Math 308 Project

Interactive Mathematics

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Proofs: #10 - Three Circles,
#9 - Symmetries in a Triangle,
#3 - Yet another Three Circles

Note: Figures are best viewed when magnified 5x.

Raymond Lee

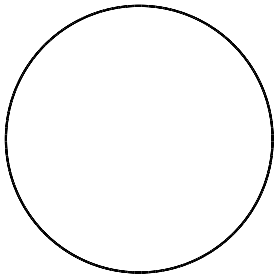
#74444993

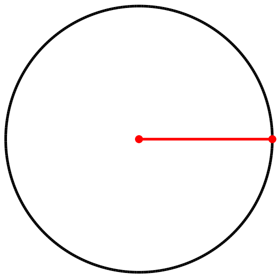
Proof: #10 - Three Circles

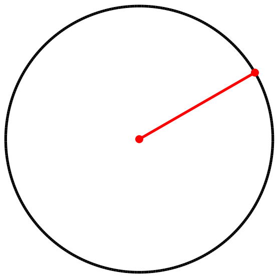
If three circles having the same radius pass through a point, the circle through their other three points of intersection also has the same radius.

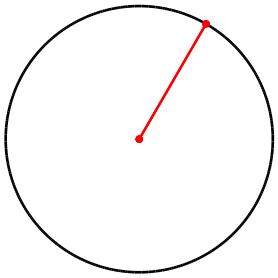
Note: Figures are best viewed when magnified 5x.

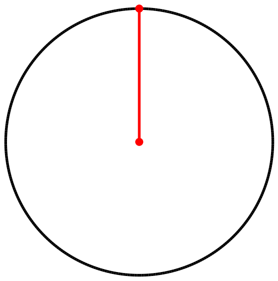
Circle has radii r - Denoted by red line.

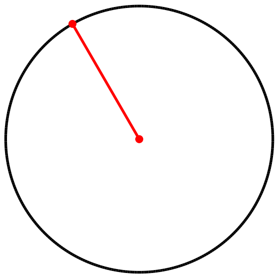


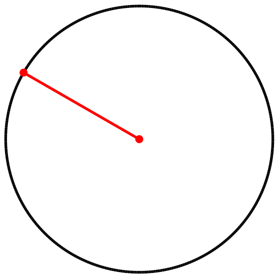


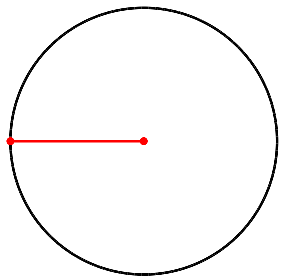


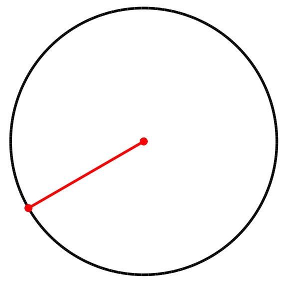


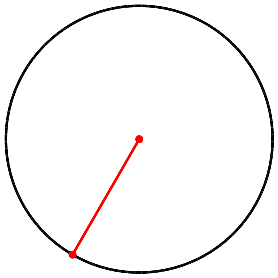


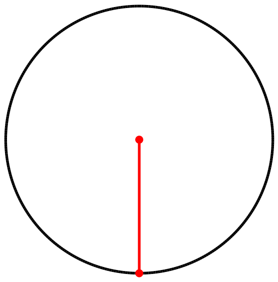


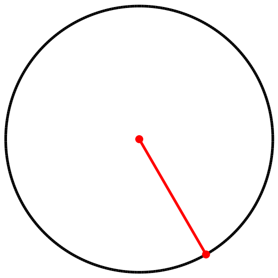


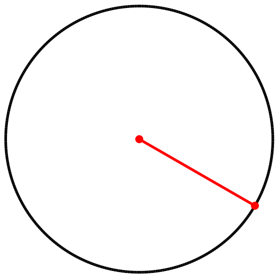


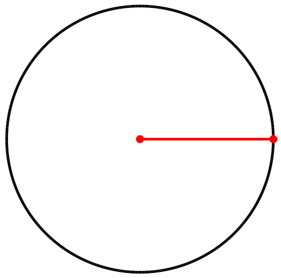


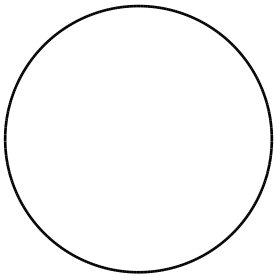




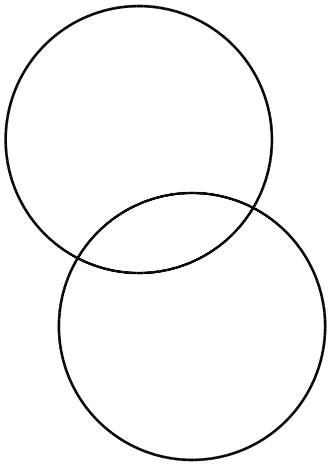


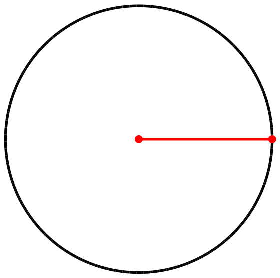


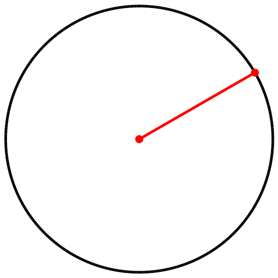


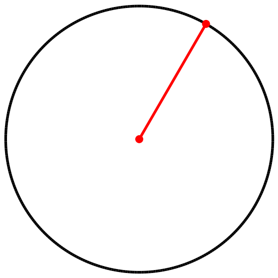


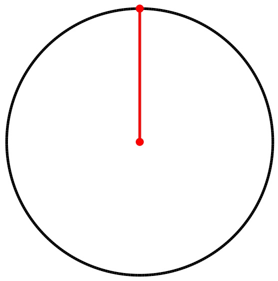
Both Circles have radii r - Denoted by red line.

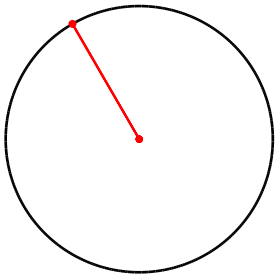


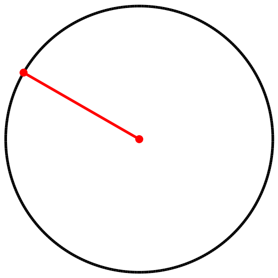


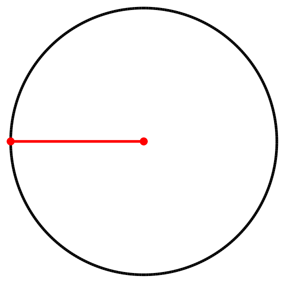


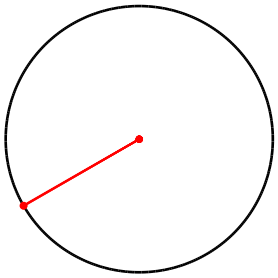


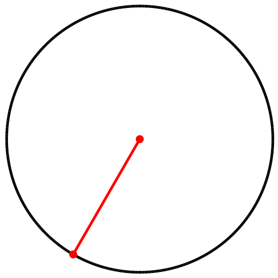


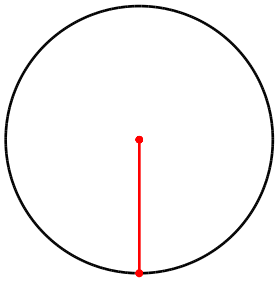


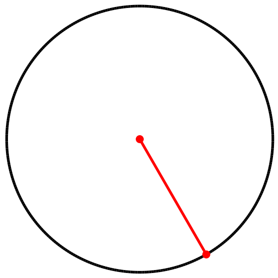


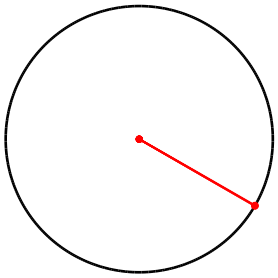


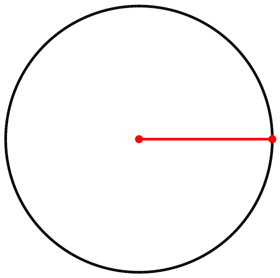


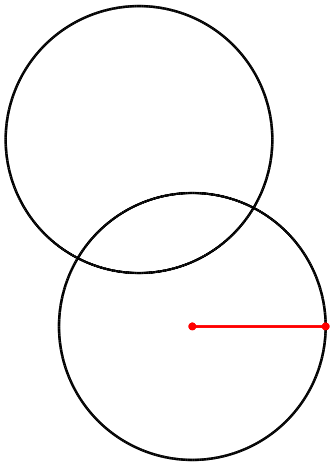


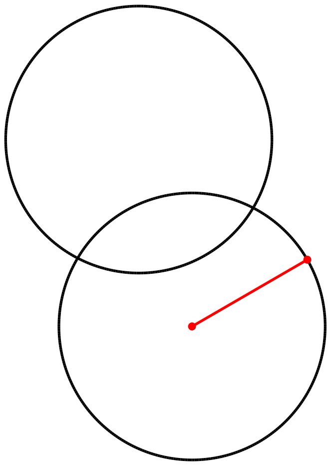


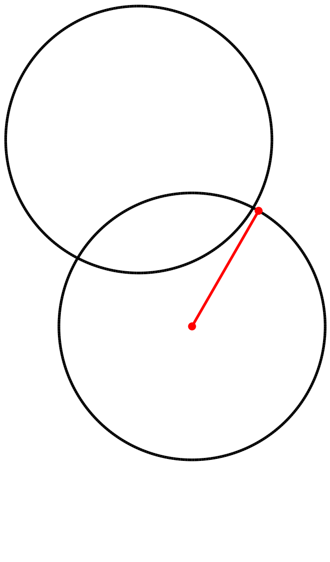


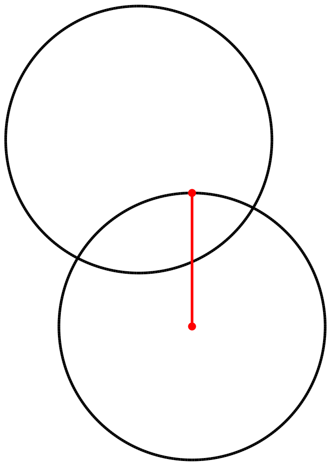


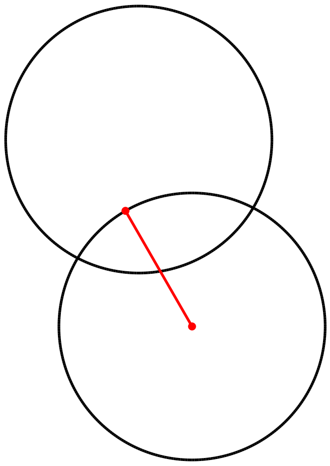


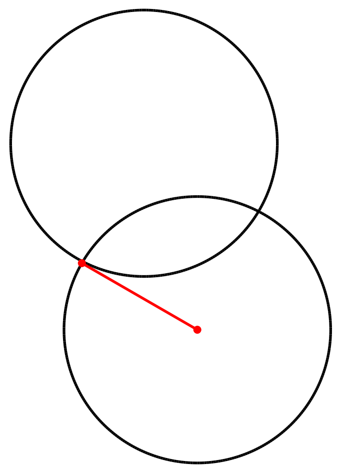


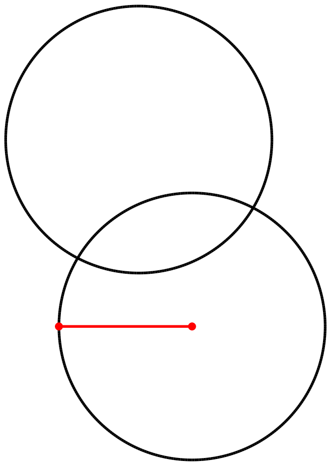


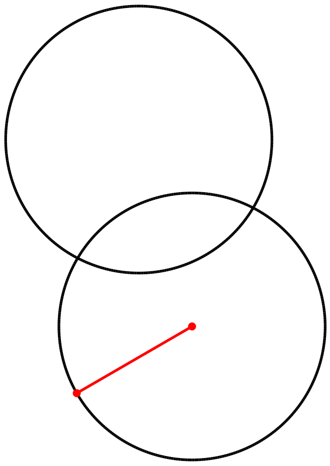


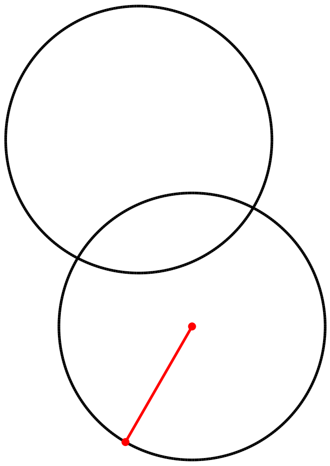


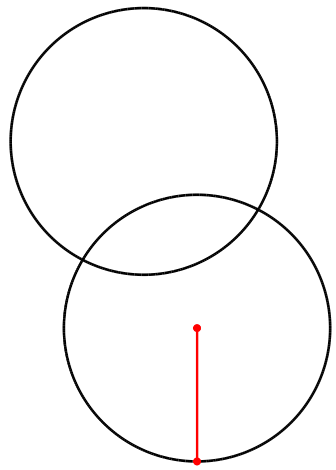


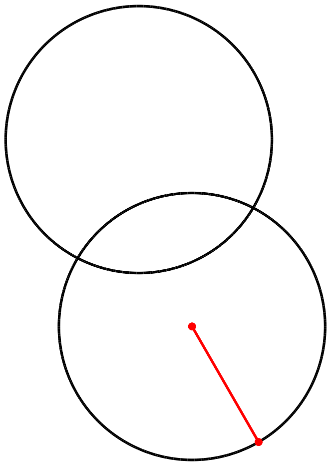


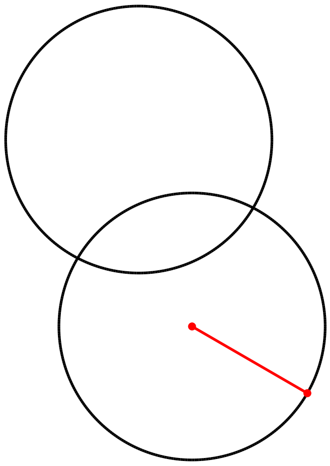


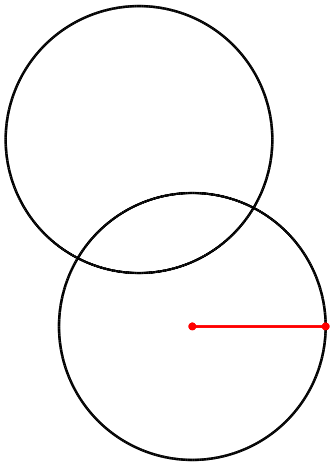


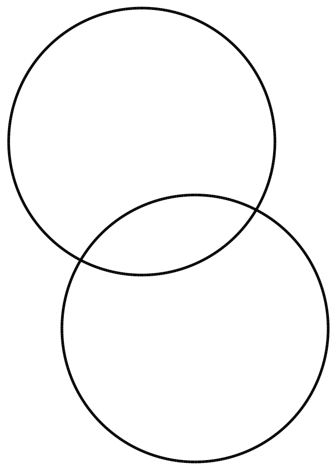




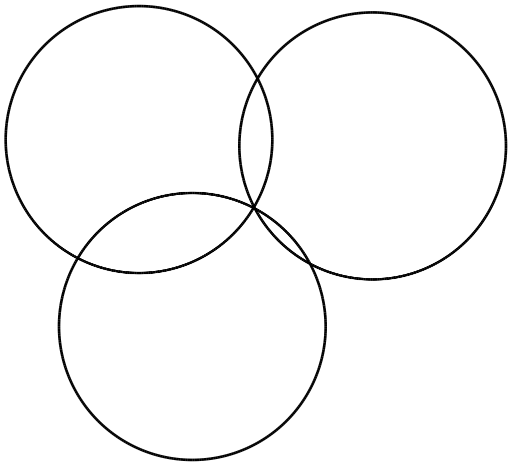


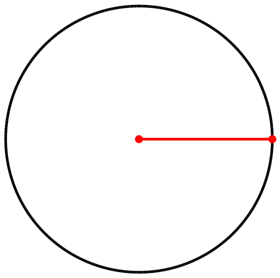


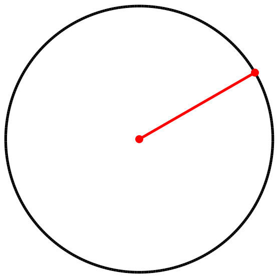


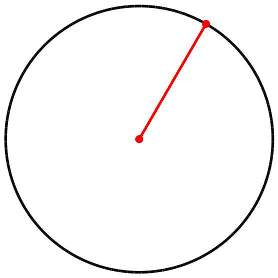


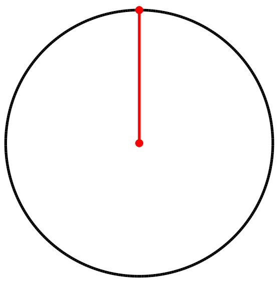
All 3 Circles have radii r - Denoted by red line.

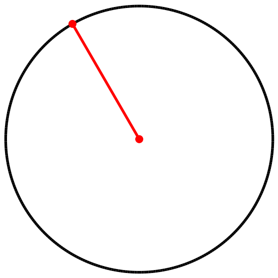


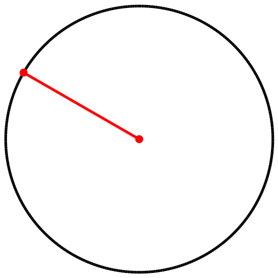


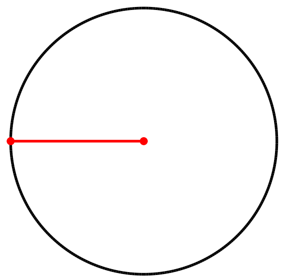


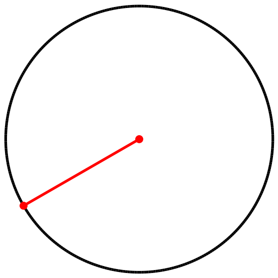


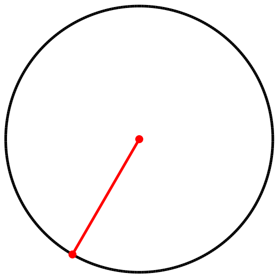


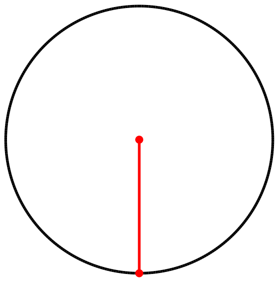


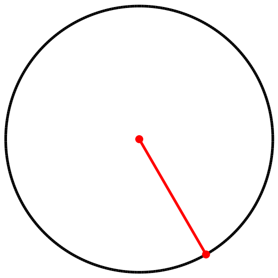


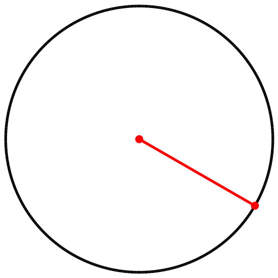


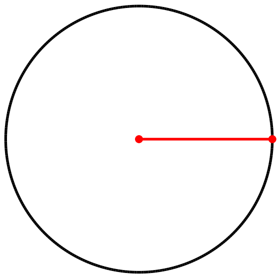


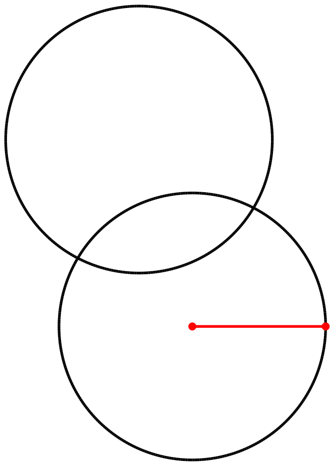


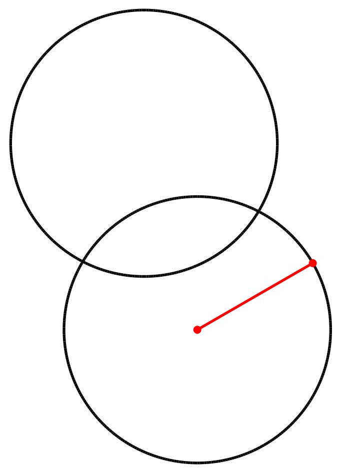


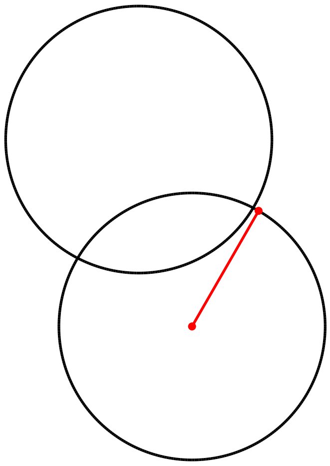


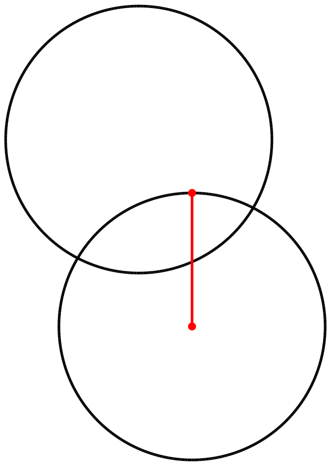


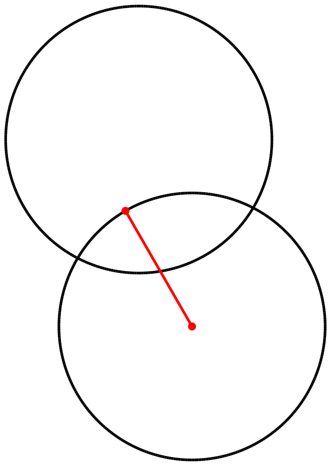


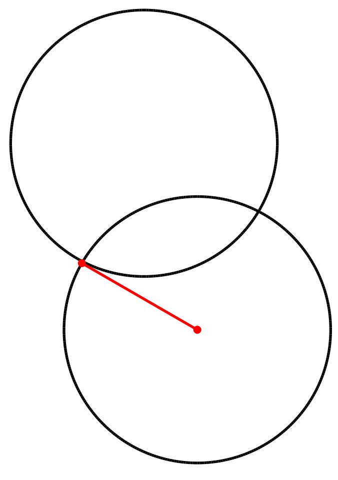


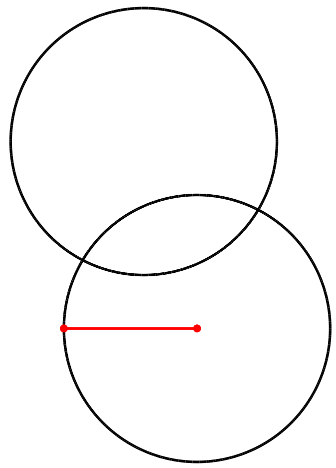


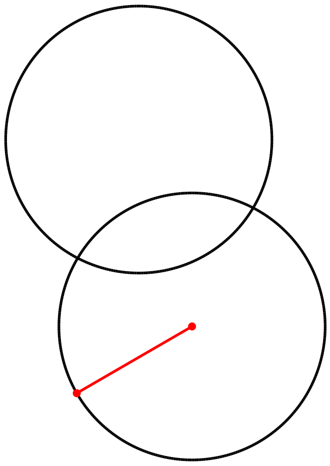


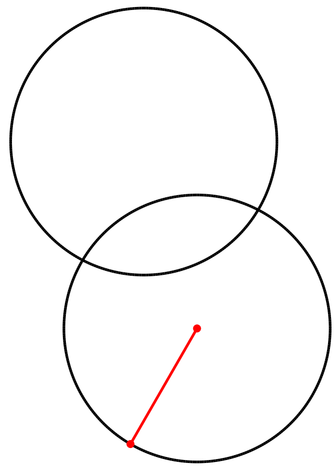


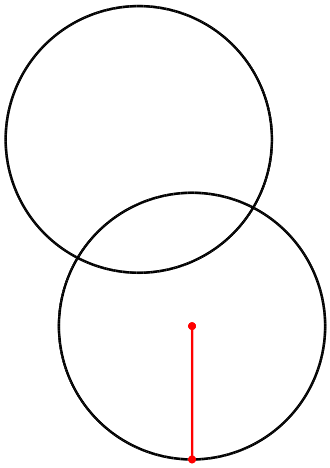


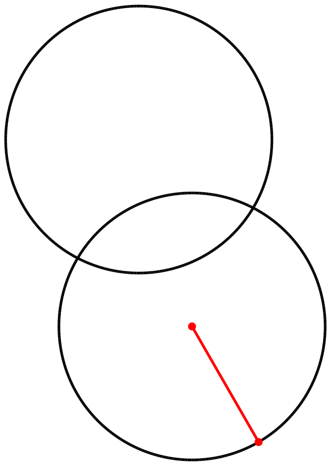


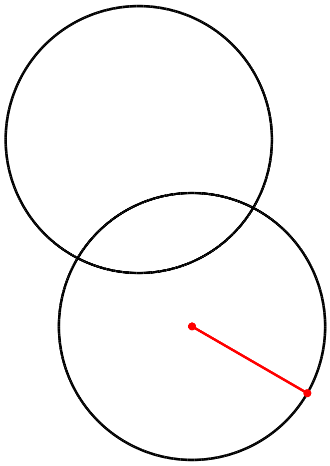


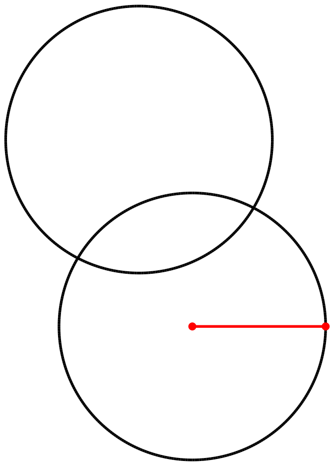


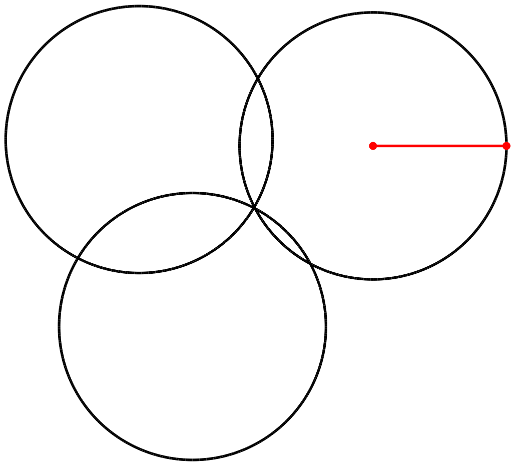


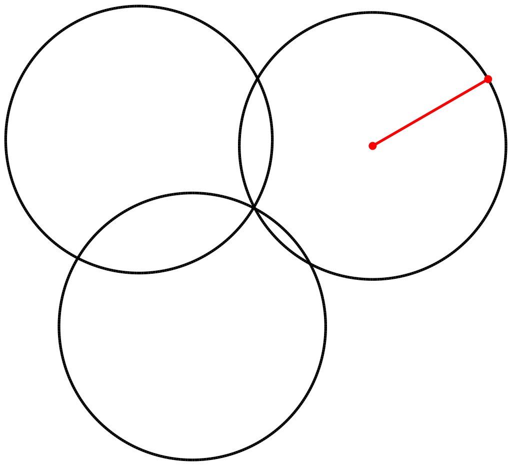


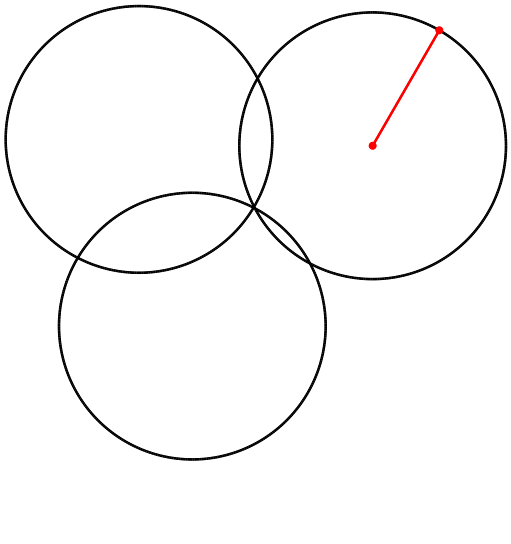


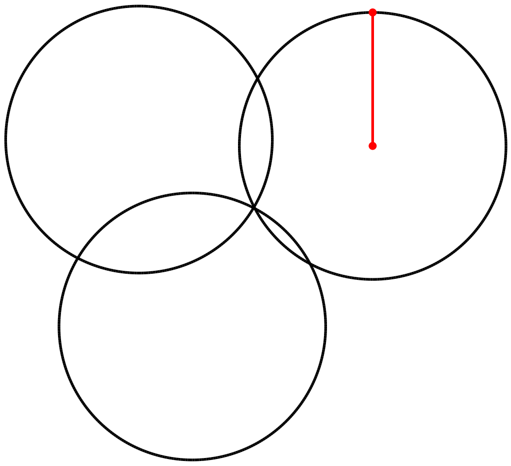


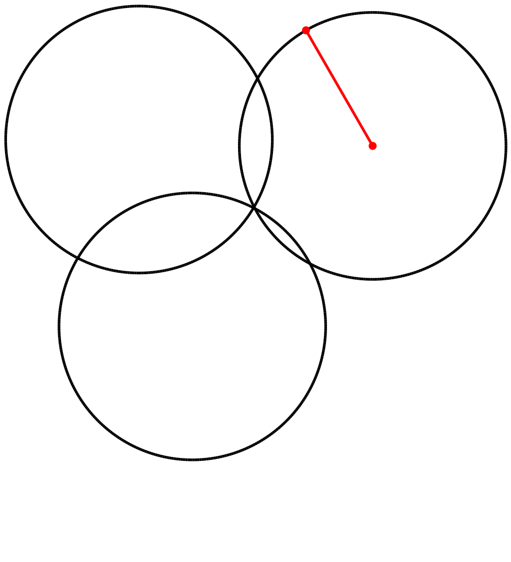


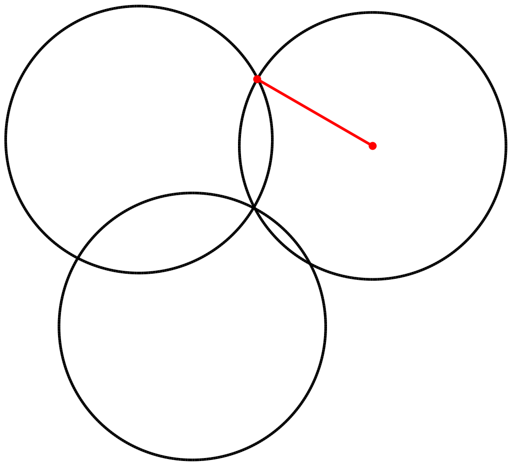


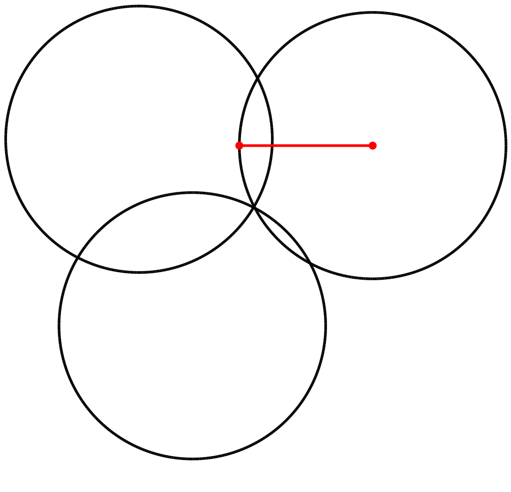


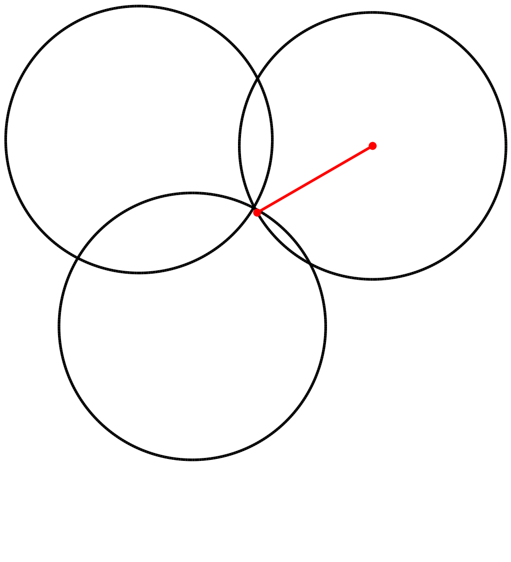


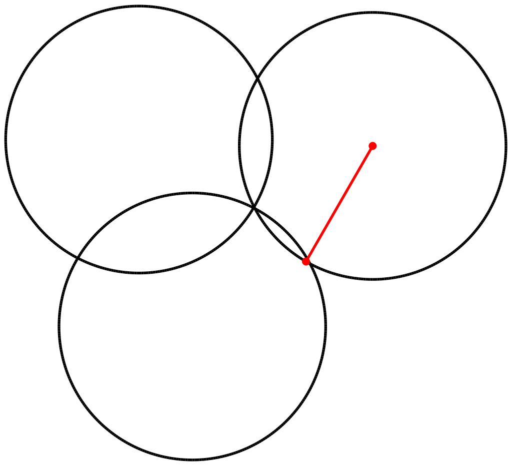


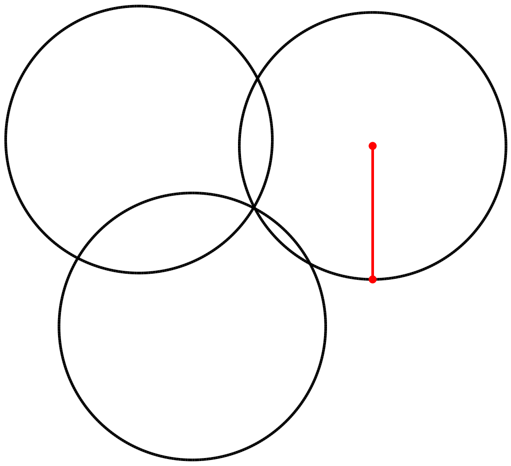


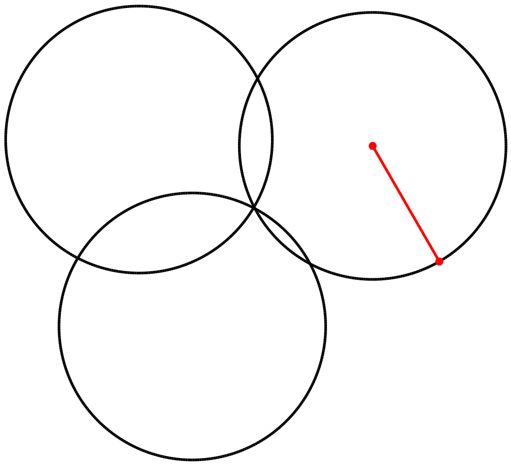


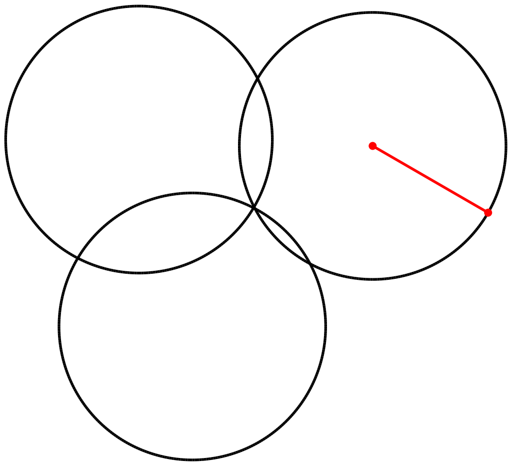


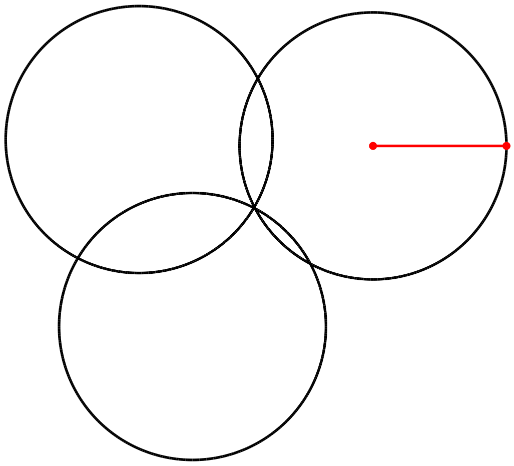


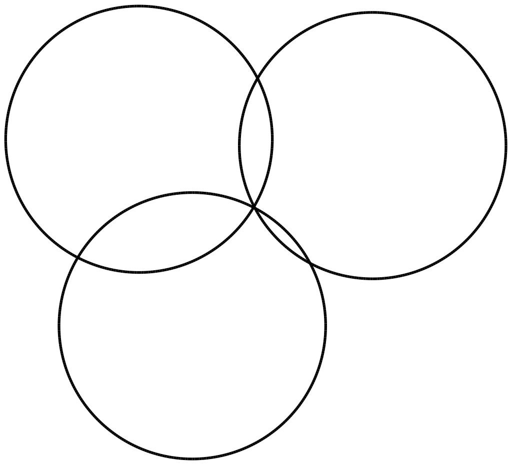




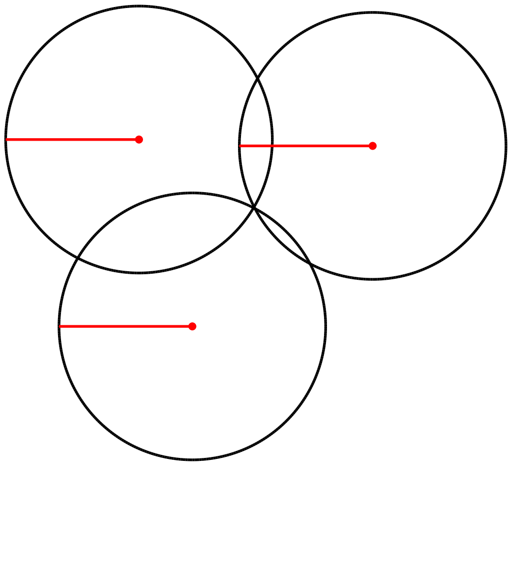


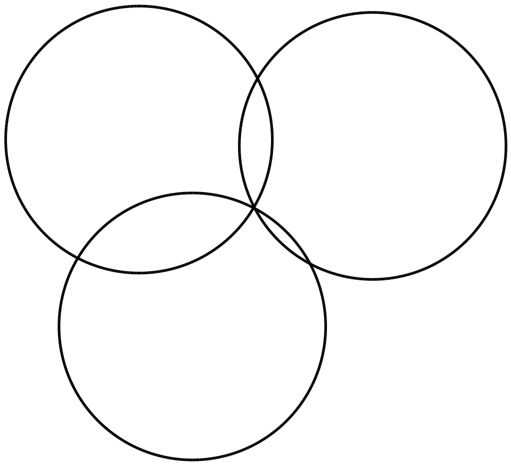




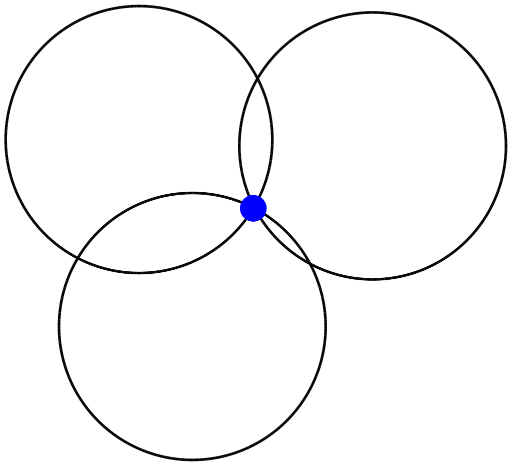


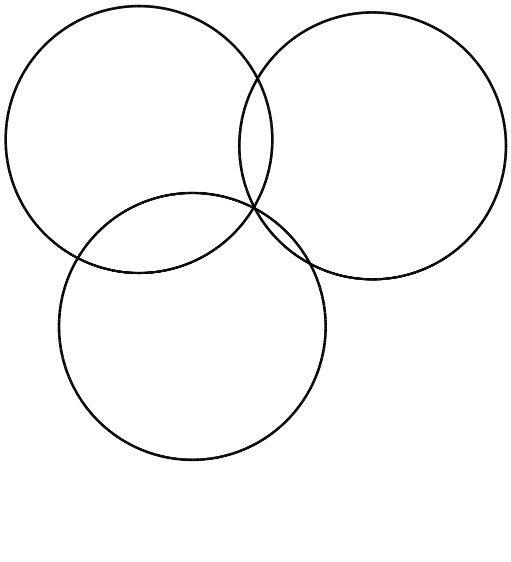
All 3 Circles have radii r - Denoted by red line.



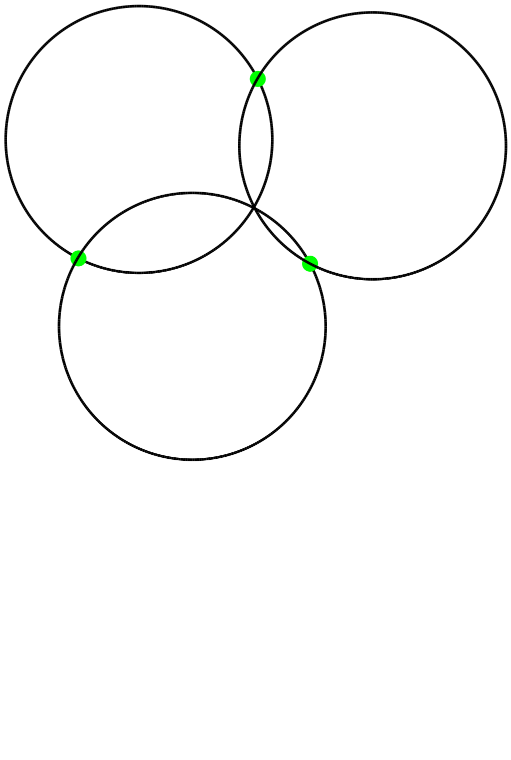


All 3 Circles intersect at blue dot.

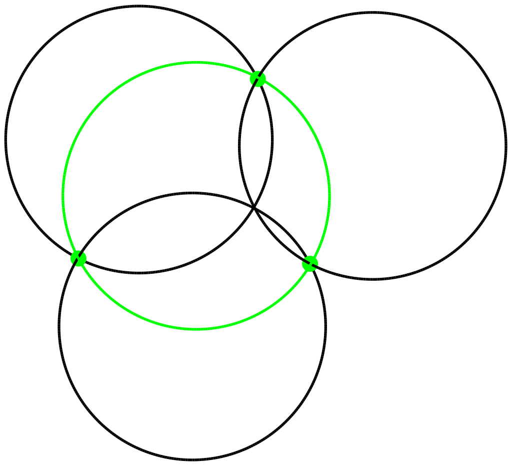


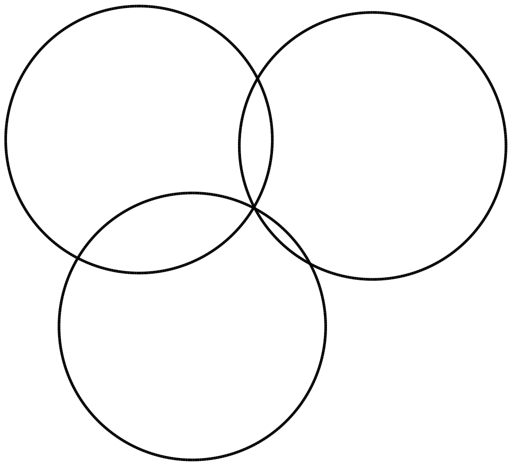


Other points of intersection - Denoted by green dots.

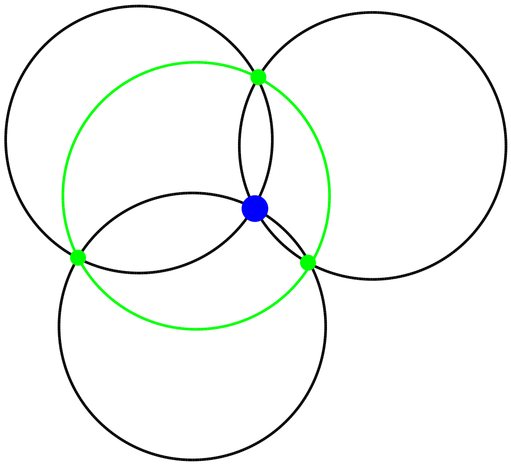


Other points of intersection form new green circle.

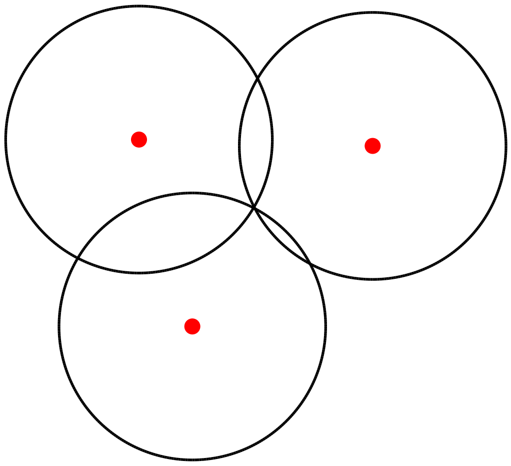




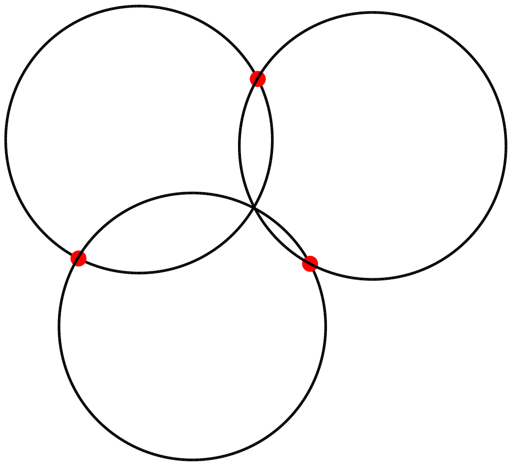
The 3 circles (black), along with the point where all three circles intersect (blue), and the new circle formed by connecting the other points of intersection (green dots and green circle).



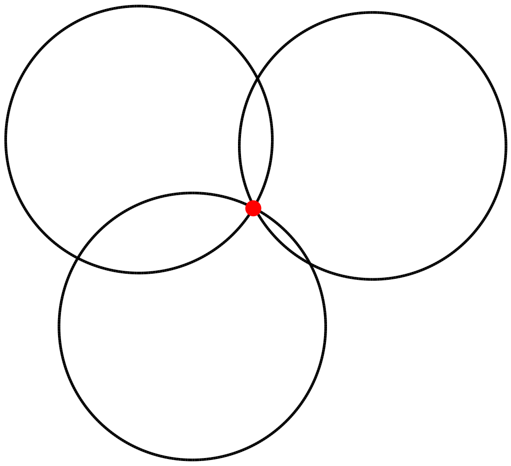
The centre of the three circles - Denoted by a red dot.



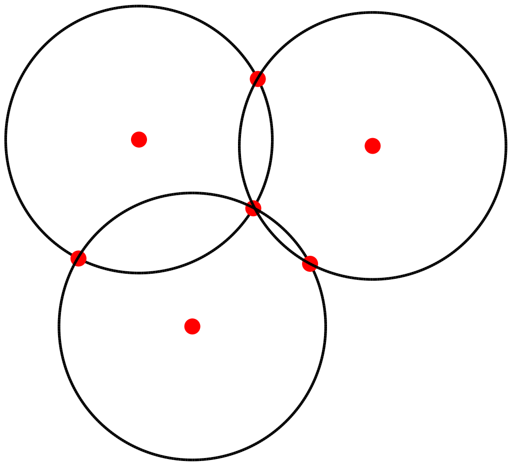
The points of intersection - Denoted by a red dot.



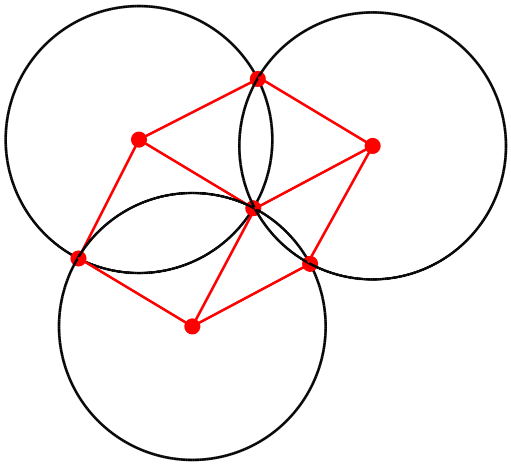
The point where all 3 circles intersect (red dot).



All the points - the centre, the points of intersection, and the point where all 3 circles intersect (red dots).

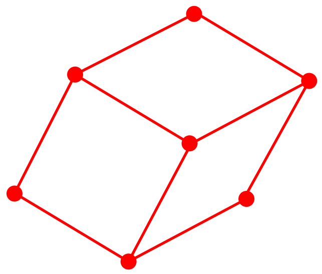


Connect the dots to form radii of circle.



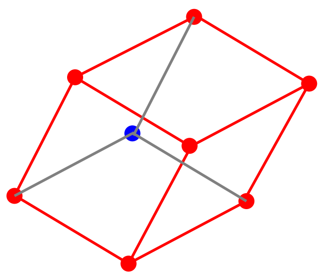
Notice: We have 9 equal segments b/c each segment. is equal to the radii - and radii of all 3 circles are equal.

Take away the 3 circles.



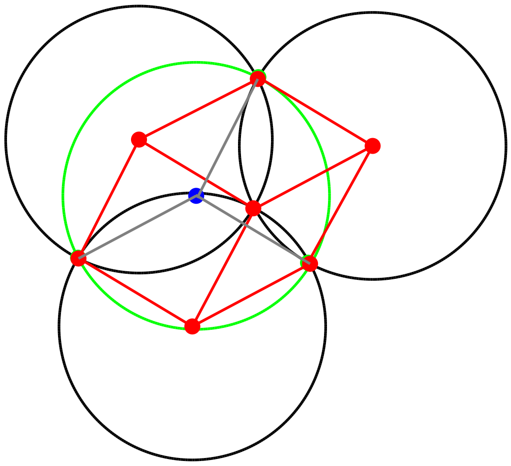
The nine equal segments appear to form a well known shape - it's a projection of a parallelepiped on a plane.

The 3 hidden edges of the parallelepipe shown in gray.



Now we have 12 equal segments where each point is equidistant from its adjacent point.

Now, if we put together all the pictures:
the 3 circles, the projection of the parellelepiped
on the plane, and all the points of intersection



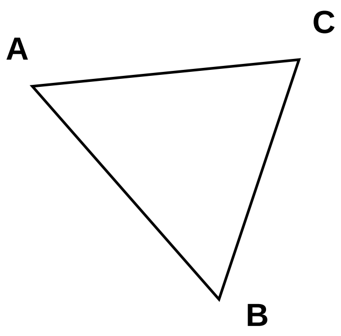
Notice:
1. the radii of the 3 circles are also the
9 equal segments of the parellelepiped on
the plane (red dots and lines).
2. the radii of the green circle are also the hidden
edges of the parellelepiped on the plane (gray lines).
3. the centre of the green circle is the hidden point
of the parellelepiped on the plane (gray dot).

Proof: #9 - Symmetries in a Triangle

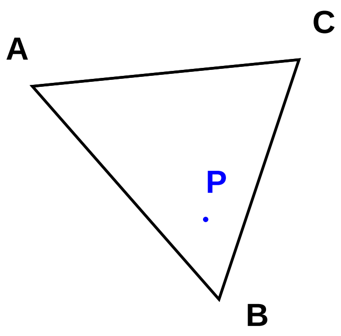
If ABC is a triangle and P any point in the plane distinct from A,B,c and if X,Y,Z are the symmetries of P with respect to the midpointst BC,CA,AB respectively, show that AX, BY, CZ concur.

Note: Figures are best viewed when magnified 5x.

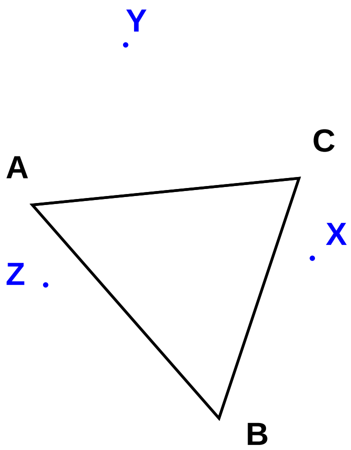
The triangle ABC.



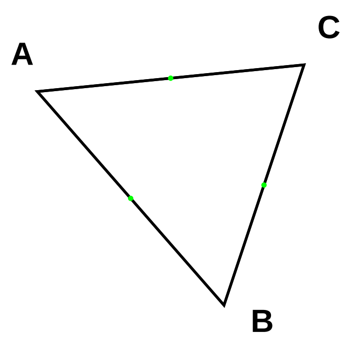
The triangle ABC with point P in the plane.



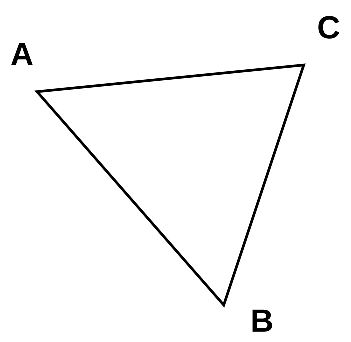
The triangle ABC with points X,Y,Z.



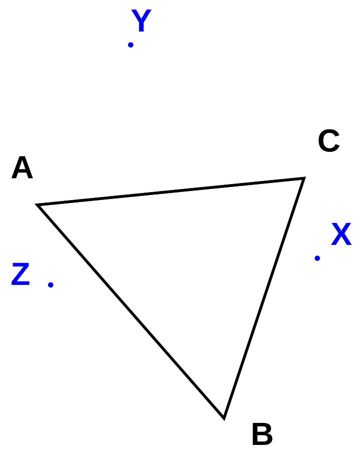
The triangle ABC with midpoints of each edge (green dot).



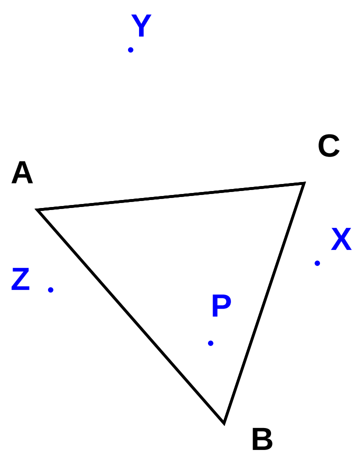
The triangle ABC.



The triangle ABC with points X,Y,Z.

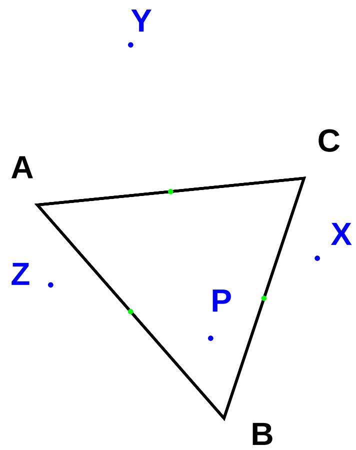


The triangle ABC with points X,Y,Z and P in the plane.



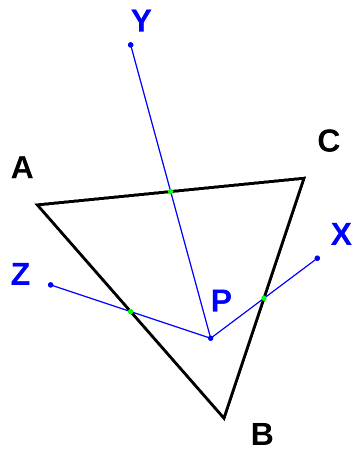
Note: Point P is distinct from A,B,C and X,Y,Z are symmetries to P with respect to midpoints of each edge.

The triangle ABC with points X,Y,Z and P in the plane, and midpoints of each edge (green dots).



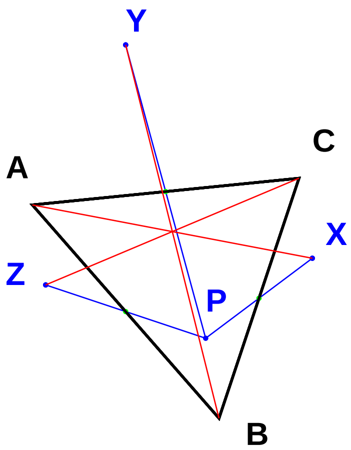
Note: Point P is distinct from A,B,C and X,Y,Z are symmetries to P with respect to midpoints of each edge.

The triangle ABC with points X,Y,Z and P in the plane, and midpoints of each edge (green dots).
Lines Px, PY and Pz are shown in blue.



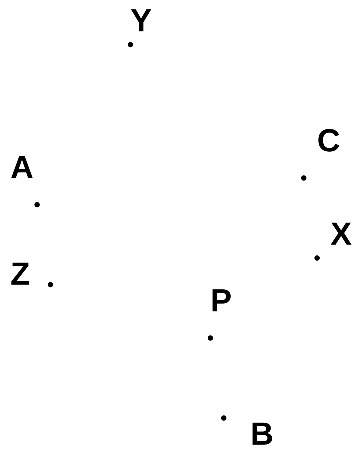
Note: Point P is distinct from A,B,C and X,Y,Z are symmetries to P with respect to midpoints of each edge.

The triangle ABC with points X,Y,Z and P in the plane, and midpoints of each edge (green dots).
Lines PX, PY and PZ are shown in blue.
Lines AX, BY and CZ are shown in blue.

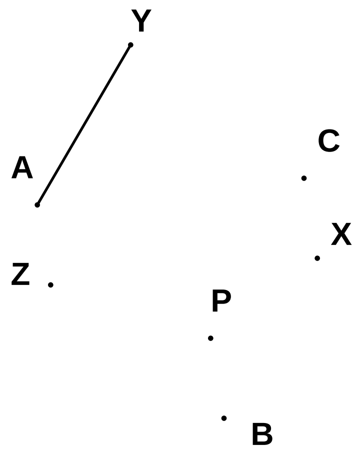


Note: Point P is distinct from A,B,C and X,Y,Z are symmetries to P with respect to midpoints of each edge.

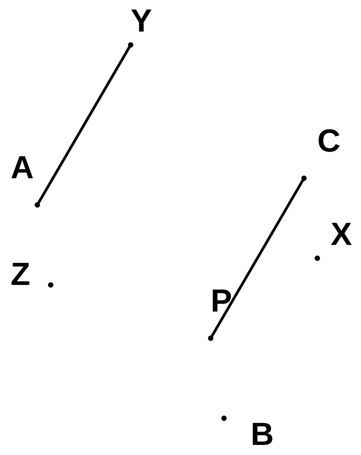
We take away triangle ABC and the lines to be left with just the points A,B,C,P,X,Y,Z.



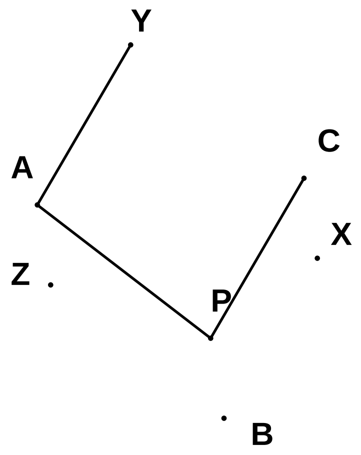
Join Point A,Y to form line AY (black line).



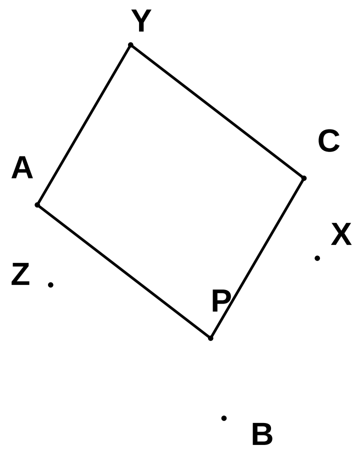
Join Point C,P to form line CP.
We have line AY,CP (black lines).



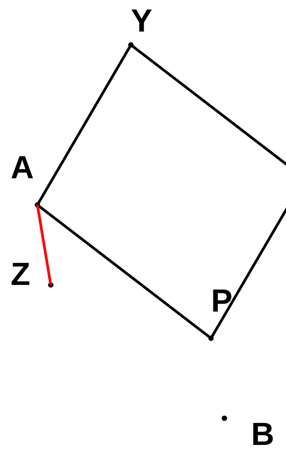
Join Point A,P to form line AP.
We have line AY,CP,AP (black lines).



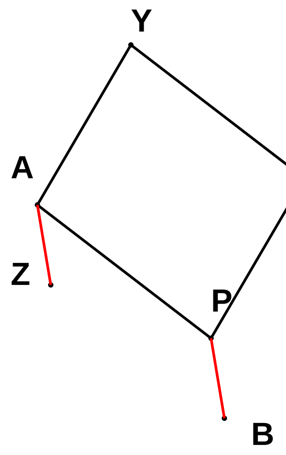
Join Point C,Y to form line CY.
We have line AY,CP,AP,CY (black lines).



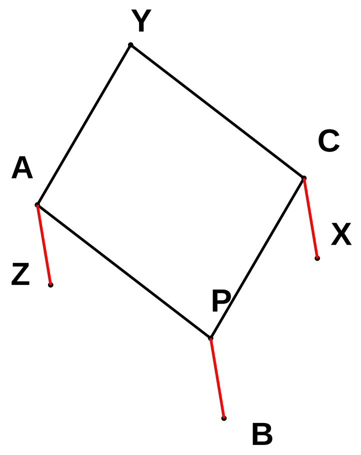
Join Point A,Z to form line AZ (red line).



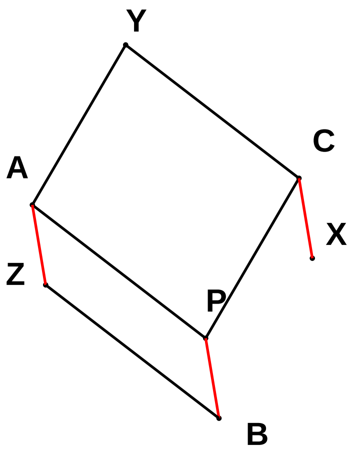
Join Point B,P to form line BP.
We have line AZ,BP (red lines).



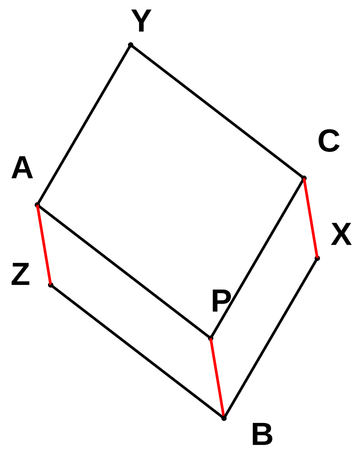
Join Point C,X to form line CX.
We have line AZ,BP,CX (red lines).



Join Point B,Z to form line BZ (black line).

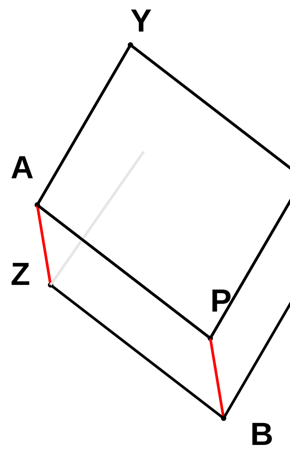


Join Point B,X to form line BX.
We have line BZ,BX (black lines).

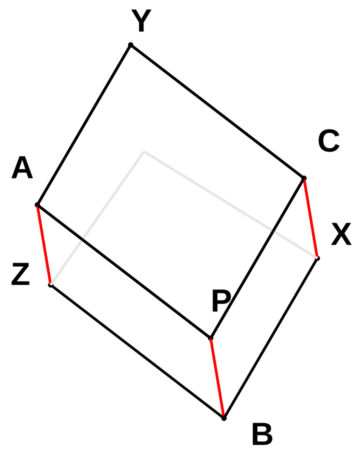


Note: We can imagine from a 3D perspective, a parallelepipe forms. We know need to show that lines AX,BY,CZ concur, which amounts to saying these lines are diagonals of the figure

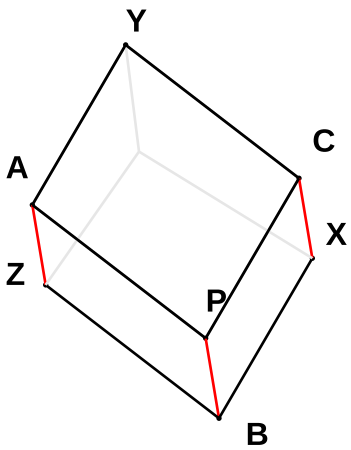
Show the hidden line of the parallelpipe (gray line).



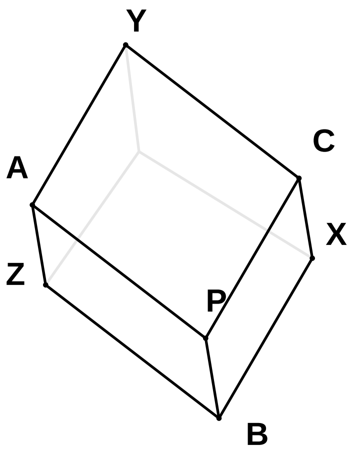
Show the hidden line of the parallelpipe (gray line).



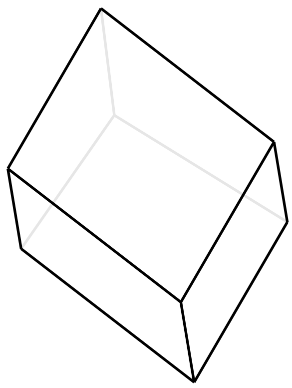
Show the hidden line of the parallelpipe (gray line).



The parallelpipe with labels.

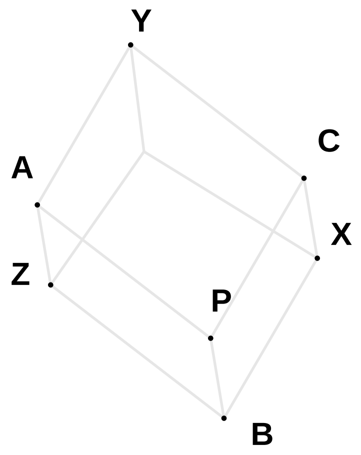


The parallelpipe without labels.

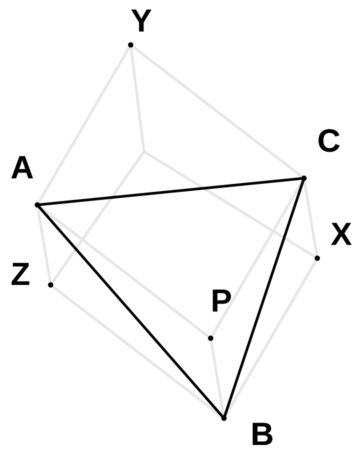


The parallelpipe in gray.

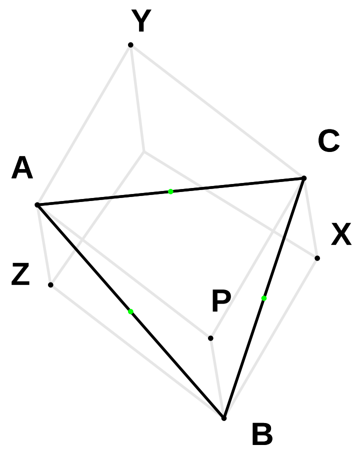
The parallelpipe in gray with labels.



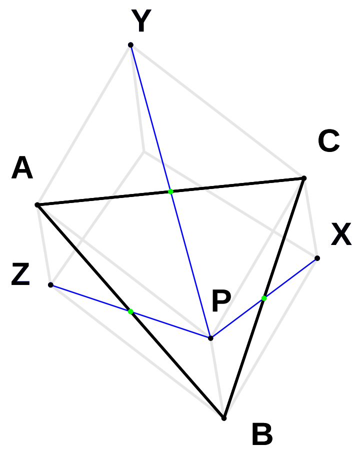
The parallelpipe in gray with labels along with the original triangle ABC.



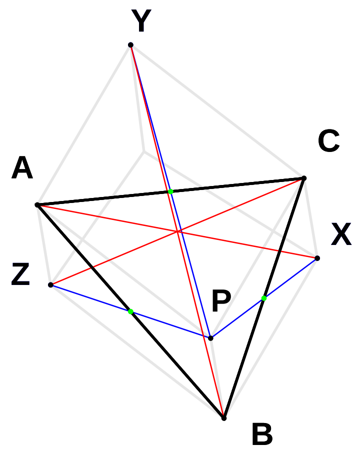
The parallelpipe in gray with labels along with the original triangle ABC and midpoints.



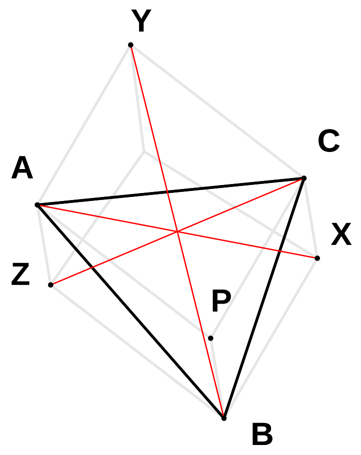
The parallelpipe in gray with labels along with the original triangle ABC, midpoints, and lines PX,PY,PZ.



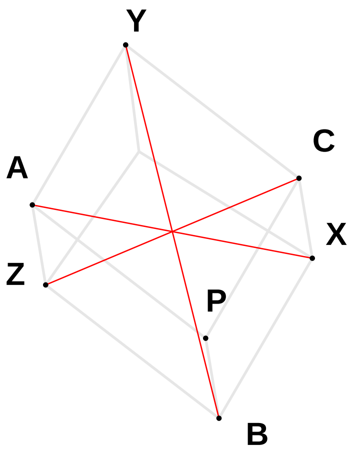
The parallelpipe in gray with labels along with the original triangle ABC, midpoints, and lines PX,PY,PZ,AX,BY,CZ.



The parallelpipe in gray with labels along with the original triangle ABC, and lines AX,BY,CZ.

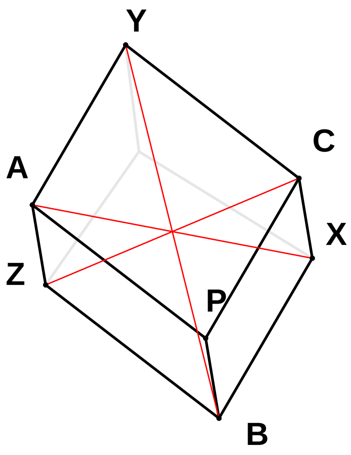


The parallelpipe in gray with labels along with the original triangle ABC, and lines AX,BY,CZ.



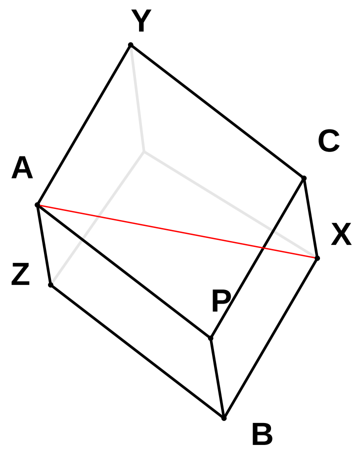
Note: Lines AX,BY,CZ are the diagonals of the parallelepipe figure.

The parallelpipe with labels along with the original triangle ABC, and lines AX,BY,CZ.

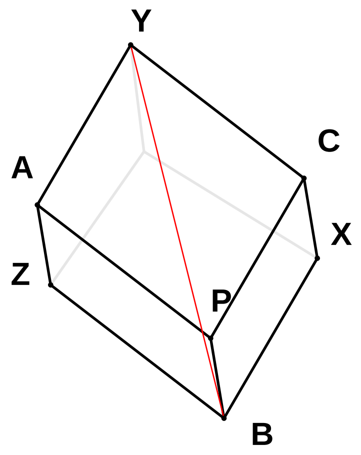


Note: Lines AX,BY,CZ are the diagonals of the parallelepipe figure.

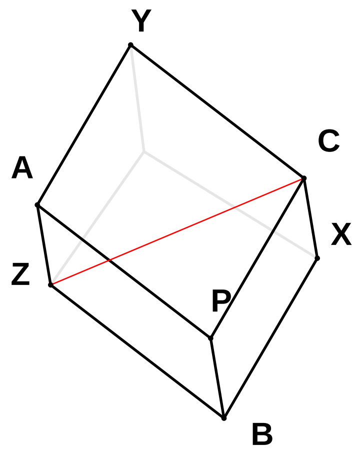
The parallelpipe with labels along with the diagonal AX.



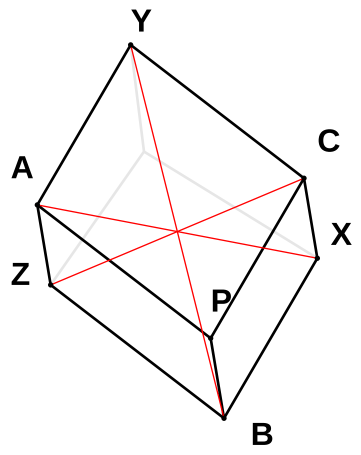
The parallelpipe with labels along with the diagonal BY.



The parallelpipe with labels along with the diagonal CZ.



The parallelpipe with labels along with the lines AX,BY,CZ.



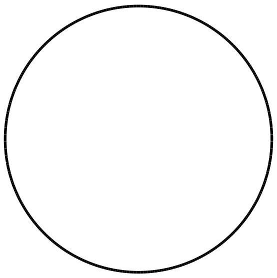
Note: Lines AX,BY,CZ are the diagonals of the parallelepipe figure and thus, AX,BY,CZ concur.

Proof: #3 - Yet another Three Circles

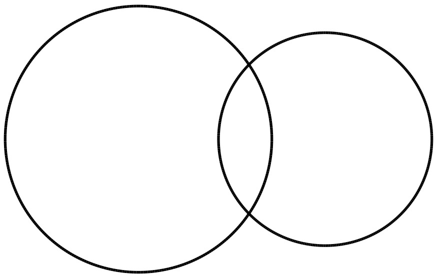
Three circles intersect pariwise but let's assume there is no point shared by all three of them. There are three pairs of circles. Two circles in any pair share a chord. The problem is to prove that the three chords meet at a point.

Note: Figures are best viewed when magnified 5x.

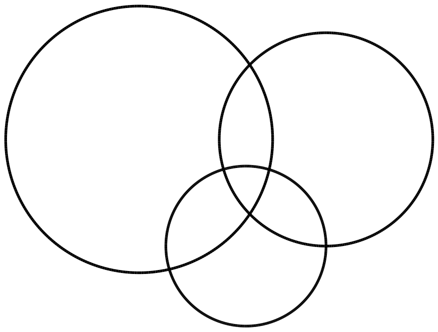
The 1st circle.



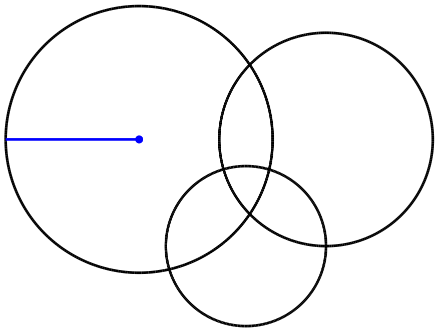
The 1st circle and the 2nd circle.



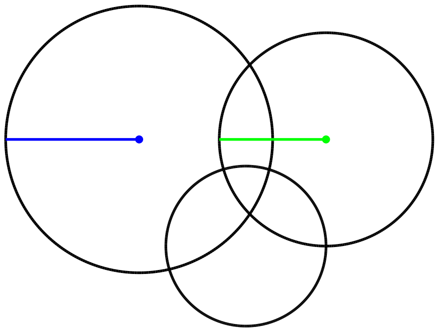
The 1st circle,2nd circle,and 3rd circle.



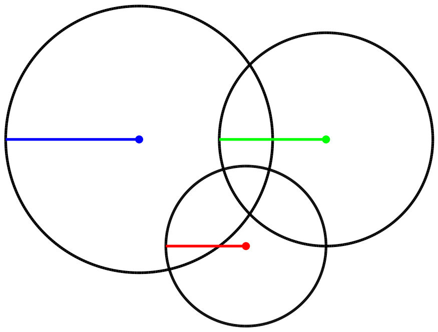
The 1st circle of radius r (blue).



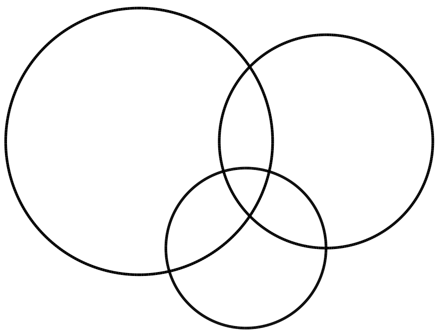
The 1st circle of radius r(blue),
the 2nd circle of radius s(green).



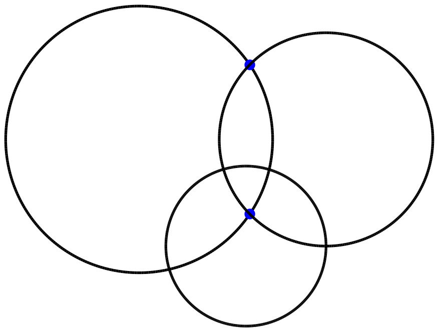
The 1st circle of radius r(blue),
the 2nd circle of radius s(green),
the 3rd circle of radius t(red),



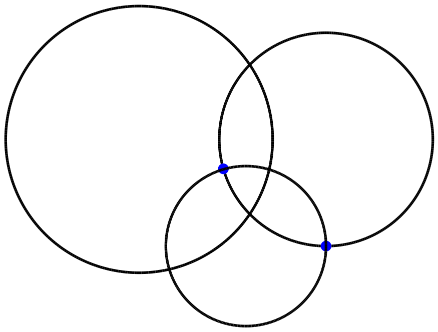
The 3 different circles.



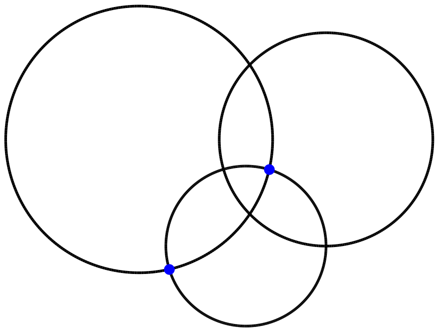
The points of intersection between 2 circles.



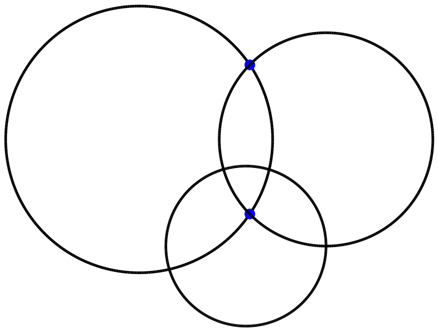
The points of intersection between 2 circles.



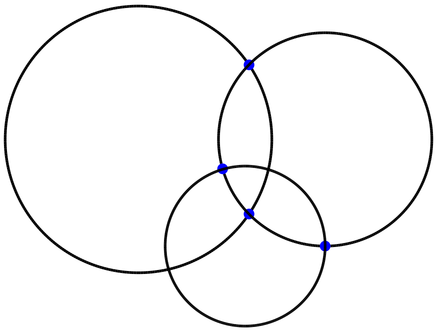
The points of intersection between 2 circles.



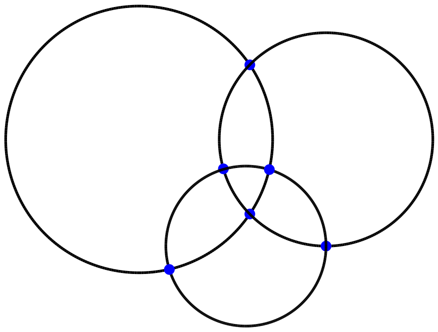
The points of intersection between 2 circles.



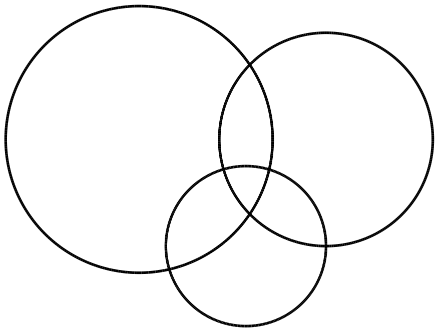
The points of intersection between 2 circles.



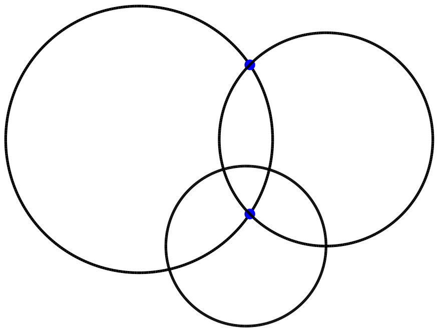
The points of intersection between 2 circles.



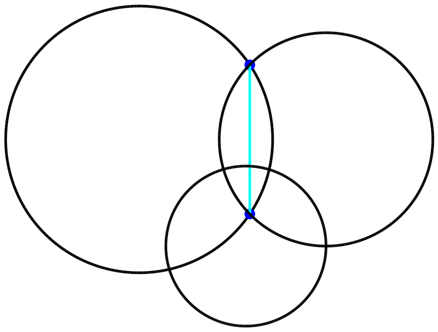
The 3 different circles.



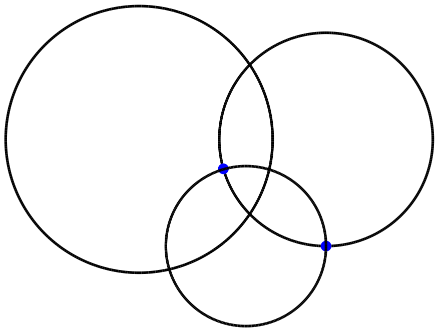
A line forms when we join the intersecting points.



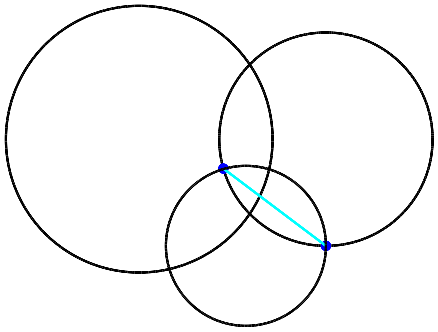
A line forms when we join the intersecting points.



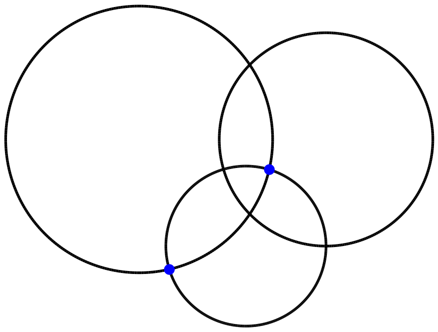
A line forms when we join the intersecting points.



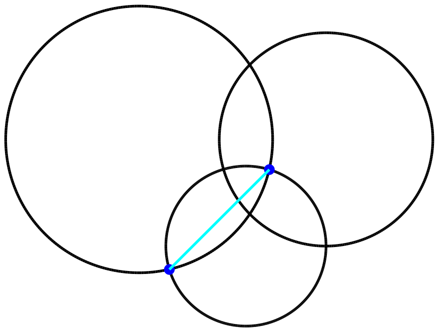
A line forms when we join the intersecting points.



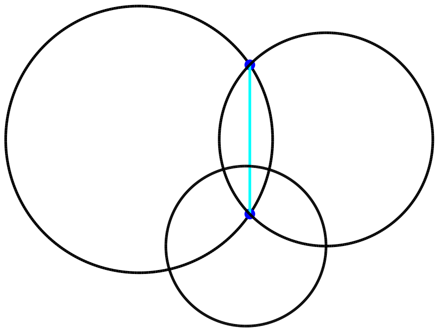
A line forms when we join the intersecting points.



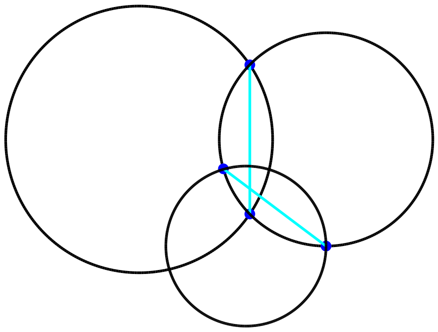
A line forms when we join the intersecting points.



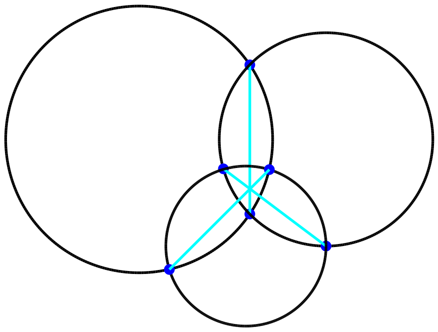
The 3 lines joining points of intersection between 2 circles.



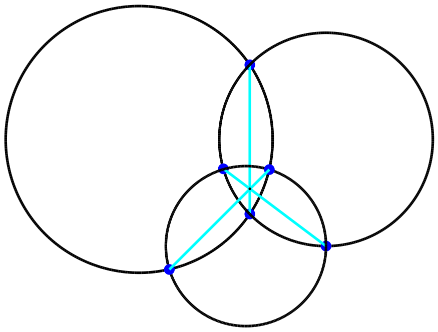
The 3 lines joining points of intersection between 2 circles.



The 3 lines joining points of intersection between 2 circles.

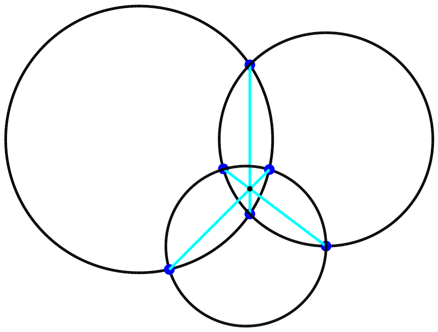


The 3 lines intersect at the black dot.



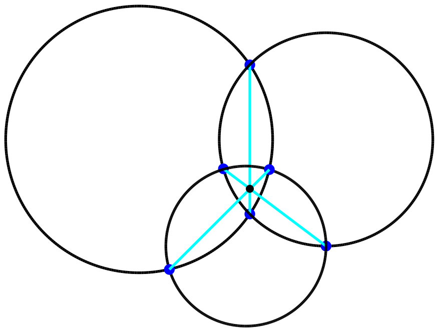
Note: Since the three lines intersect at the black dot, that means the 3 chords meet at a point.

The 3 lines intersect at the black dot.



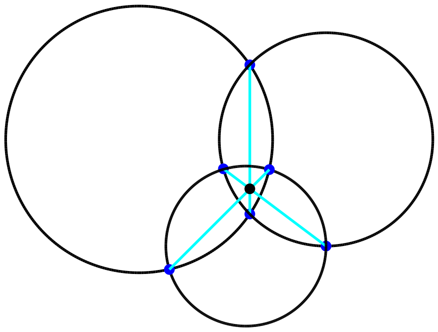
Note: Since the three lines intersect at the black dot, that means the 3 chords meet at a point.

The 3 lines intersect at the black dot.



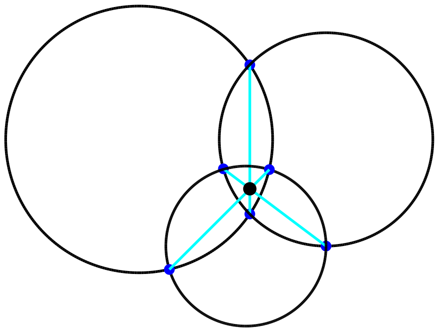
Note: Since the three lines intersect at the black dot, that means the 3 chords meet at a point.

The 3 lines intersect at the black dot.



Note: Since the three lines intersect at the black dot, that means the 3 chords meet at a point.

The 3 lines intersect at the black dot.



Note: Since the three lines intersect at the black dot, that means the 3 chords meet at a point.

The End.