Mathematics 308 — Fall 2003

Sixth homework — due Monday, November 24

1. Place the eye at $(0, 0, 5)$. Use @ps3d.inc@ to draw several sequences of 10 pictures showing a regular tetrahedron of radius 1, centre at $(0, 0, -1)$, pole in direction $[0, 1, 0]$, spinning around the axis $[1, 0, 1]$. Do (a) Wire frame; (b) solid; (c) shaded and green. (Do this in stages. First figure out the vertices of the tetrahedron if its centre were at $0, 0, 0$). Then draw it still. Then make up a list of faces of the tetrahedron, along with tangent half-plane. Then to shade, loop through faces.)

2. Same place for the eye. Draw the right hand figure from Euclid XII.5 for the pyramid with corners at $(0, 0, 0)$, $(0, 0, -1)$, $(1, 0, -1)$, $(0, -1, -1)$, and with the slices solid. Show it rotated into many positions, so the slices have to be drawn in different orders.

3. Place the eye at $(0, 0, 5)$. Start with the cube of side 1 centred at the origin. Translate its centre to $(0, -1, -1)$, and then rotate it around the axis through its centre and parallel to the z-axis by $45°$. Plot and draw accurately by hand what you see if it is drawn in perspective.

4. Assume the eye at $(0, 0, a)$. It turns out that all the lines in space with a given direction, say (X, Y, Z), intersect at one point when drawn in perspective. What is that point? (Hint: describe, and in particular find parametrizations of, all these lines.)

5. Rotation around the axis $[2, 1, 1]$ and through an angle of $45°$ takes $[1, 0, 0]$ to what?

6. So far I have only discussed rotations around the origin, or around axes through the origin. But in 2D, for example, we can also speak of rotatons around other points. Rotation through θ around $(1, 1)$ will be an affine transformation. What is its formula? (I.e. what are the six numbers describing it?) Write a PostScript procedure `affine-rotate` with two arguments $P = (x, y)$ and θ, and rotates the coordinate system through θ around P. Demonstrate it.

7. Write a PostScript procedure `reflect` with a single argument $f = [A, B, C]$ that has the effect of reflecting the coordinate system in the line $Ax + +By + C$.

8. Place a light at the point $(0, 5, -1)$, and a point object at $(1, 2, -4)$. The light casts a shadow of the point on the plane $y = -2$. Where, exactly? Where, if the point object is at (x, y, z)?