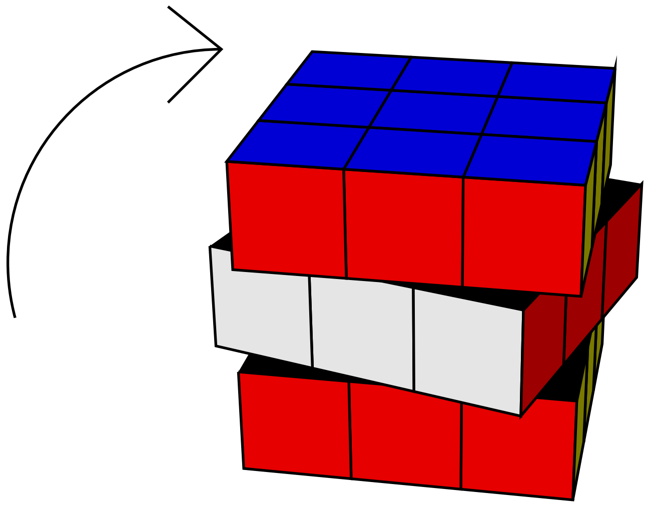
Ts



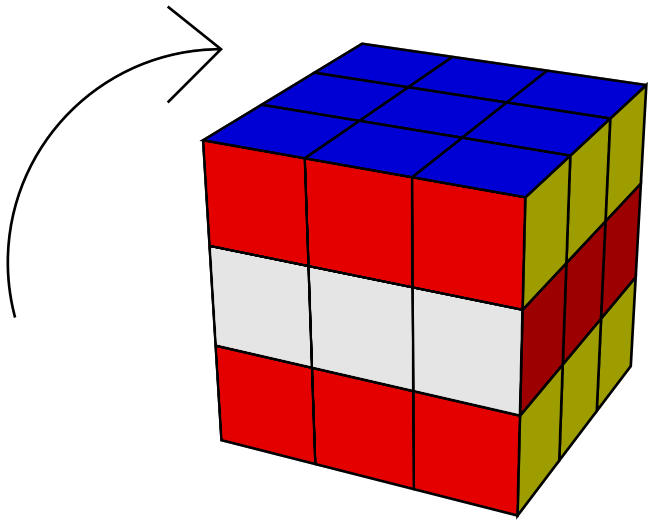
Ts



Ts

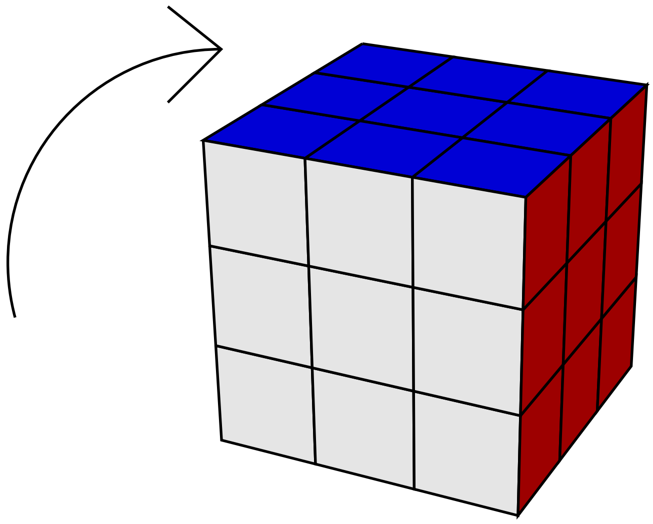


Ts

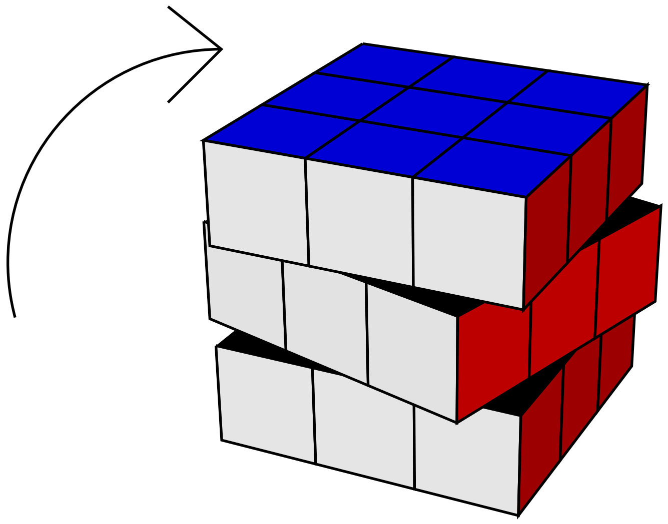


Ts

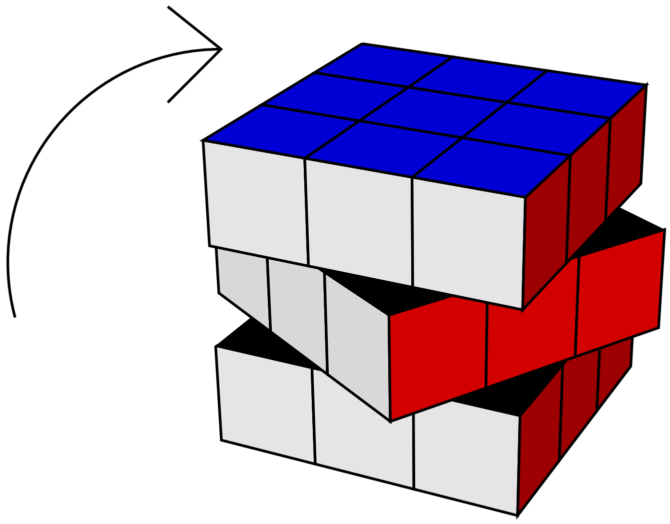
And so it follows that we can do an opposite twist where the middle turns and not the faces. We use the small letter m to represent such a twist. For example, Tm is the twist of the slice between the top and down faces, clockwise according to the top face:



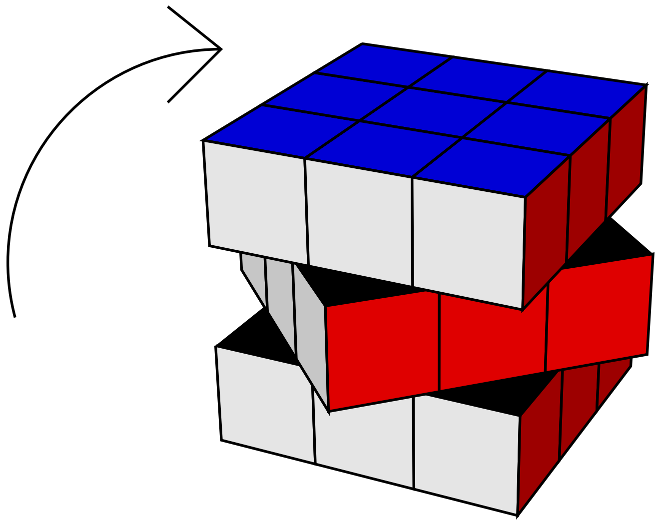
Tm



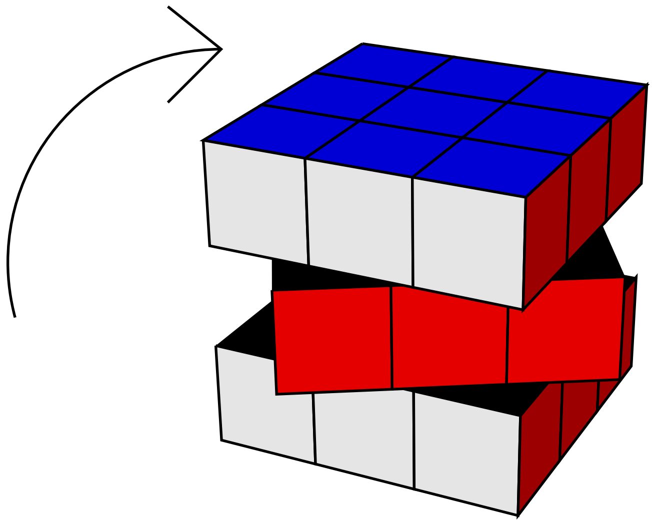
Tm



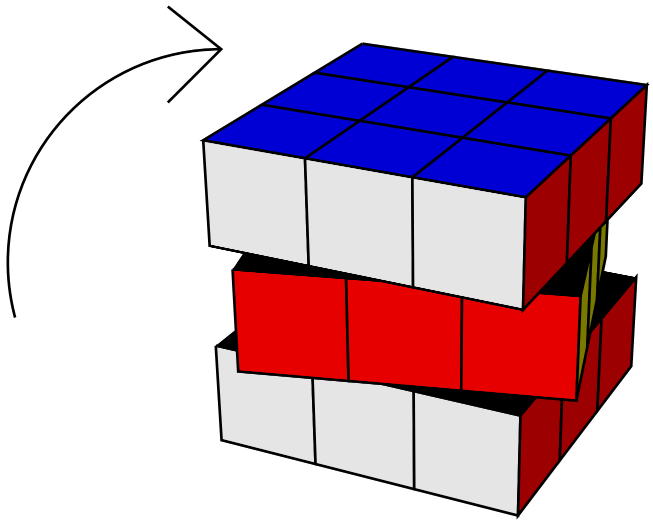
Tm



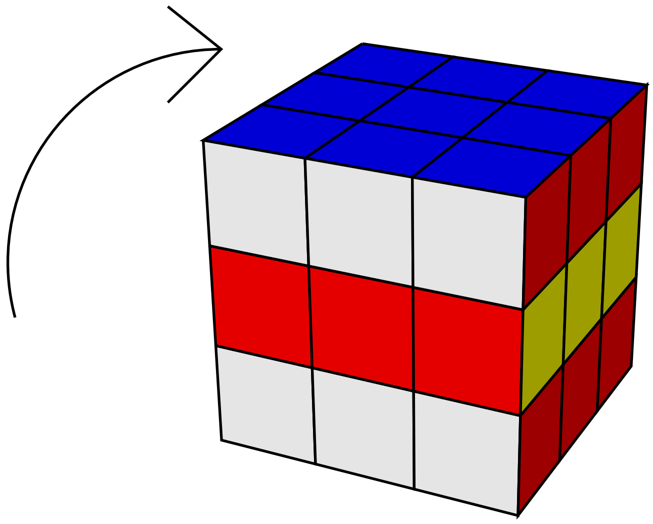
Tm



Tm



Tm



Tm

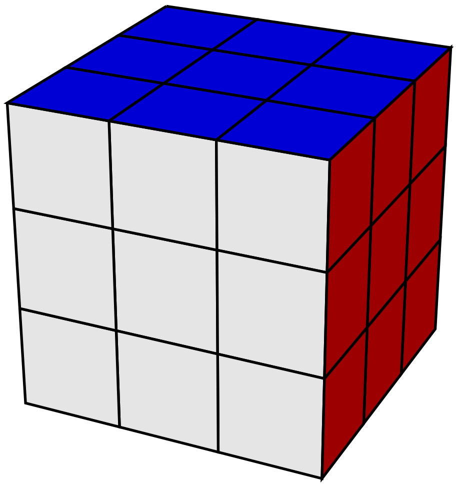
Here is a summary of all the twists we can do around the axis through the top and down faces:

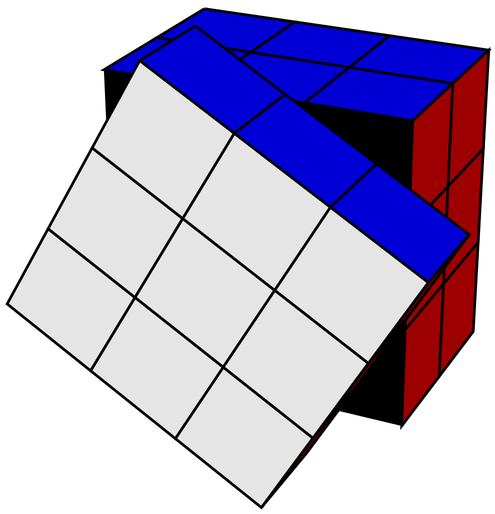
T	**T^2**	**T'**
D	**D^2**	**D'**
Ts	**Ts^2**	**Ts'**
Tm	**Tm^2**	**Tm'**

Note that Ds would be the same as Ts', and Dm the same as Tm'.

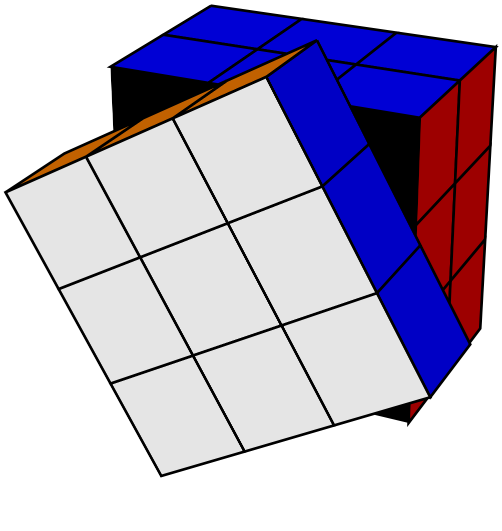
And remember we also saw how to describe how these twist change the colour of the facets by using mapping notation.

Twists around the other two axes are named in the same way. For example:

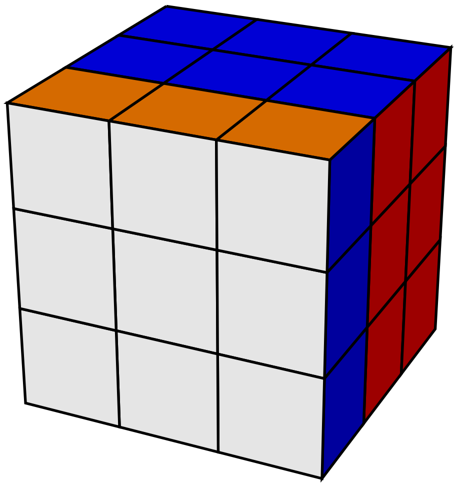




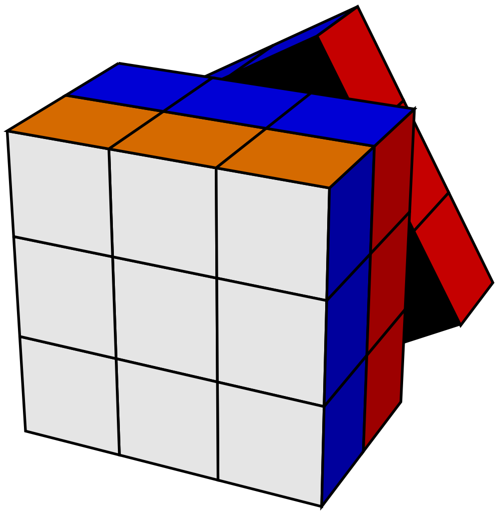
F



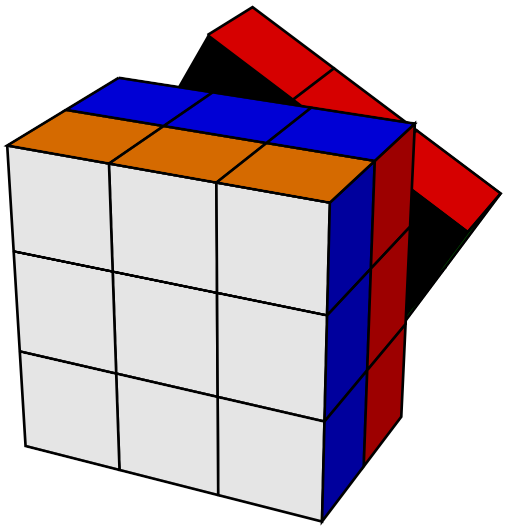
F



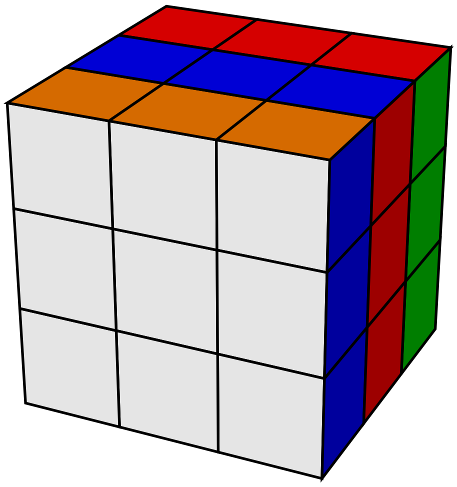
F



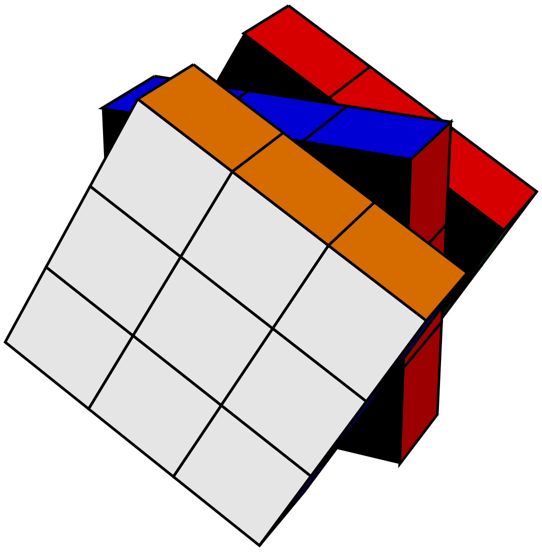
B



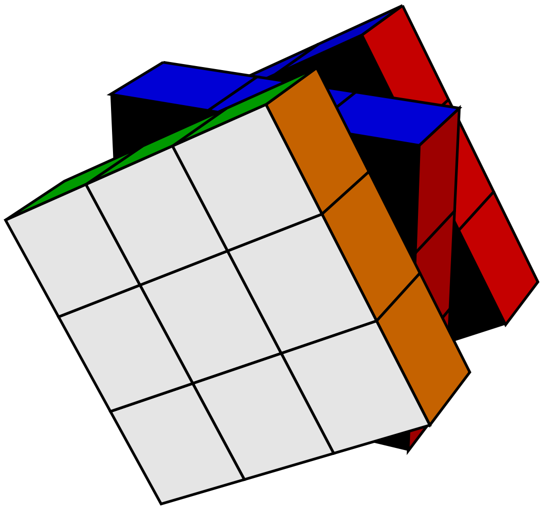
B



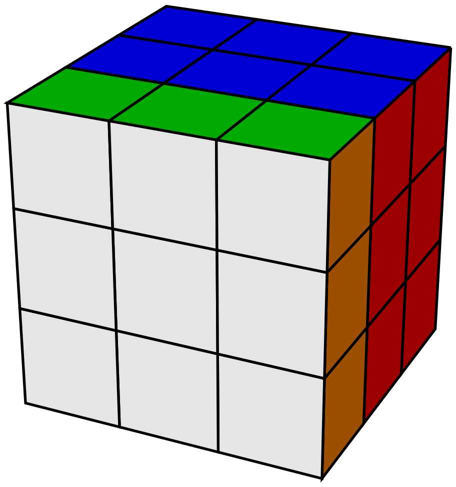
B



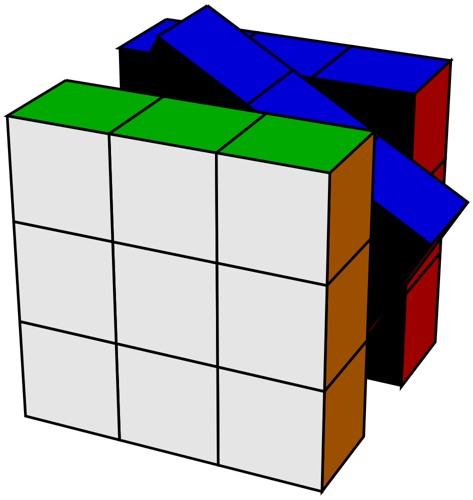
Fs



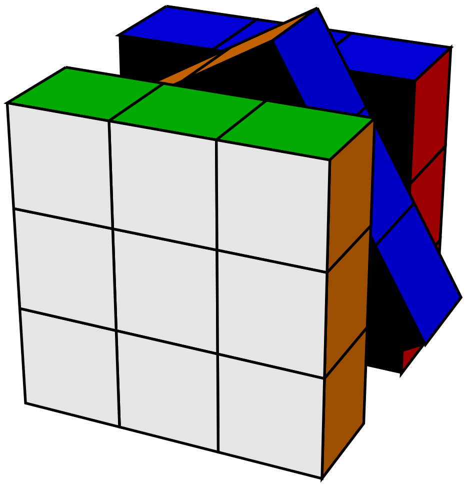
Fs



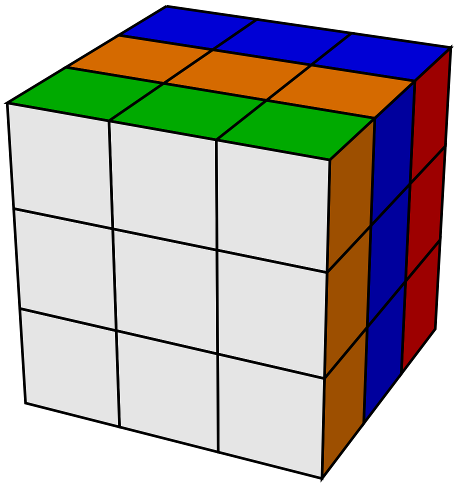
Fs



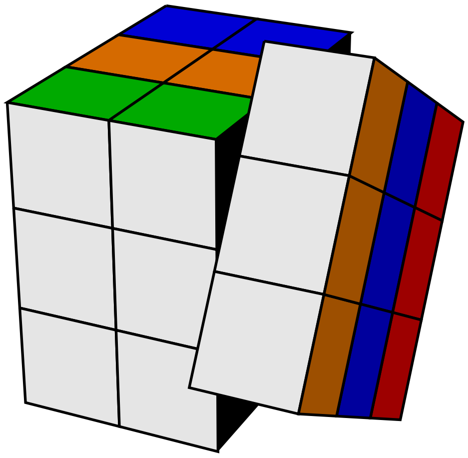
Fm



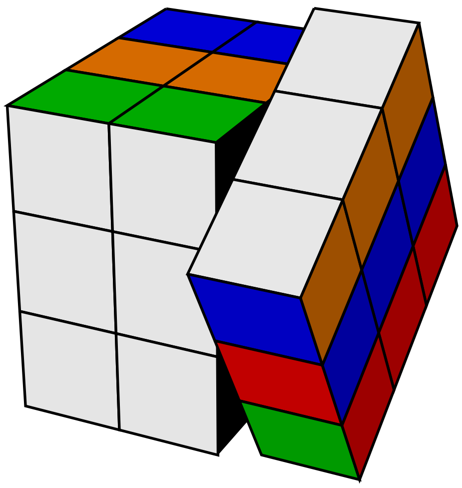
Fm



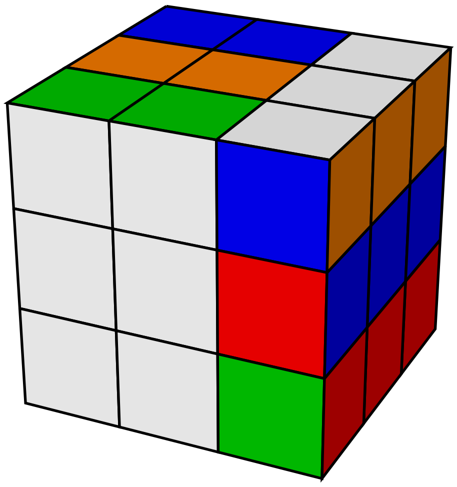
Fm



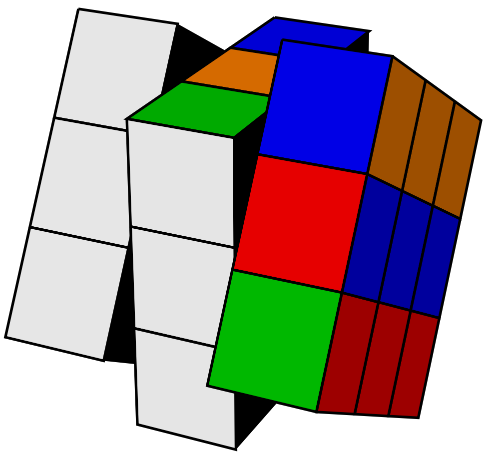
R



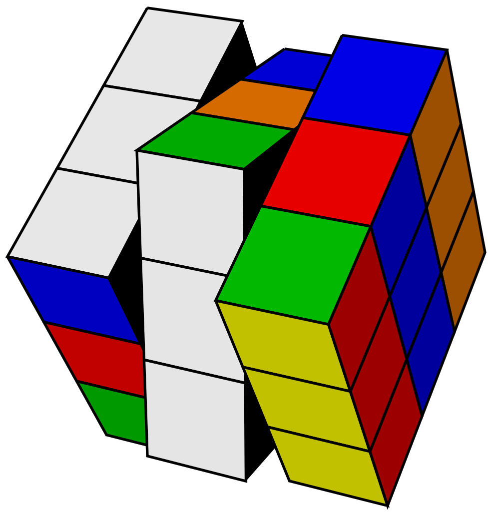
R



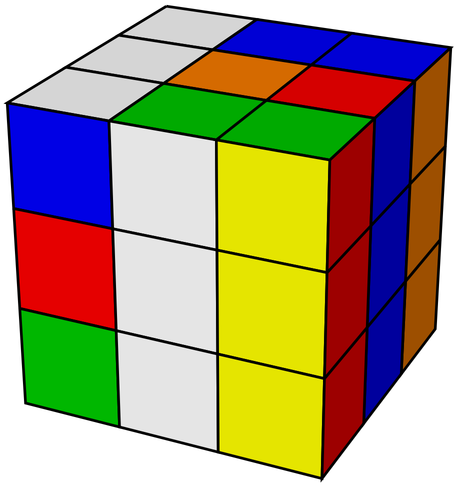
R



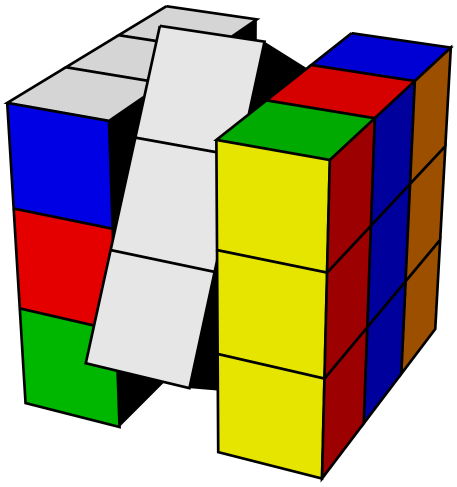
Rs



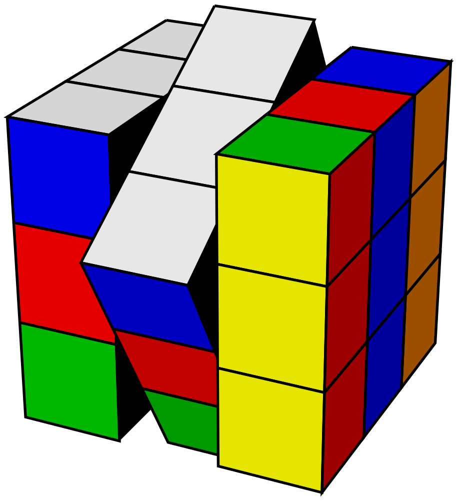
Rs



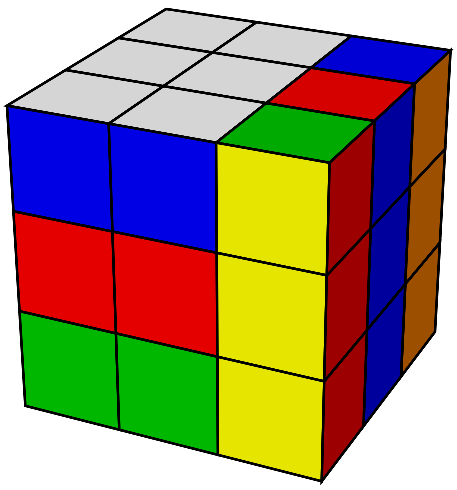
Rs



Rm



Rm



Rm

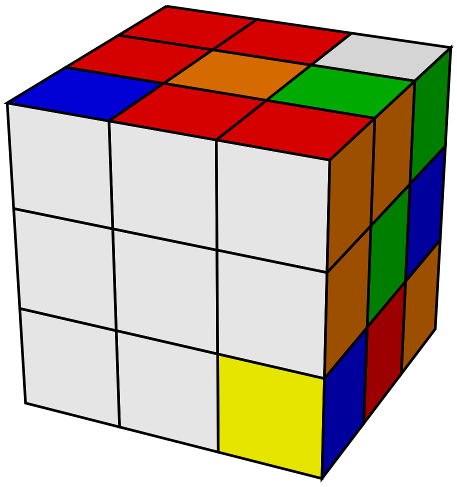
And... I think you get the picture.

Here is a summary of all twists in our notation:

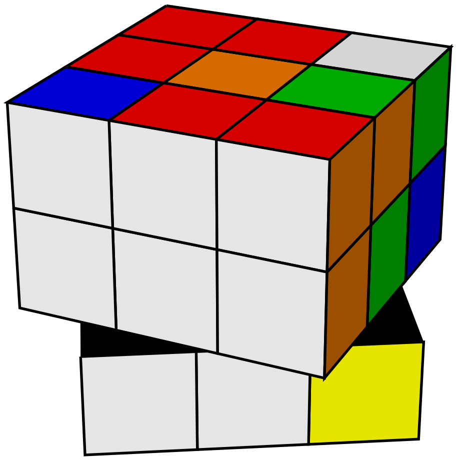
T	**T^2**	**T’**	**D**	**D^2**	**D’**
Ts	**Ts^2**	**Ts’**	**Tm**	**Tm^2**	**Tm’**
F	**F^2**	**F’**	**B**	**B^2**	**B’**
Fs	**Fs^2**	**Fs’**	**Fm**	**Fm^2**	**Fm’**
R	**R^2**	**R’**	**L**	**L^2**	**L’**
Rs	**Rs^2**	**Rs’**	**Rm**	**Rm^2**	**Rm’**

And remember we also saw how to describe how these twist change the colour of the facets by using mapping notation.

With this notation we are able to describe sequences of moves in a short form notation. For example, if our cube looks like the one below, the sequence DB'D' will bring the missing white facet to the front.

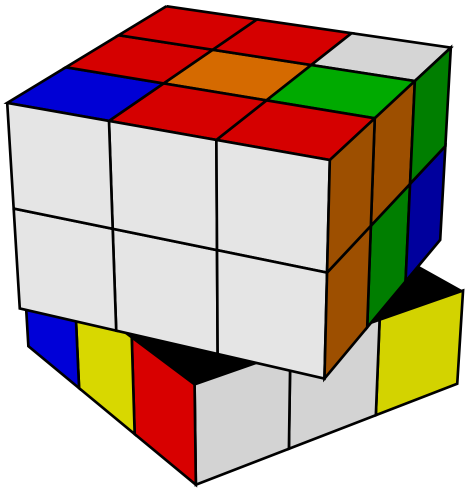


DB'D' will bring the missing white facet to the front.



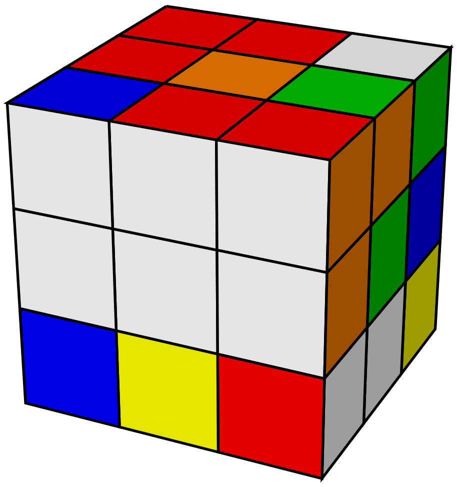
D

DB'D' will bring the missing white facet to the front.



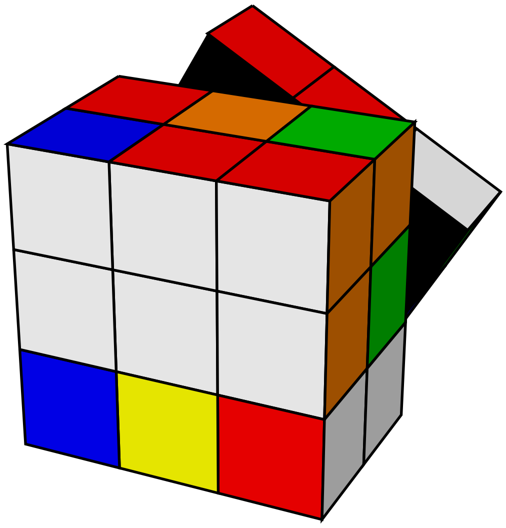
D

DB'D' will bring the missing white facet to the front.



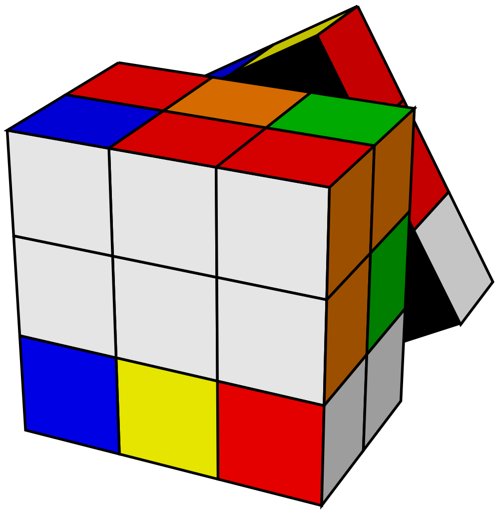
D

DB’D’ will bring the missing white facet to the front.



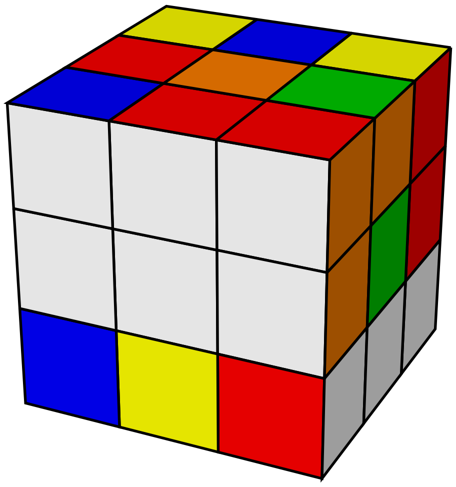
B’

DB’D’ will bring the missing white facet to the front.



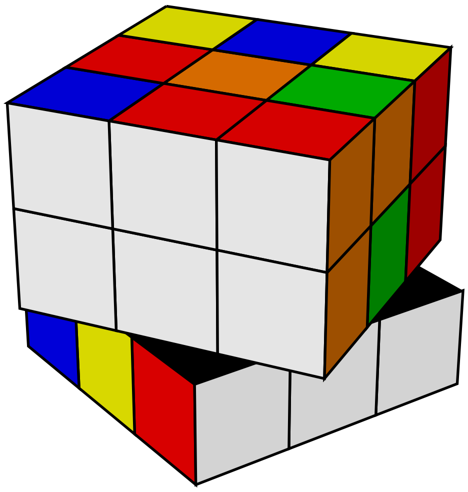
B’

DB’D’ will bring the missing white facet to the front.



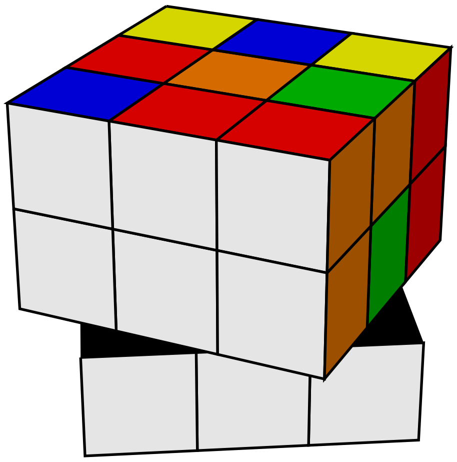
B’

DB’D’ will bring the missing white facet to the front.



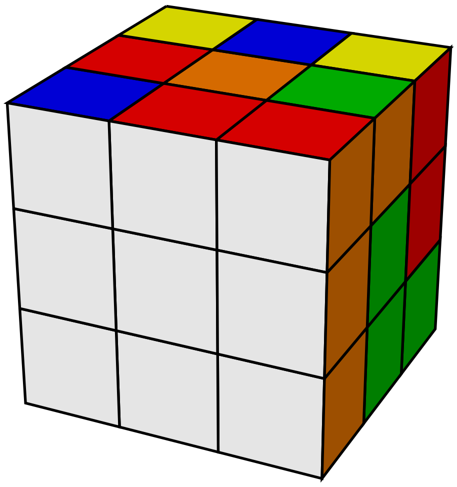
D’

DB’D’ will bring the missing white facet to the front.



D’

DB’D’ will bring the missing white facet to the front.



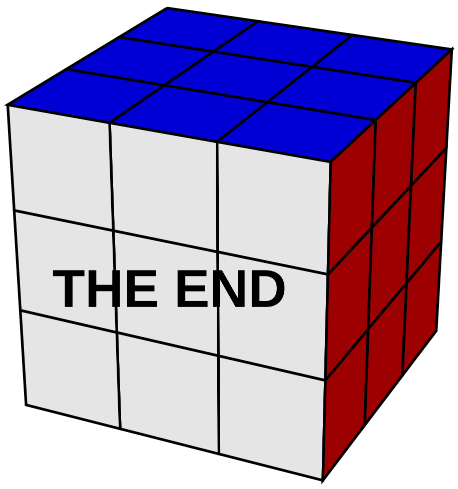
D’

A list of such sequences and their results can then be used to solve the cube.

One solution strategy works in three stages:

1. **Solve the top face and the top edges**
2. **Solve the middle edges**
3. **Solve the bottom corners and edges**

There are many books and web pages with further details to sequences and solutions to the Rubik’s cube, including the book "Rubik’s Cube Compendium" by Erno Rubik, Tamas Vargan, Gerzson Keri, Gyorgy Marx, and Tamas Vekerdy.


THE END