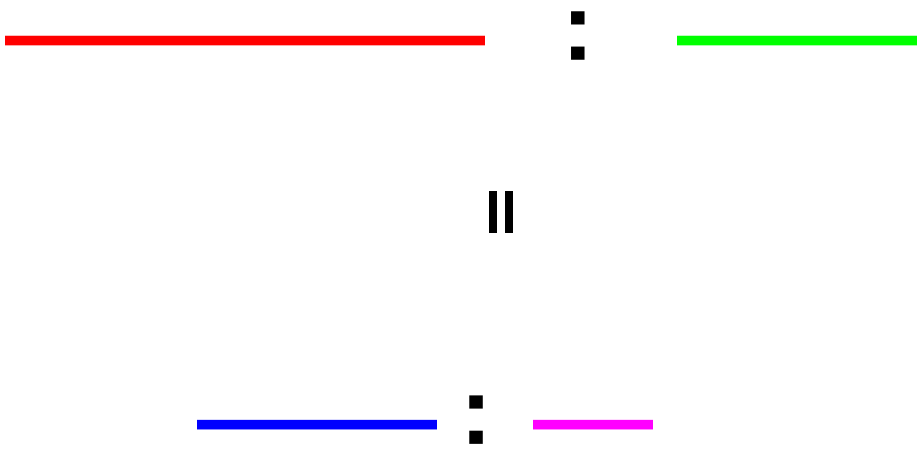
Book VI

Proposition 16

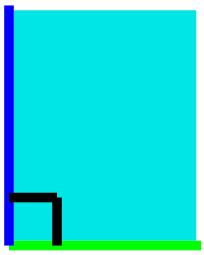
If 4 straight lines be proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means; and, if the rectangle contained by the extremes be equal to the rectangle contained by the means, the 4 straight lines will be proportional.

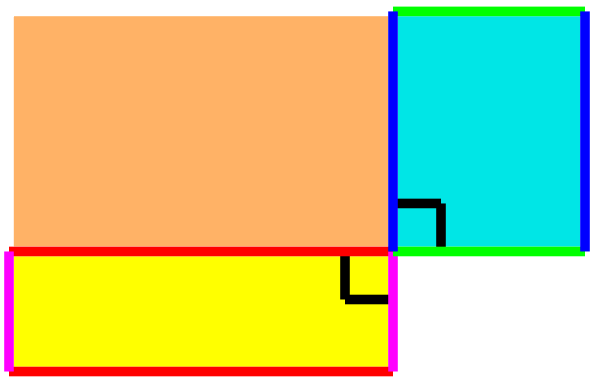
Part I

Let

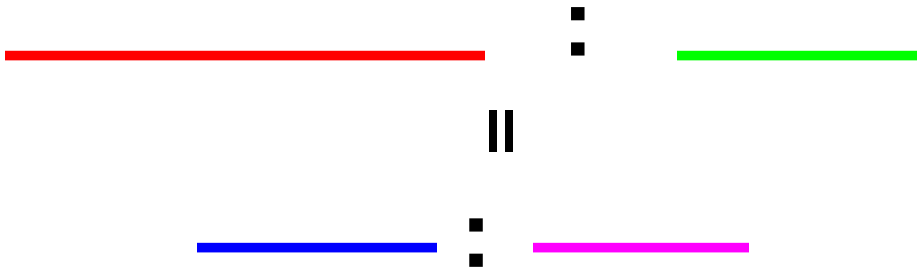


Then, we have

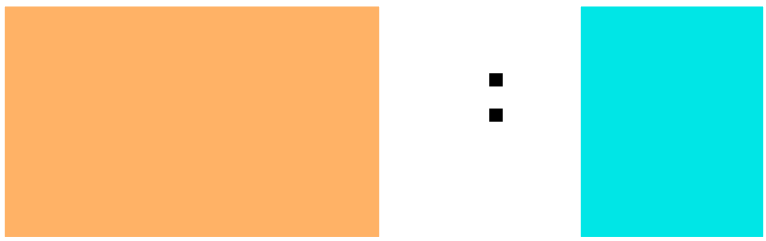




Since



we have



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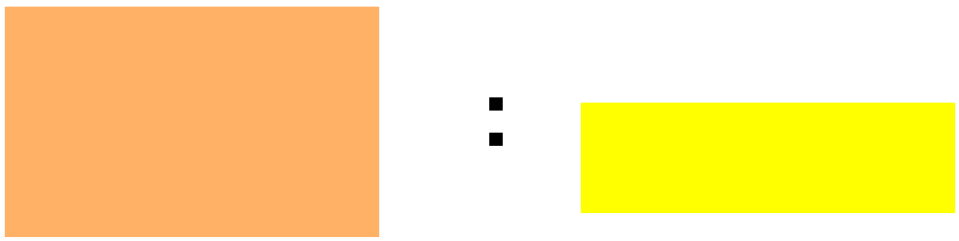
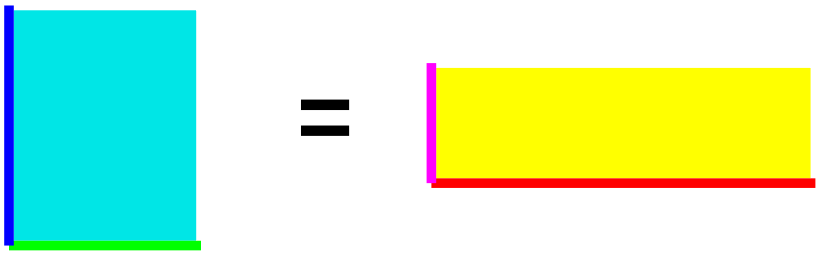
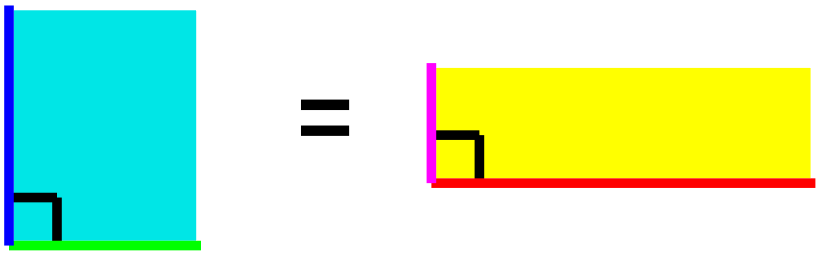


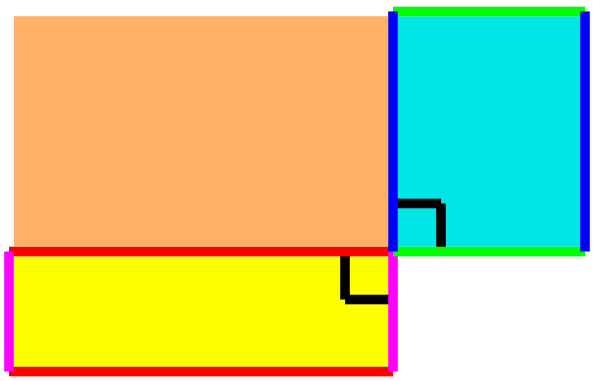
[same base]

Hence



Part II





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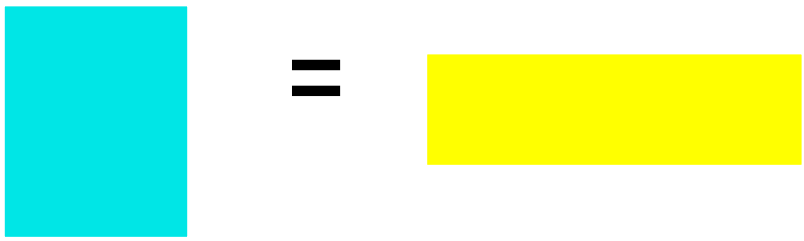
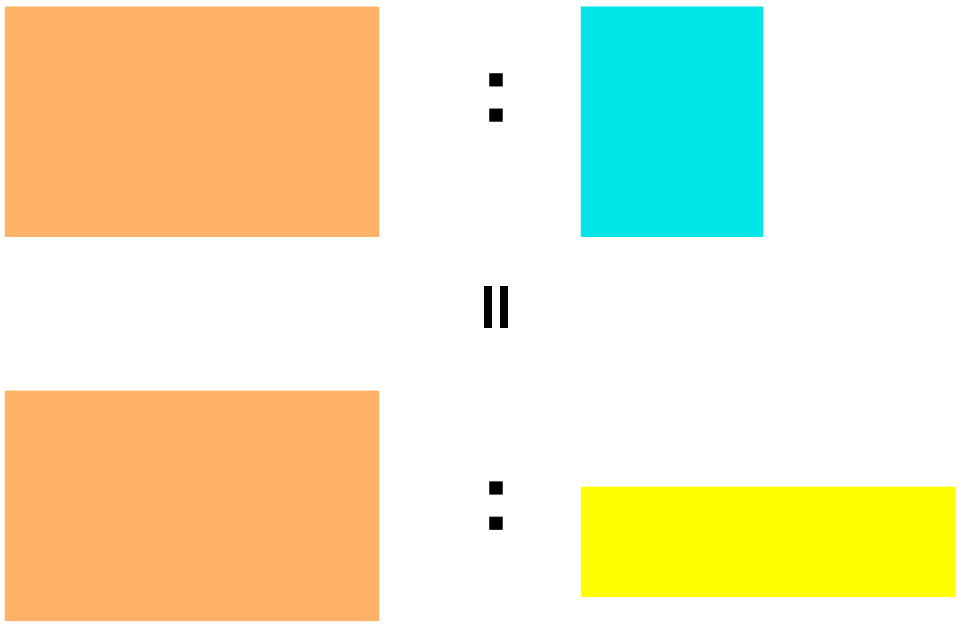
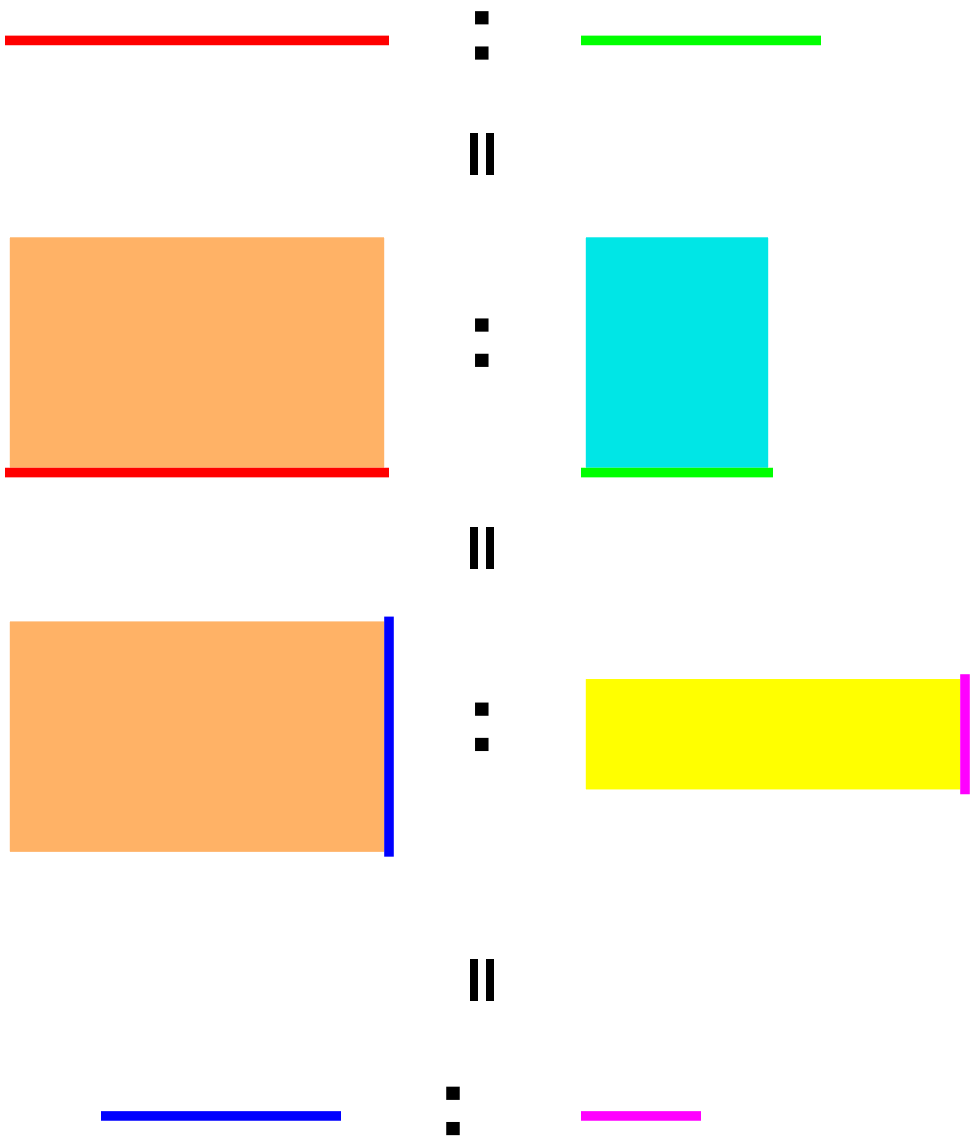


we have



Then



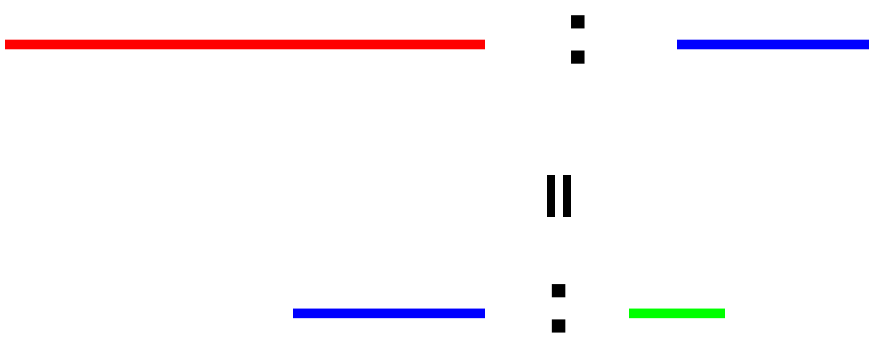
Therefore

Book VI

Proposition 17

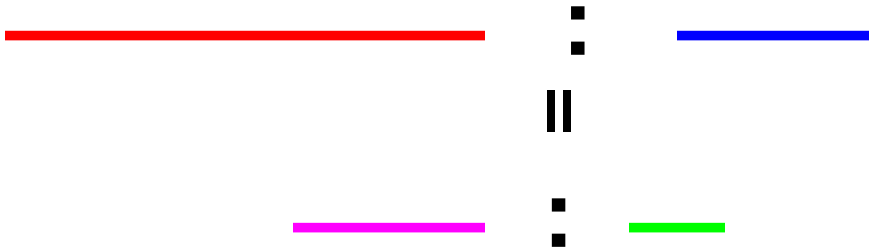
If 3 straight lines be proportional, the rectangle contained by the extremes is equal to the square on the means; and, if the rectangle contained by the extremes be equal to the square on the means, the 3 straight lines will be proportional.

Part I



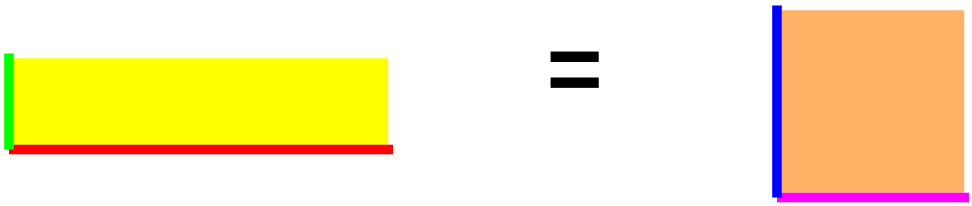
Let

We have



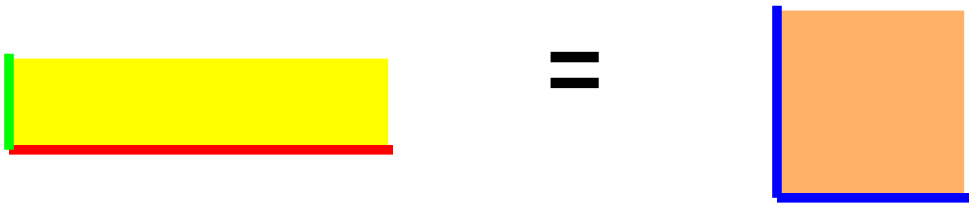
By VI.16, if 4 straight lines be proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means.

Therefore

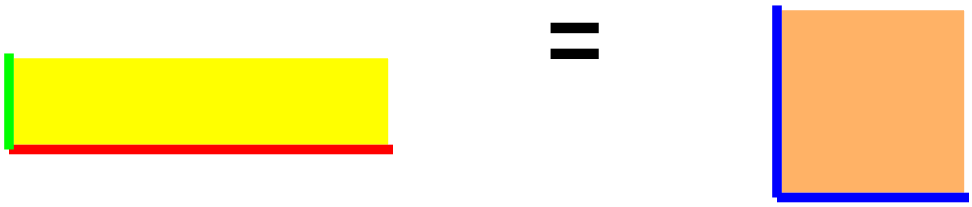


Since

we have

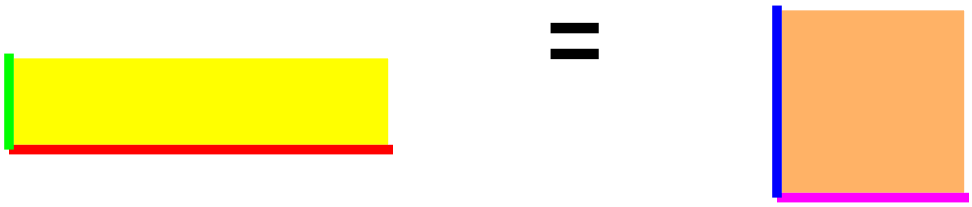


Part II



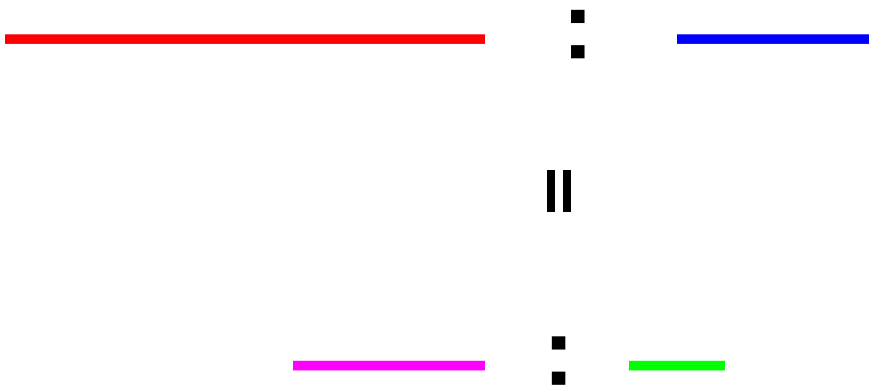
Let

We have



By VI.16, if the rectangle contained by the extremes be equal to that contained by the means, the 4 straight lines are proportional.

Therefore



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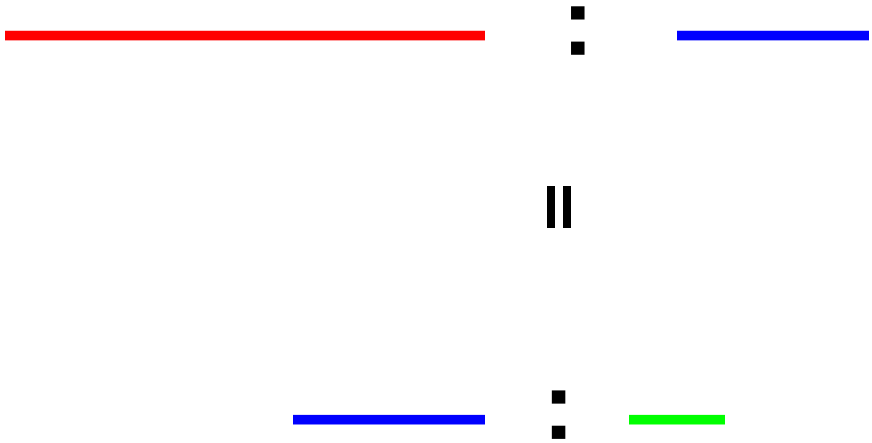
We have