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ARCHIMEDES STOMACHION

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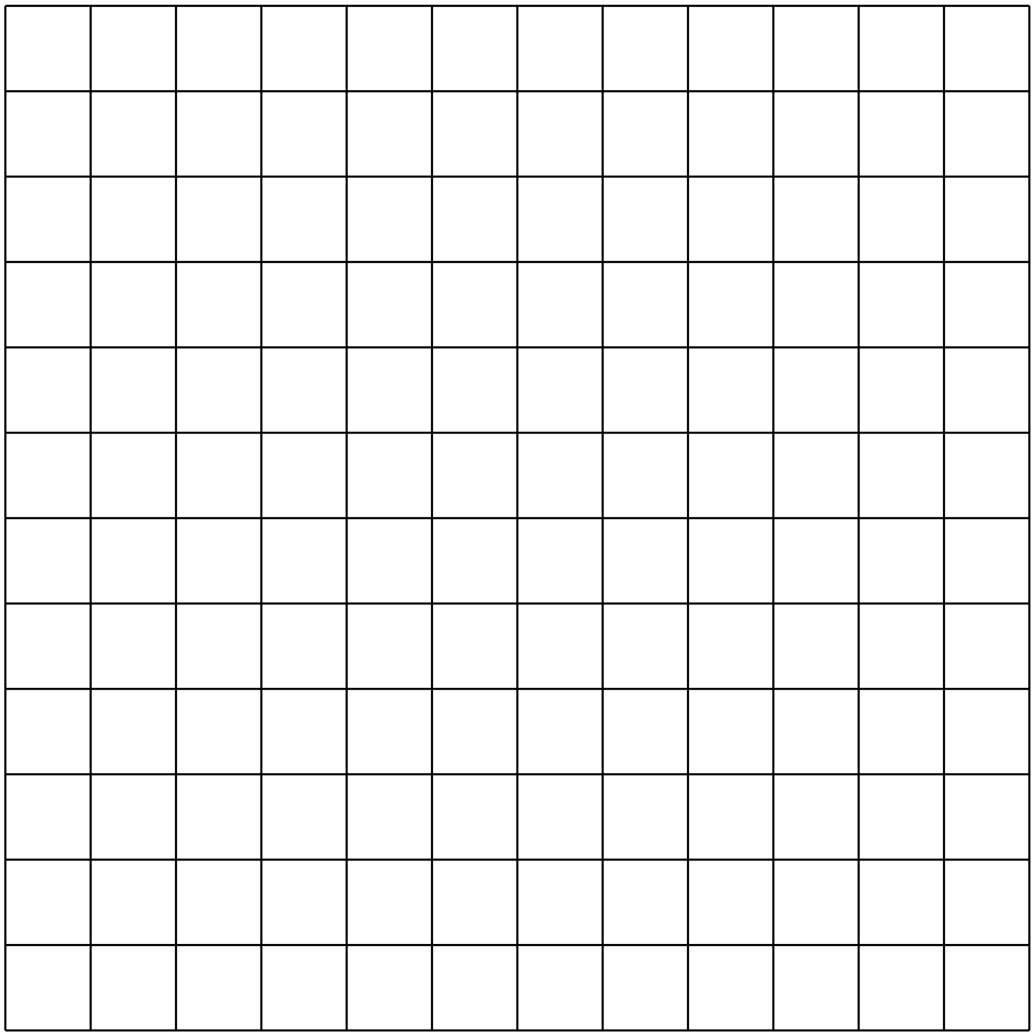
Stomachion Introduction

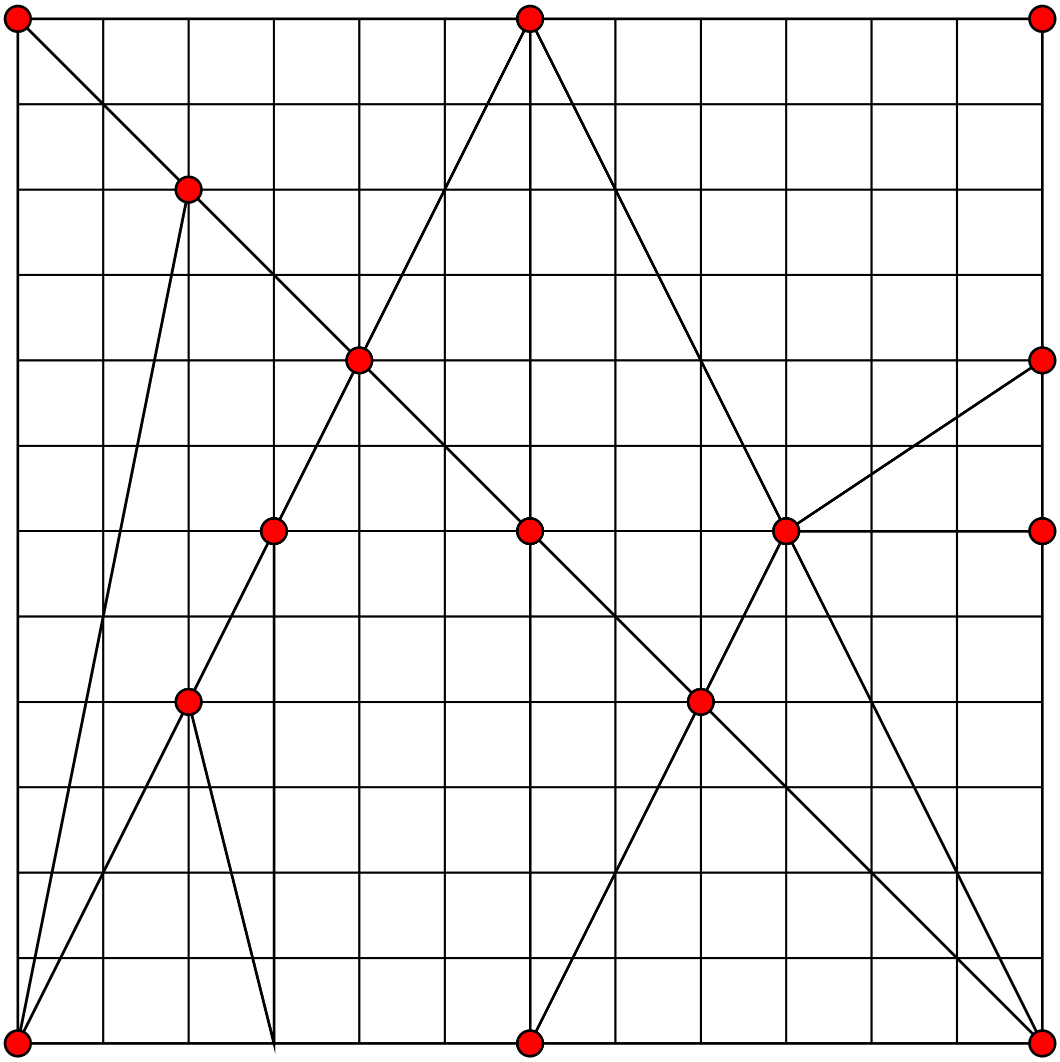
Two manuscripts in fragmentary form attributed to Archimedes survive on an ancient game that the Greeks called the Stomachion. One is an Arabic translation and the other is a Greek manuscript dating from the tenth century discovered in Constantinople in 1899. It is not known whether Archimedes invented the game {probably not} or simply explored the geometrical aspects of the game {more probable}. There are many other references to the game in the ancient literature, of which two refer to it in Latin as loculus Archimedius {Archimedes' box}. The word Stomachion has as its root the Greek word for stomach, but otherwise its meaning is unclear. The game consisted of 14 flat ivory pieces of various polygonal shapes originally forming a square. The object of the game was to rearrange the pieces to form interesting things {people, animals, objects, etc.} such as an elephant.

The Greek manuscript attributed to Archimedes is rather incomplete and is concerned with determining the relationships of various angles of the pieces. The Arabic manuscript provides more information, describing a construction of the Stomachion and determining the areas of its pieces.

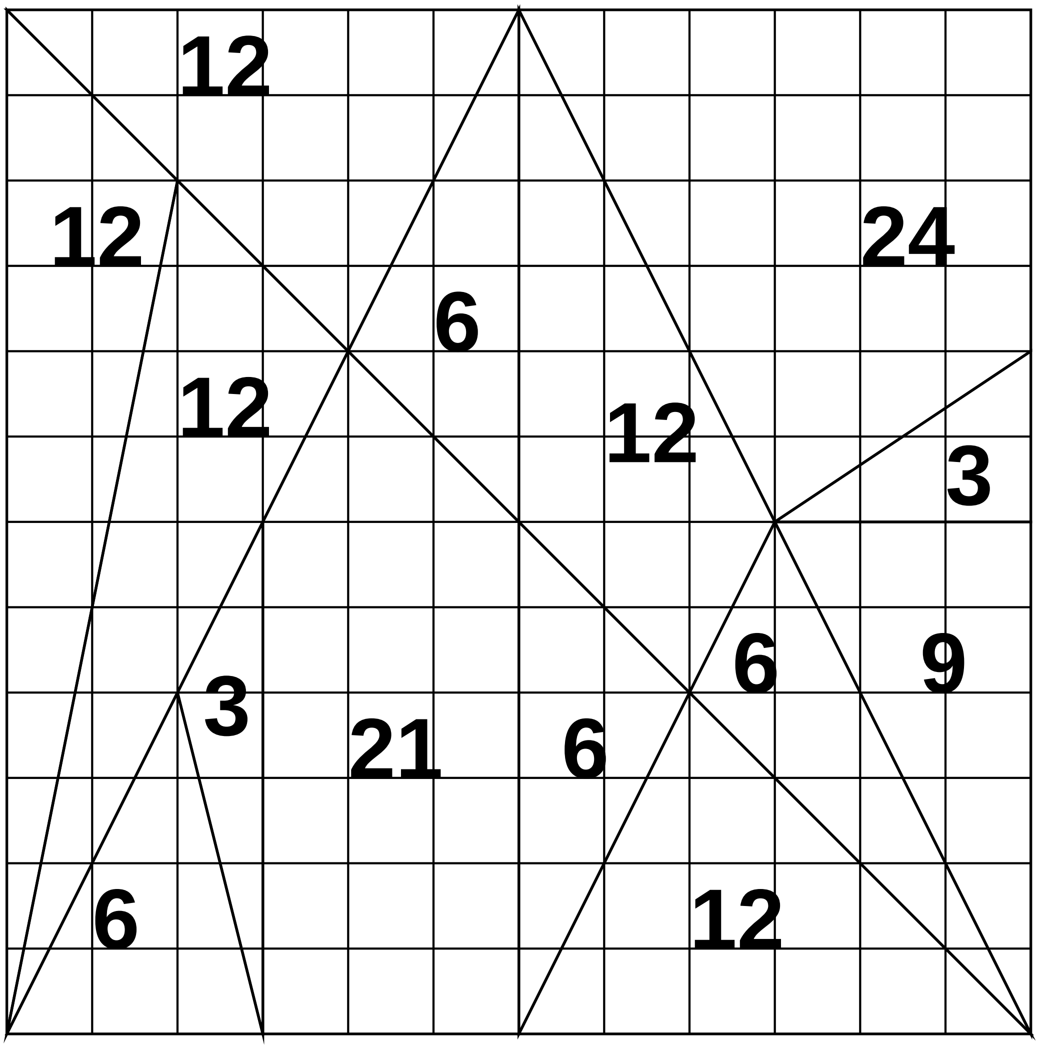


This description of the geometric construction of the Stomachion is somewhat different than that given in the Arabic manuscript, but results in the same figure.

**Start with a square containing a uniform 12 x12 grid. The intersections of the grid lines are called lattice points.
Take each small square to have unit area, so that
the original square has area 144.**

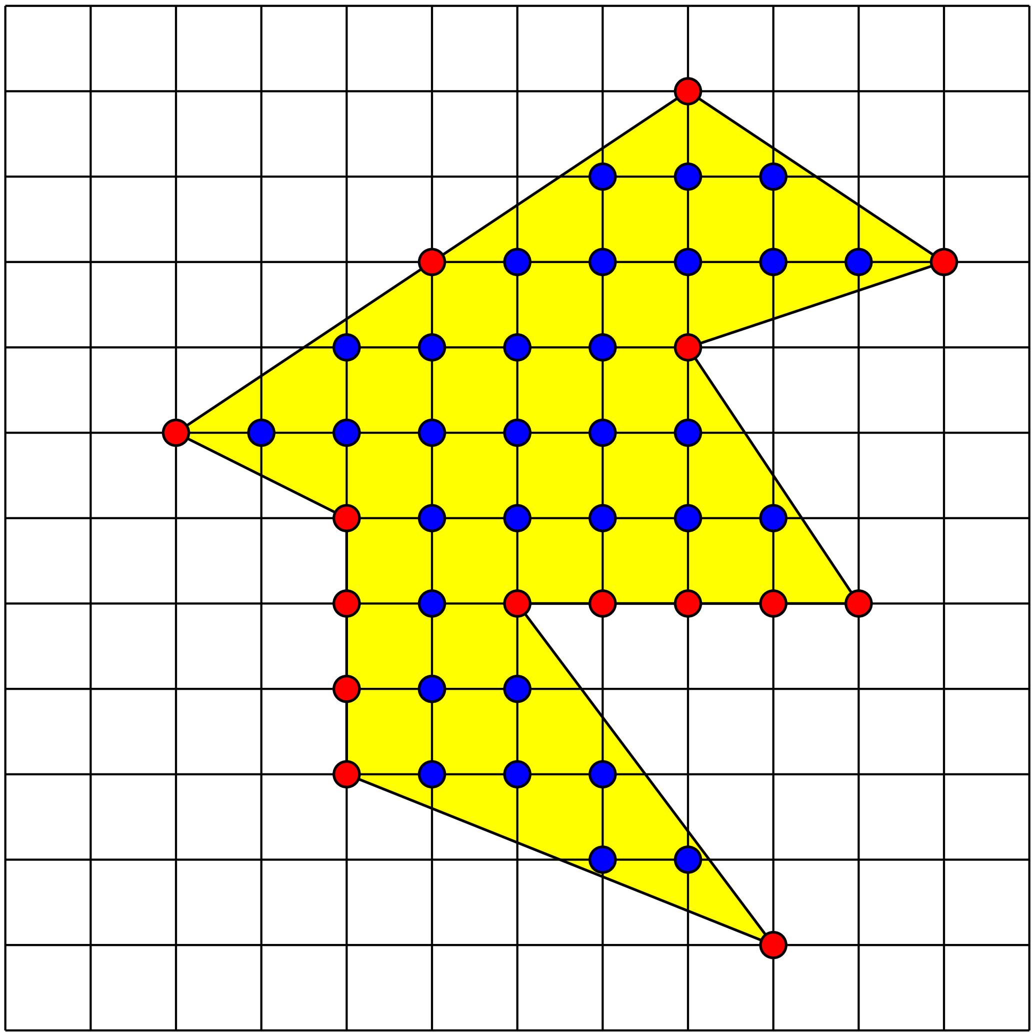


Draw lines through the indicated lattice points. The lines divide the square into 14 three-, four-, and five-sided polygons. These polygons are called lattice polygons because their vertices are at lattice points. The lattice polygons form the 14 pieces of the Stomachion.



The Arabic manuscript also contains computations for the areas of the pieces of the Stomachion. It shows that the areas of the pieces are all commensurate with the area of the square in the ratios 1:48 {2 pieces of area 3}, 1:24 {4 pieces of area 6}, 1:16 {1 piece of area 9}, 1:12 {5 pieces of area 12}, 7:48 {1 piece of area 21}, and 1:6 {1 piece of area 24}.

A contemporary approach to the computations of the areas is by the use of Pick's theorem.



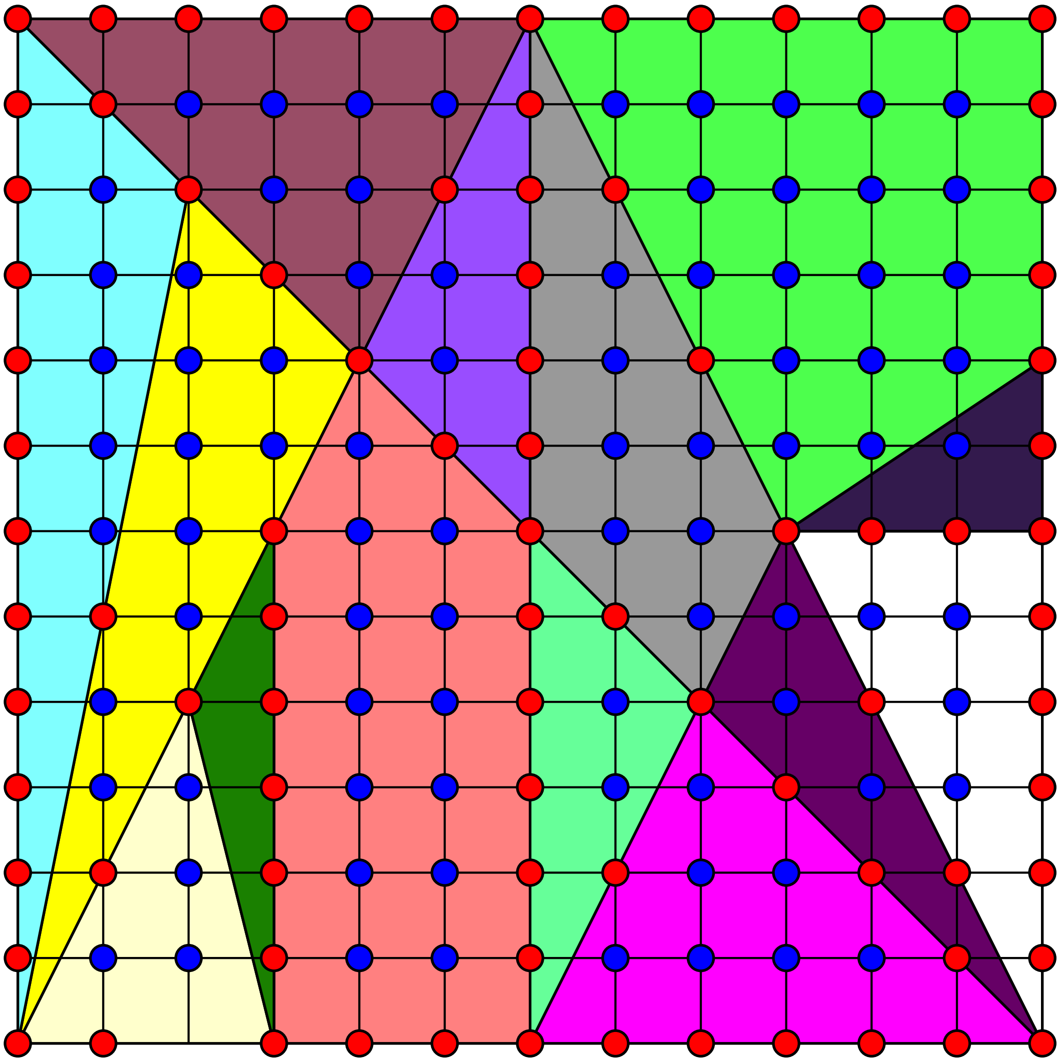
Pick's theorem provides an elegant formula for the area of a simple lattice polygon: a lattice polygon whose boundary consists of a sequence of connected nonintersecting straight-line segments. The formula is: Area = I + B/2 - 1

where I = number of interior lattice points

and B = number of boundary lattice points

For example, the area of the simple lattice polygon in the figure is:

31 + 15 /2   1 = 37.5



The interior and boundary lattice points of the fourteen pieces of the Stomachion are indicated above.

Using Pick’s theorem the areas of the fourteen pieces can be determined as in the previous example; e.g., the green piece in the upper right-hand corner has area

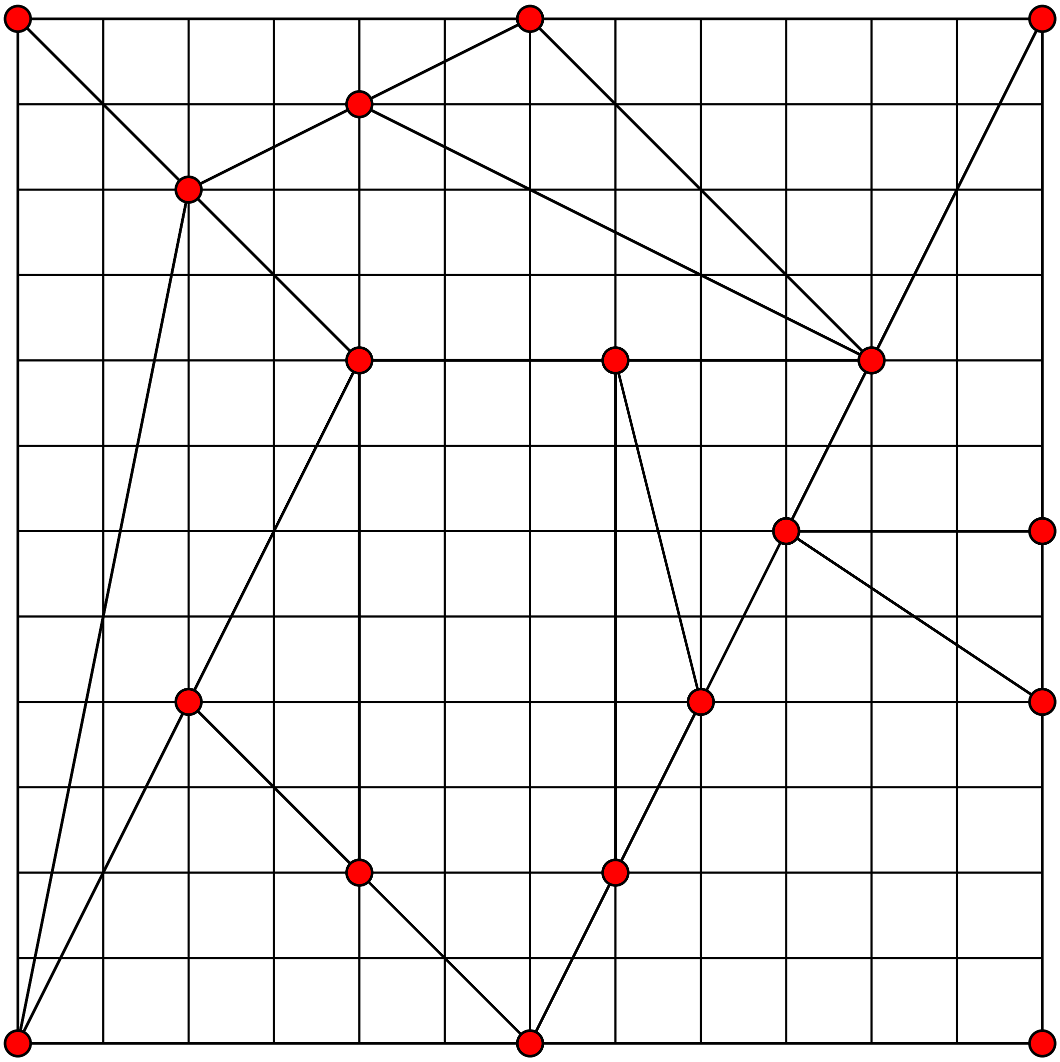
$$18 + 14/2 \quad 1 = 24$$

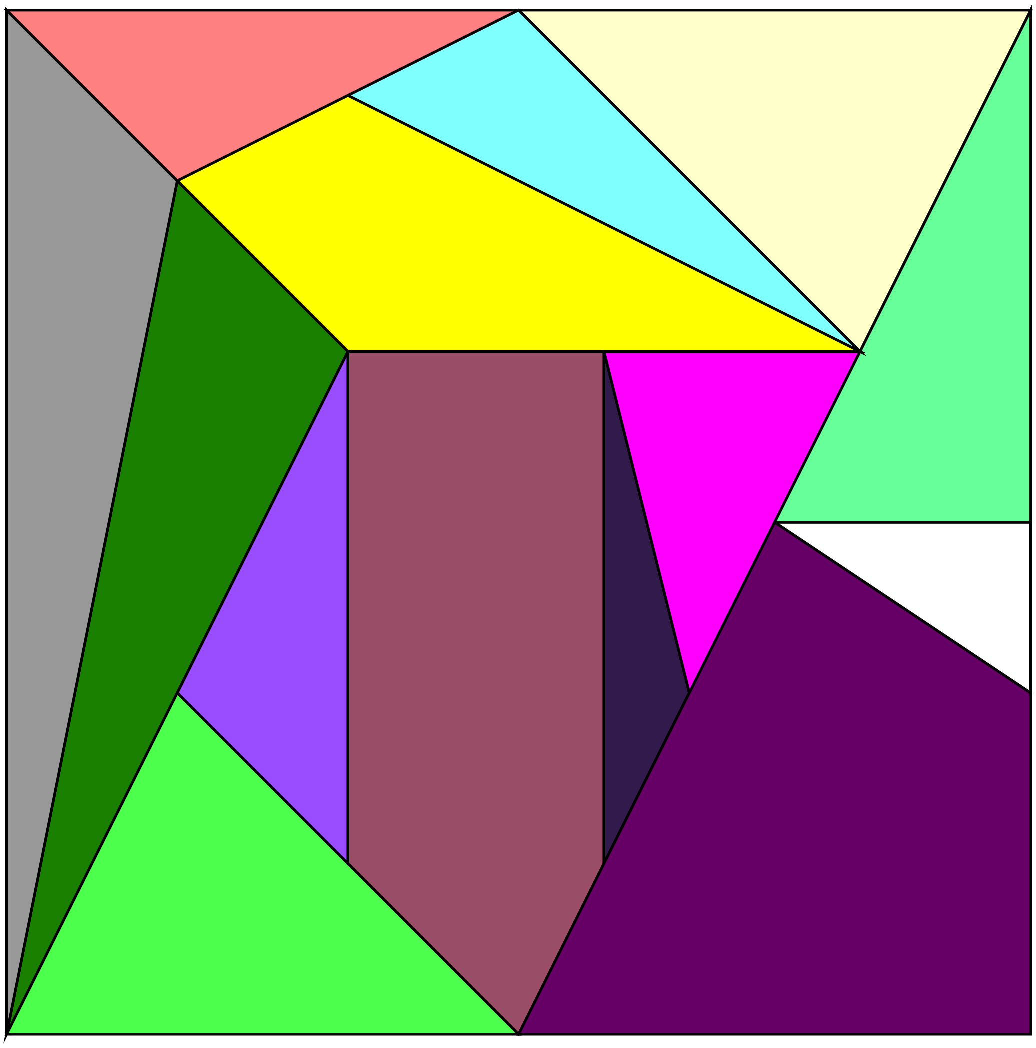
How many ways can this square be solved?

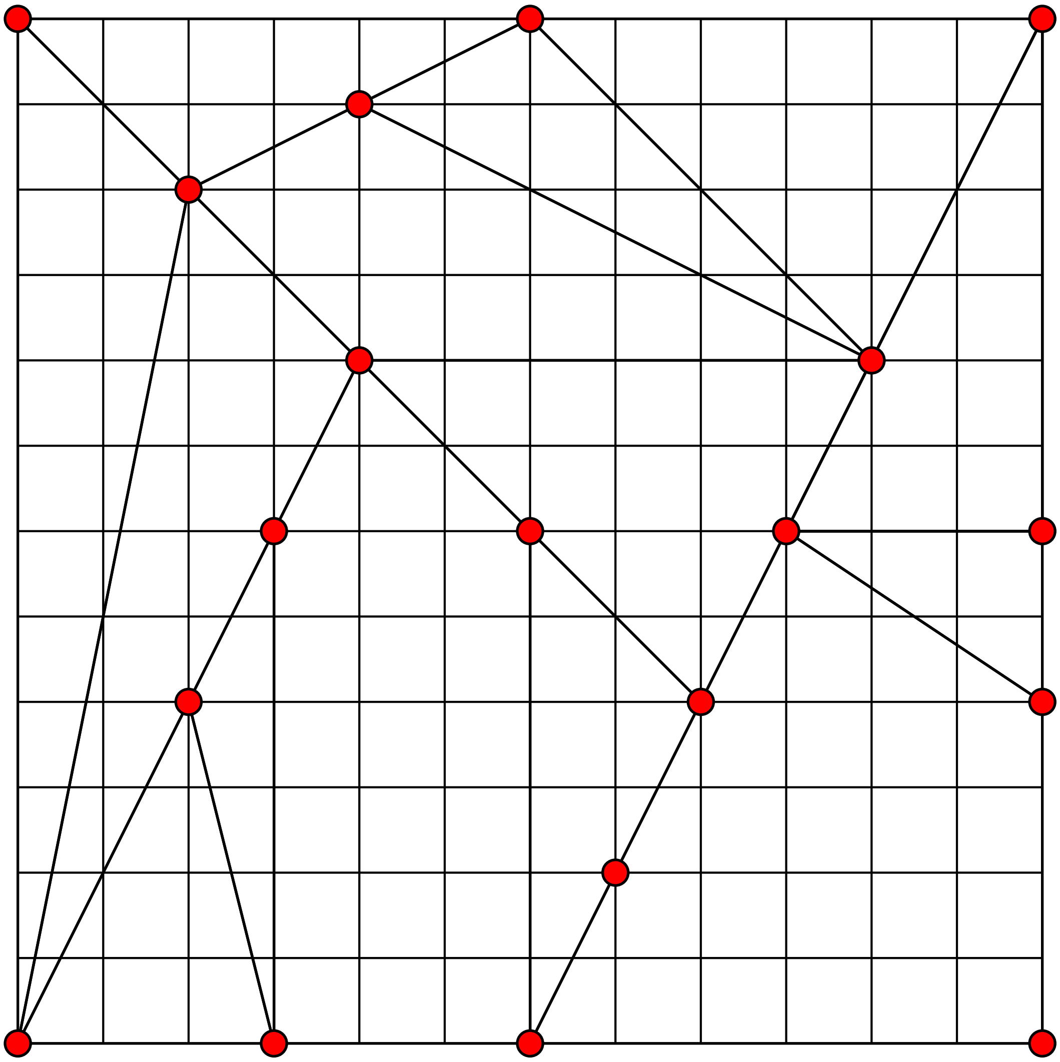
In November, 2003, Bill Cutler discovered 536 solutions.

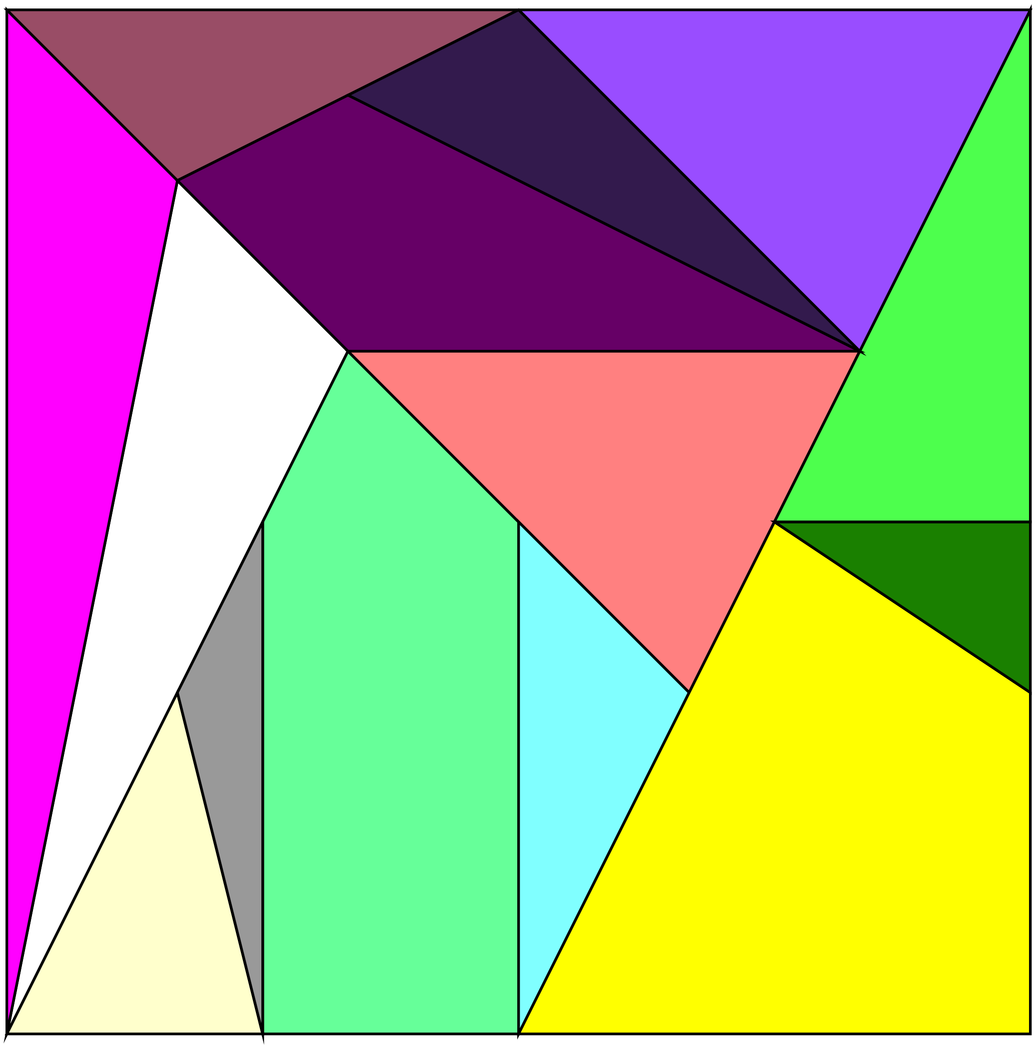
The solutions require no rotations, relfections, or switching of identical pieces.

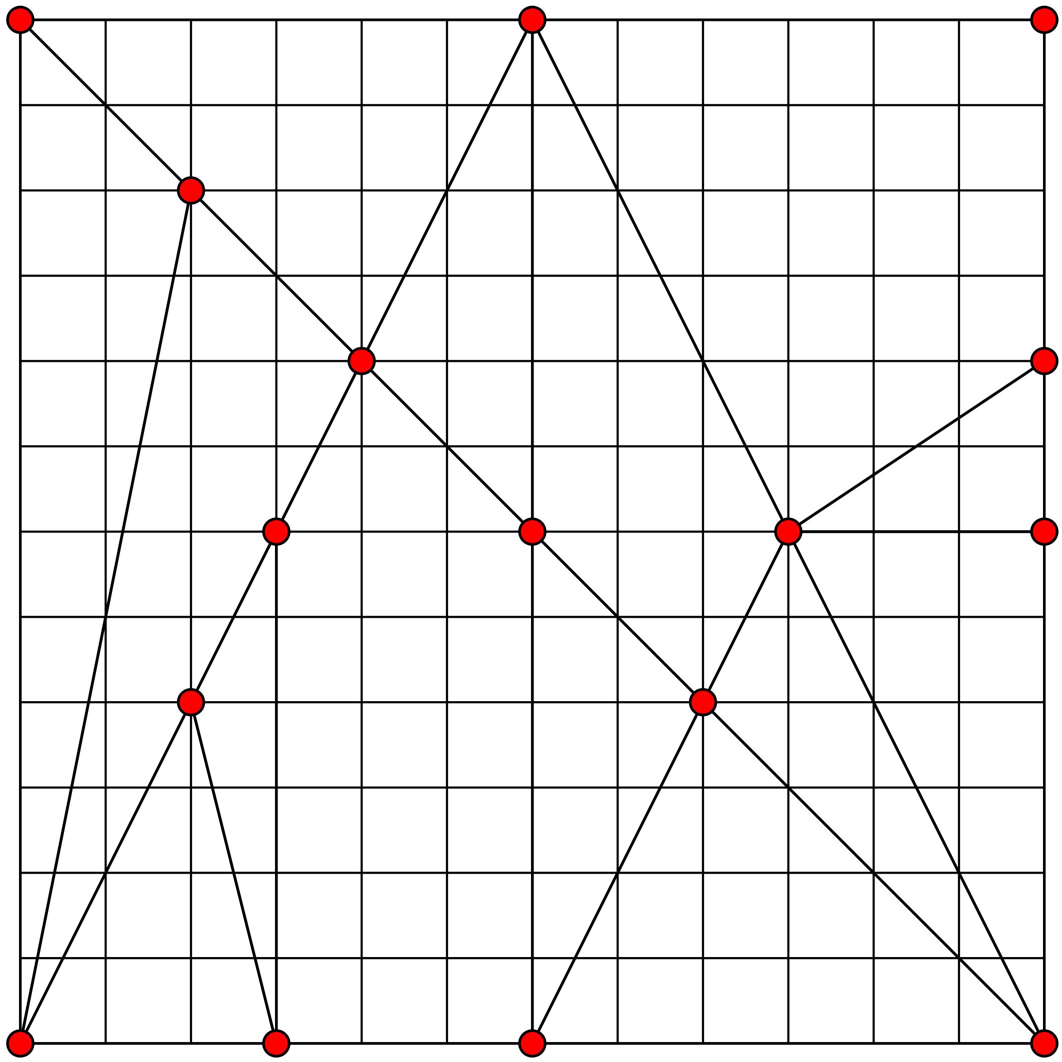
The Following are some of the 536 possible solutions:

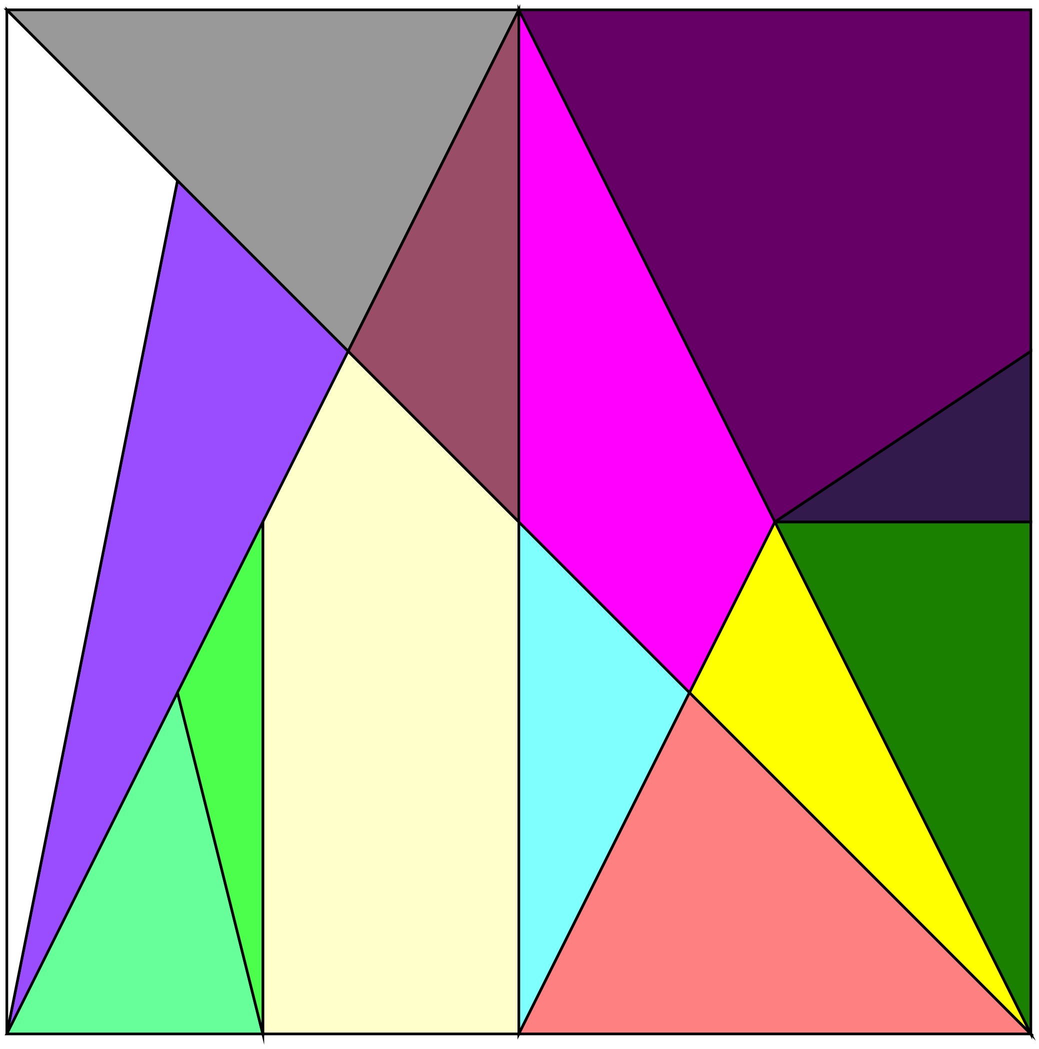


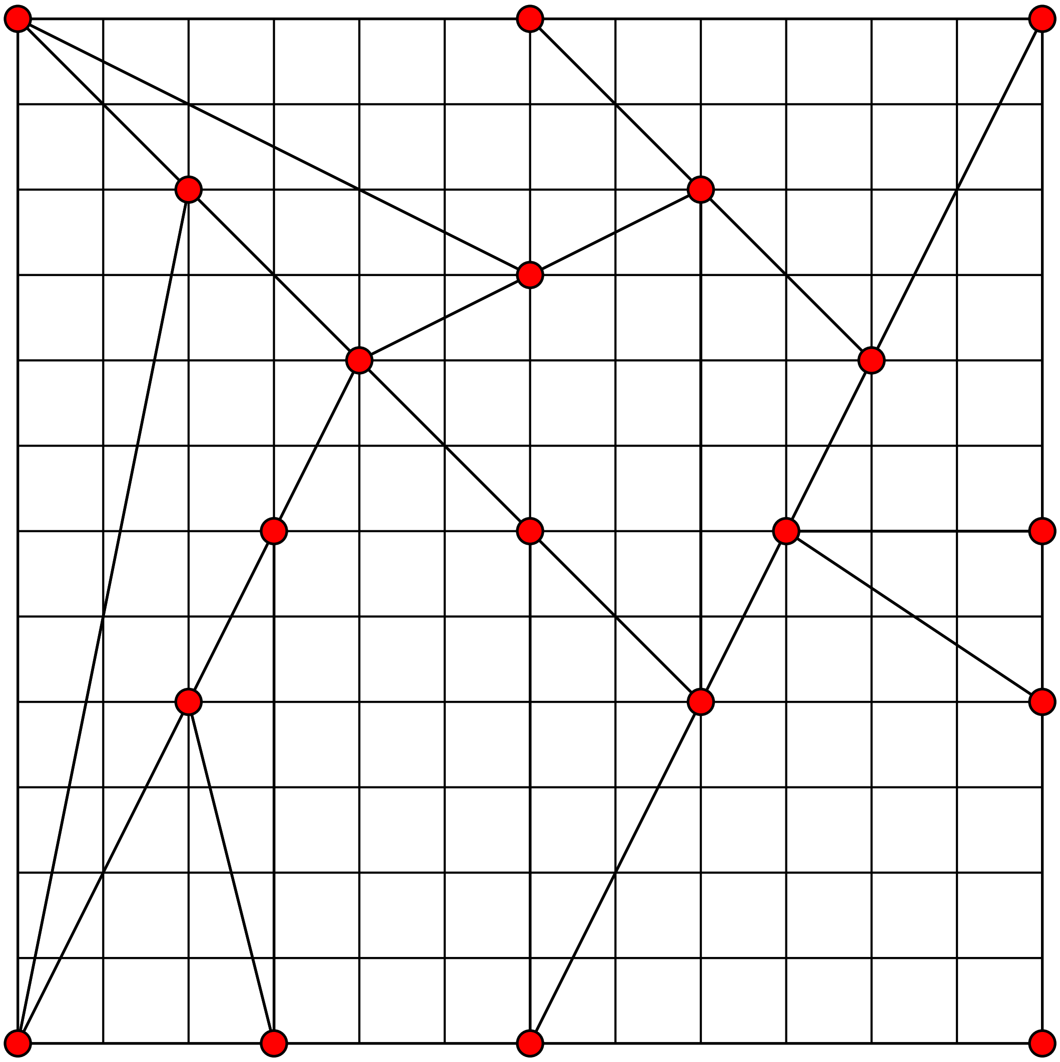


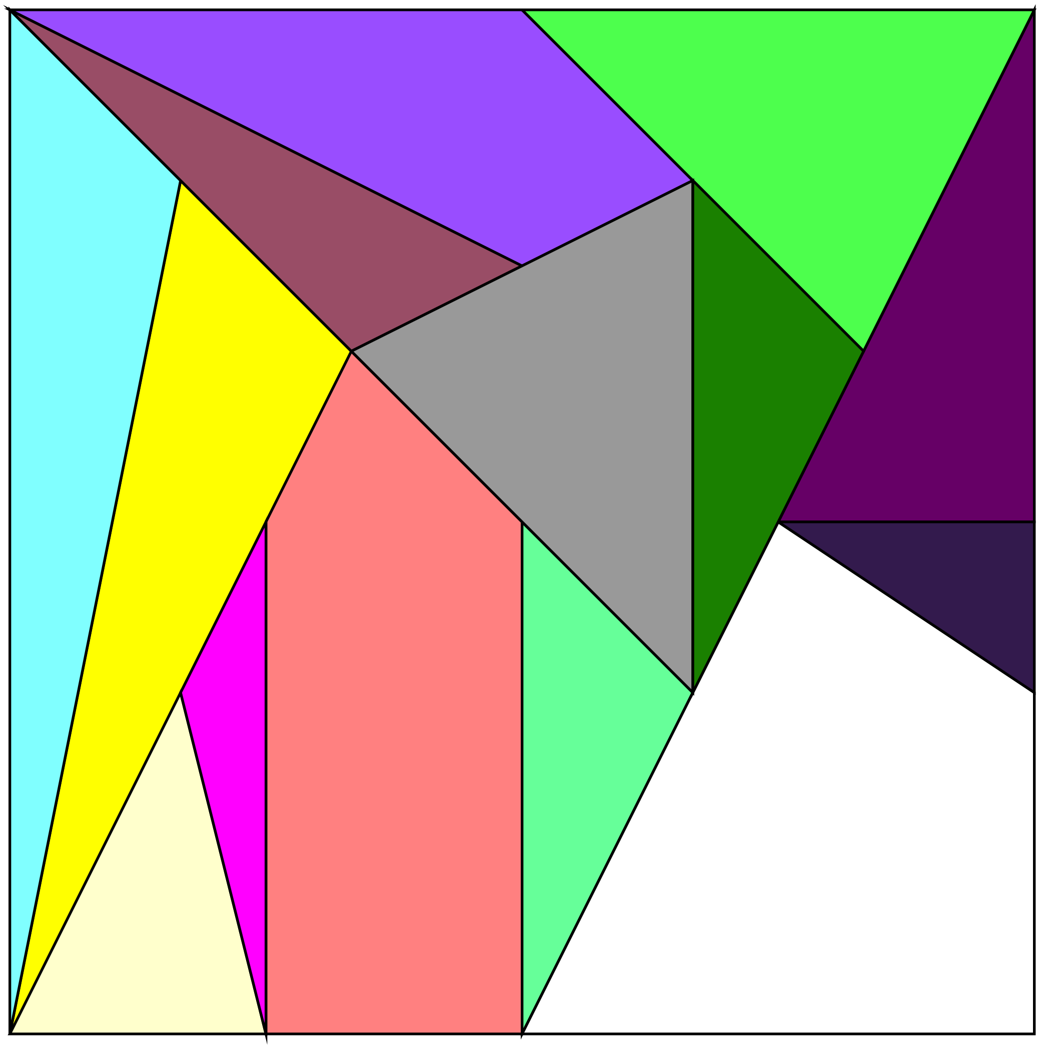


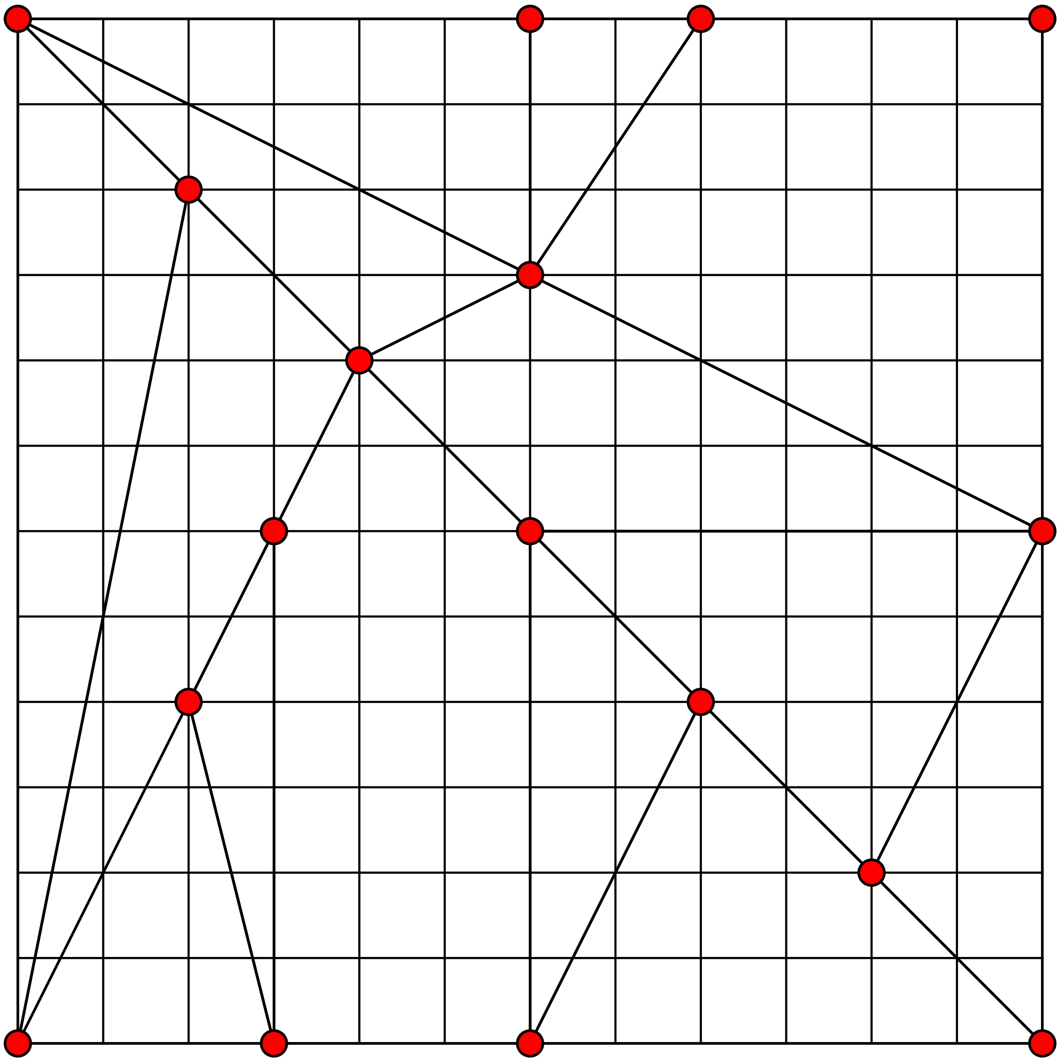


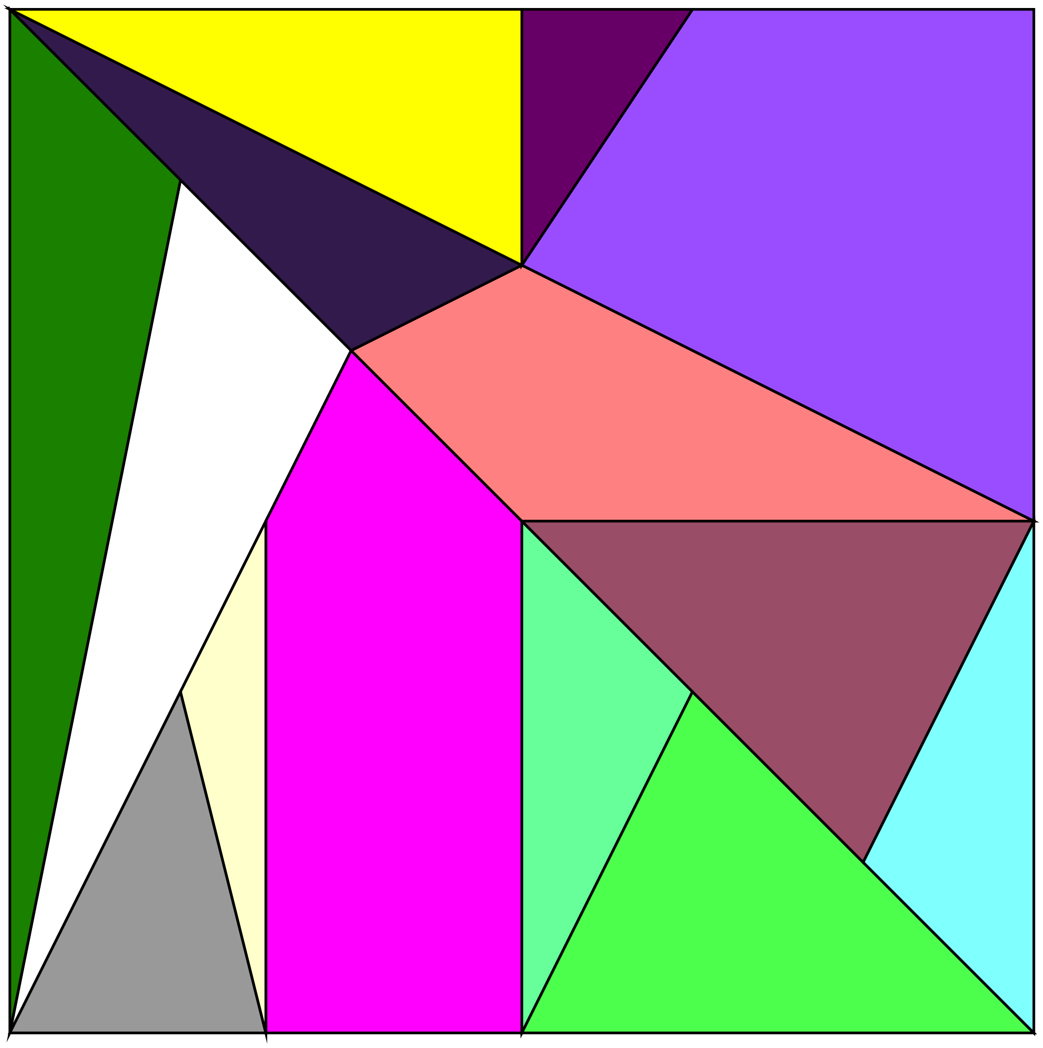


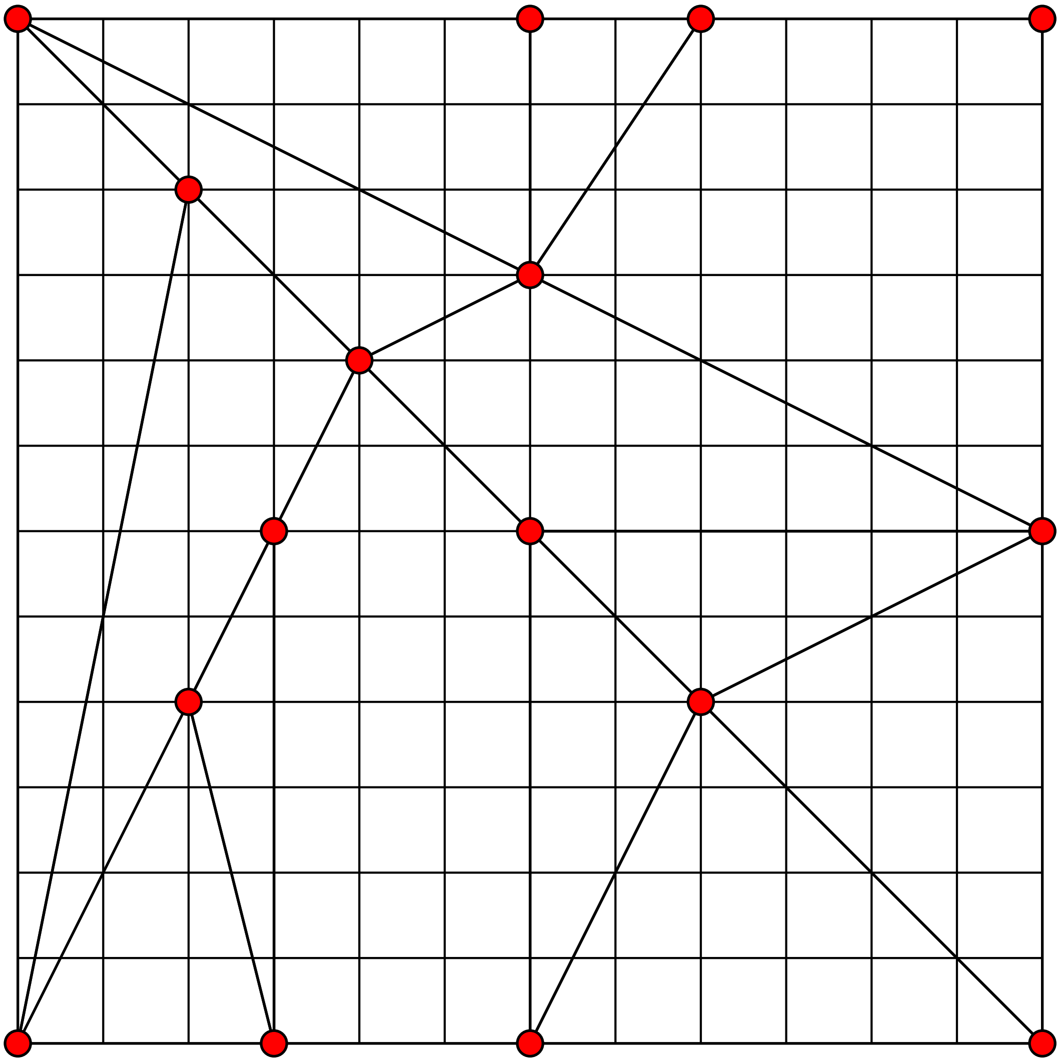


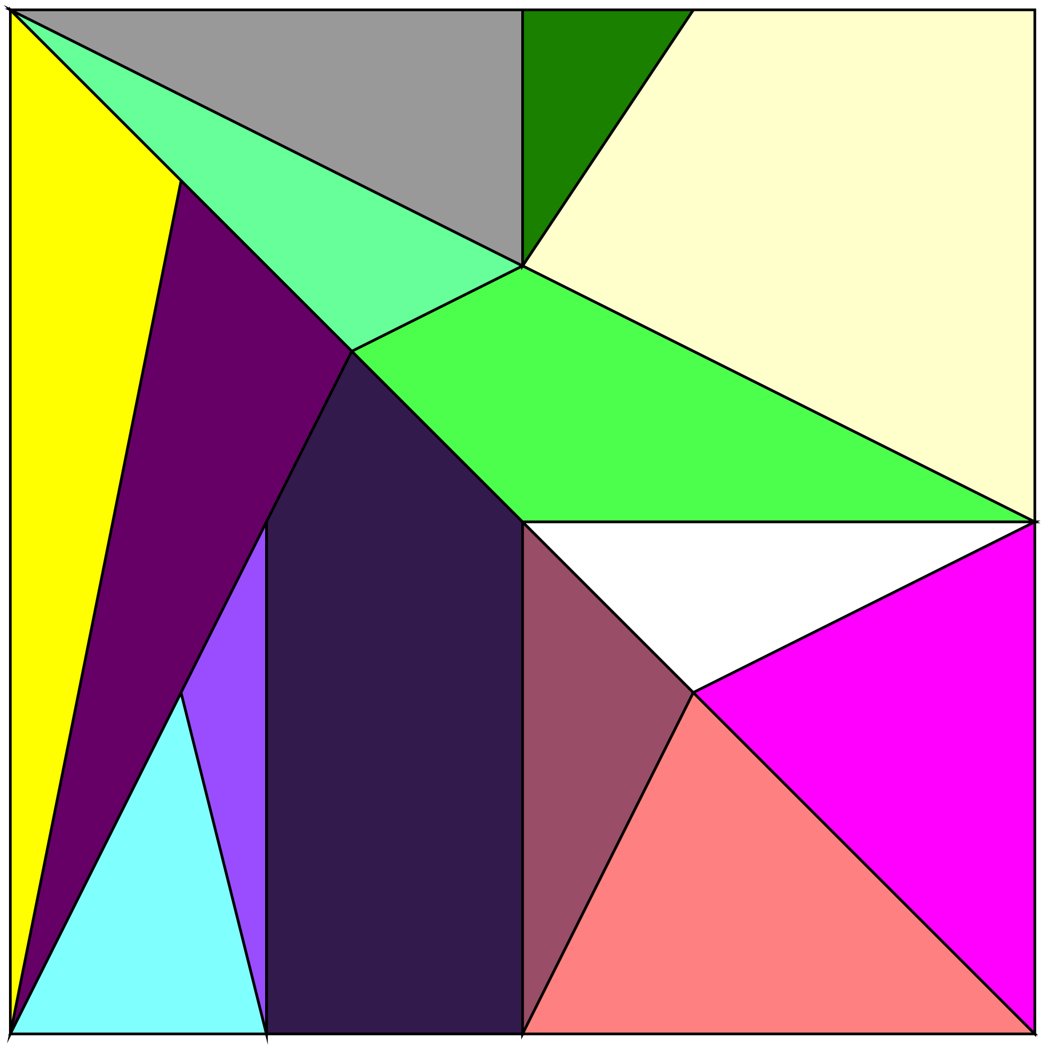


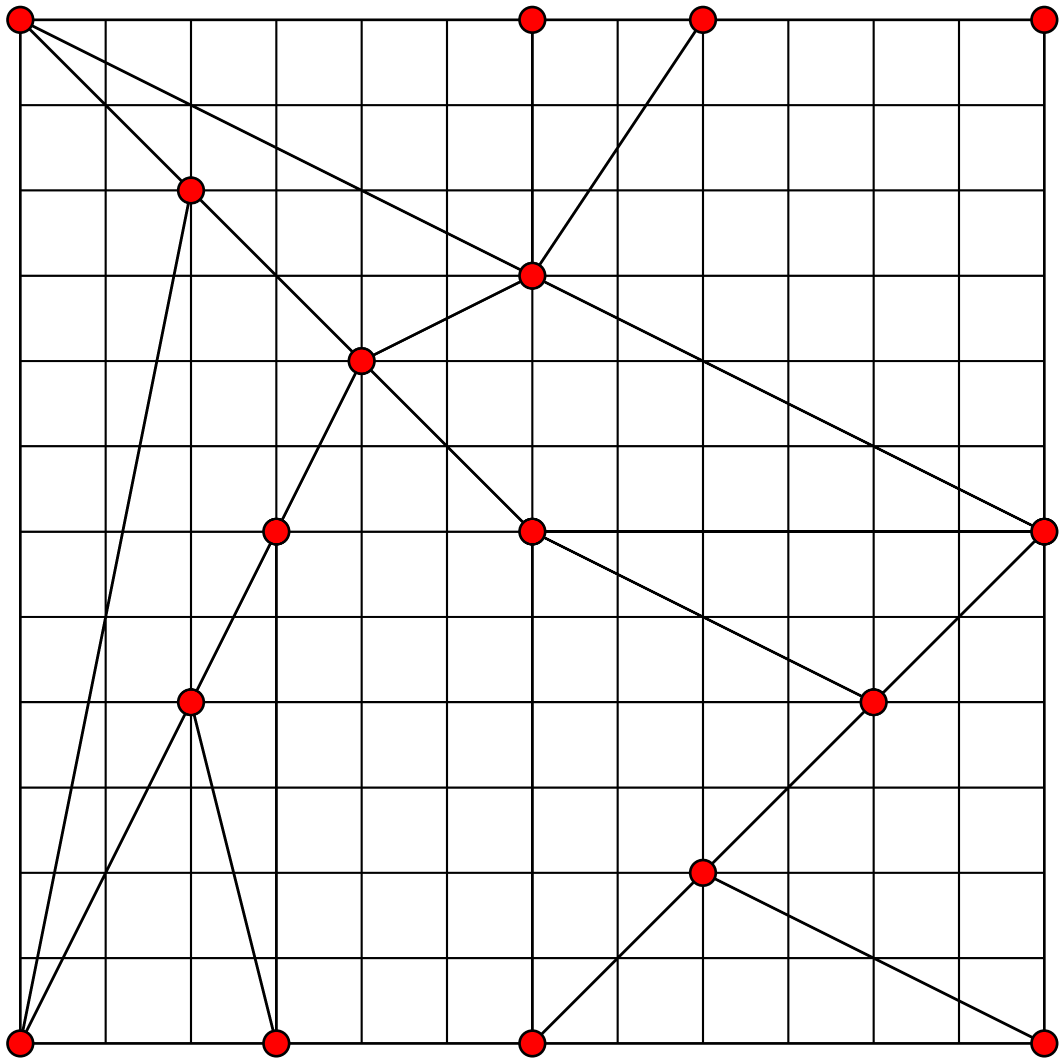


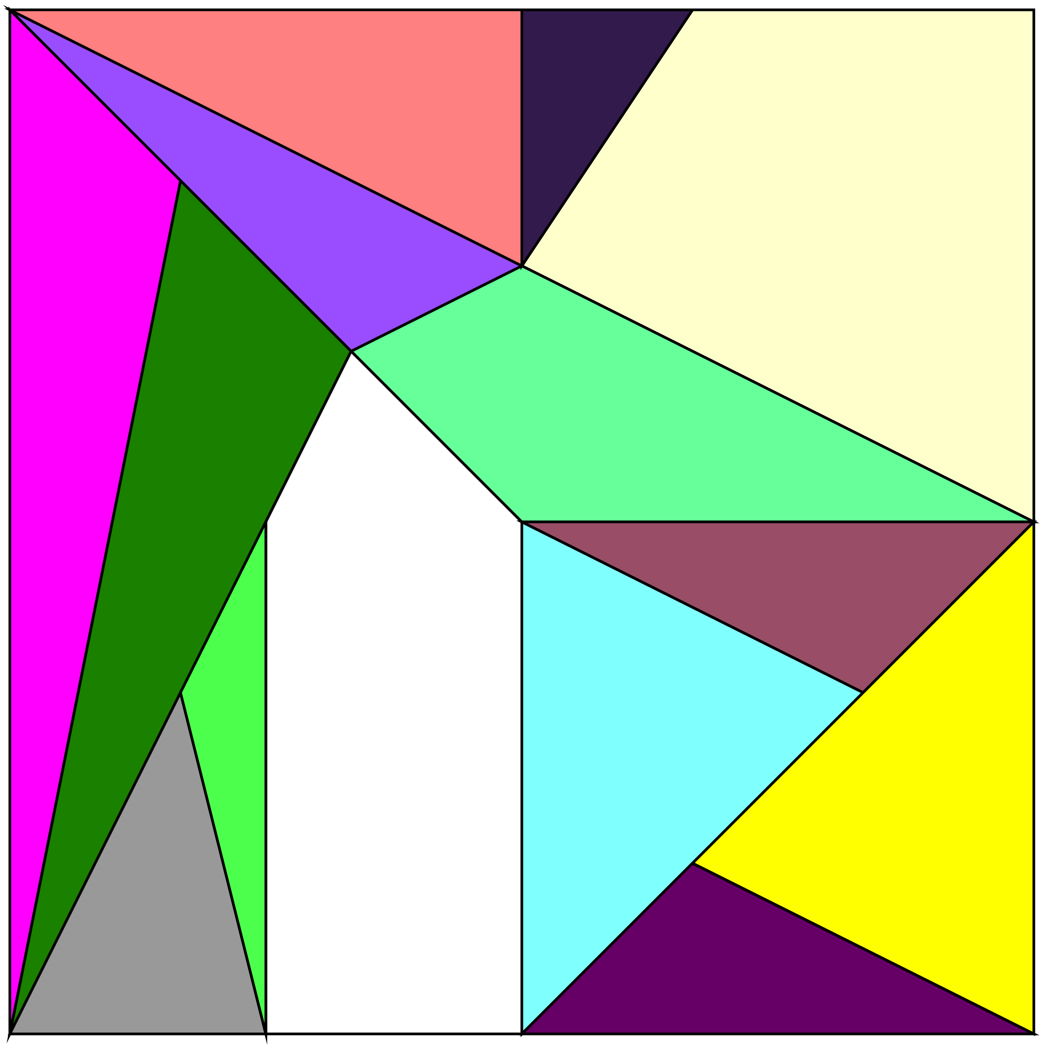


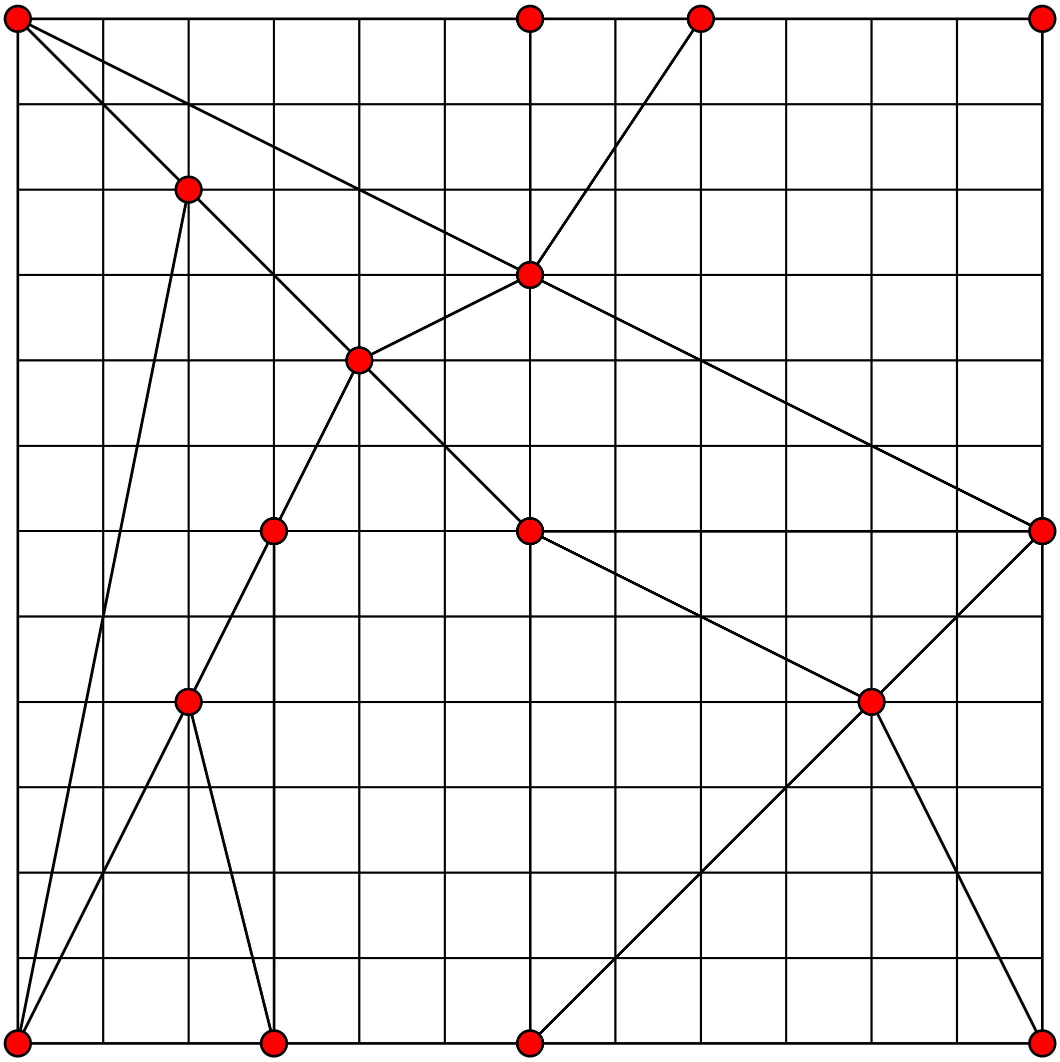


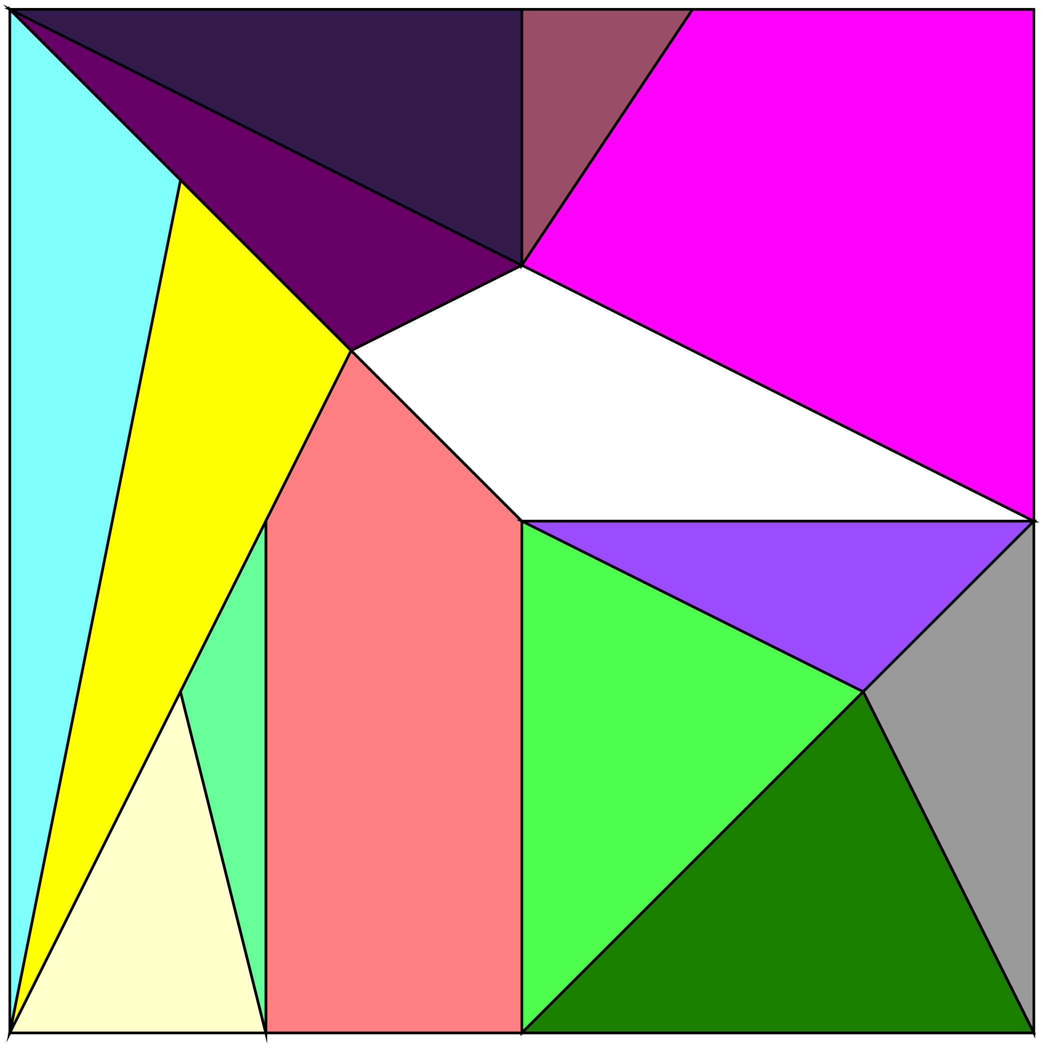


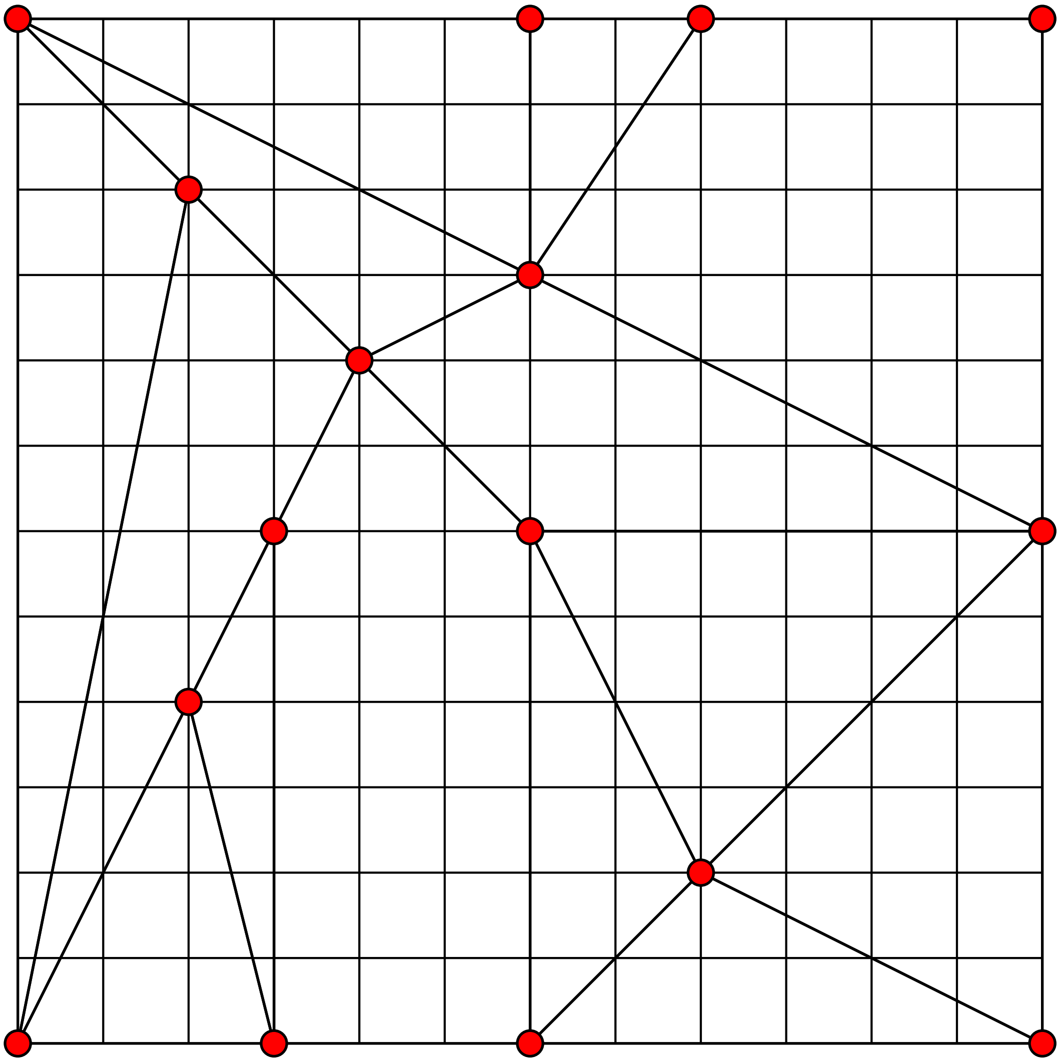


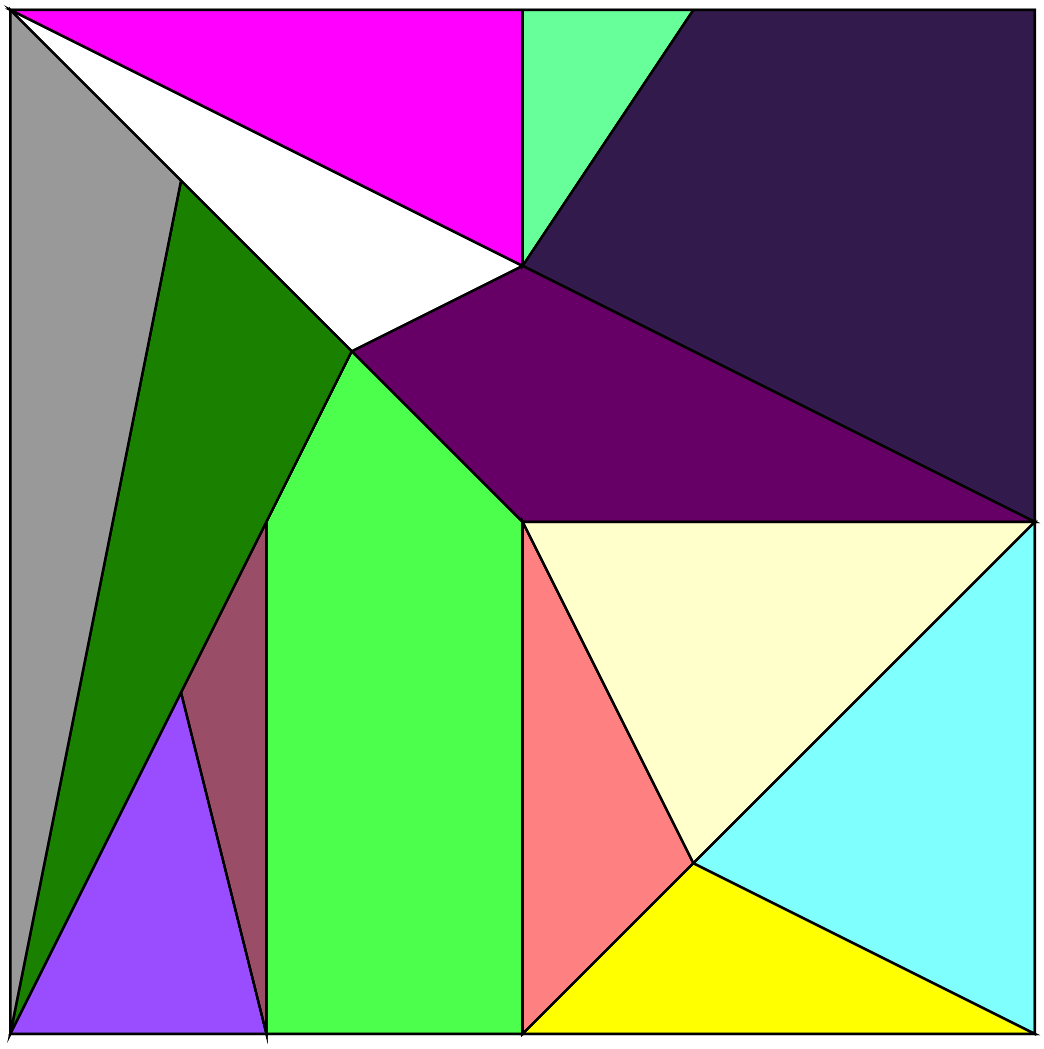


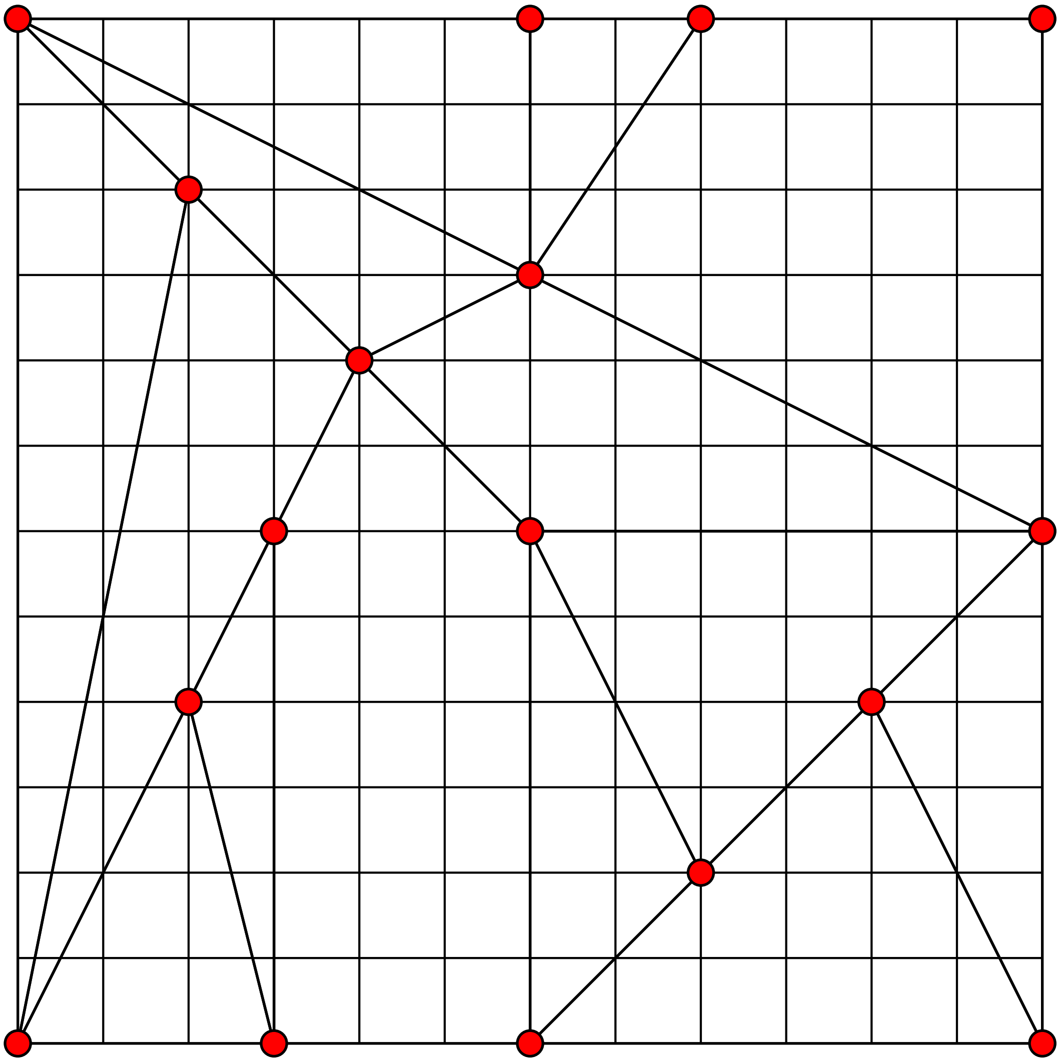


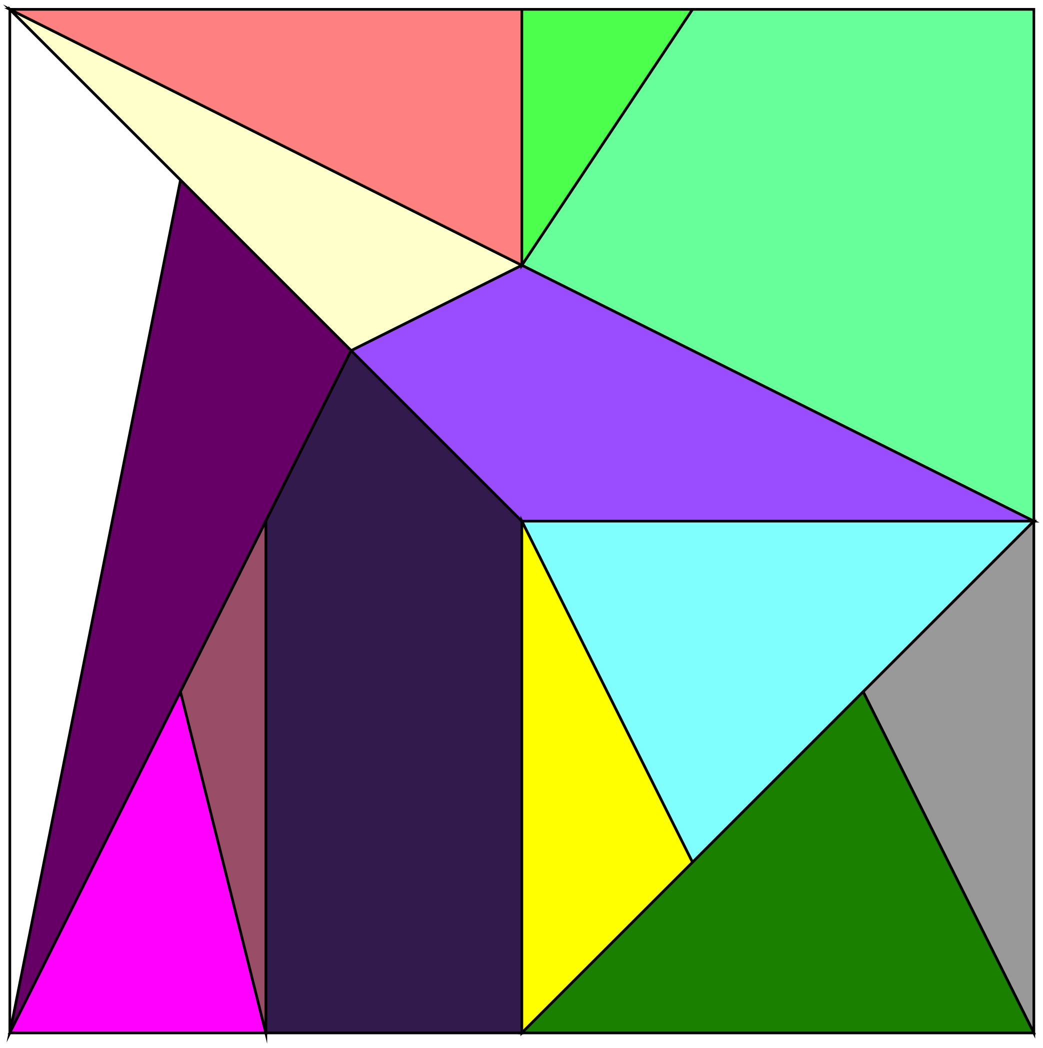


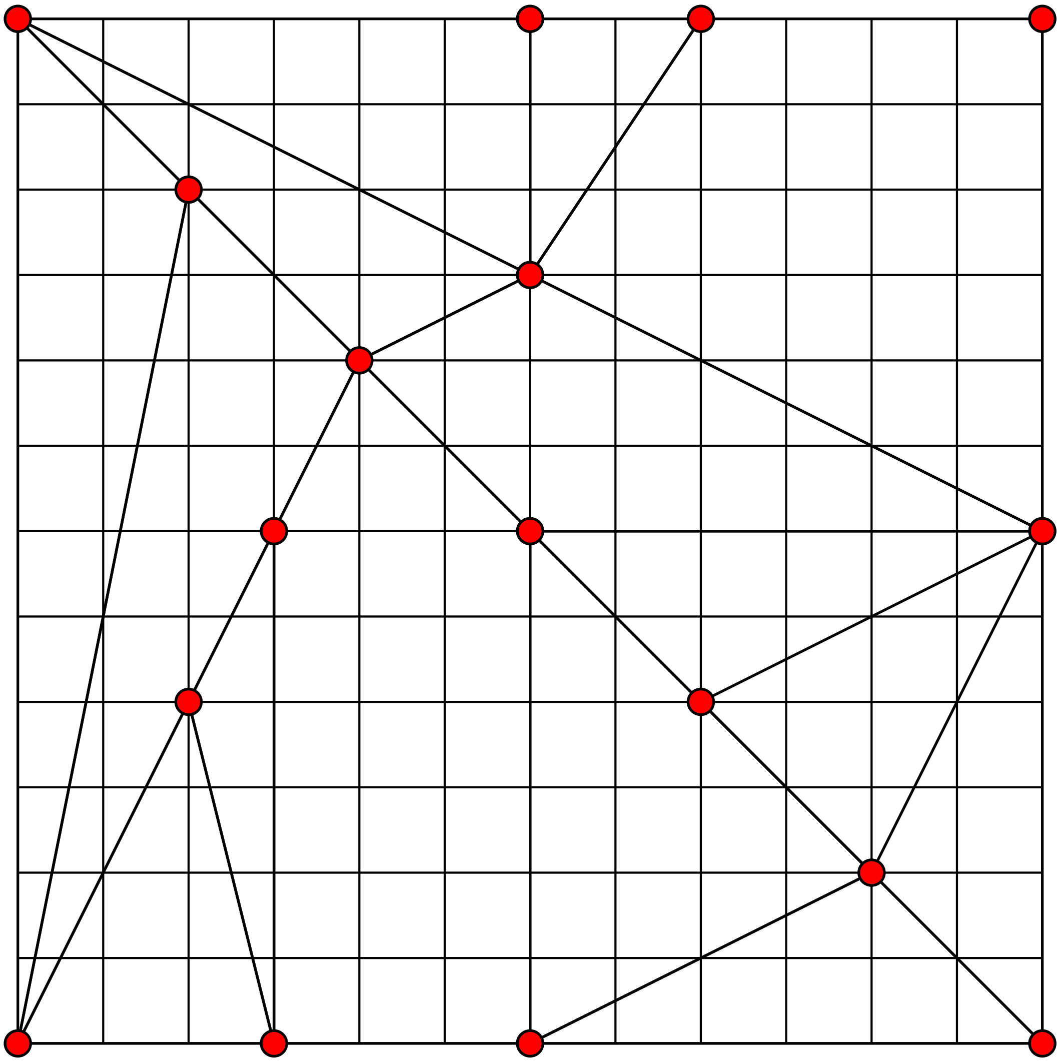


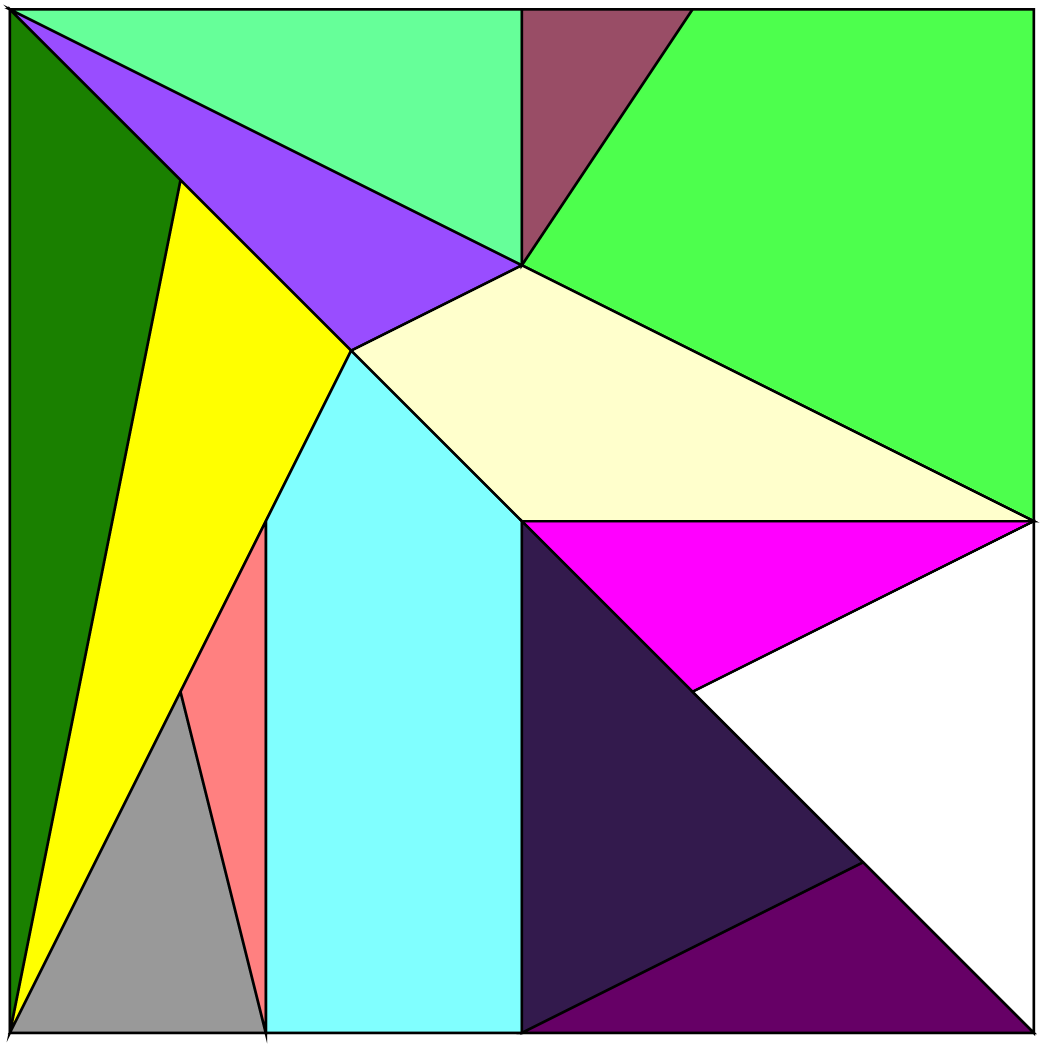


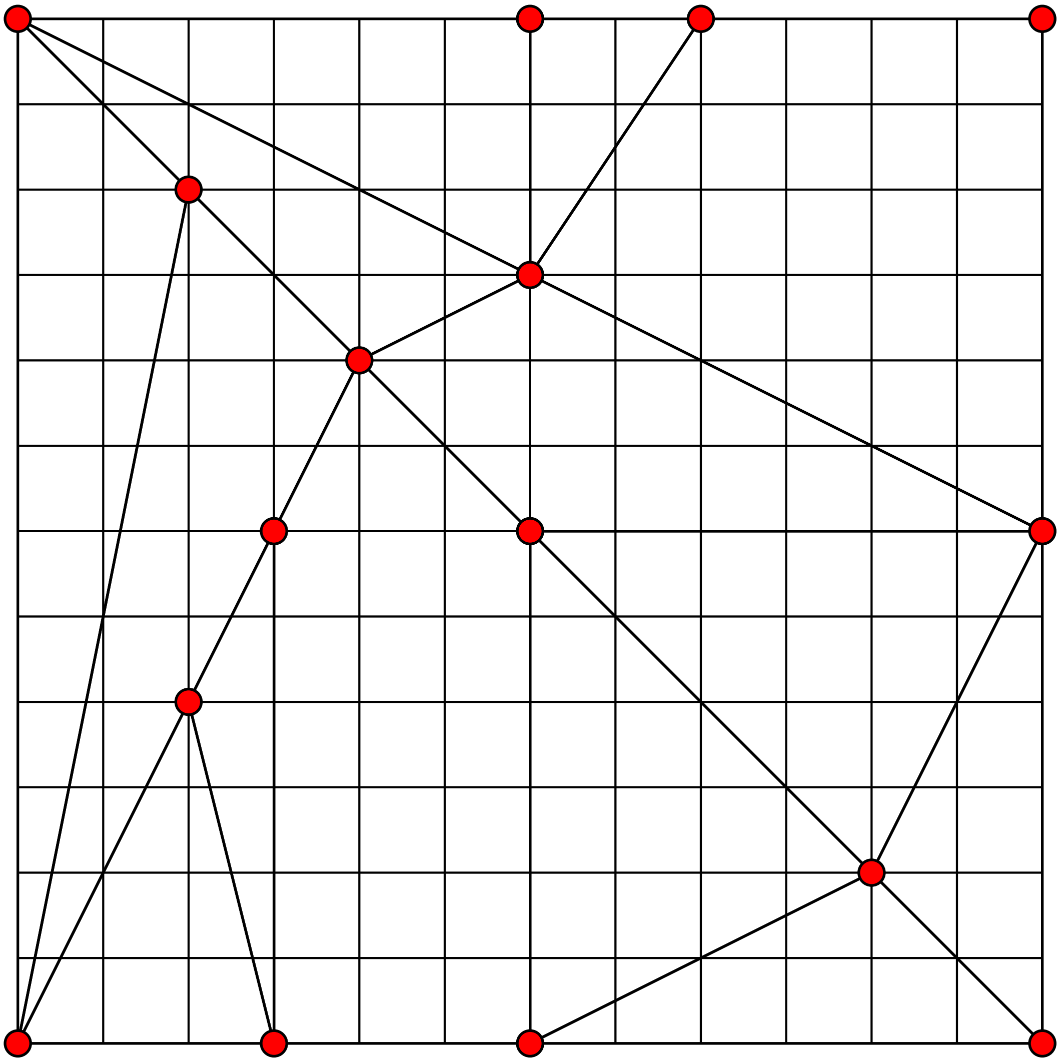


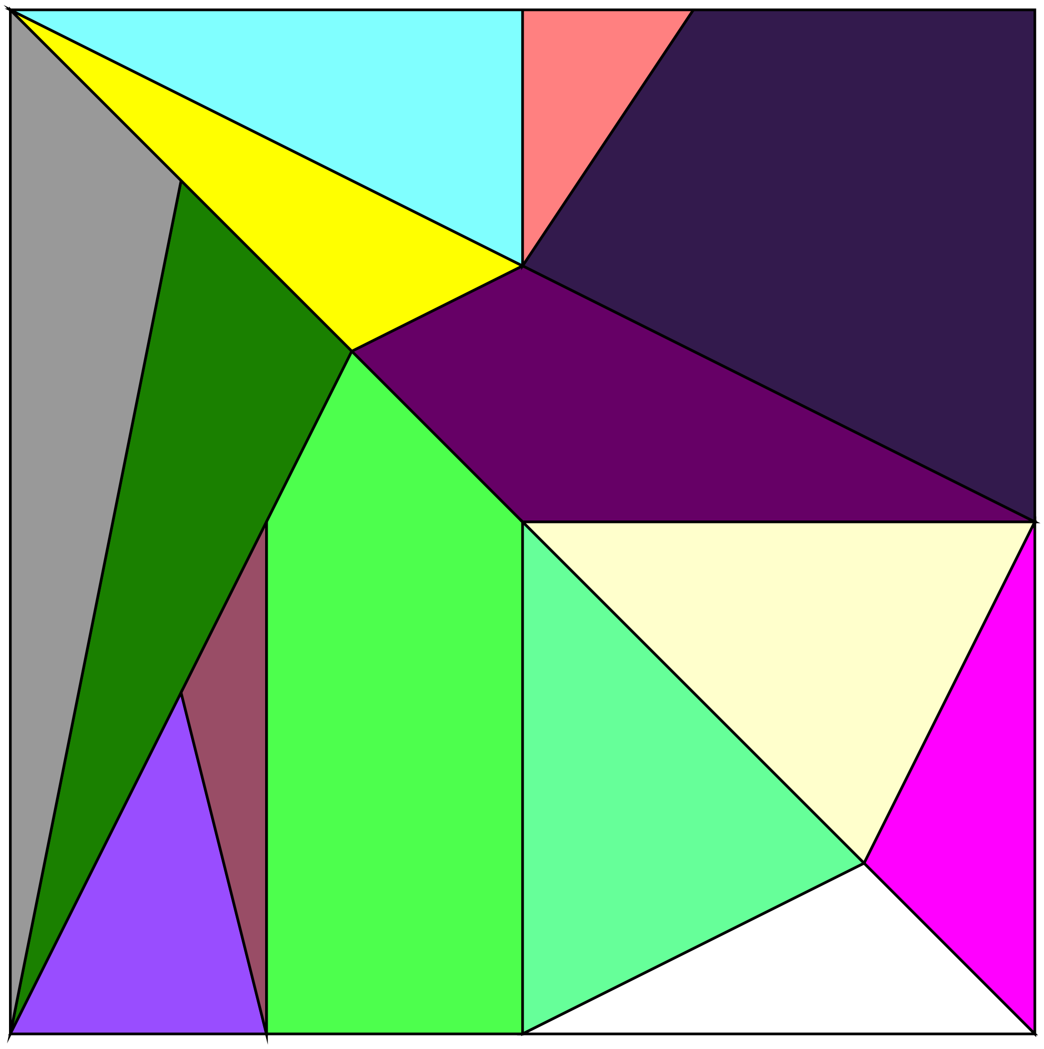


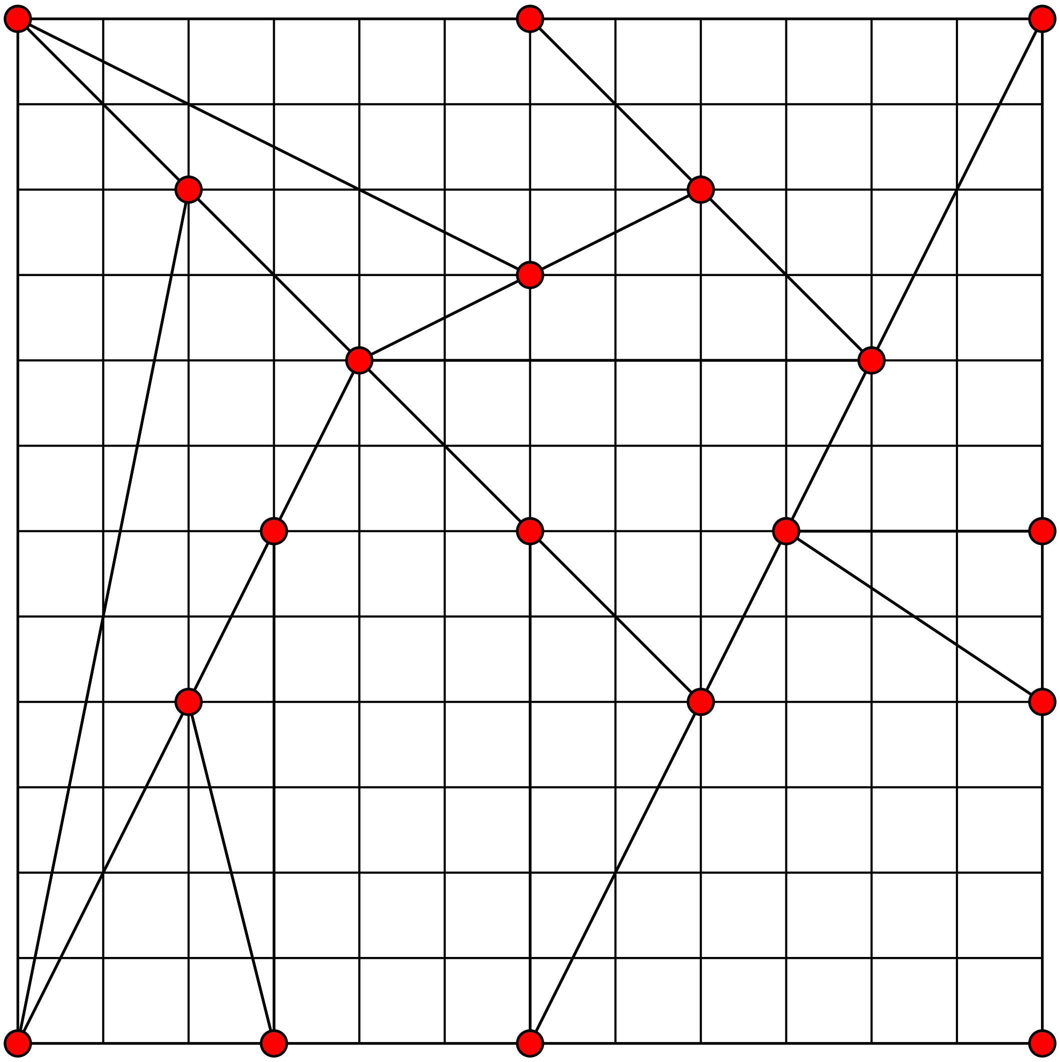


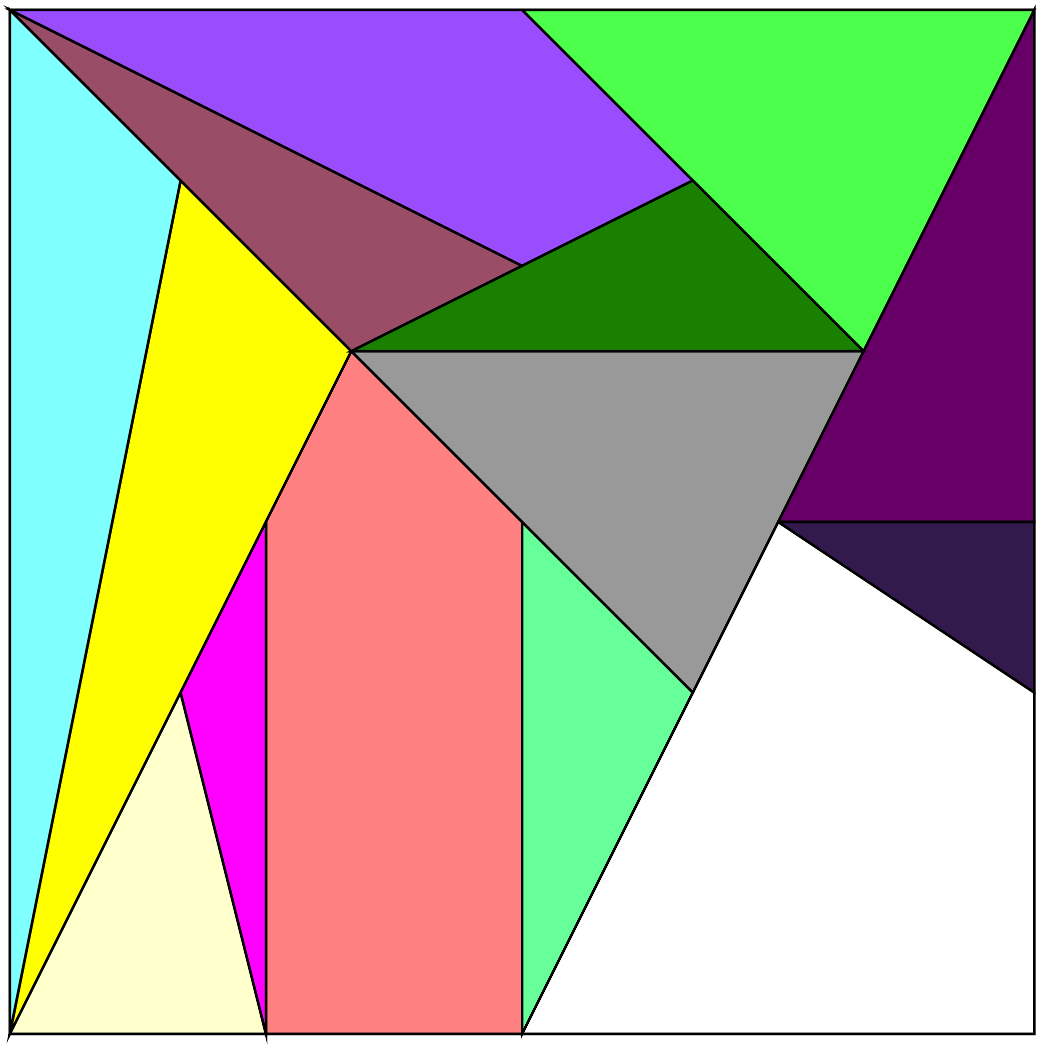


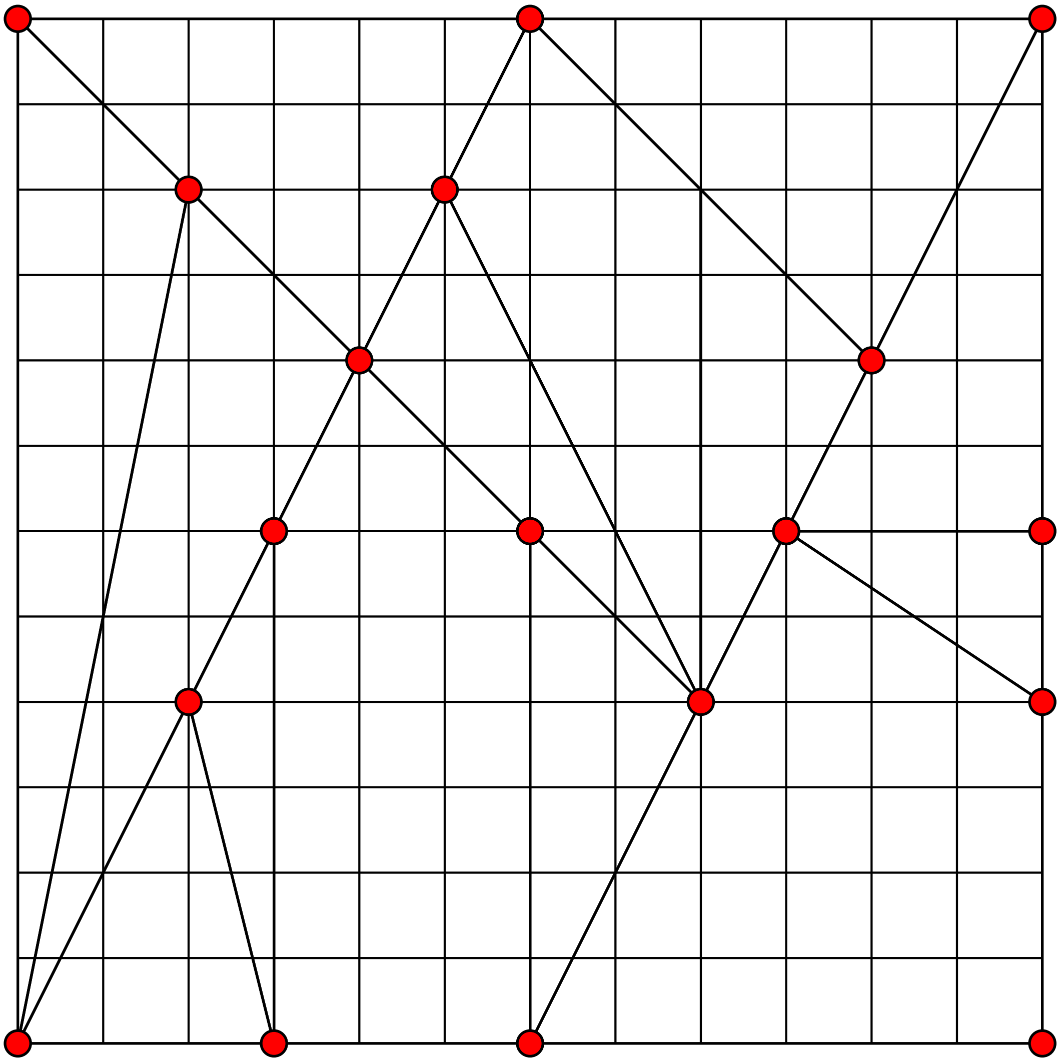


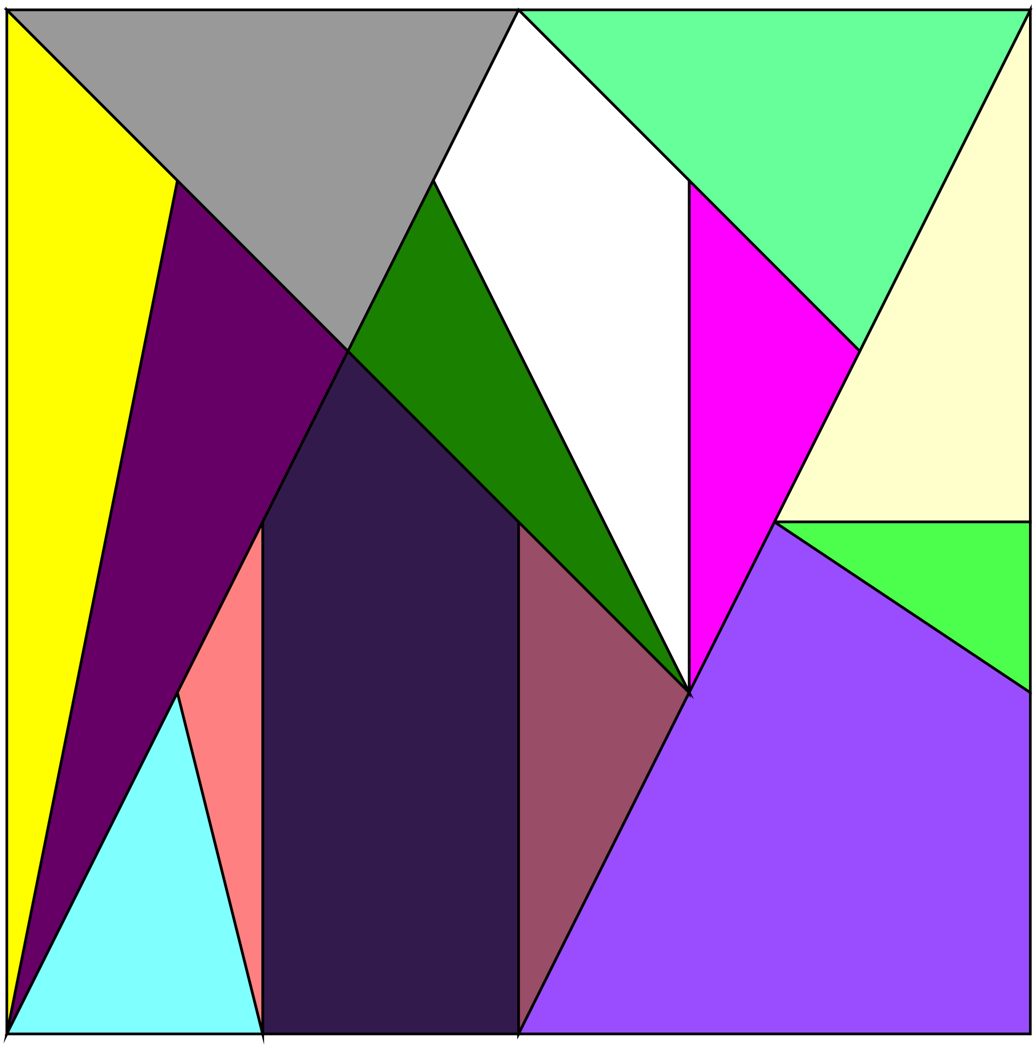


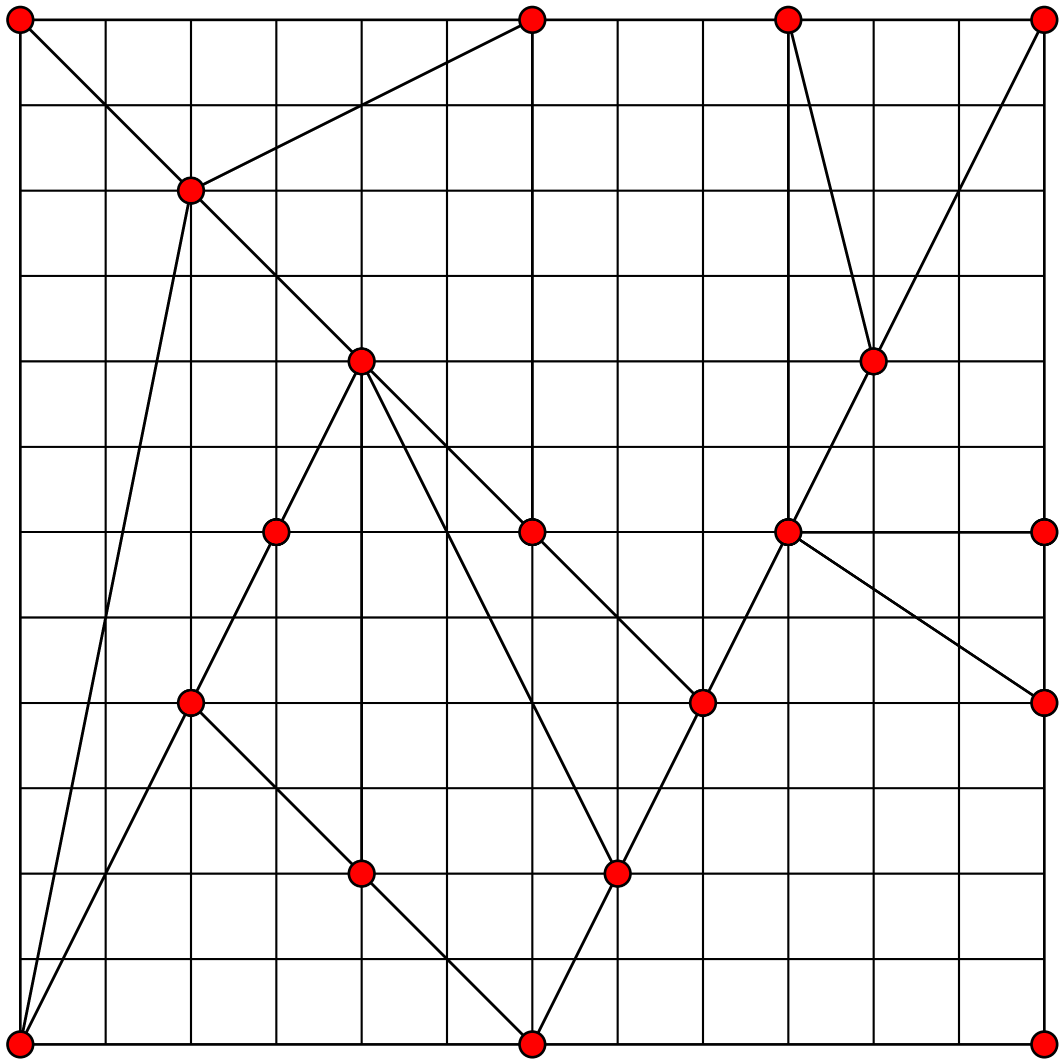


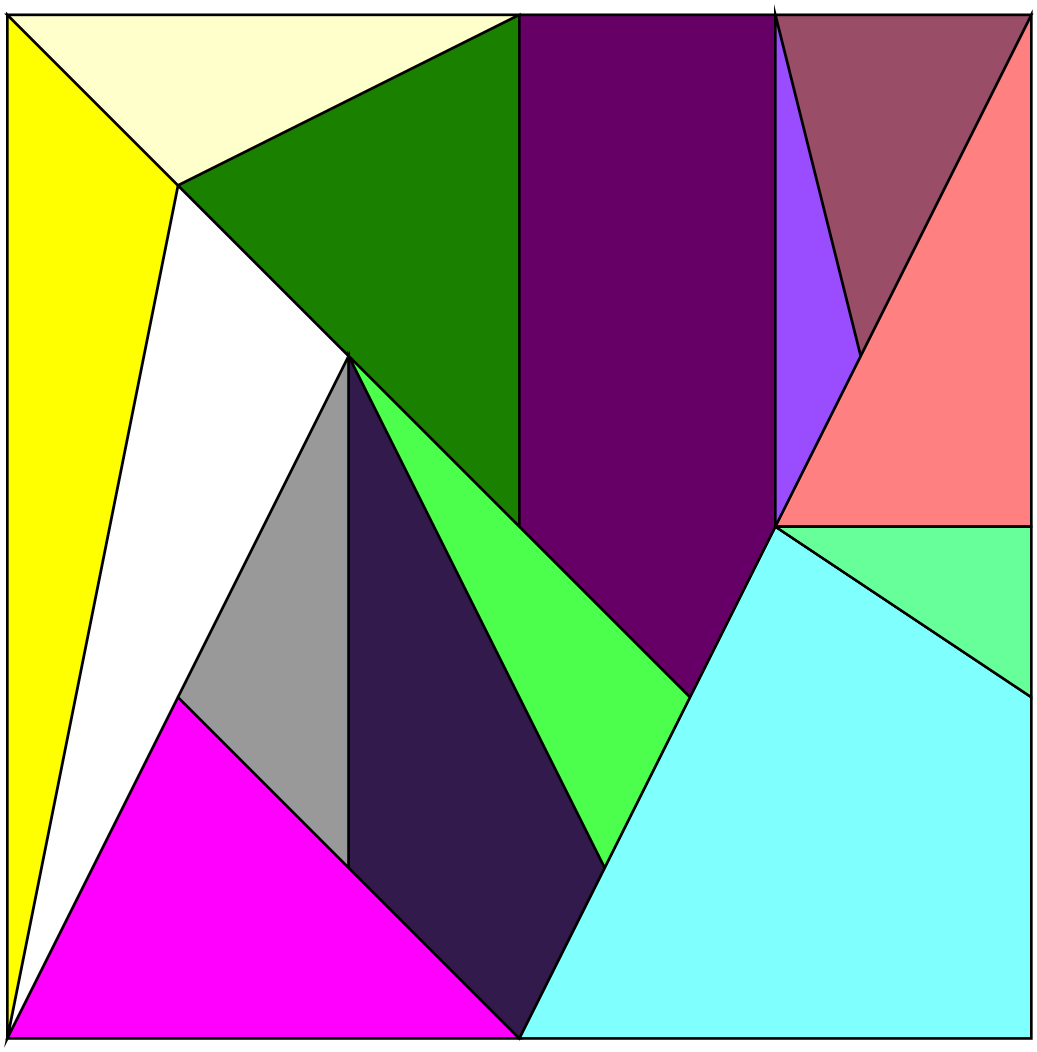


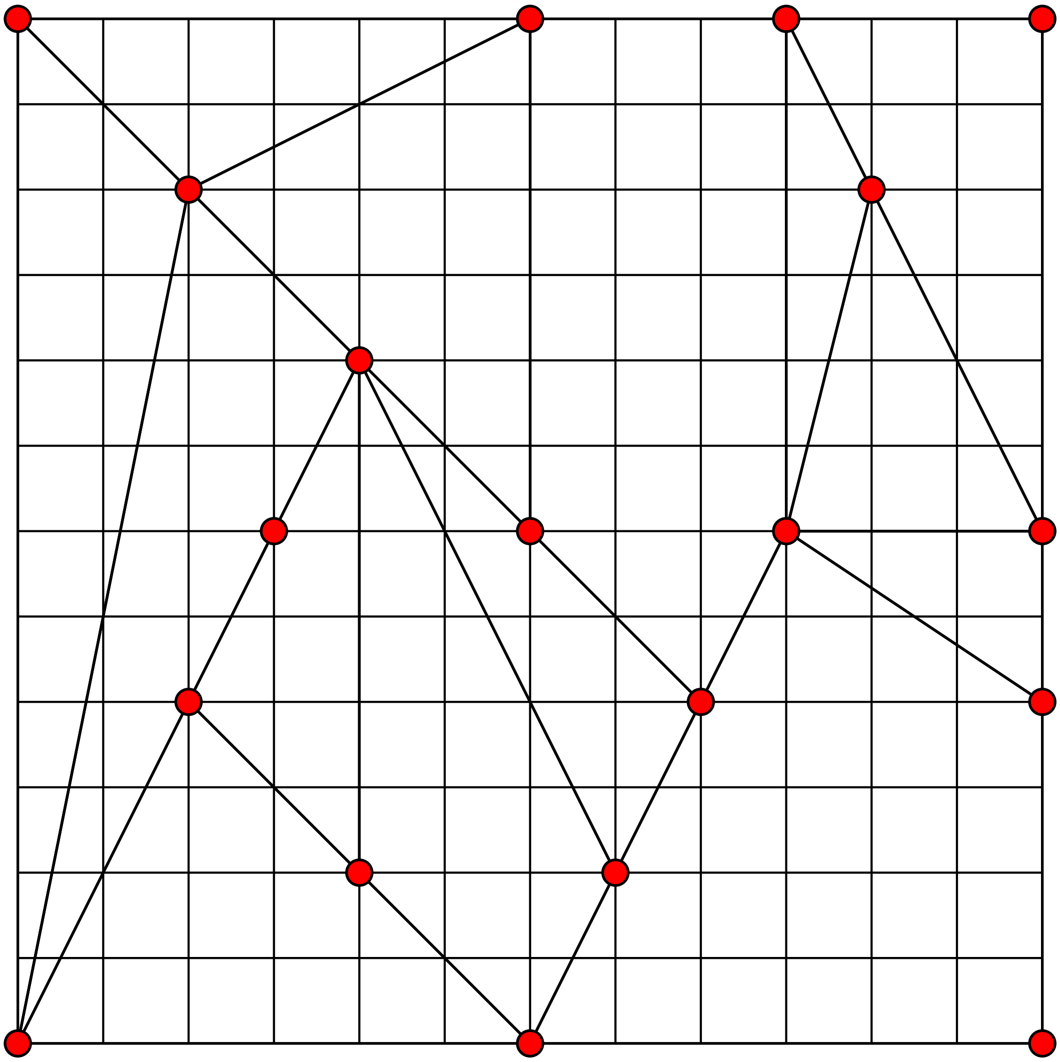


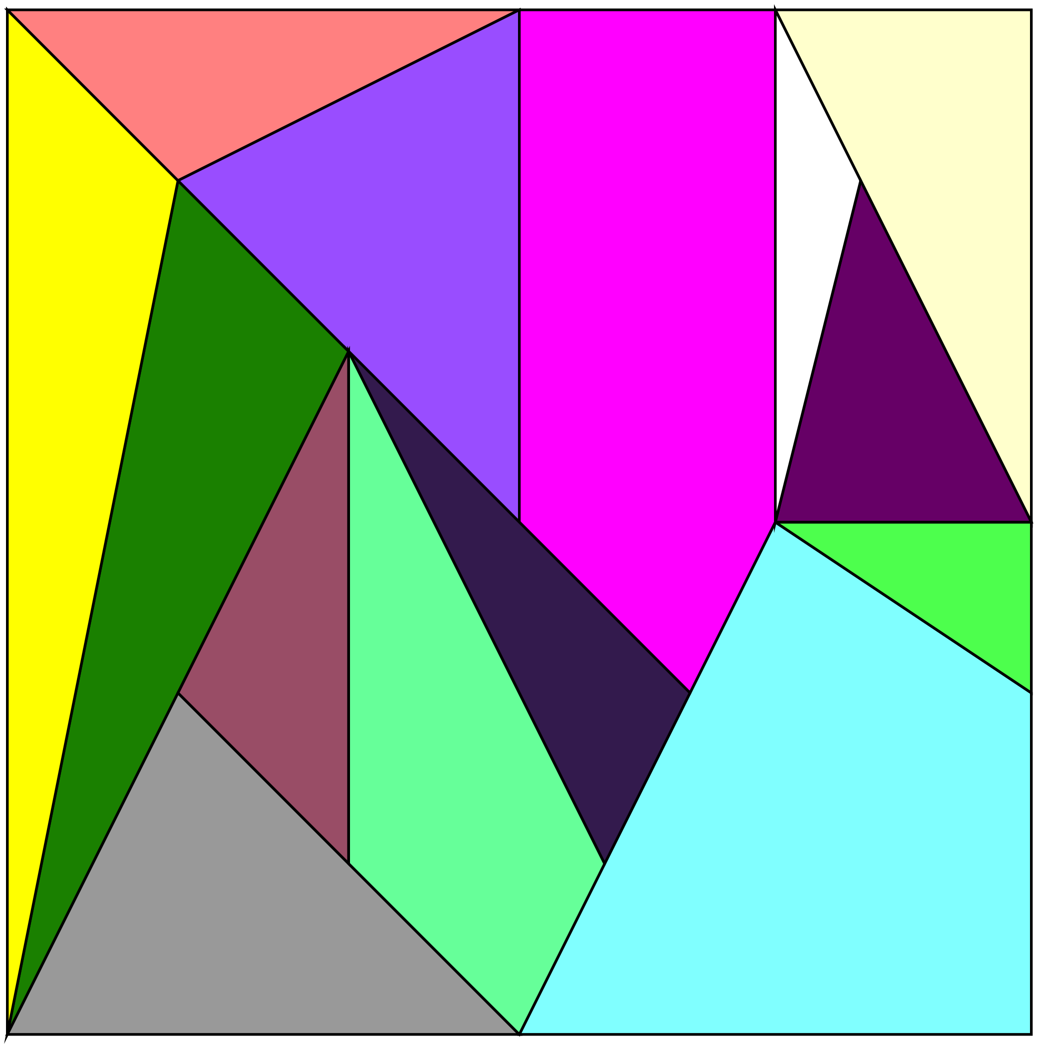


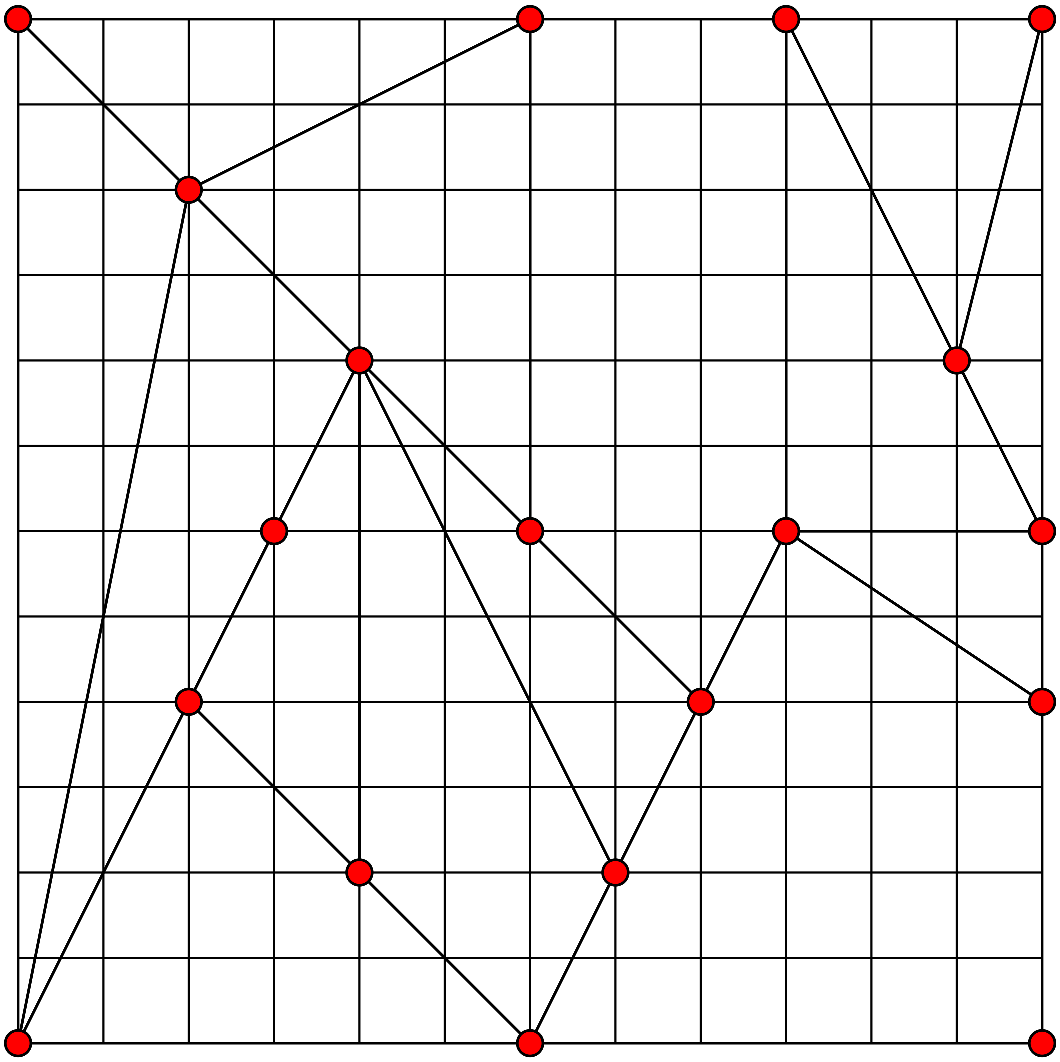


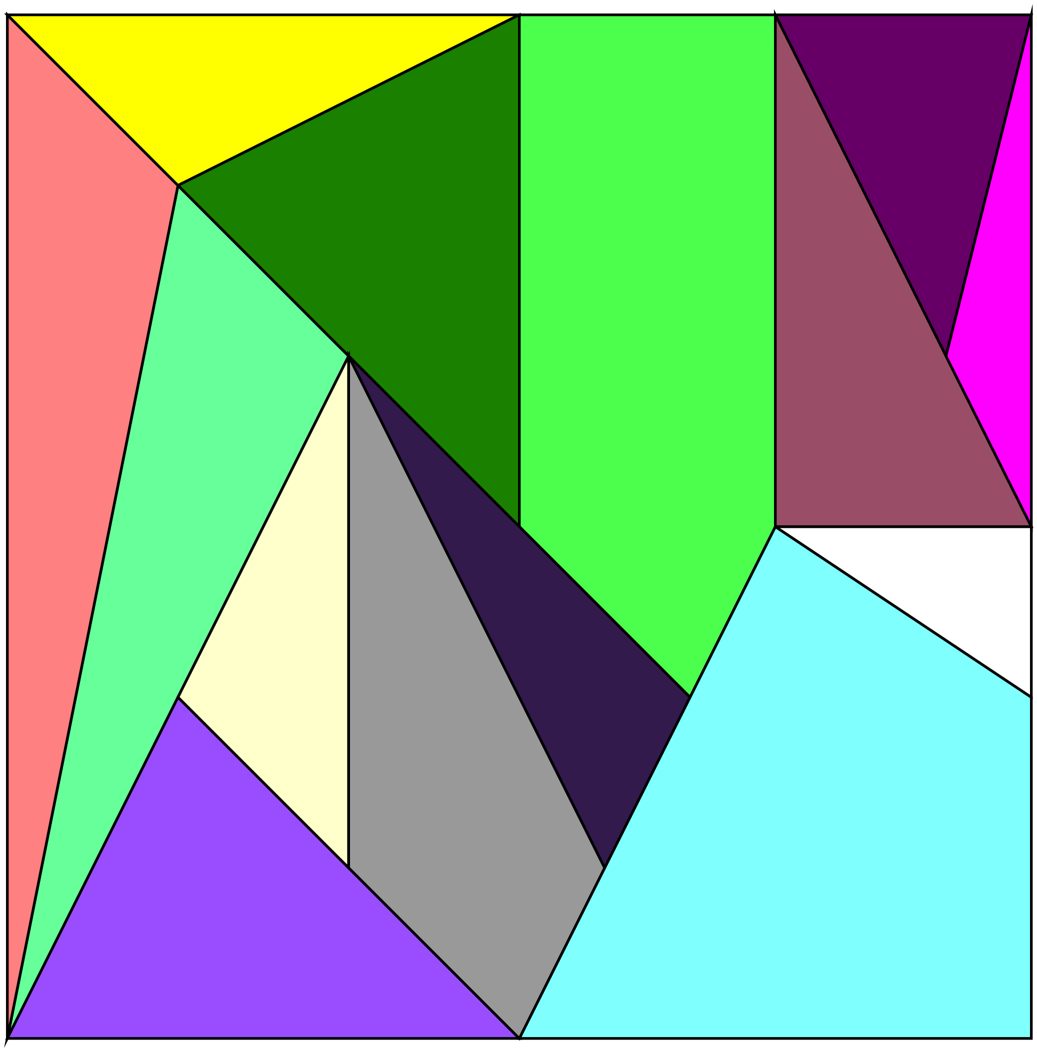


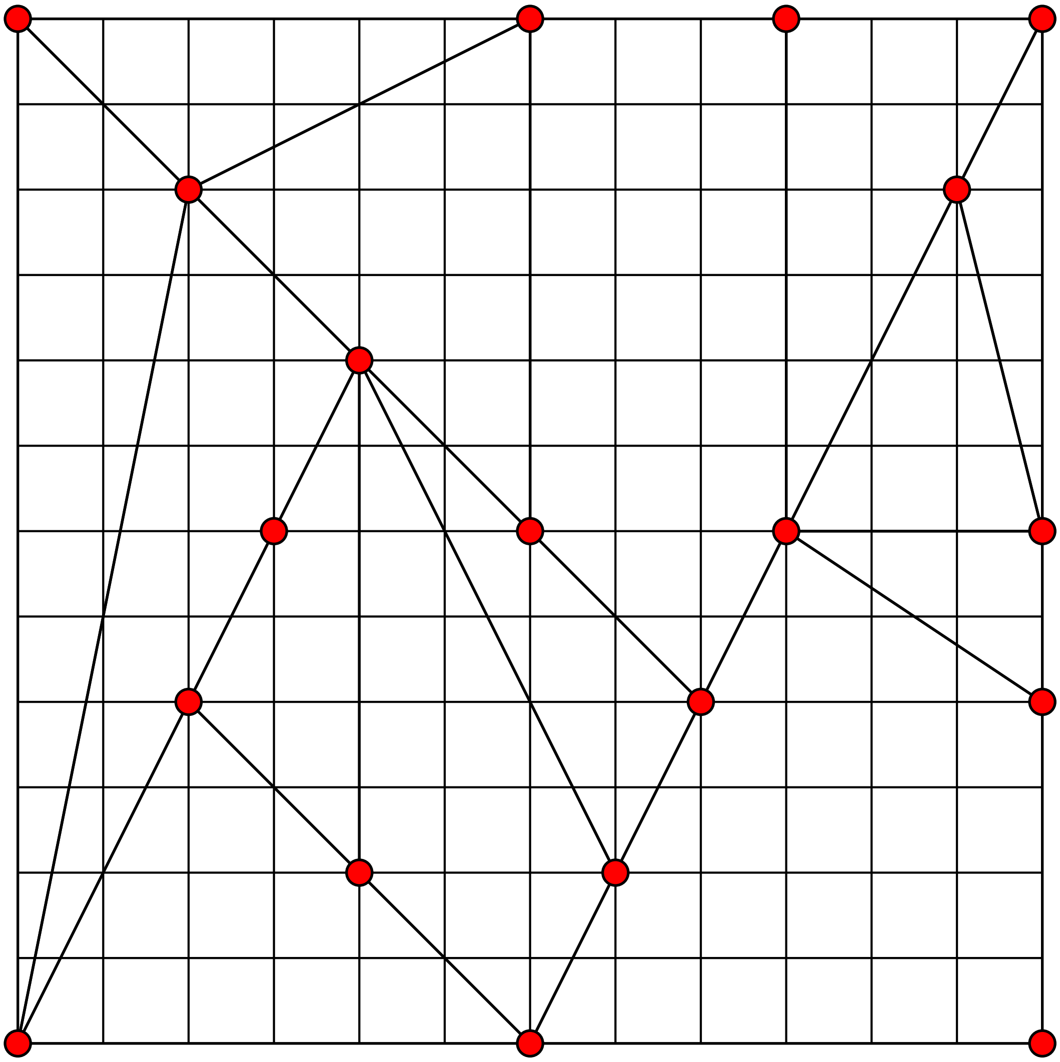


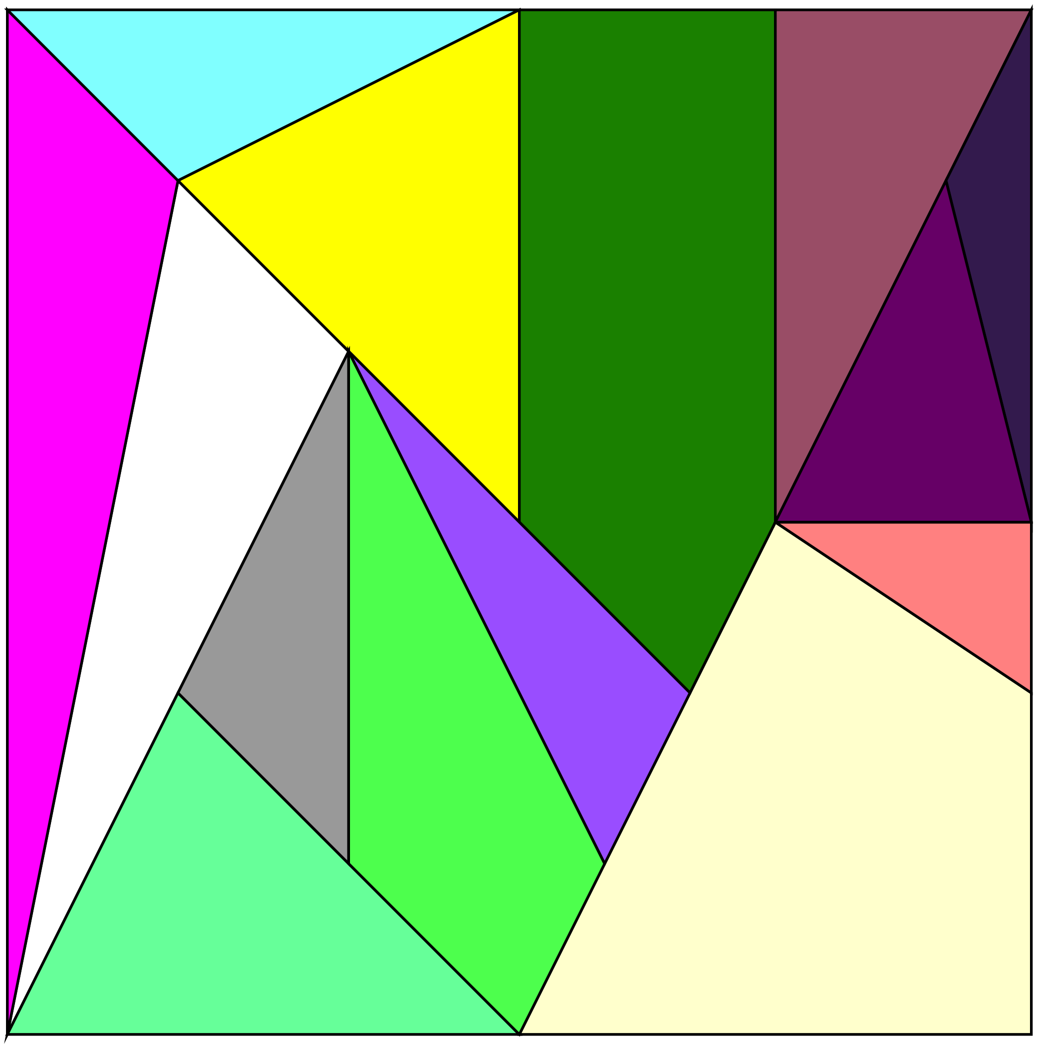


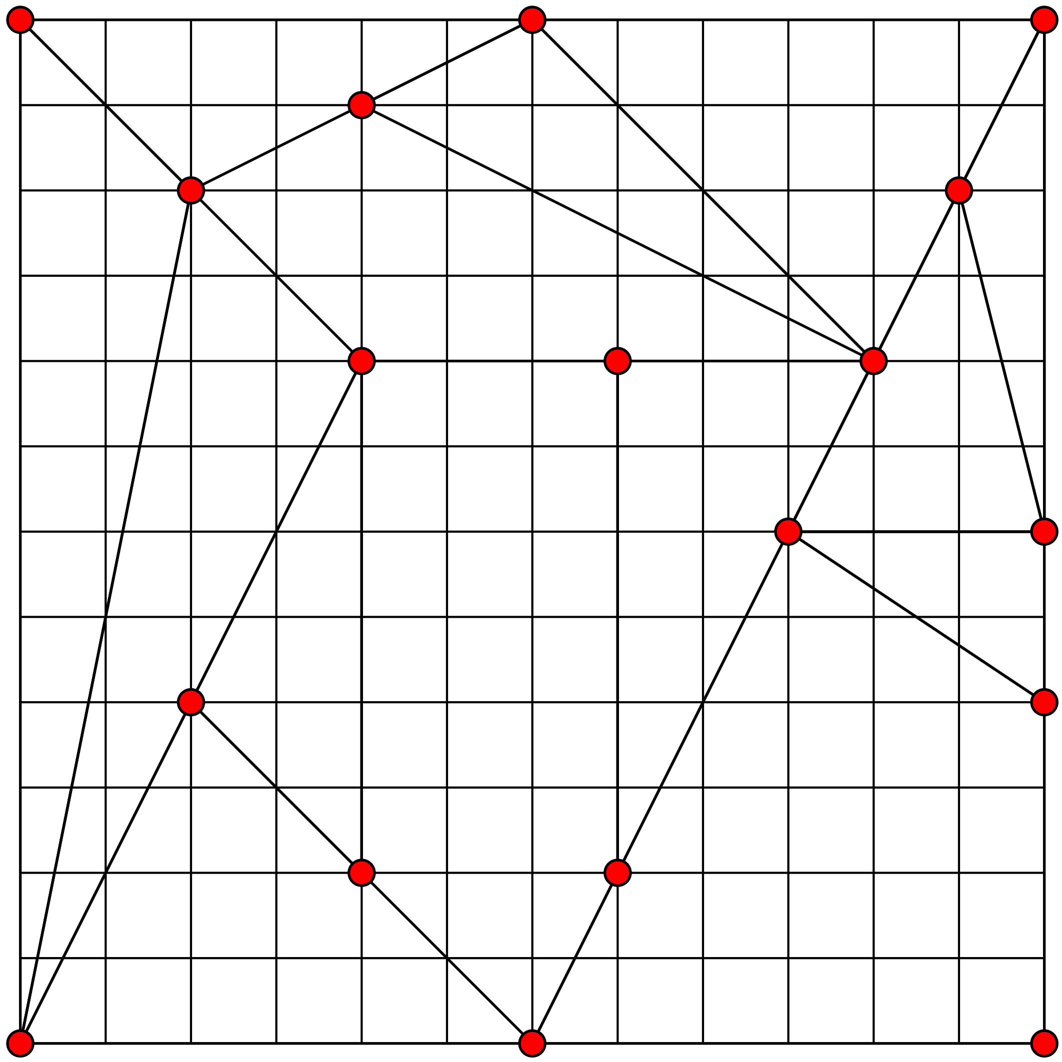


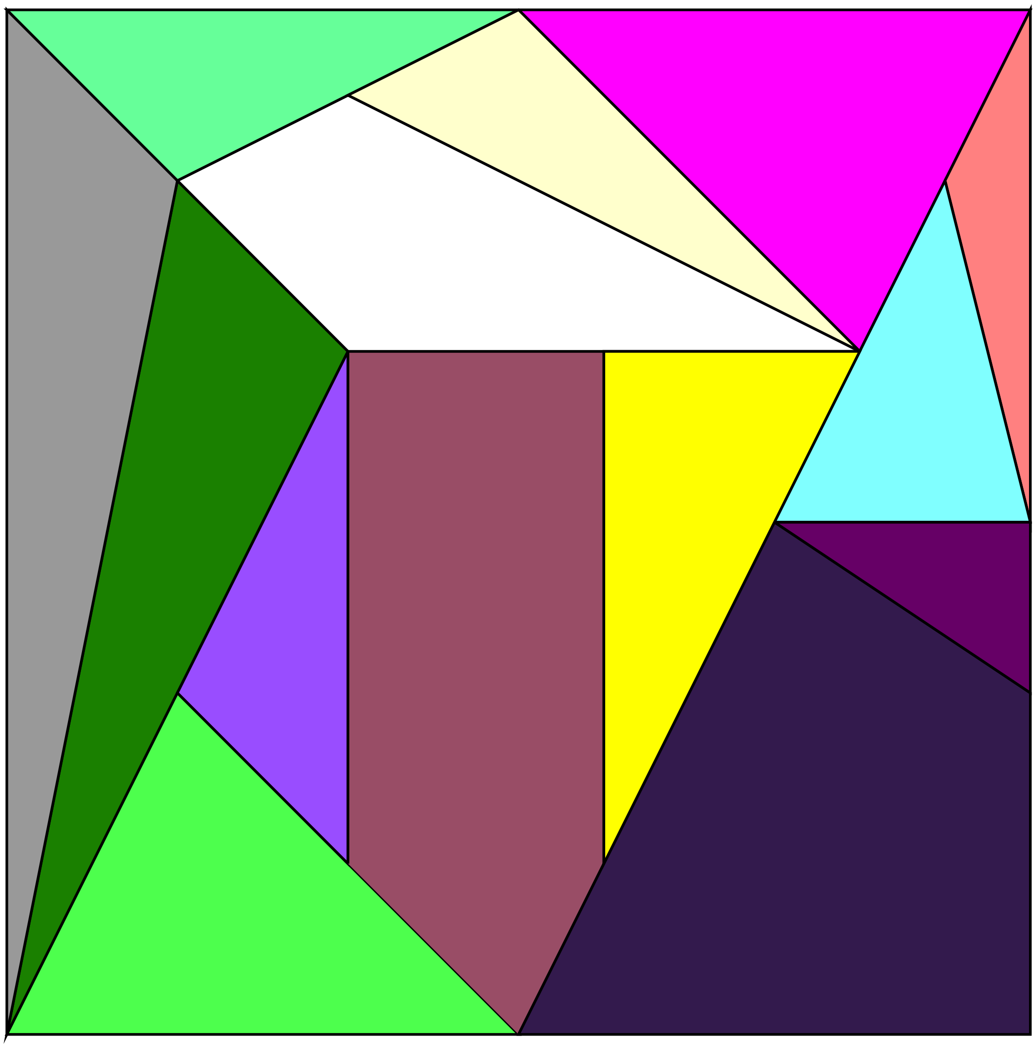


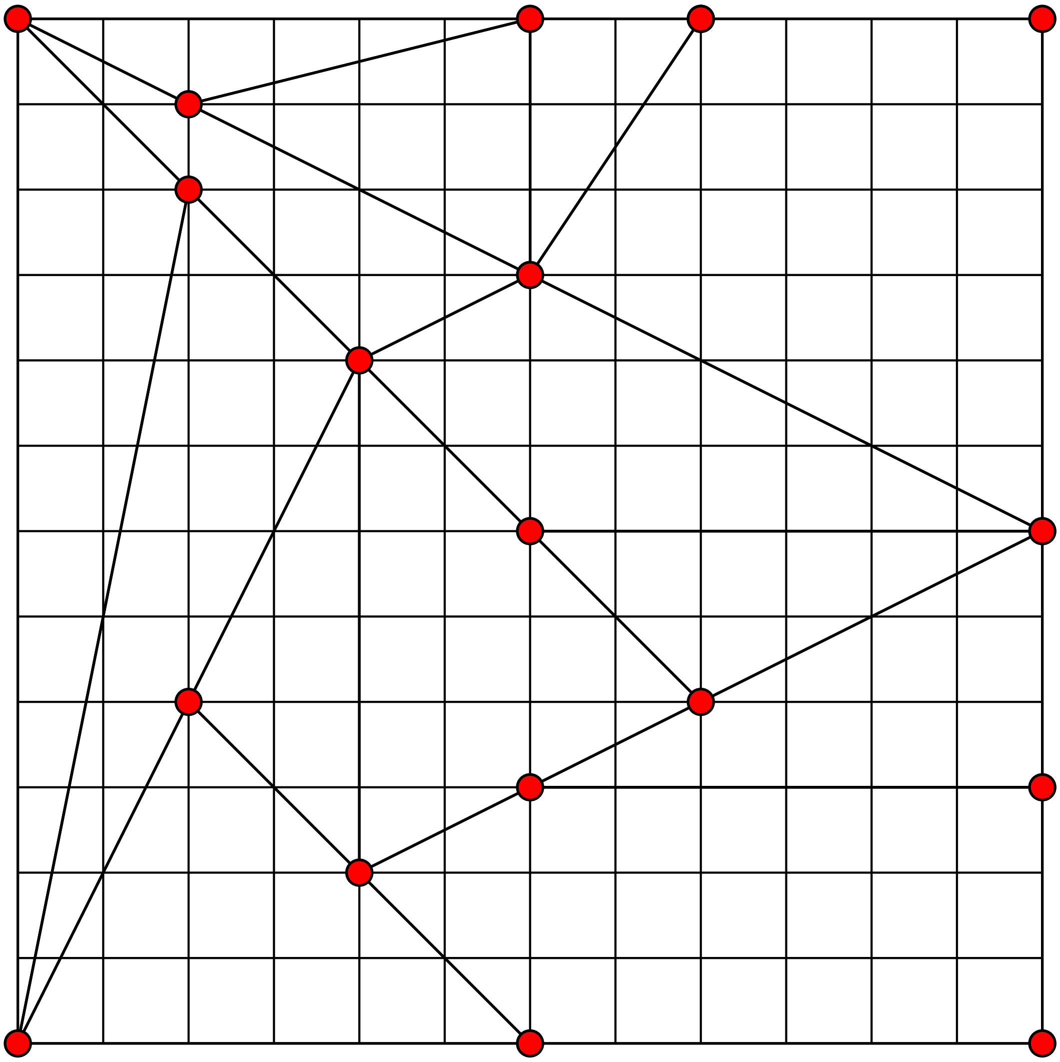


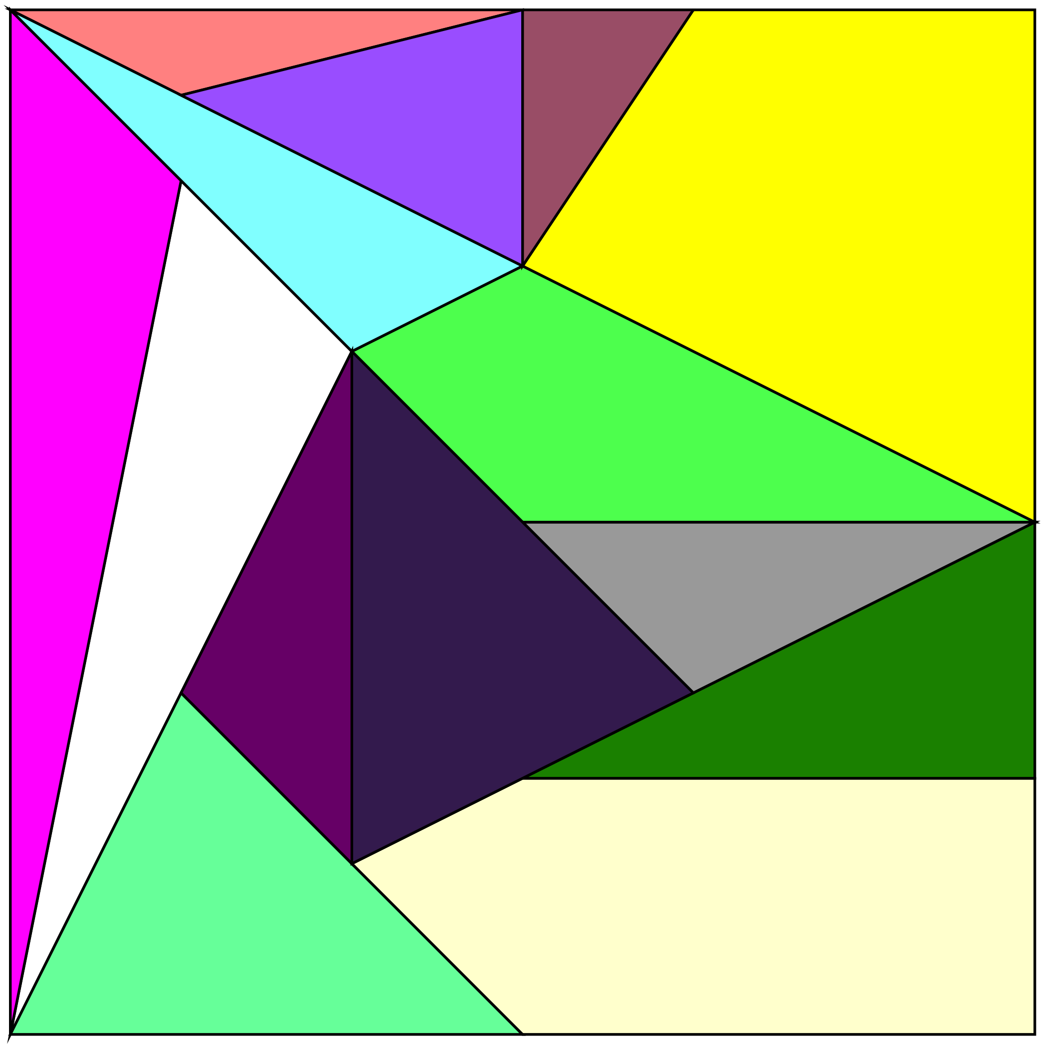


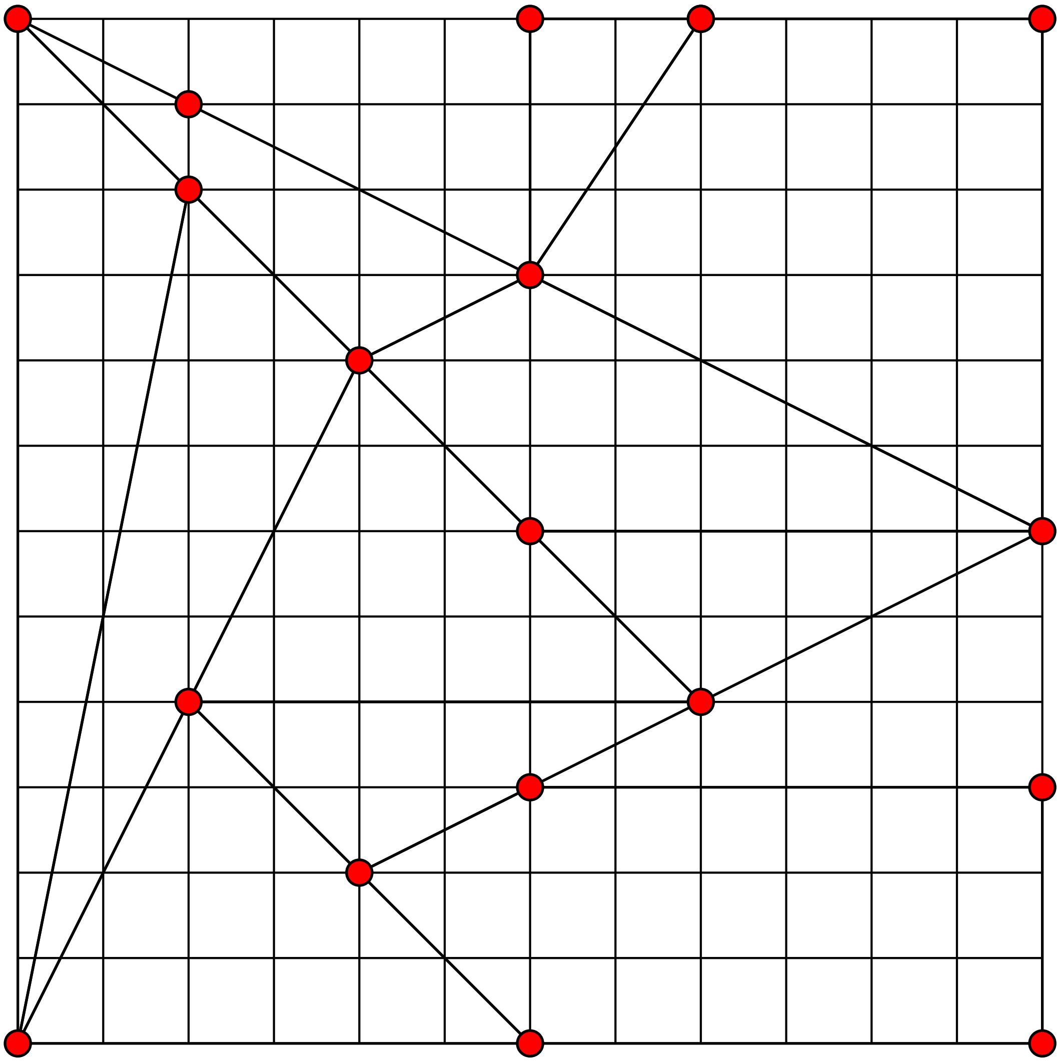


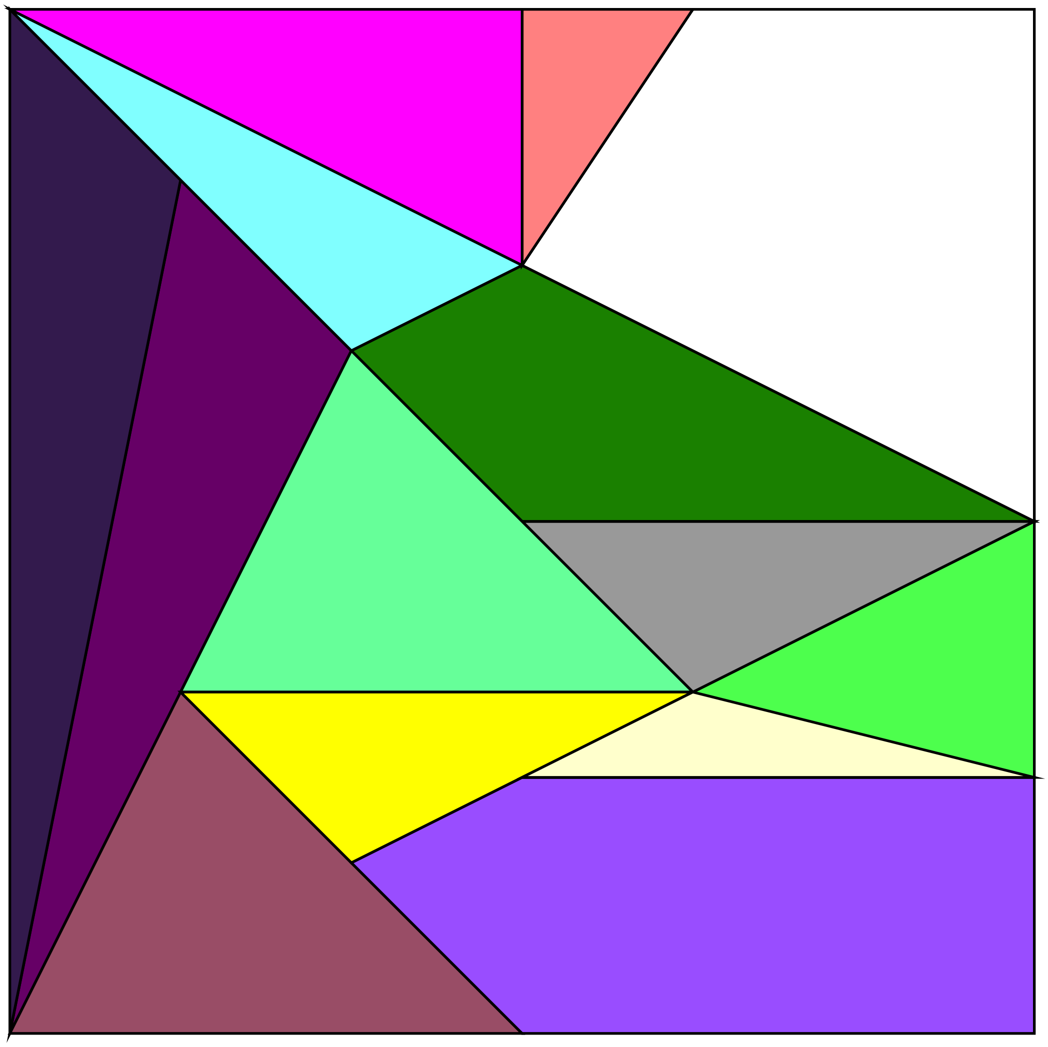


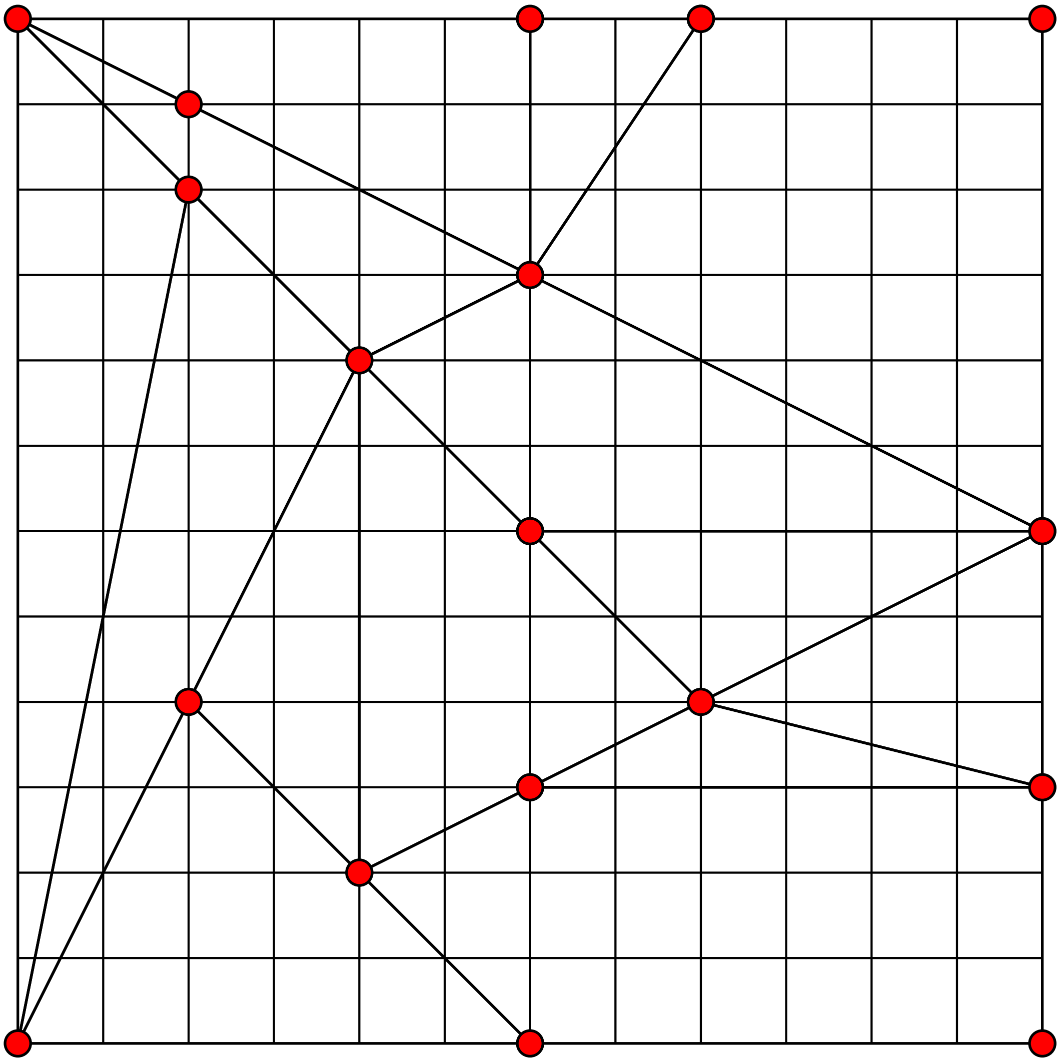


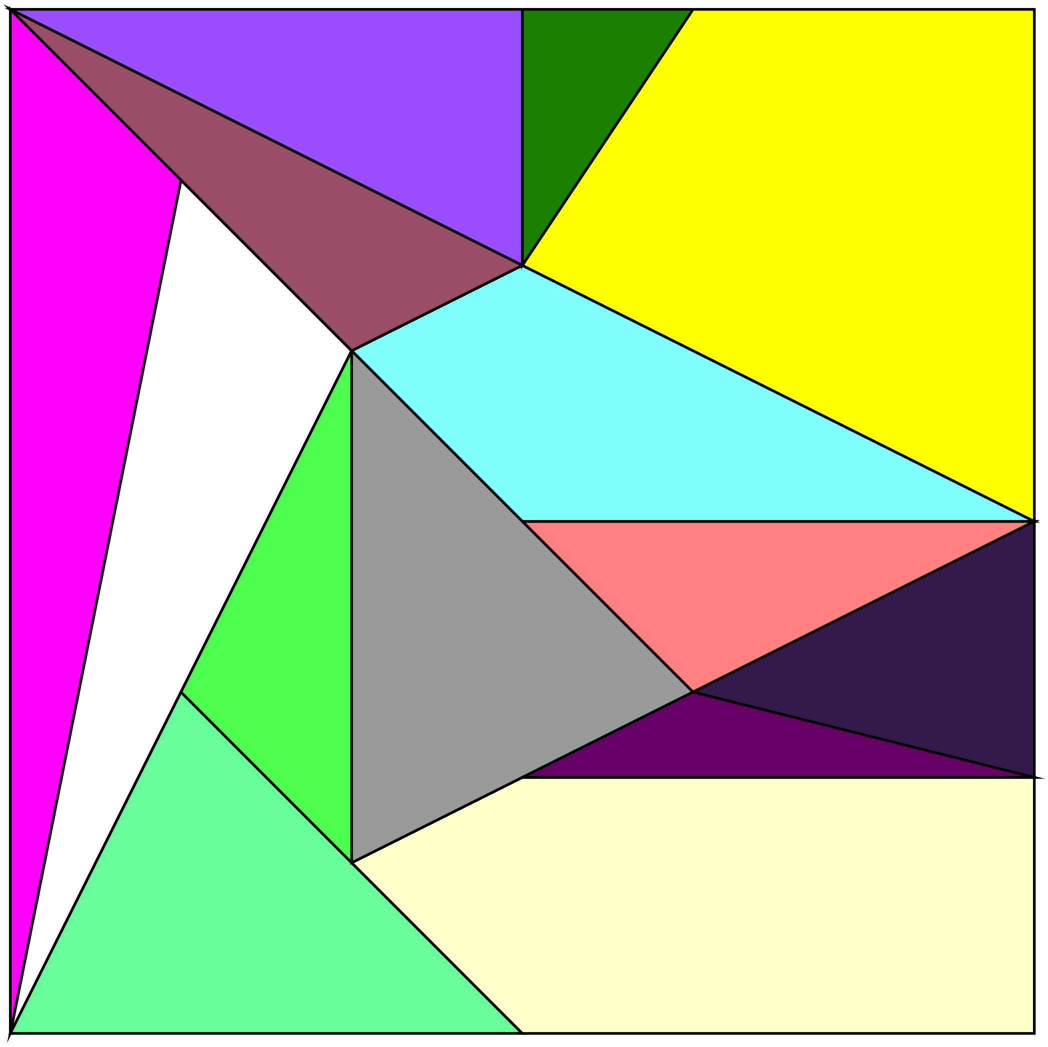


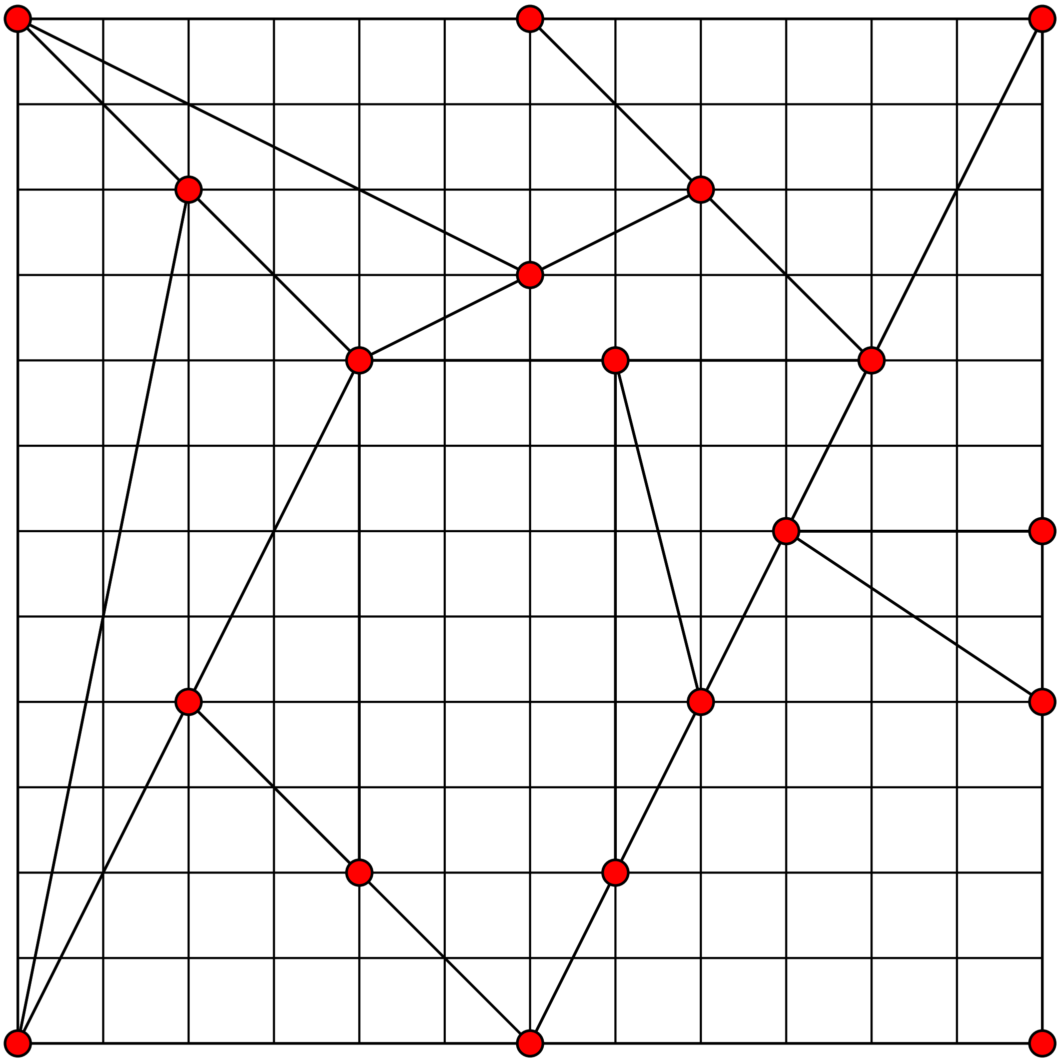


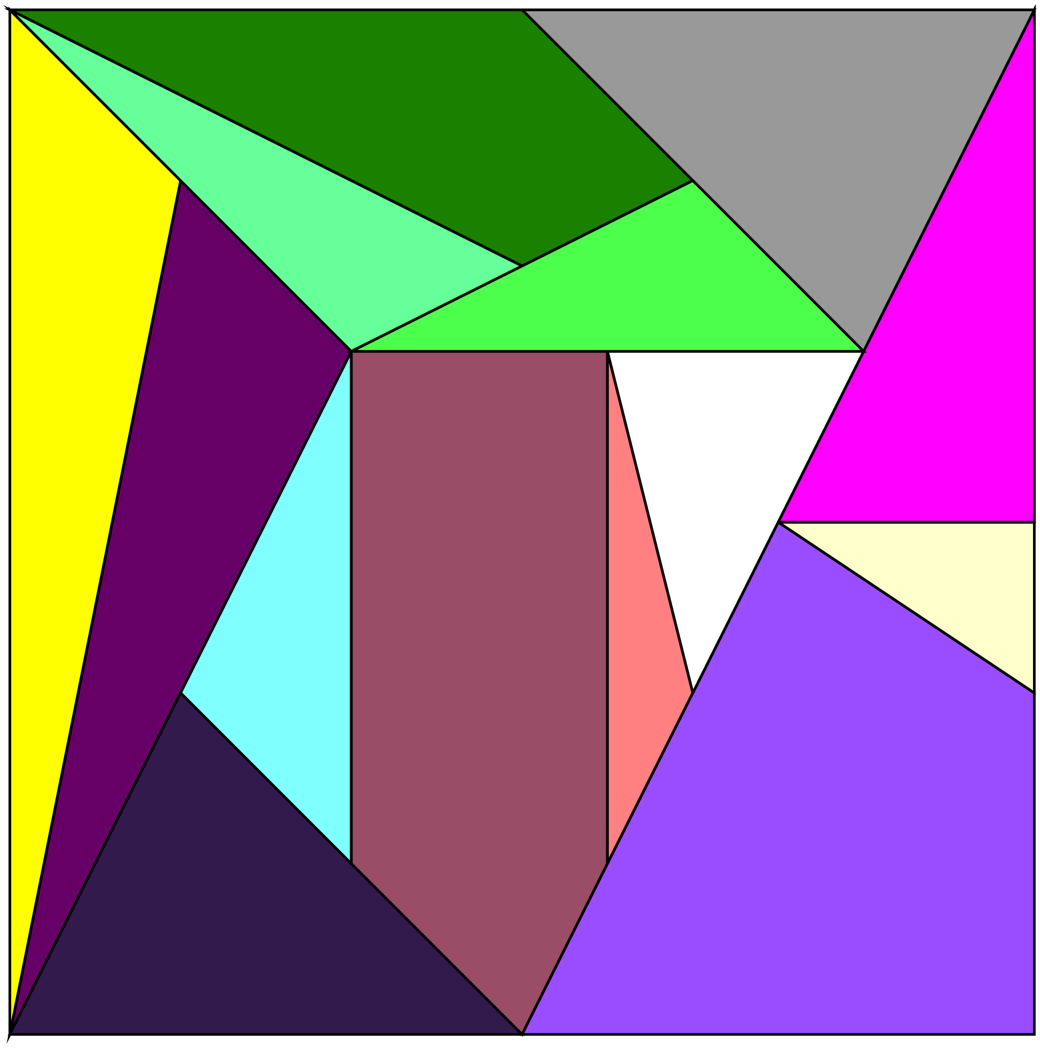


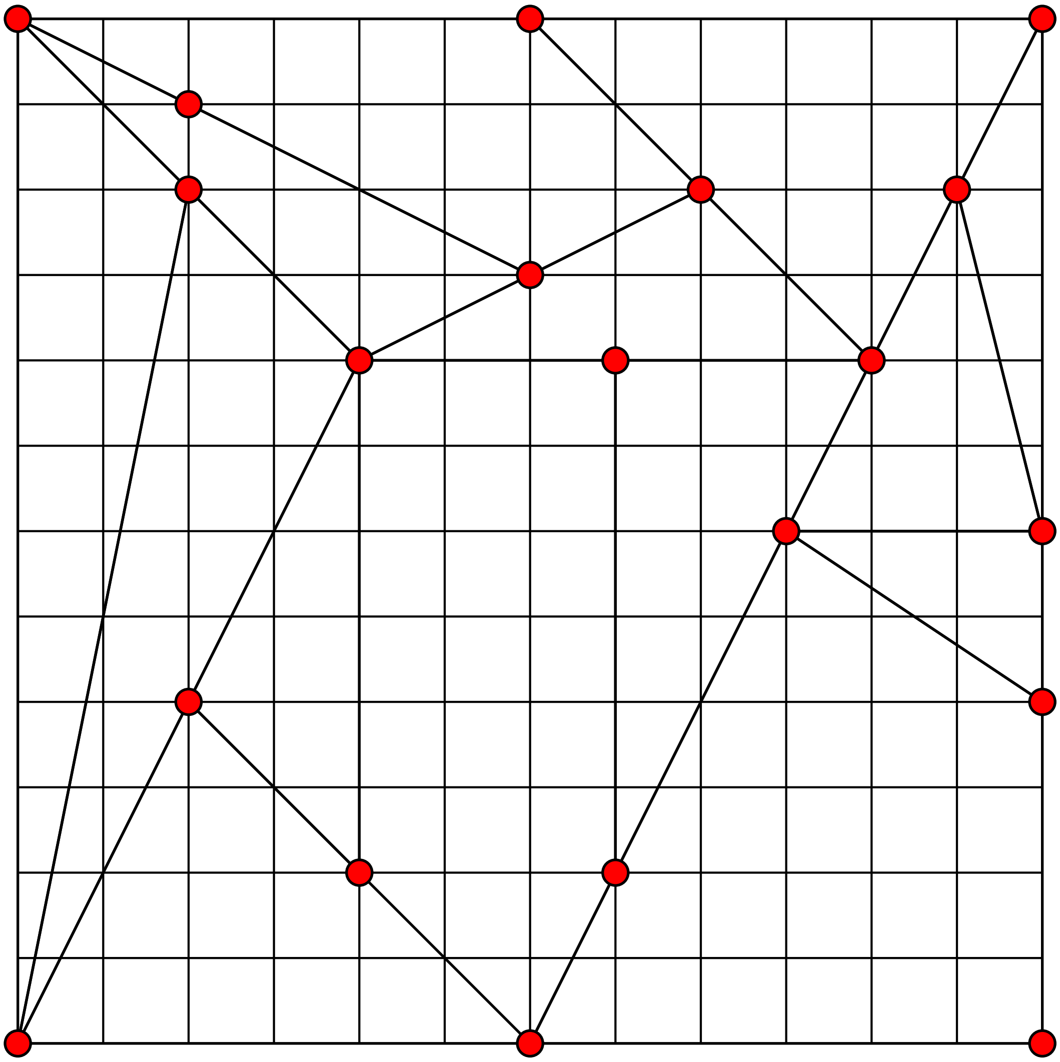


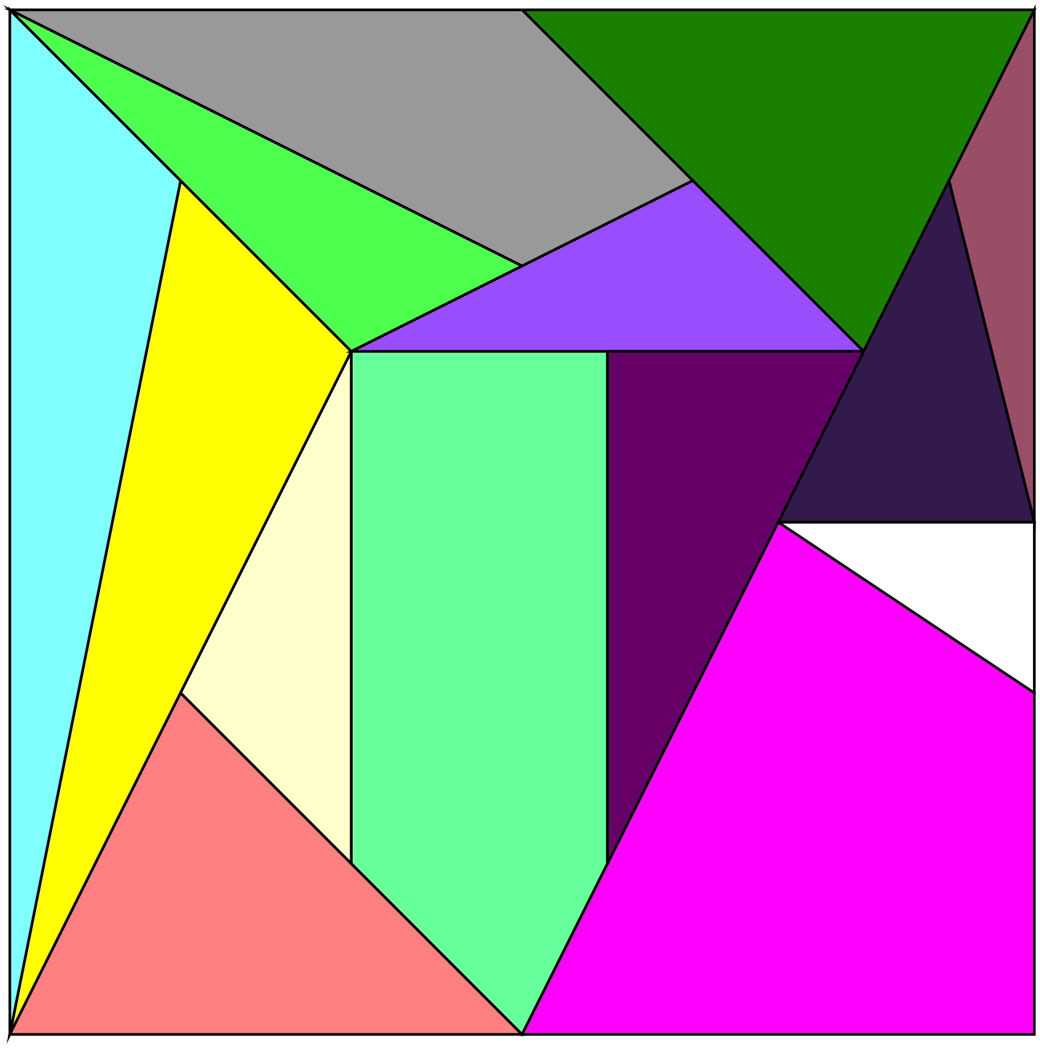


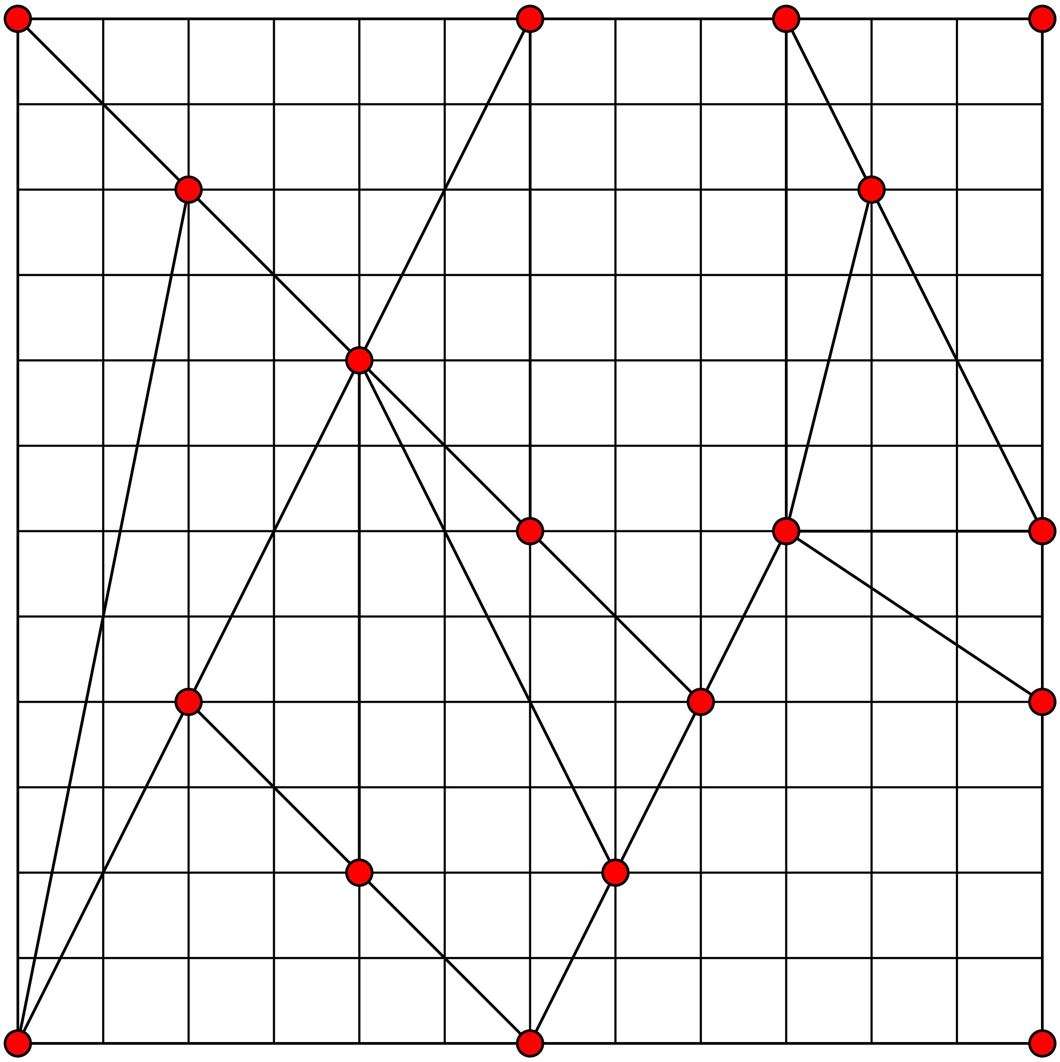


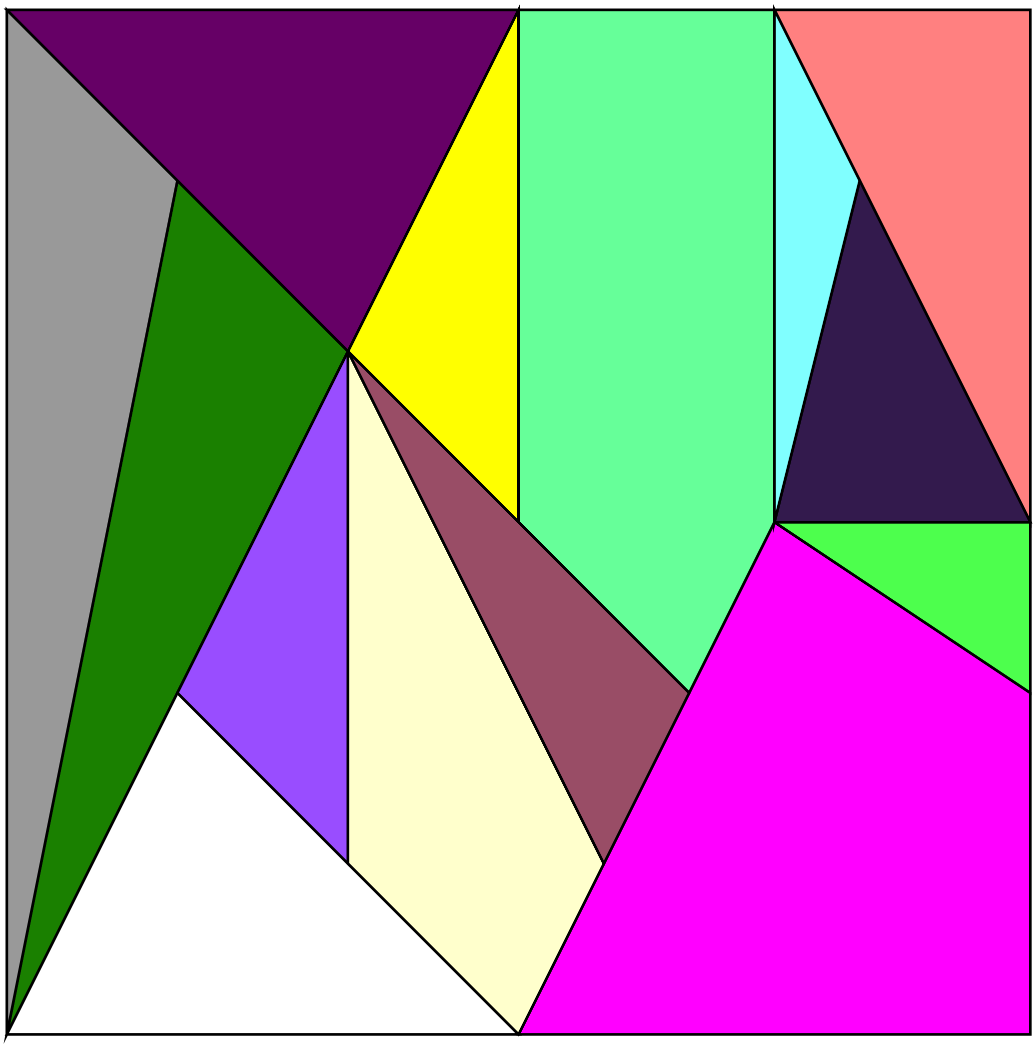


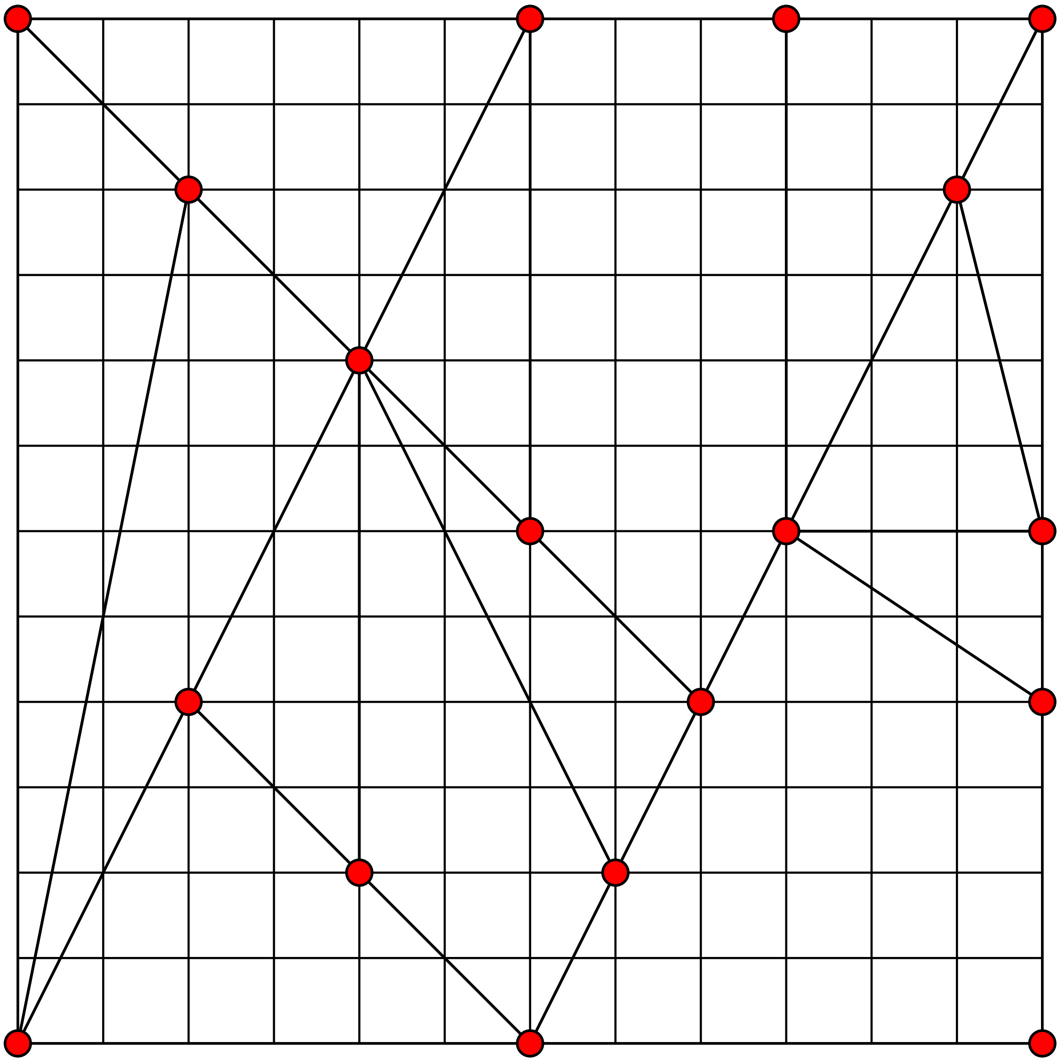


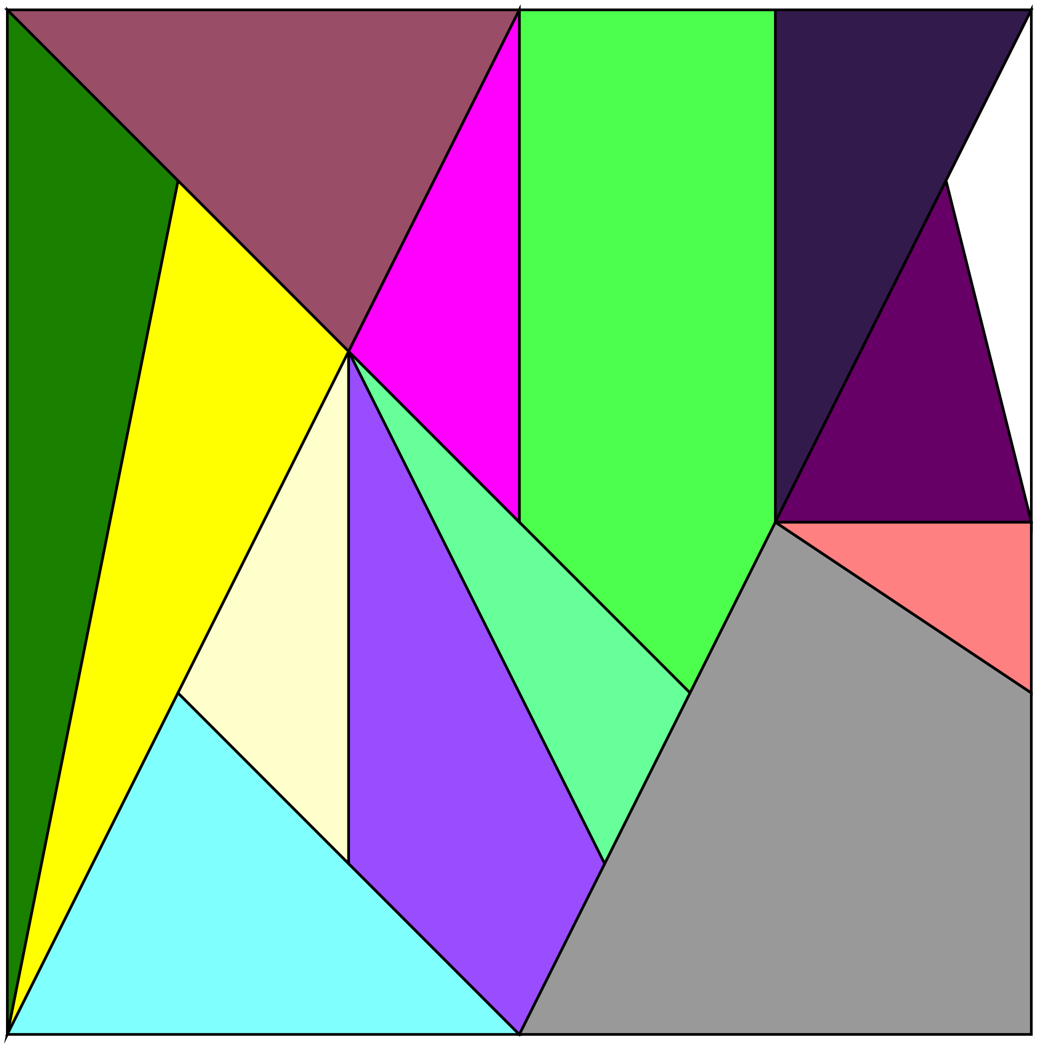


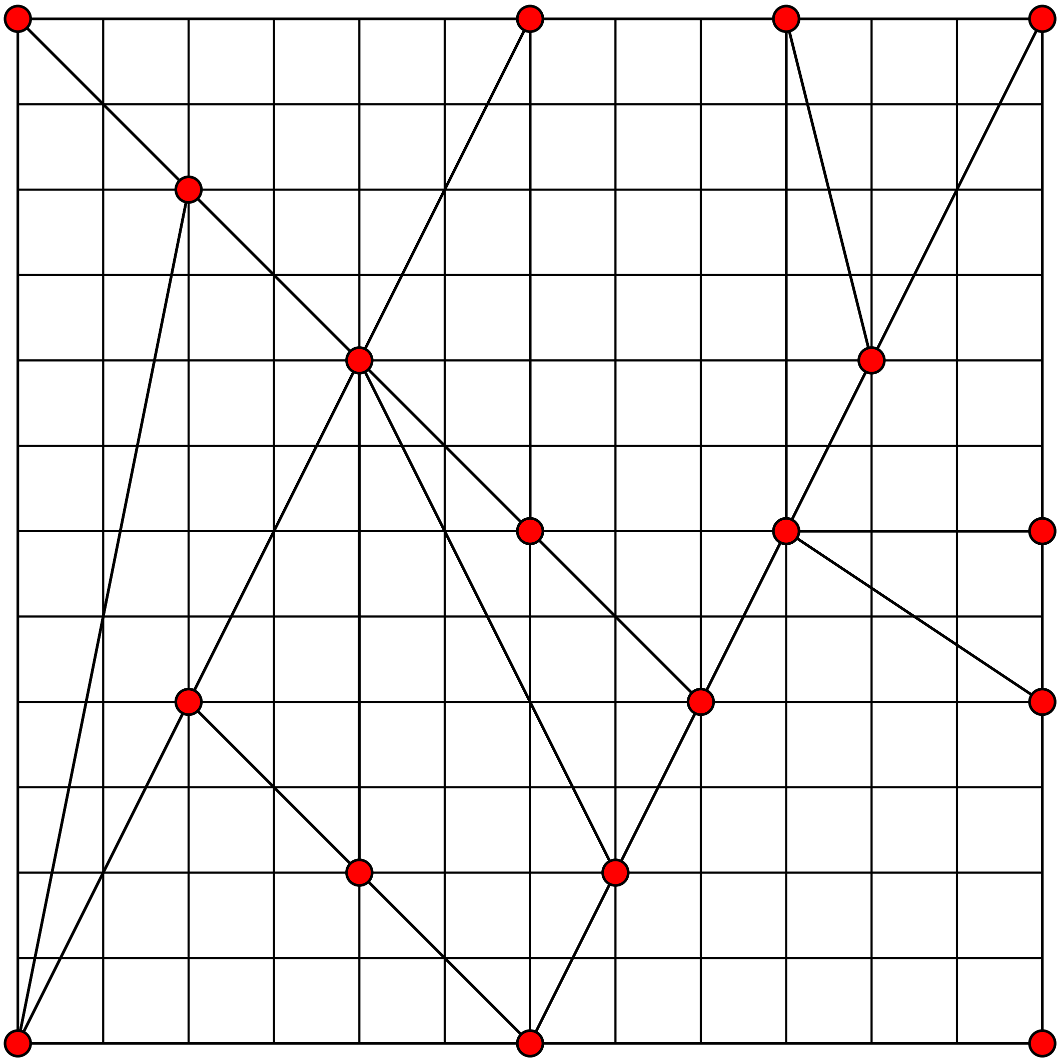


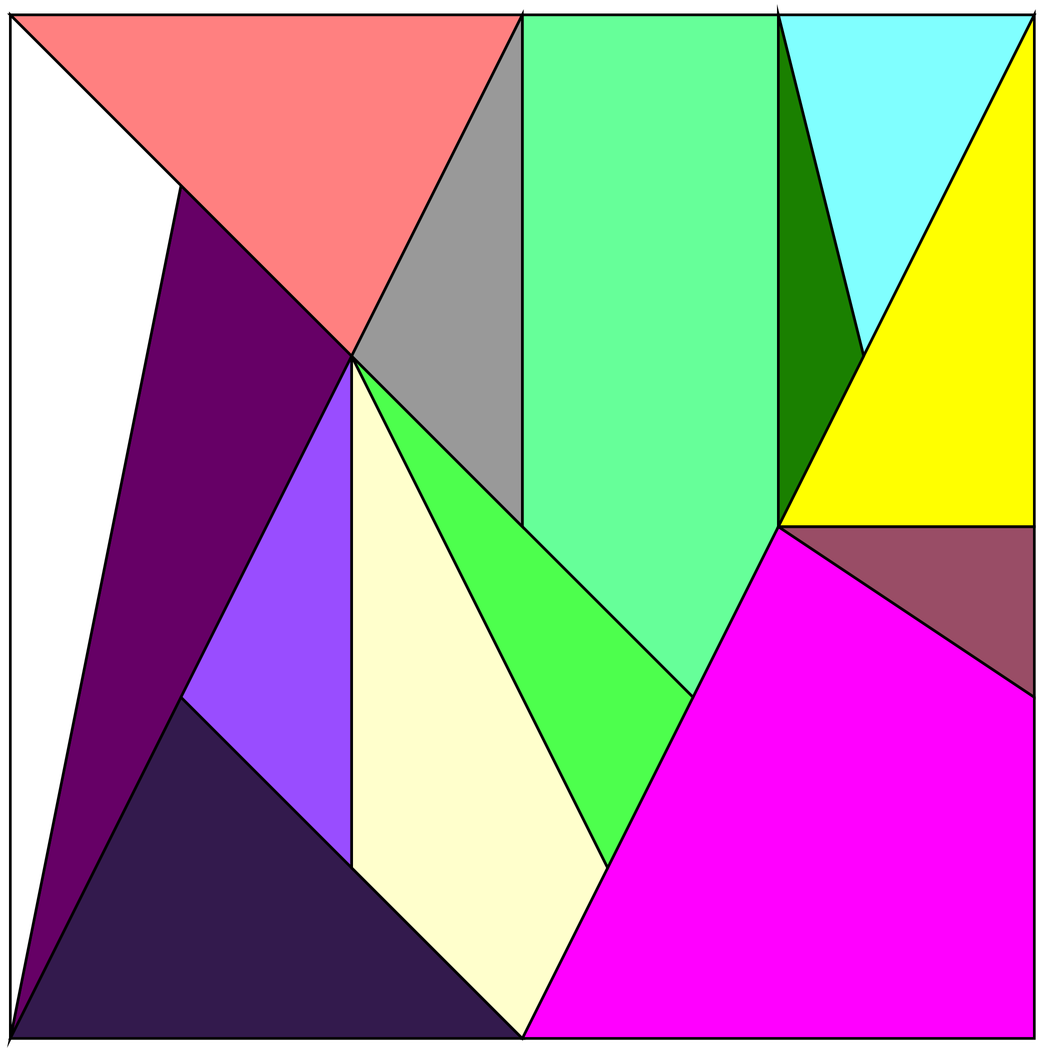


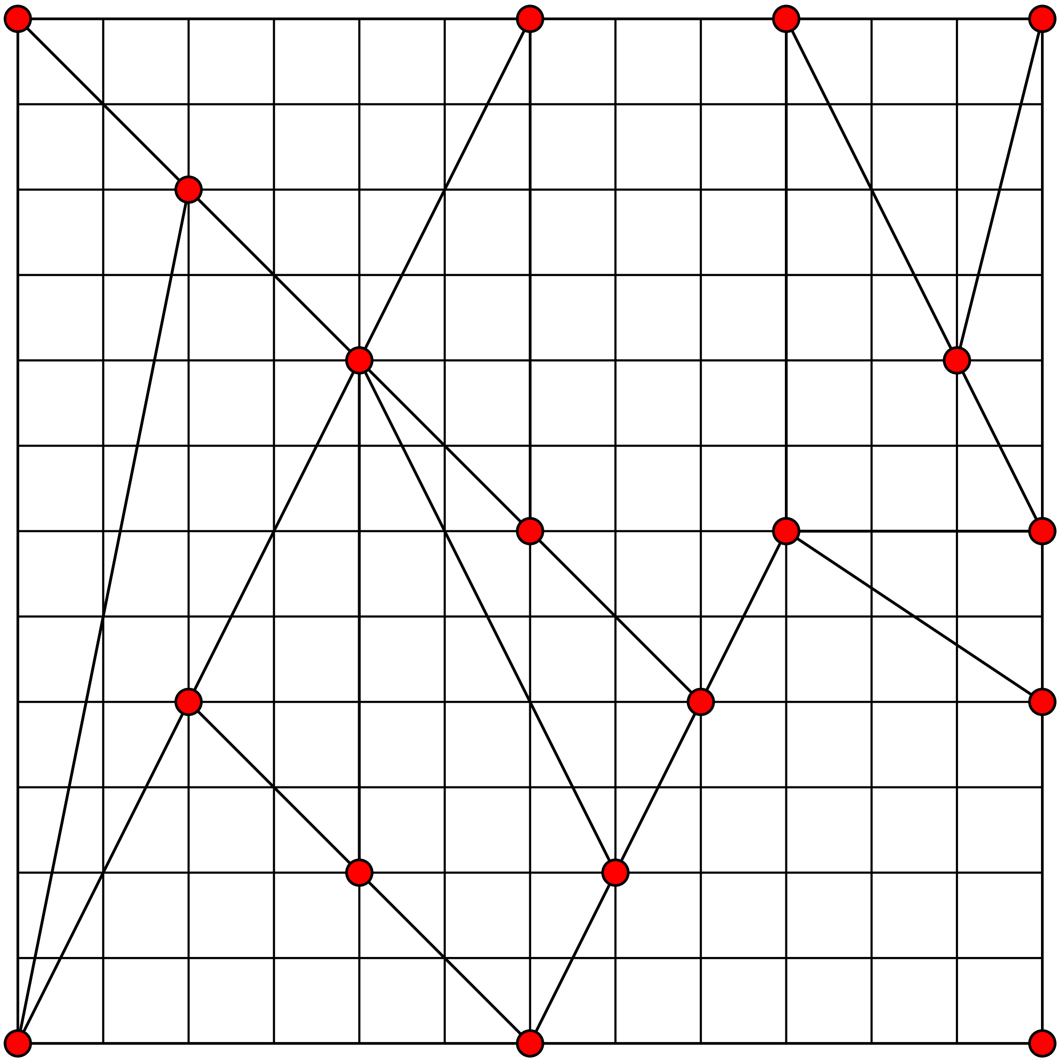


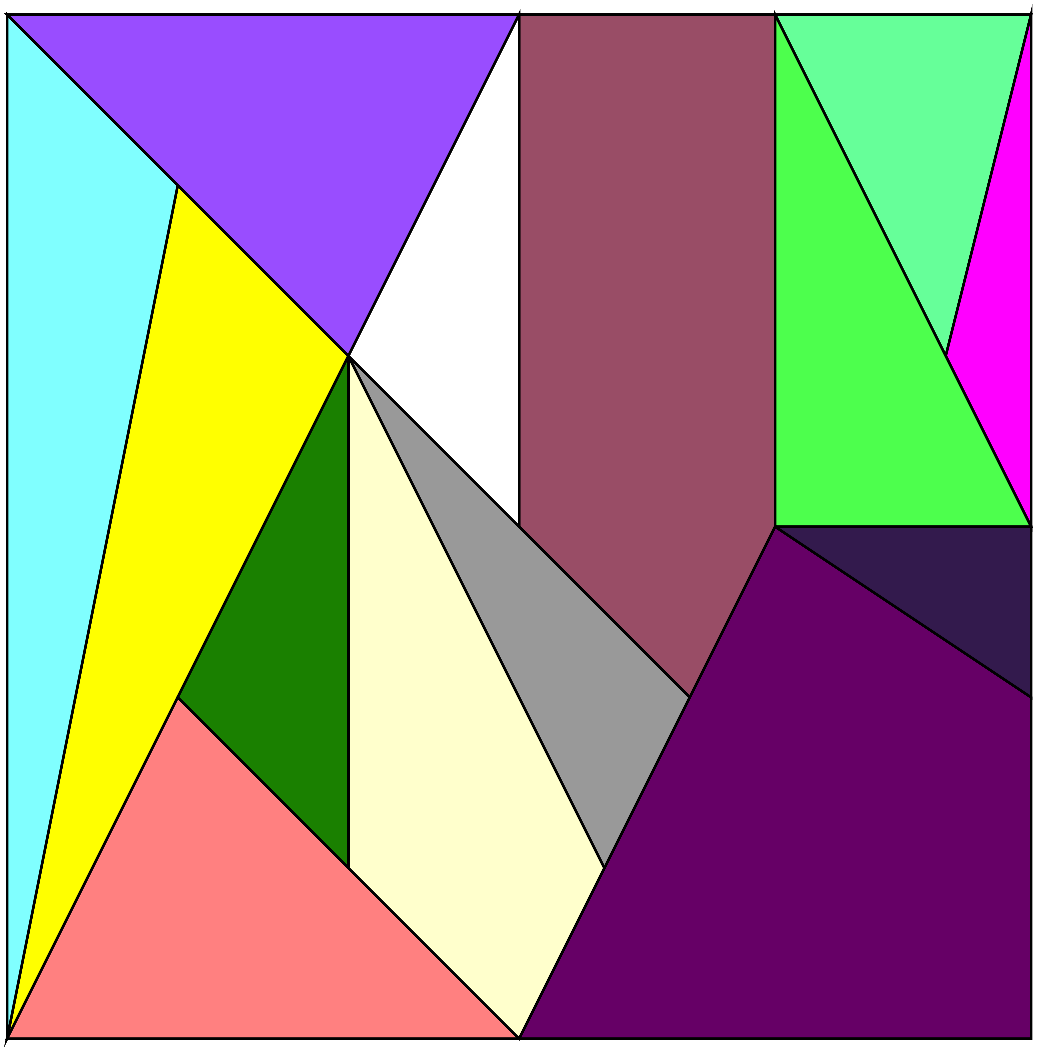


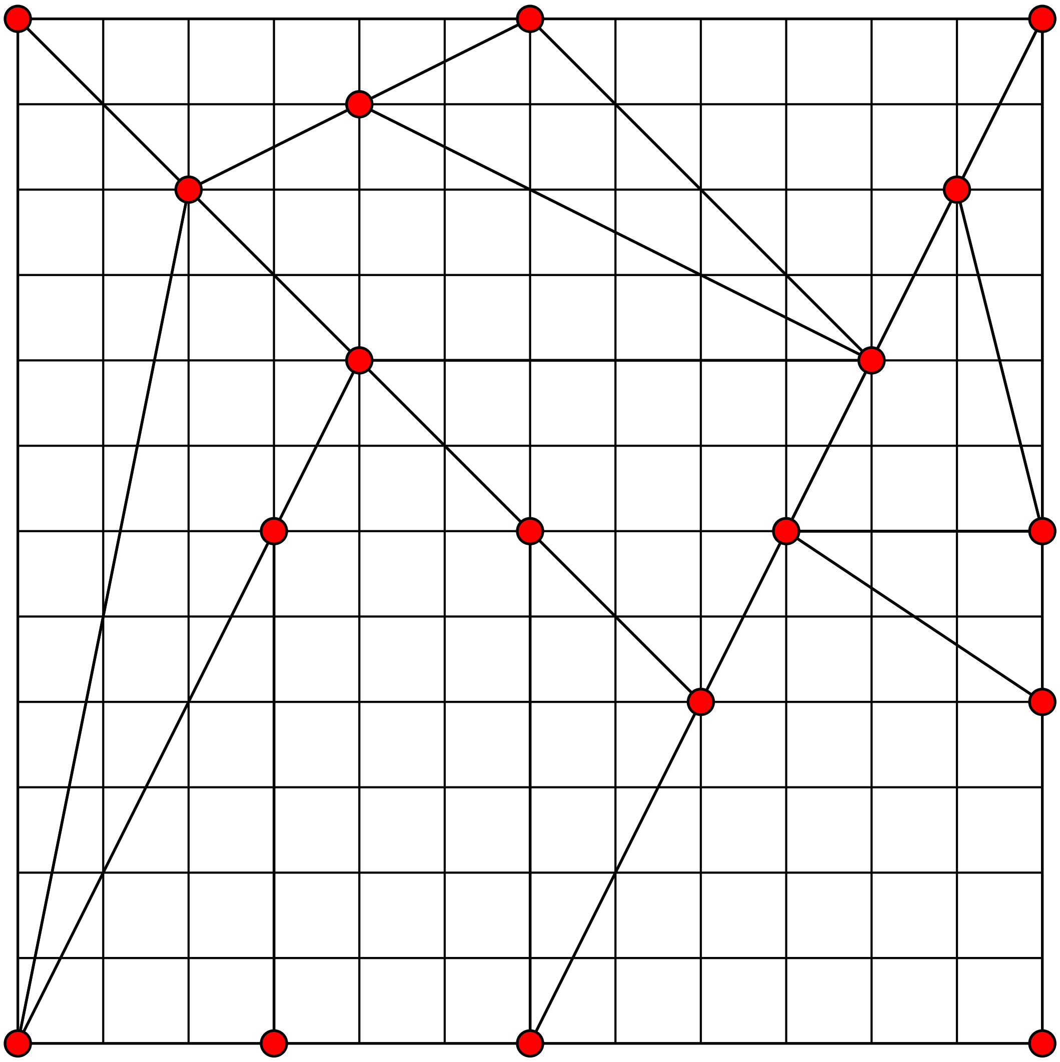


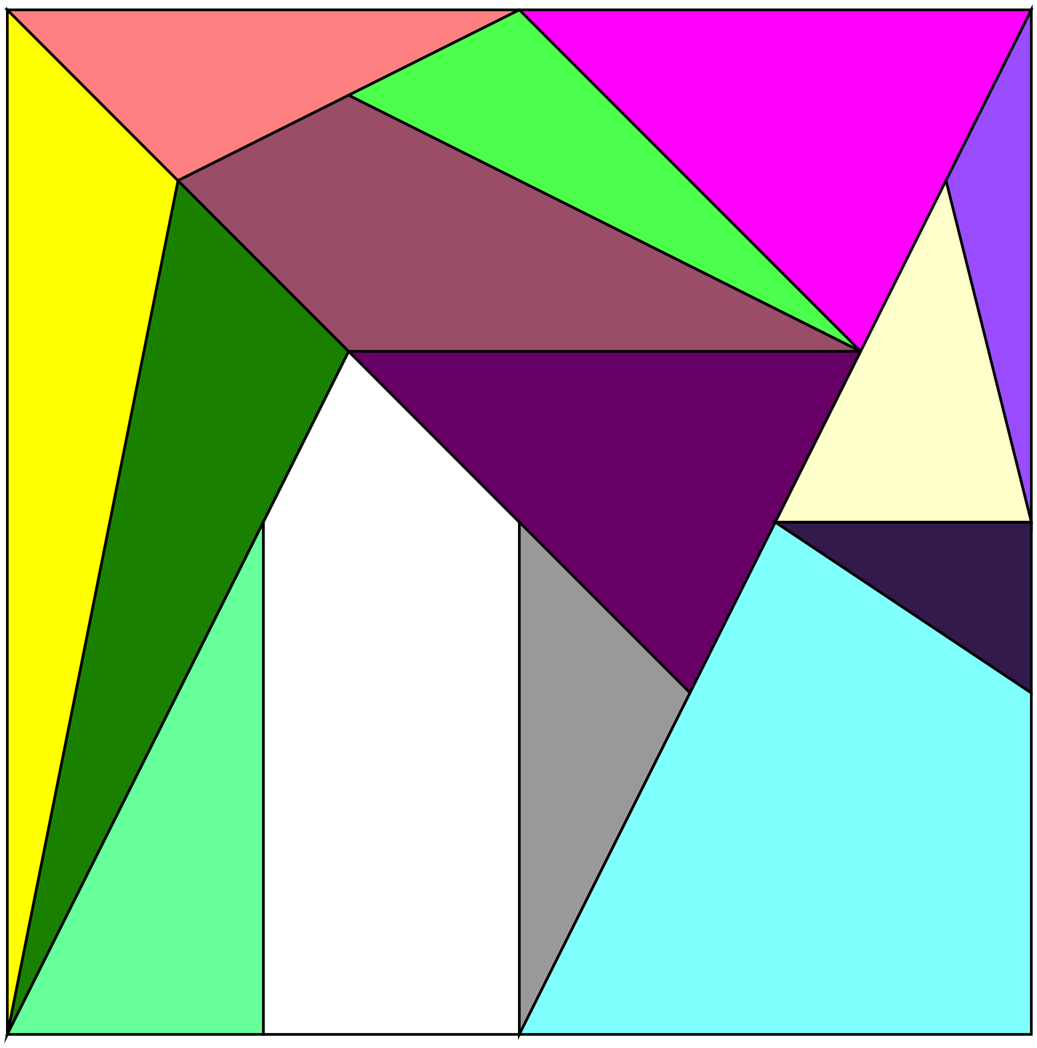


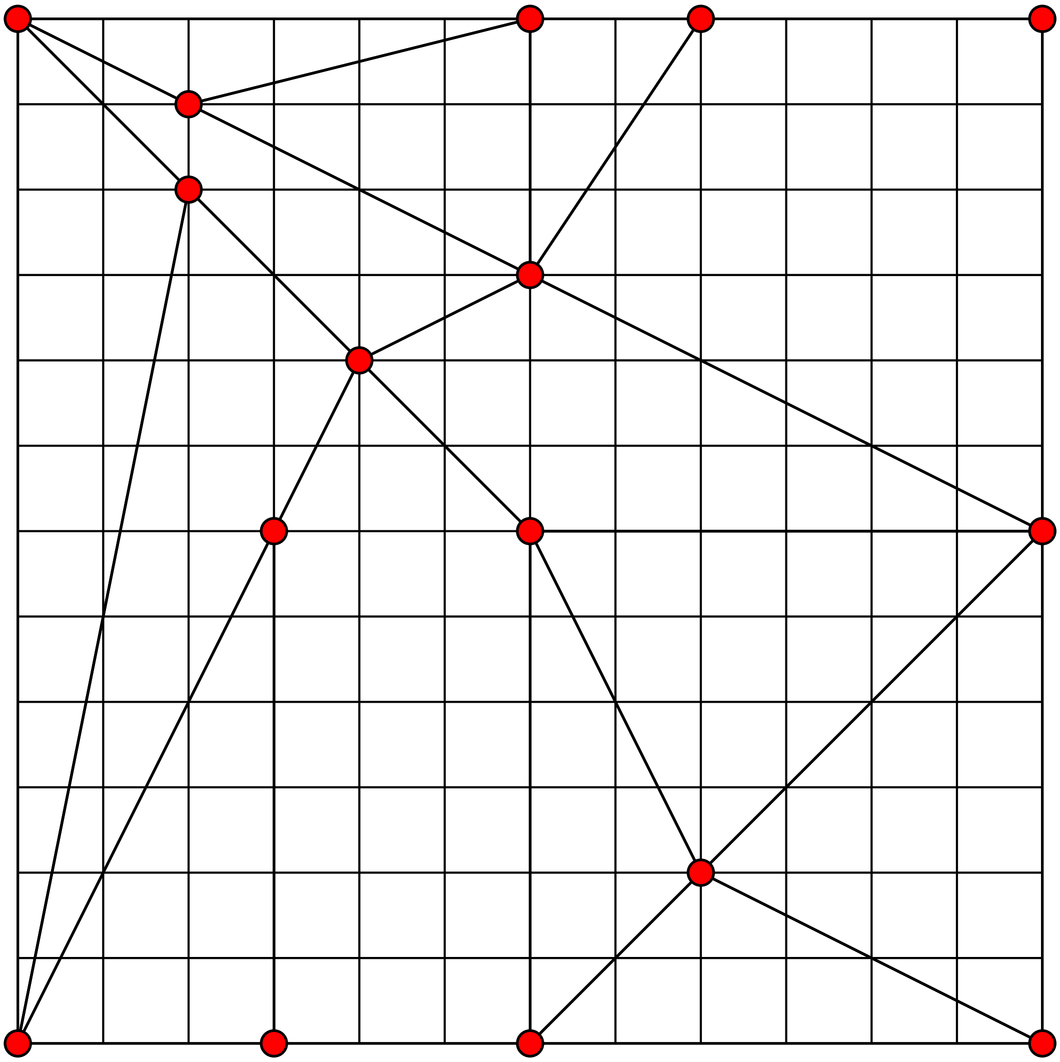


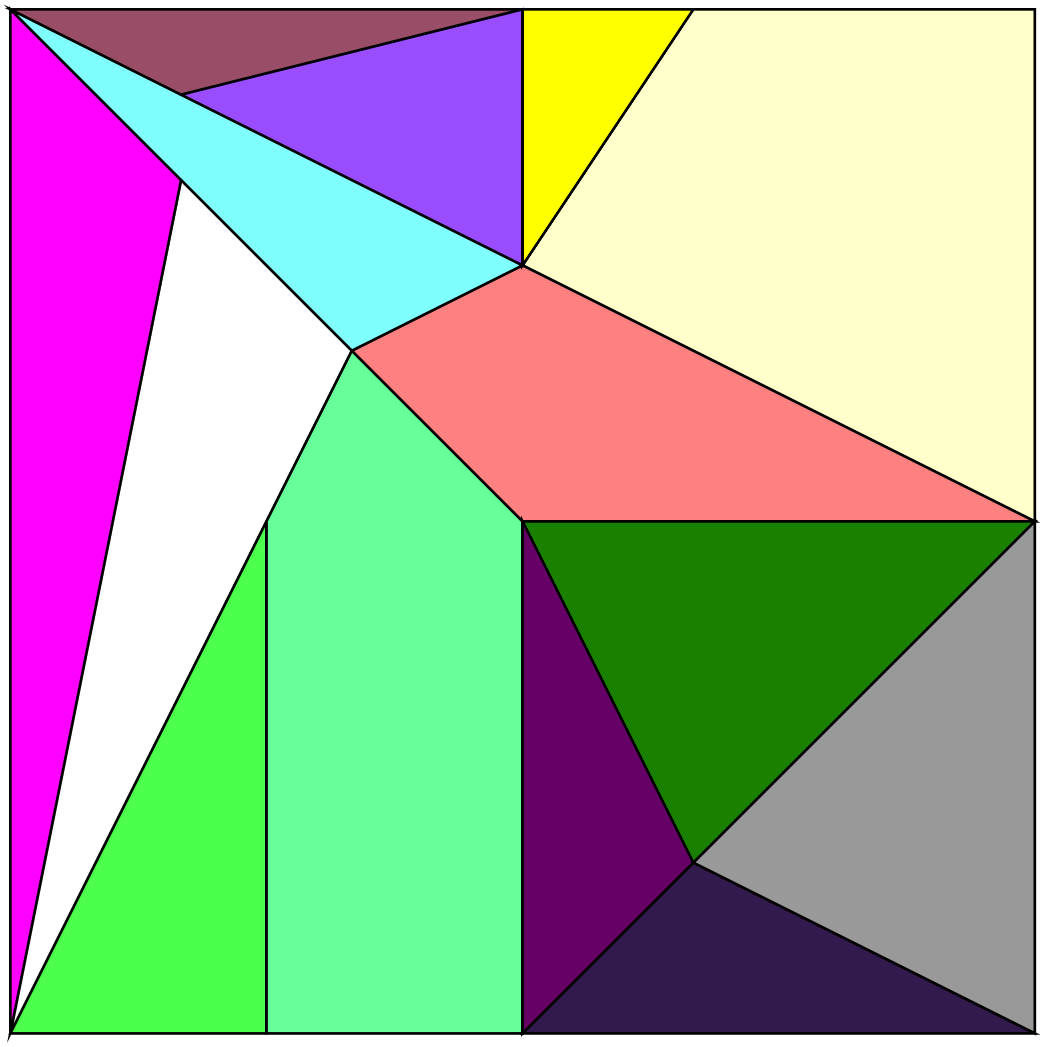


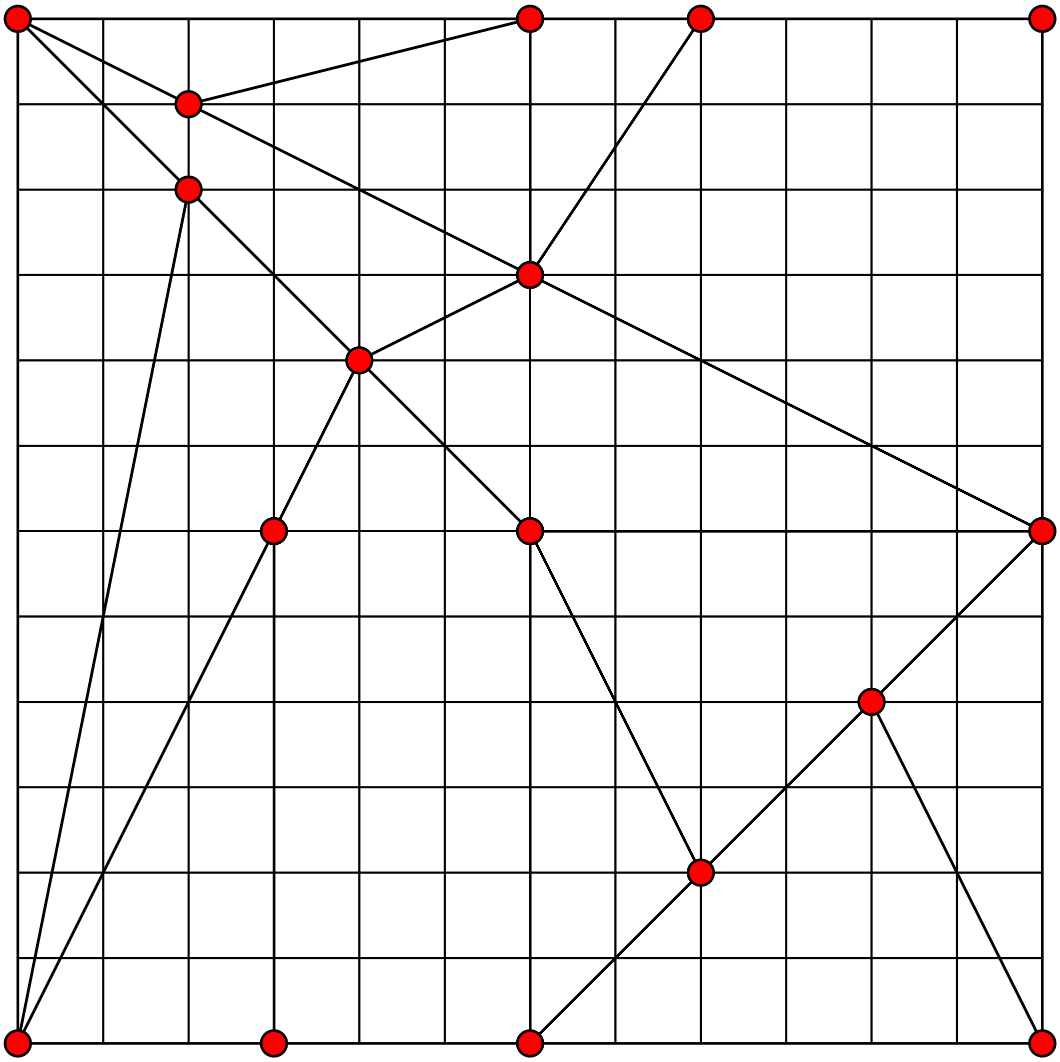


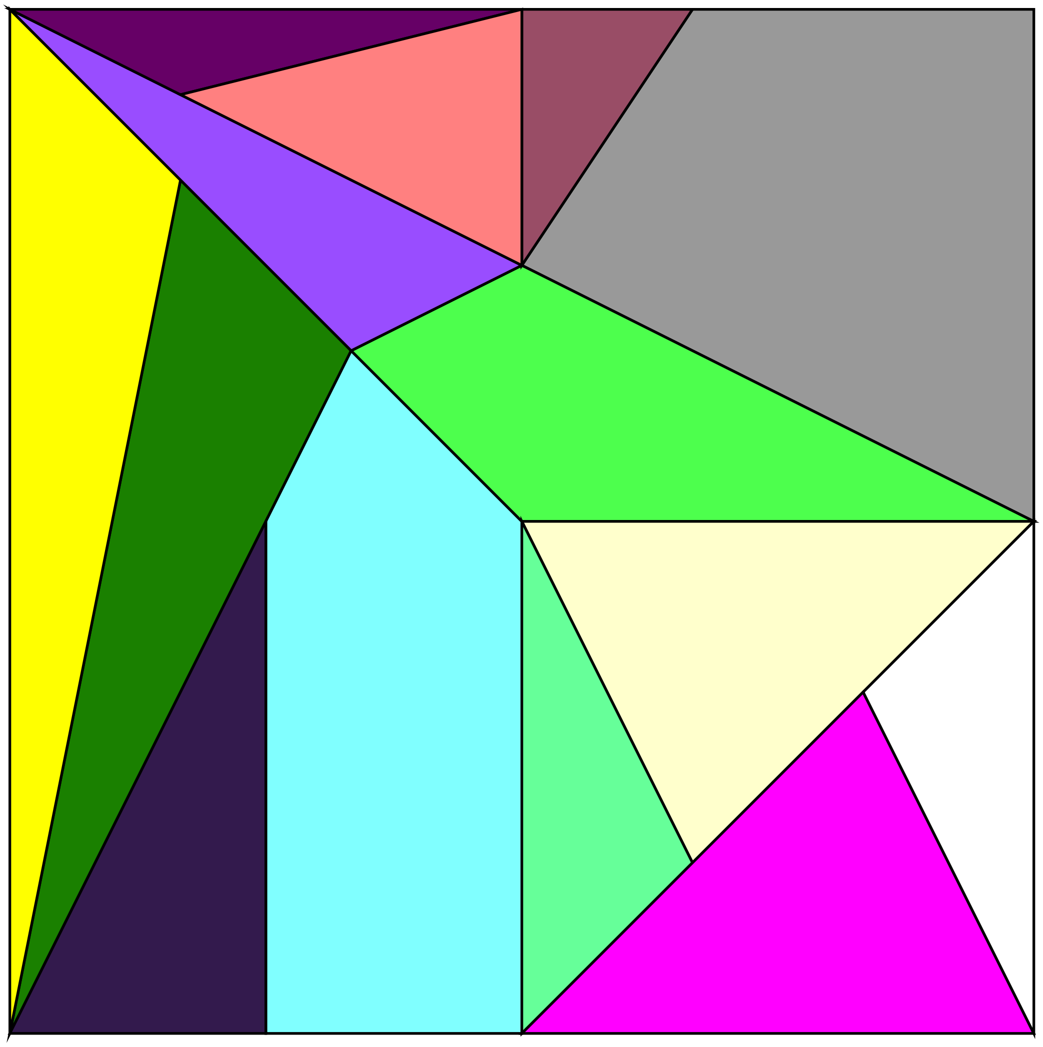


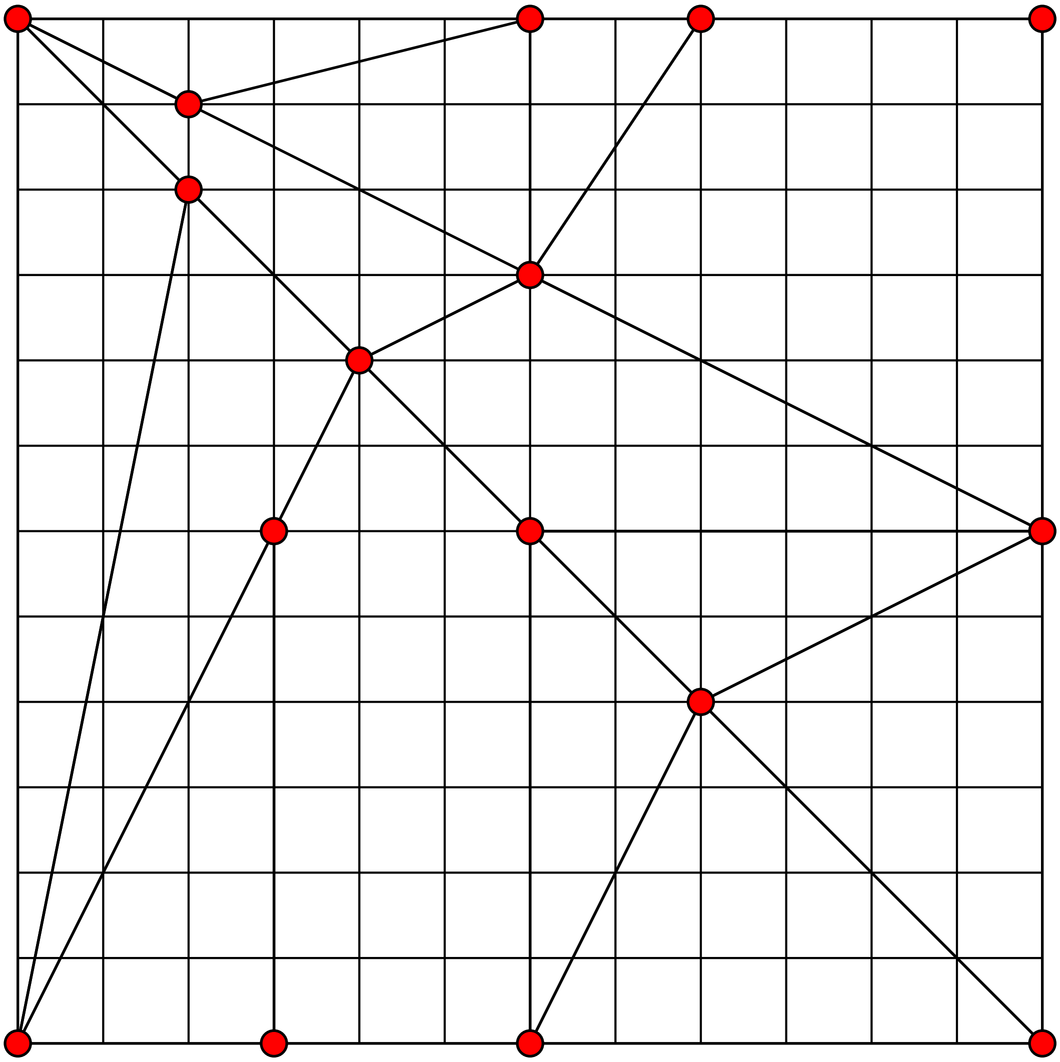


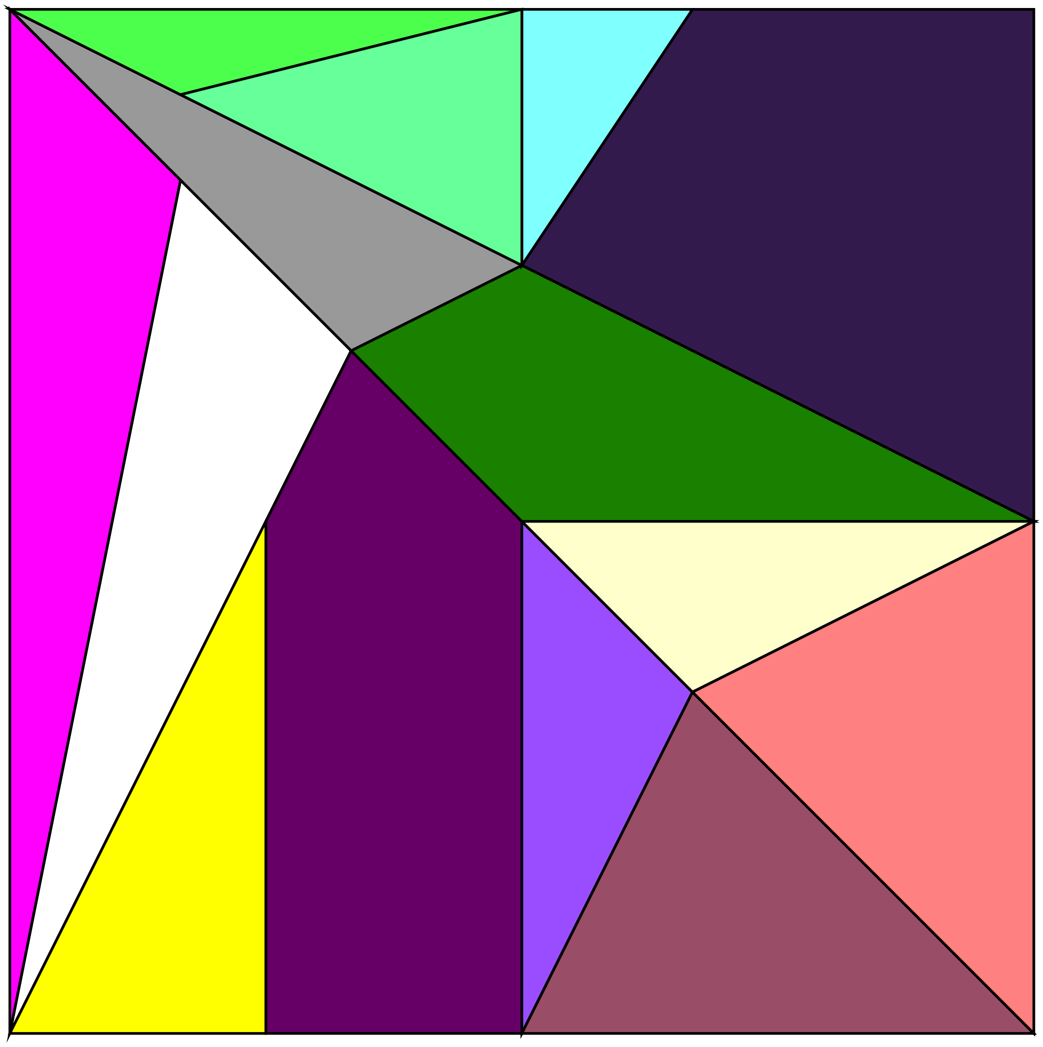


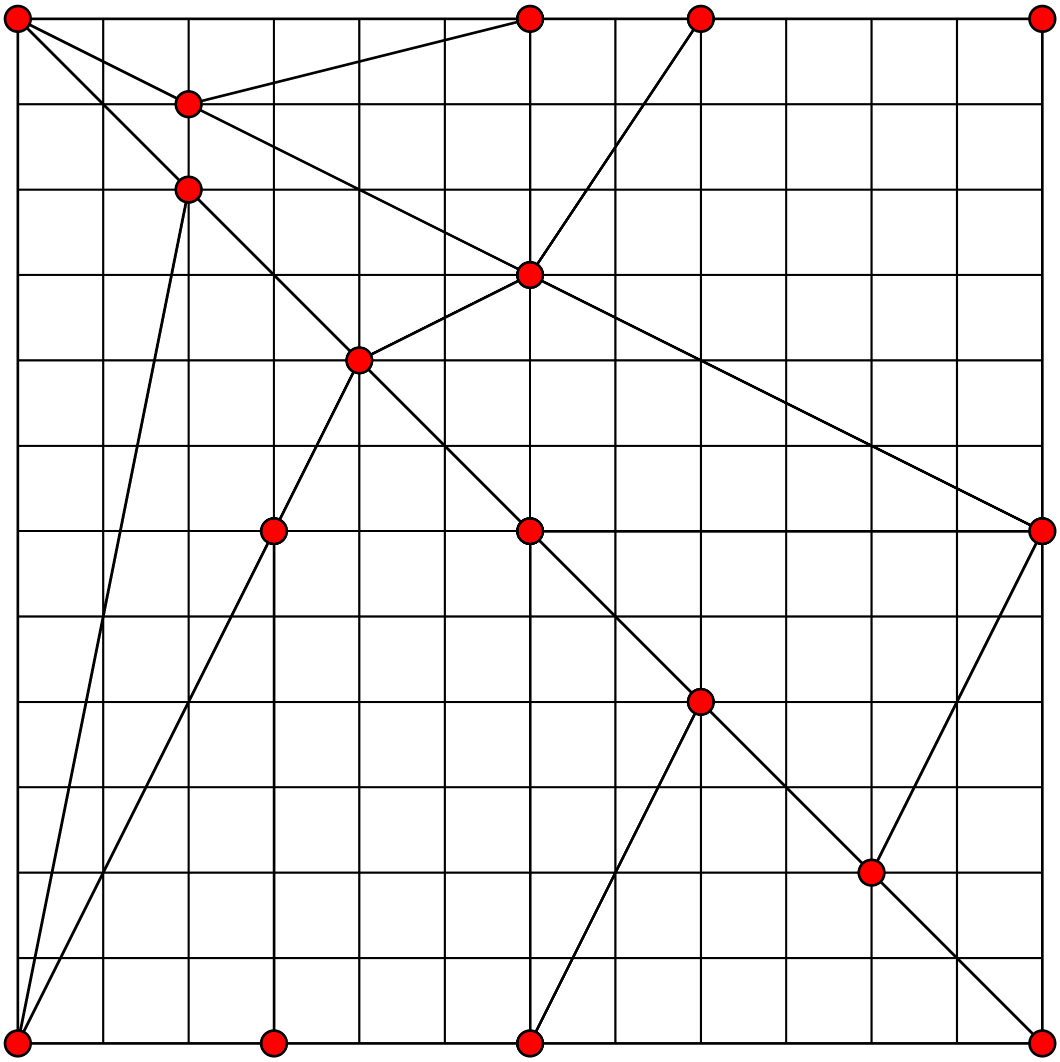


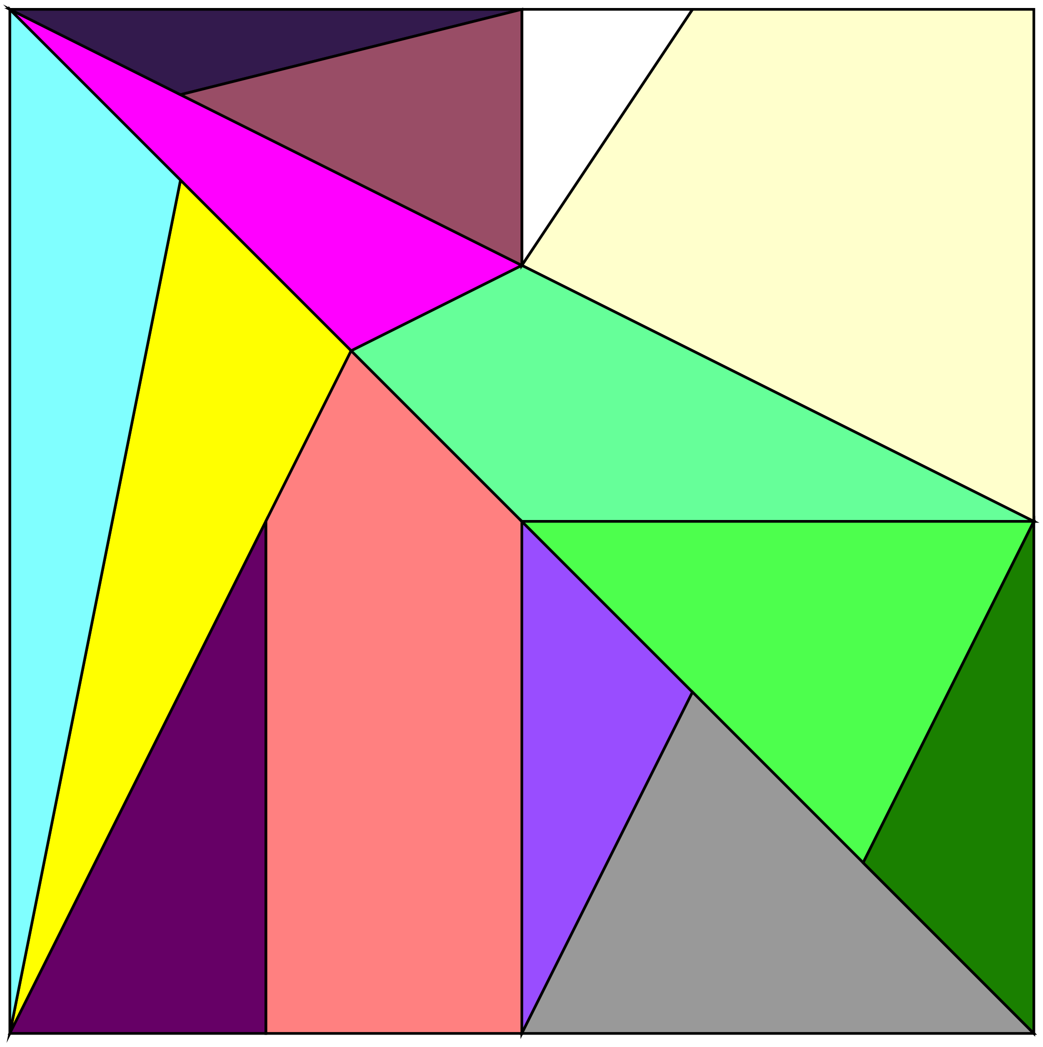


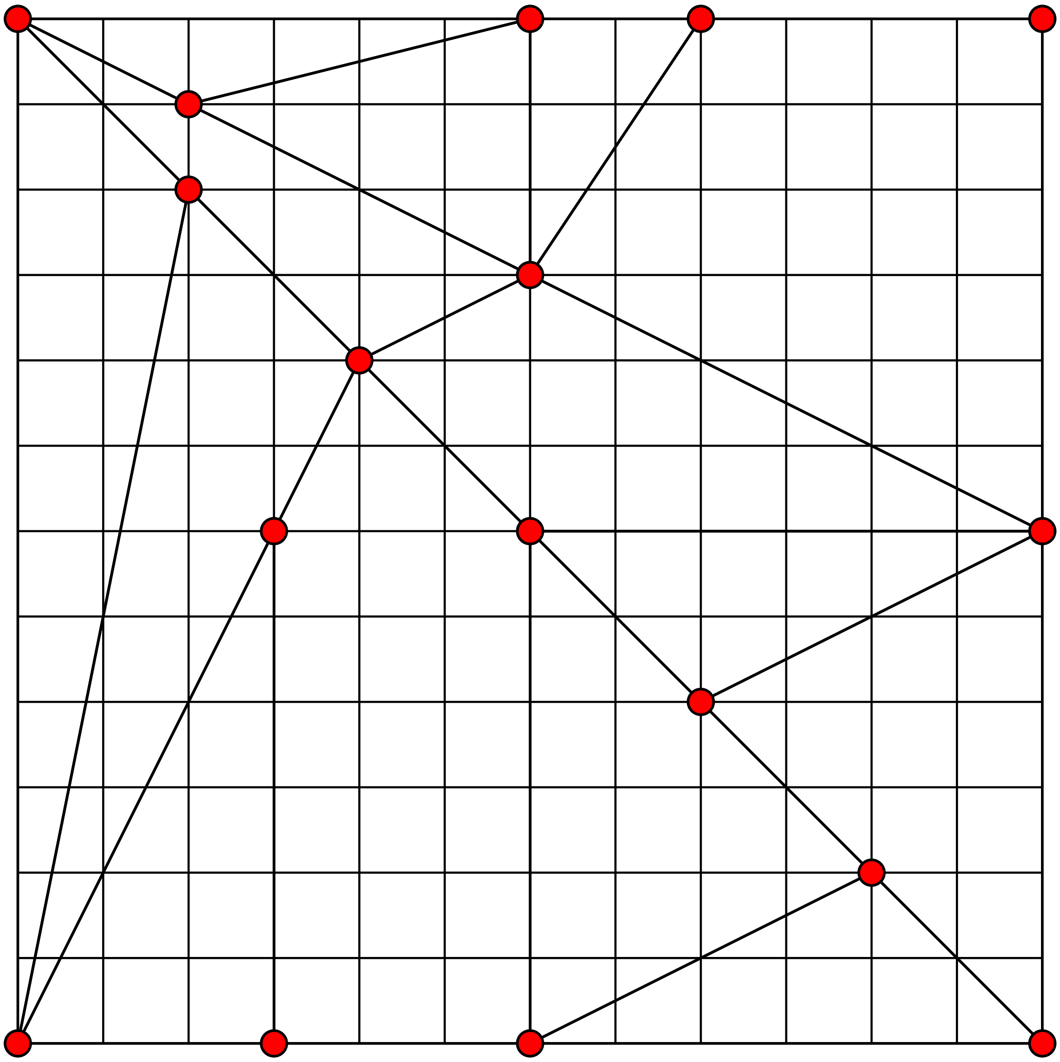


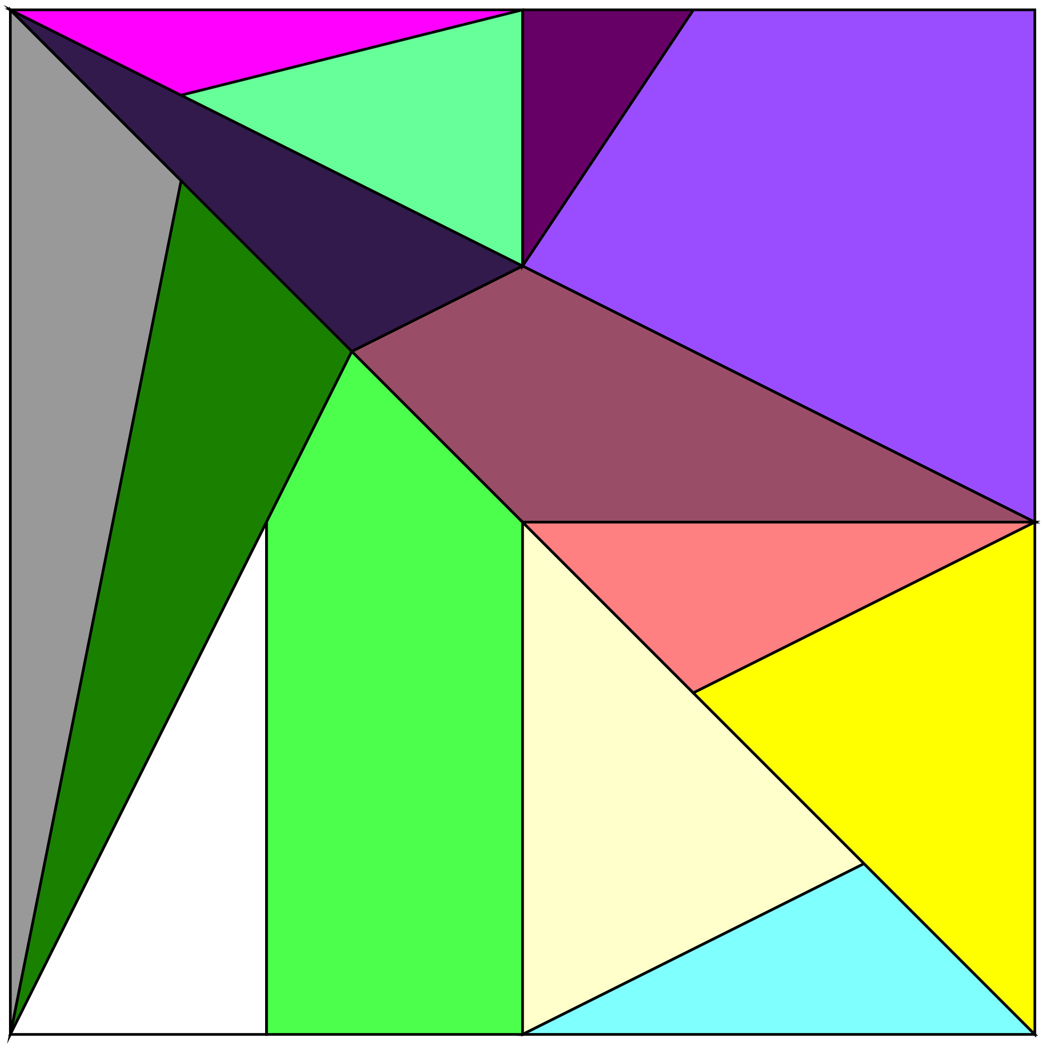


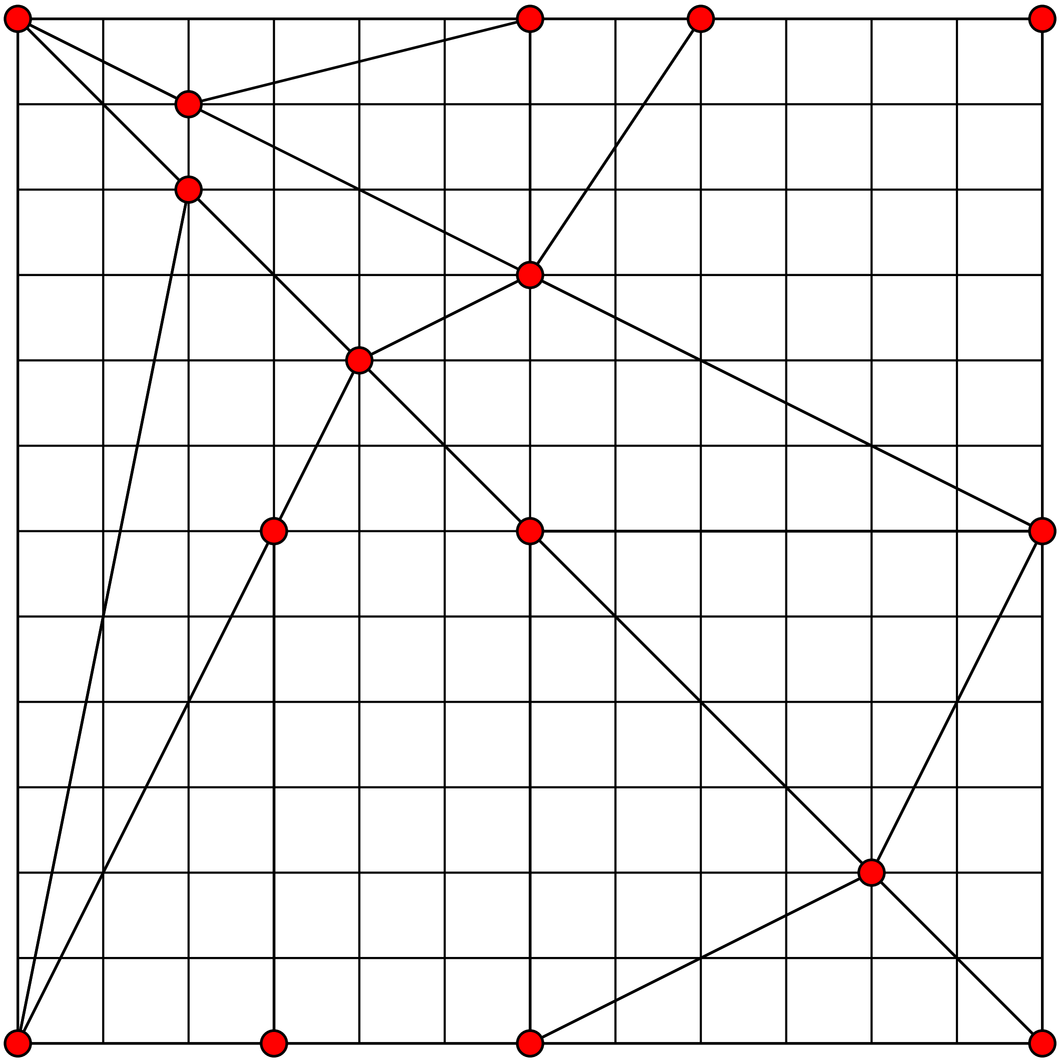


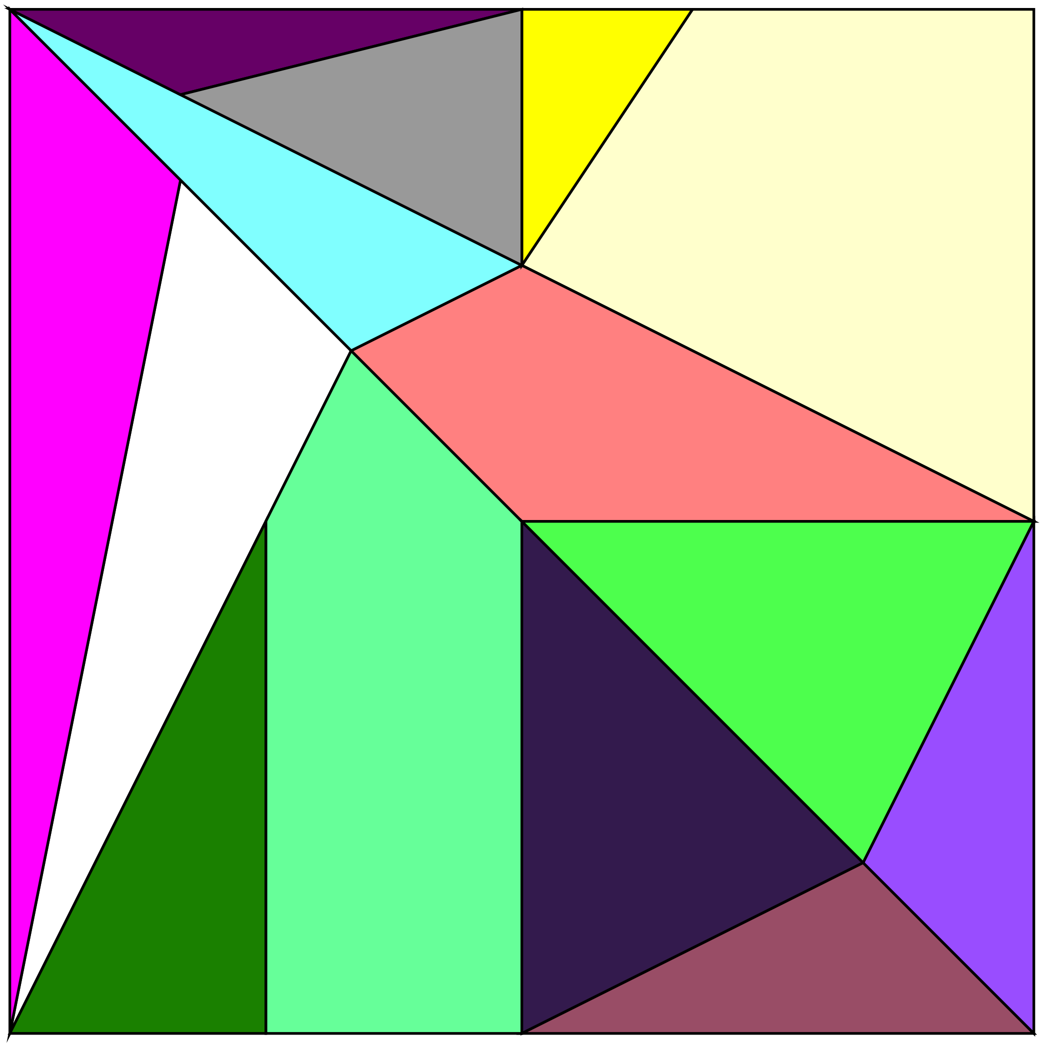


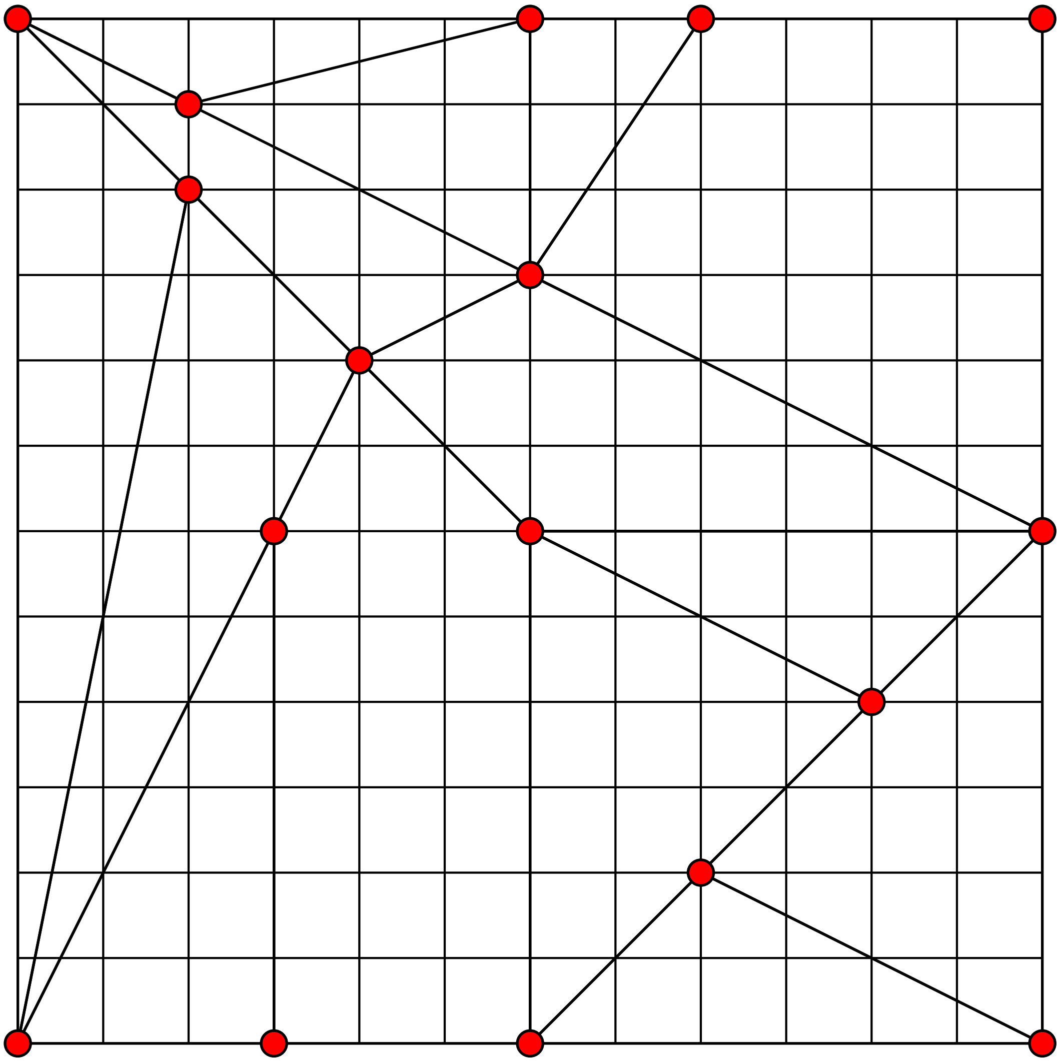


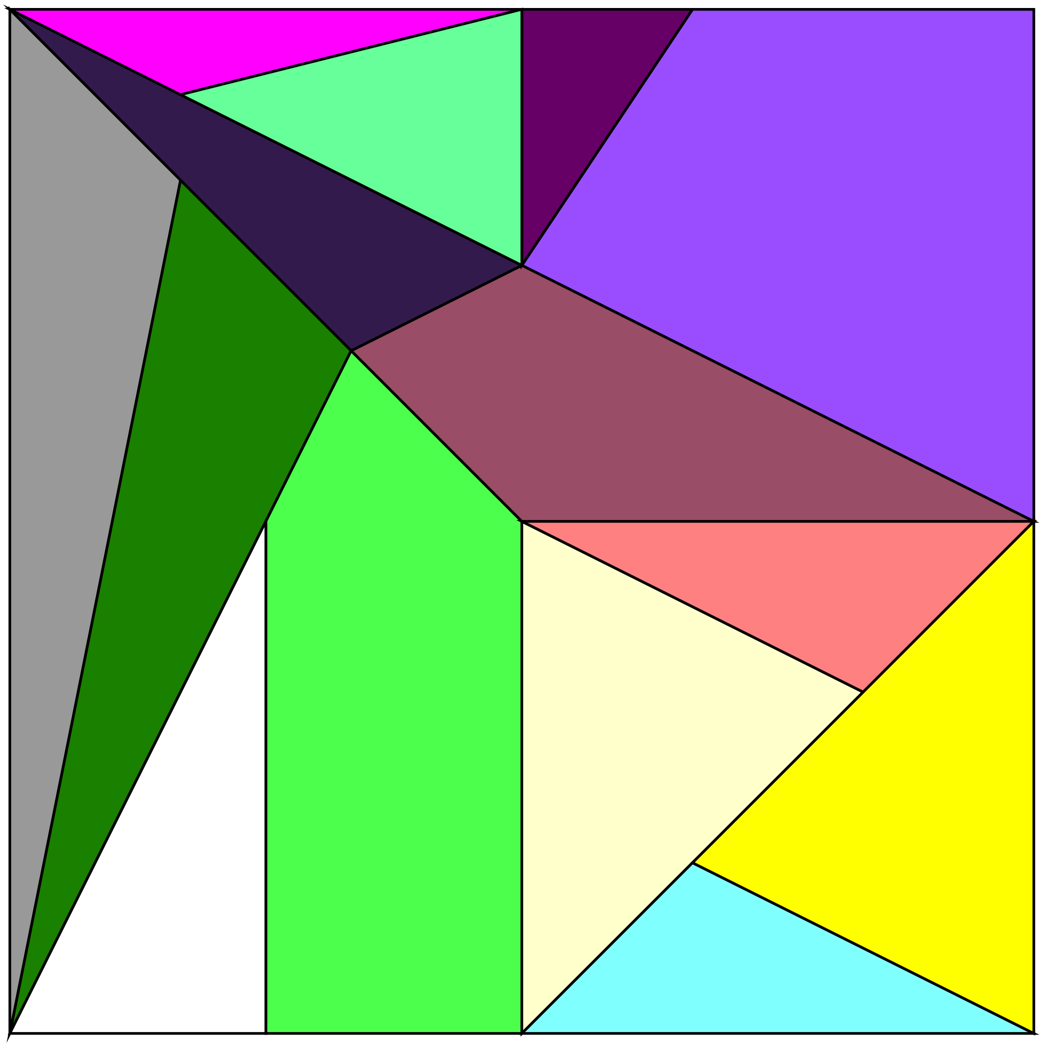


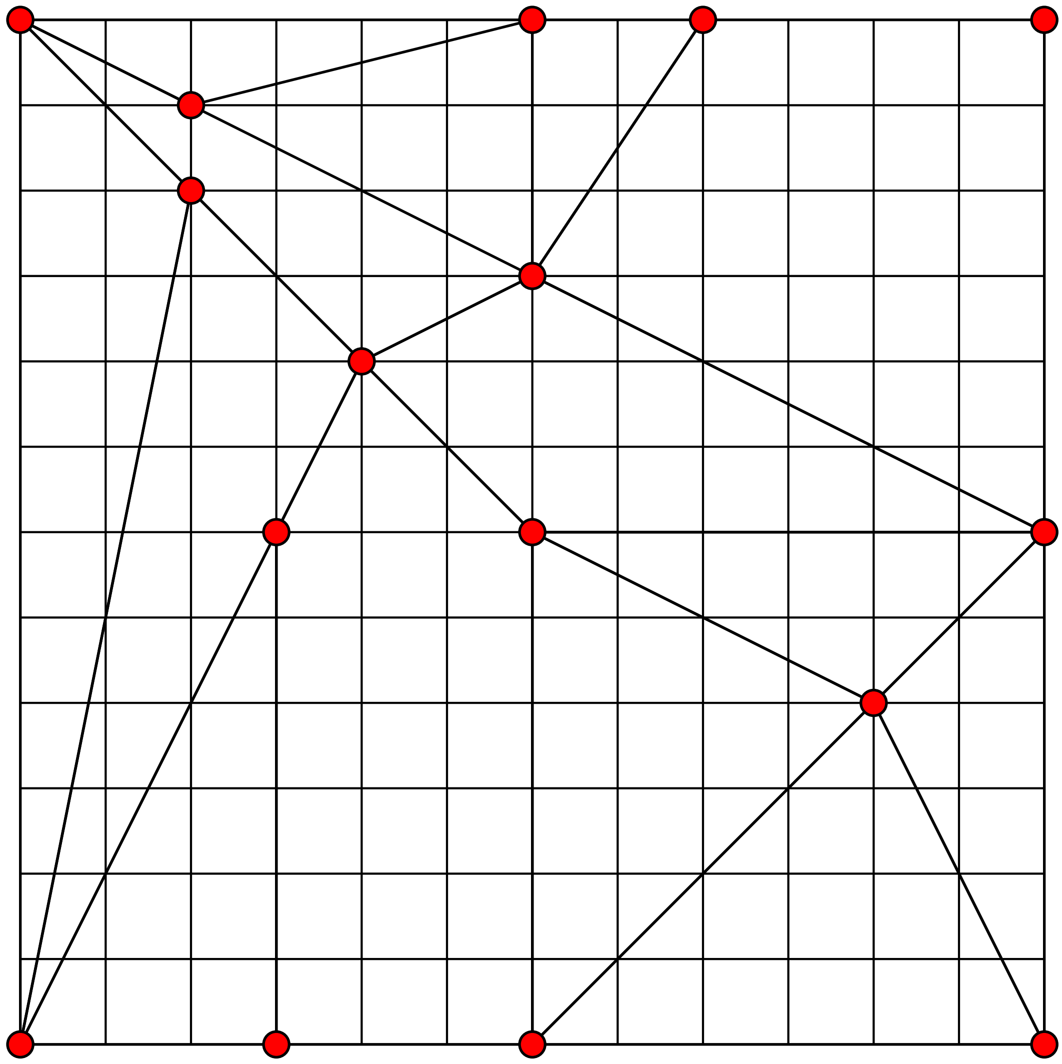


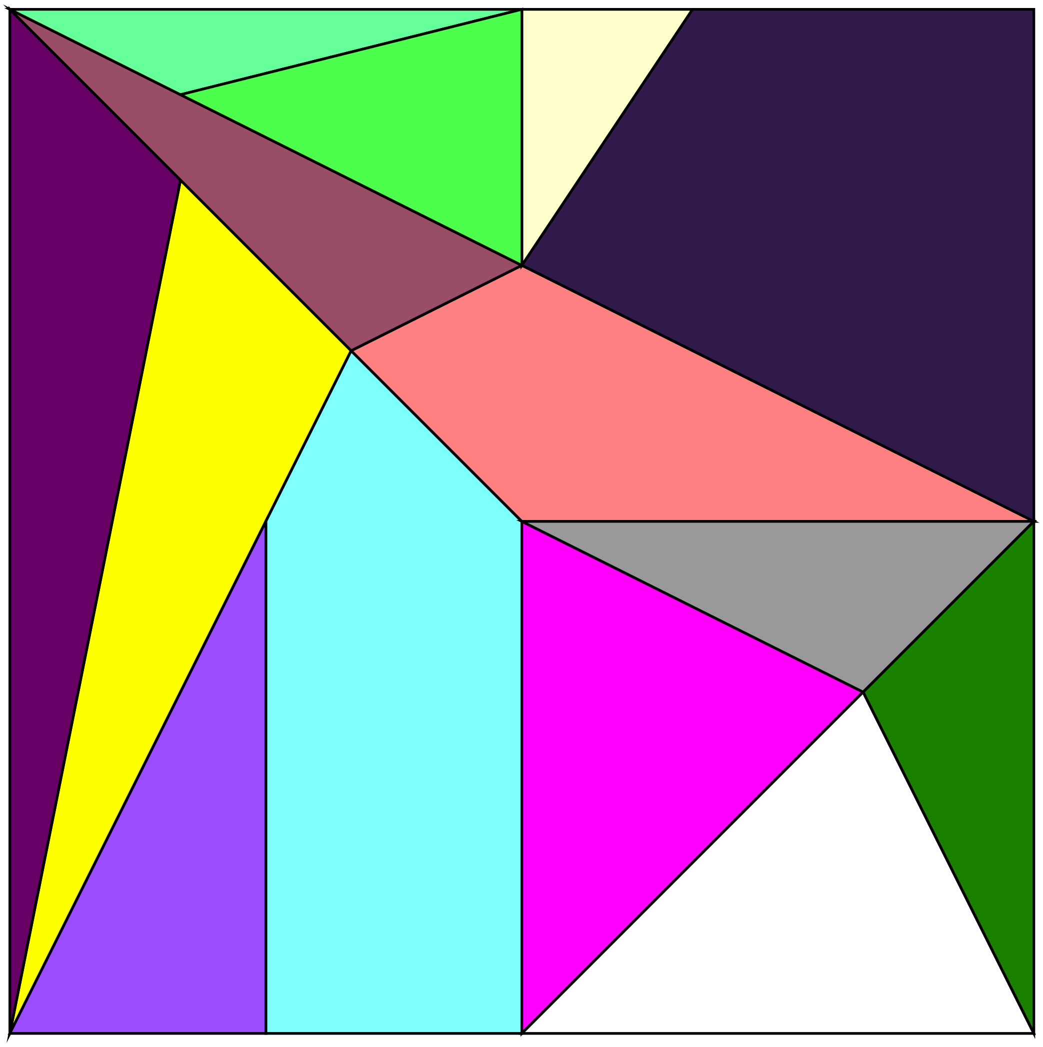


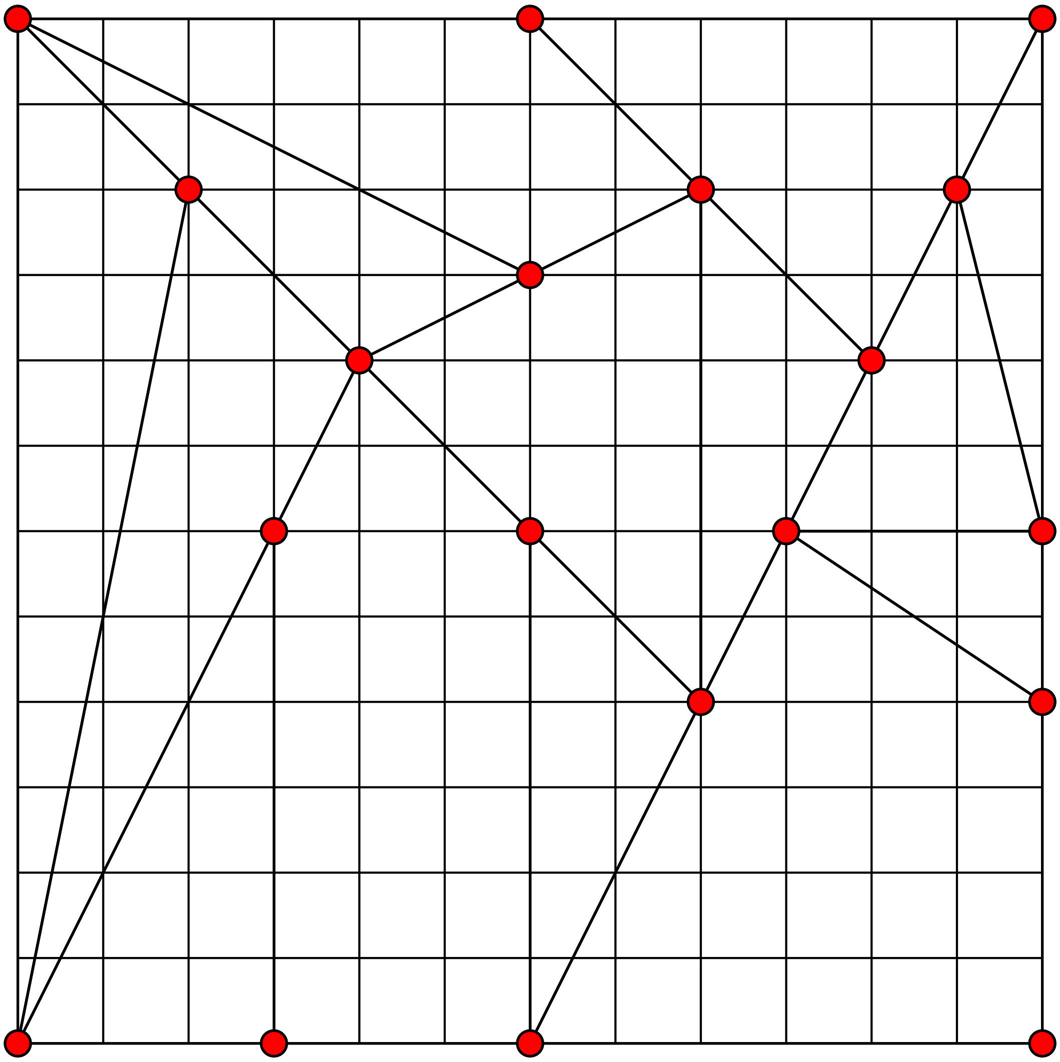


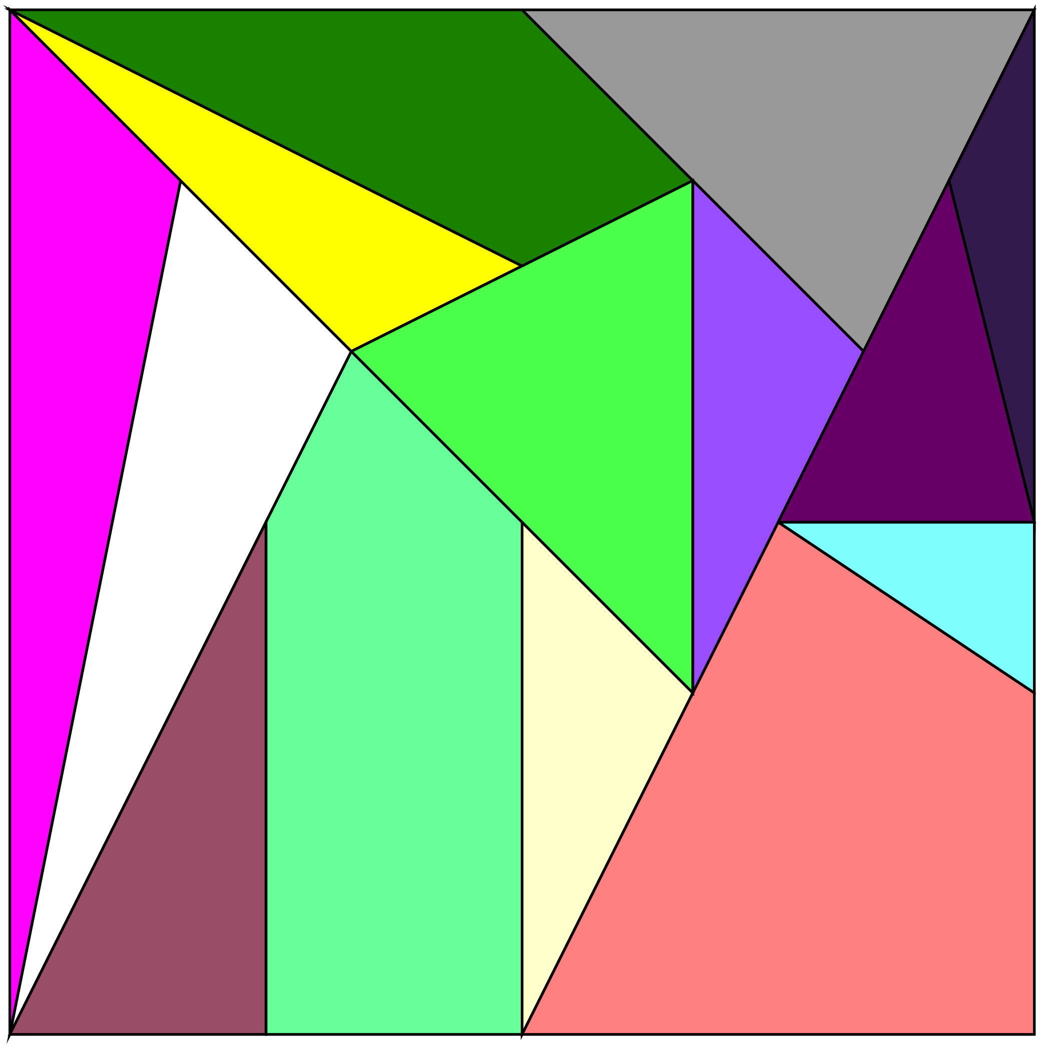


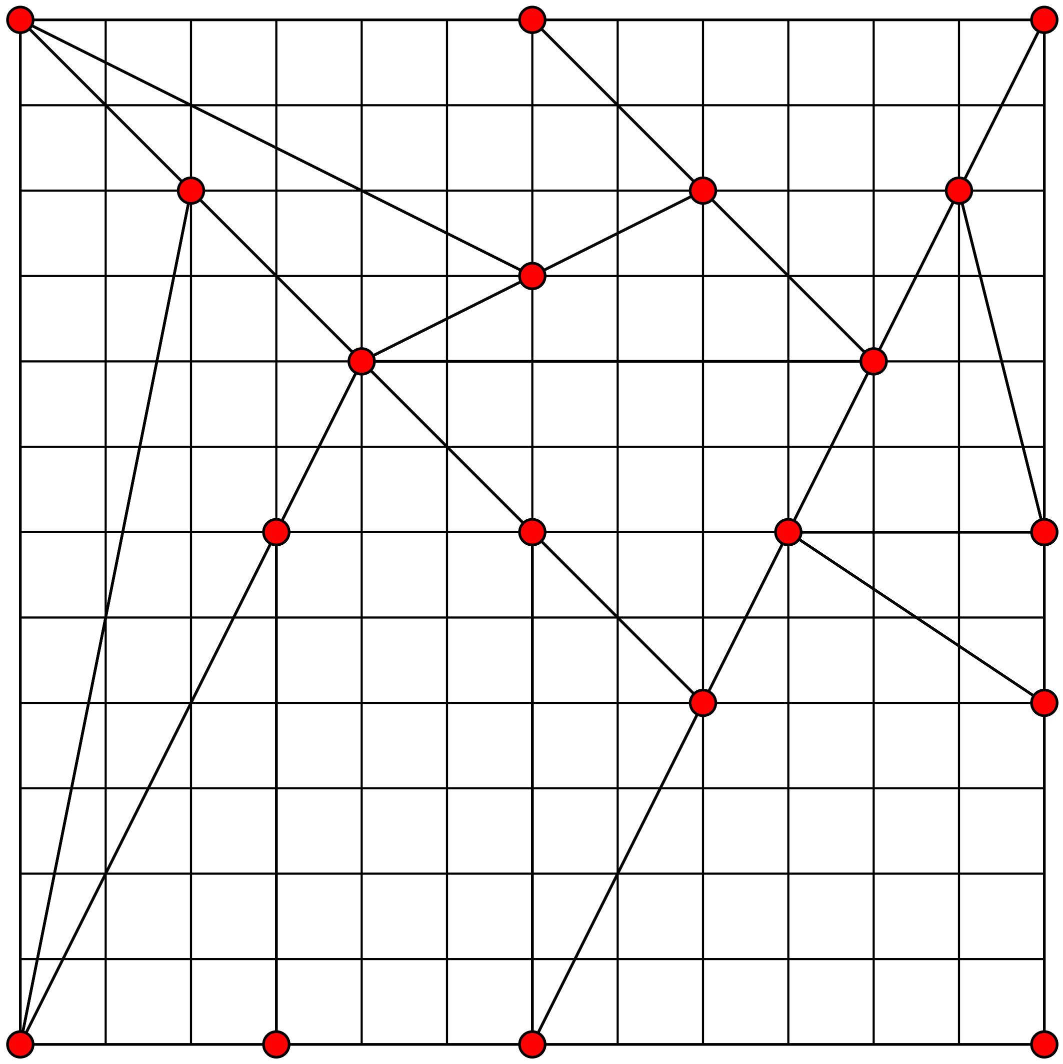


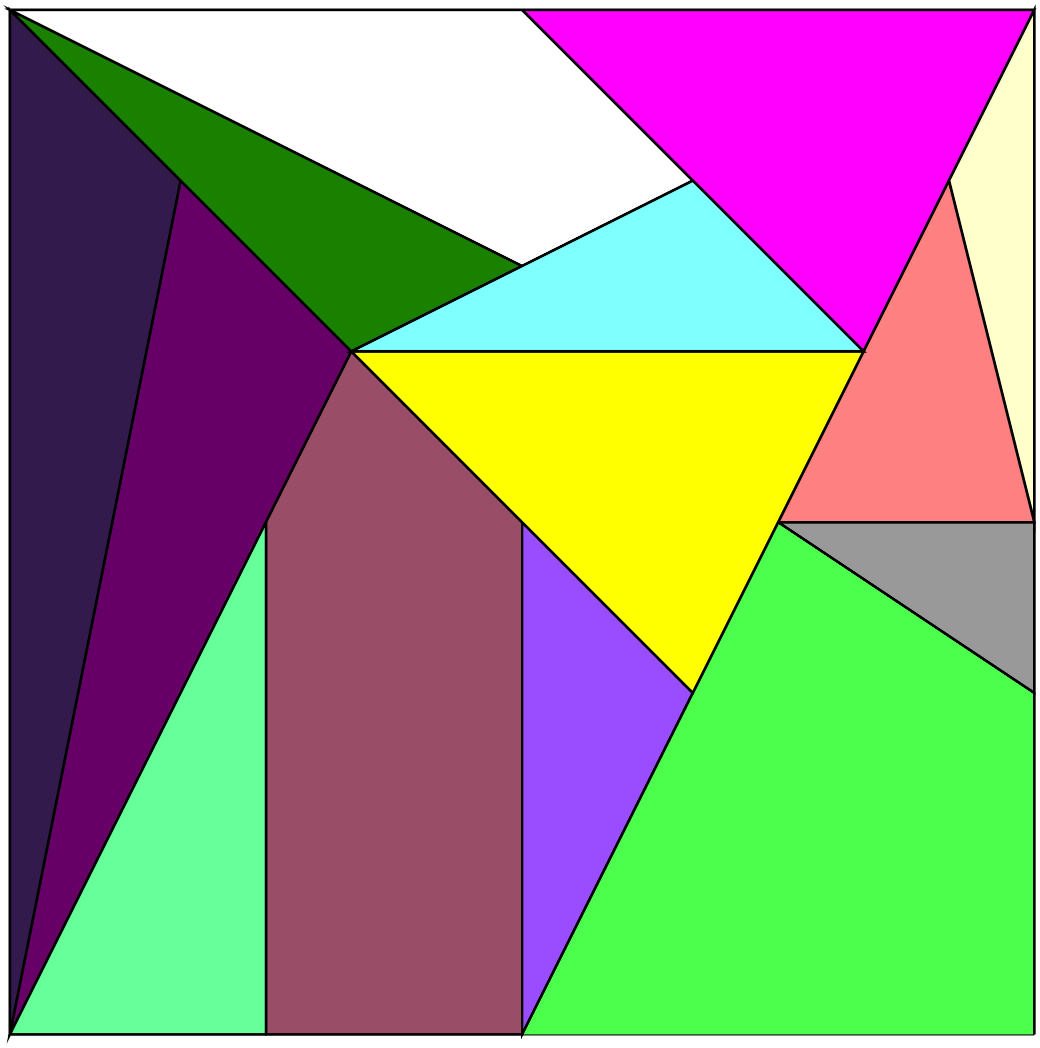


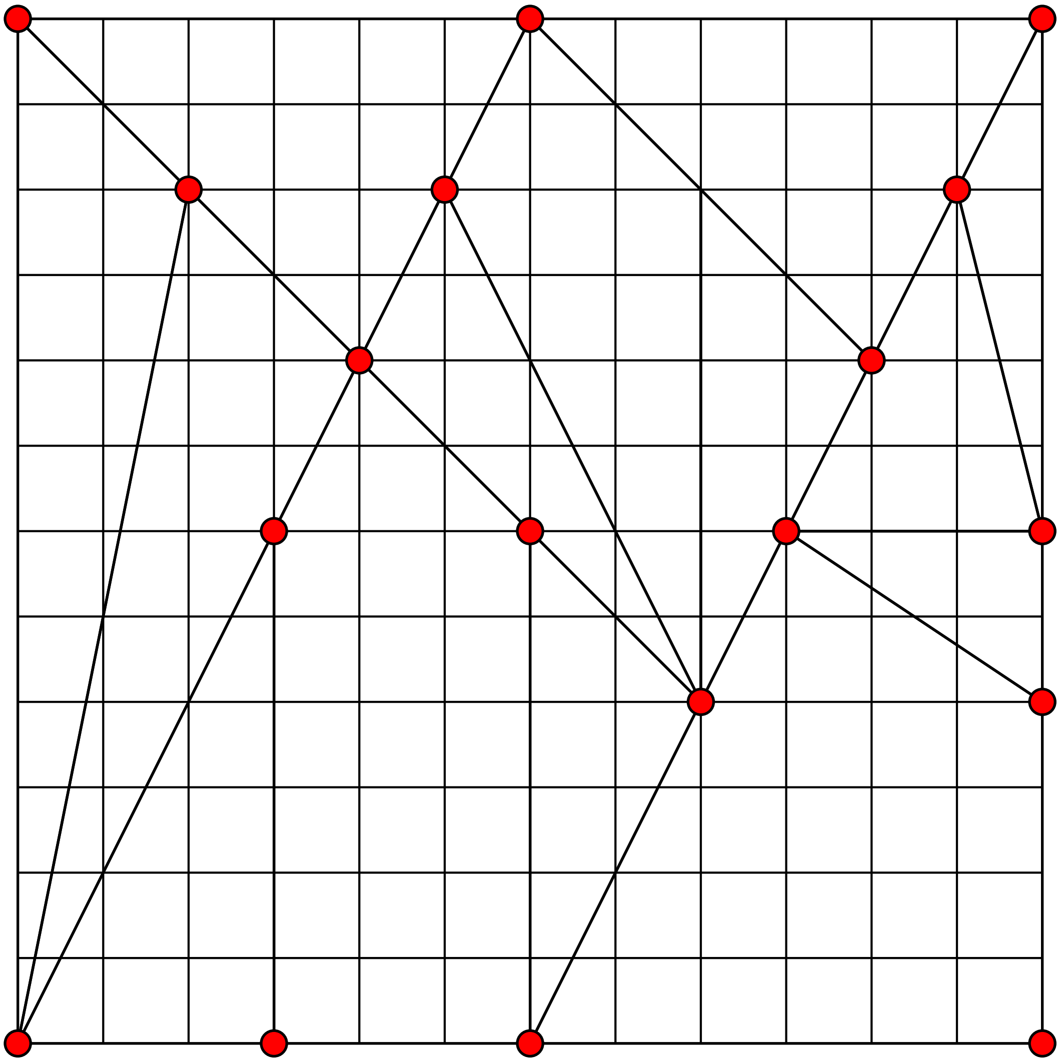


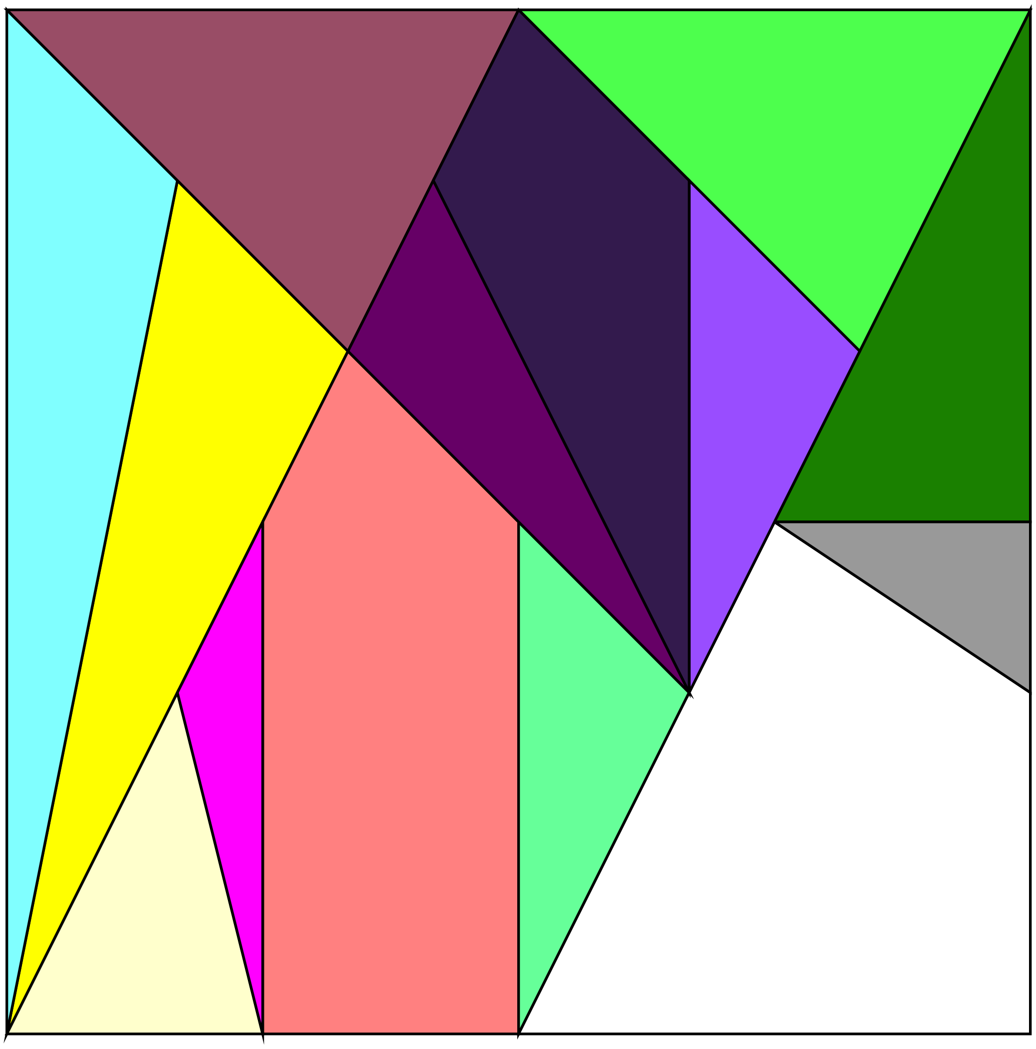












THE END.

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