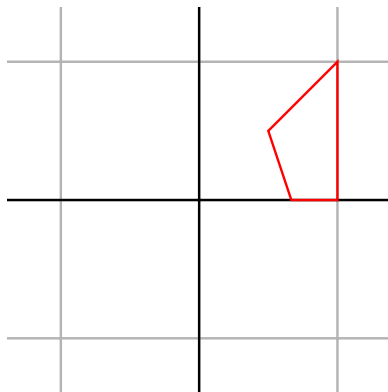


**Mathematics 308 — midterm examination solutions**

1. Let

$$M = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} .$$

If  $M$  is considered as the matrix of a projective transformation, what quadrilateral does it take the unit square with corners at  $(0, 0)$  and  $(1, 1)$  to? Draw the figure below ( $1 \times 1$  grid), and indicate exact coordinates.

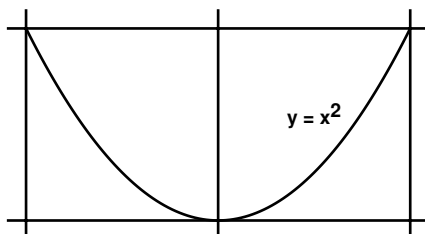


2. A certain affine function  $Ax + By + C$  is equal to  $-4$  at  $(0, 0)$  and  $7$  at  $(1, 2)$ . At what point on the line segment between these two points is it equal to  $0$ ?

$4/11$  of the way from  $(0, 0)$  to  $(1, 2)$ , which is

$$(7/11) \cdot (0, 0) + (4/11) \cdot (1, 2) = (4/11, 8/11) .$$

3. Write a complete PostScript program that produces the following picture:



Points will be given for the simplicity and readability of your code. Put in only what is necessary.

```
%!PS-Adobe-2.0
%%BoundingBox: 0 0 150 84
```

```
75 5 translate
```

```
72 dup scale
1 72 div setlinewidth
```

```
/x -1 def
/N 100 def
/dx 2 N div def
```

```
gsave
newpath
-1.1 0 moveto
```

```

1.1 0 lineto
-1.1 1 moveto
1.1 1 lineto
0 -.1 moveto
0 1.1 lineto
-1 -.1 moveto
-1 1.1 lineto
1 -.1 moveto
1 1.1 lineto
stroke
grestore

```

```

newpath
x dup dup mul moveto
N {
/x x dx add def
x dup dup mul lineto
} repeat
stroke

```

```

/Helvetica-Bold findfont
0.10 scalefont
setfont

```

```

0.36 0.5 moveto
(y = x) show
0 0.04 rmoveto
(2) show

```

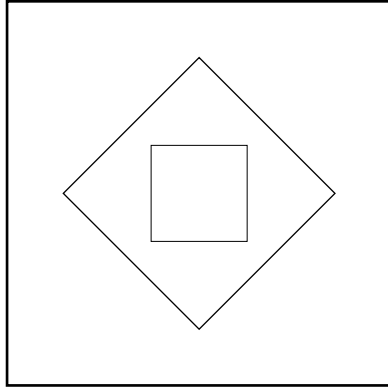
4. What does the following fragment of PostScript code draw? Assume units in inches, origin centred. Show it below. Include calculations so I can see what you've done.

```

3 {
  newpath
  -1 -1 moveto
  2 0 rlineto
  0 2 rlineto
  -1 1 lineto
  closepath
  stroke
  45 rotate
  0.5 dup scale
} repeat

```

Indicate features of the drawing carefully.



5. *On the next few pages, draw a sequence of diagrams, with as little text as possible, explaining how to construct a regular pentagon inside a given circle with ruler and compass. Explain as you do this why the construction works.*

The easiest construction is the one attributed to H. M. Taylor at the end of the selection I handed out.