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Math 308 [Casselman]

Project:

The Power of Origami

The Greek problems of antiquity were a a set of geometric problems whose solution cannot be solved using only straightedge and compass.

1. Squaring a circle
2. Doubling a cube
3. Trisecting an angle
4. Alhazen’s billiard problem

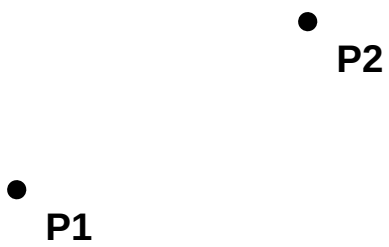
Here, we will try to solve the third problem --- angle trisection --- using paper-folding technique "origami" instead of using only straightedge and compass.

Before we proceed to the trisection, let’s see the operations allowed by origami.

The following 6 axioms are formulated by Dr. Humiaki Huzita. They are often referred to as "Huzita’s origami axioms".

AXIOM I

Given 2 points p1 and p2, we can fold a line connecting them.



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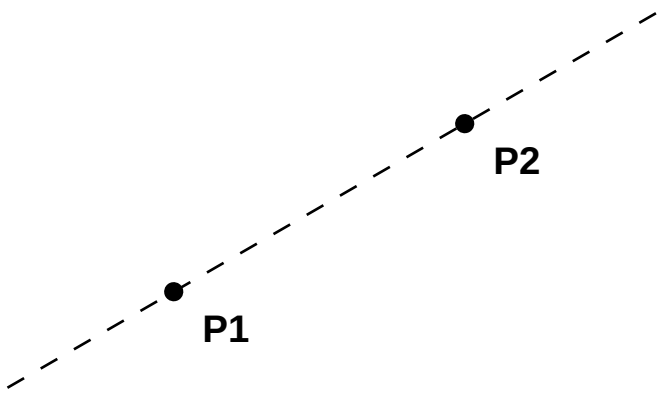
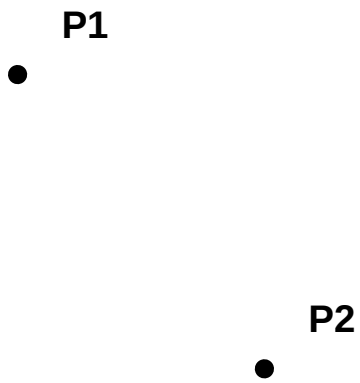


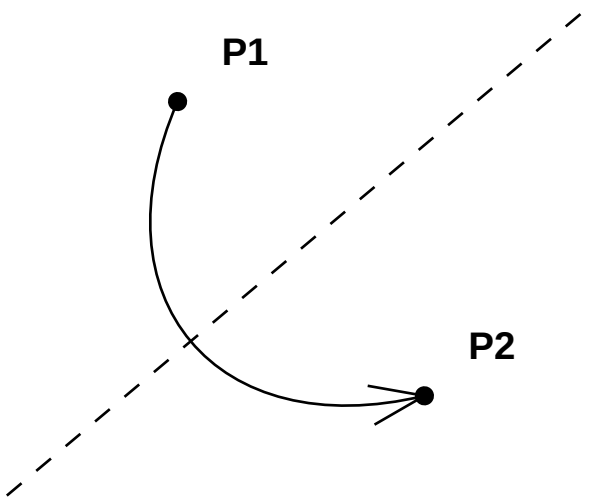
Given 2 points p1 and p2, we can fold a line connecting them.

AXIOM II

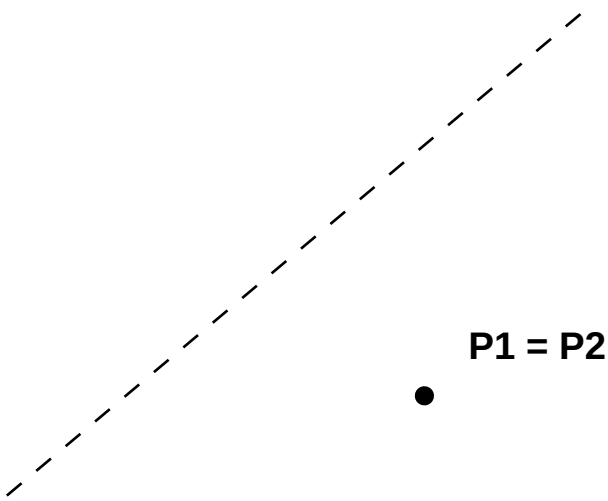
Given 2 points p1 and p2, we can fold p1 onto p2.



Given 2 points p1 and p2, we can fold p1 onto p2.



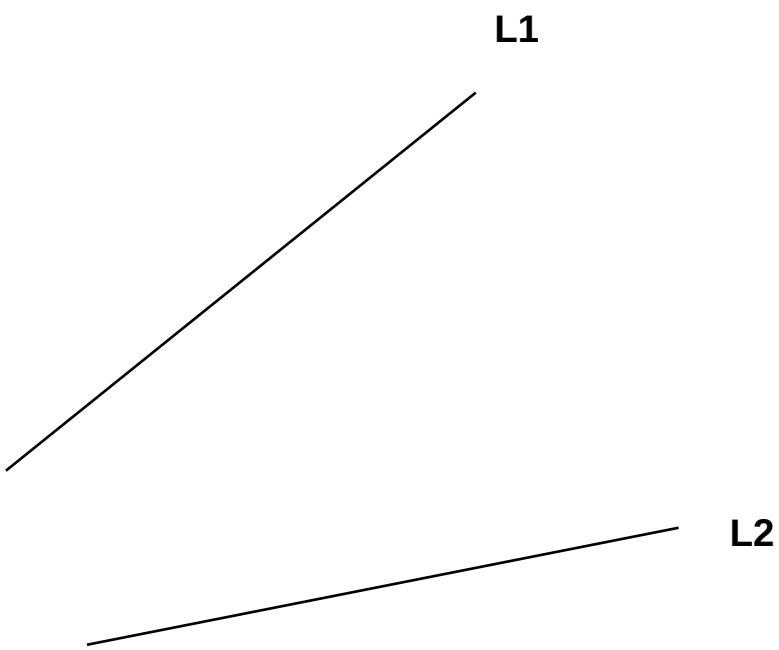
Given 2 points p1 and p2, we can fold p1 onto p2.



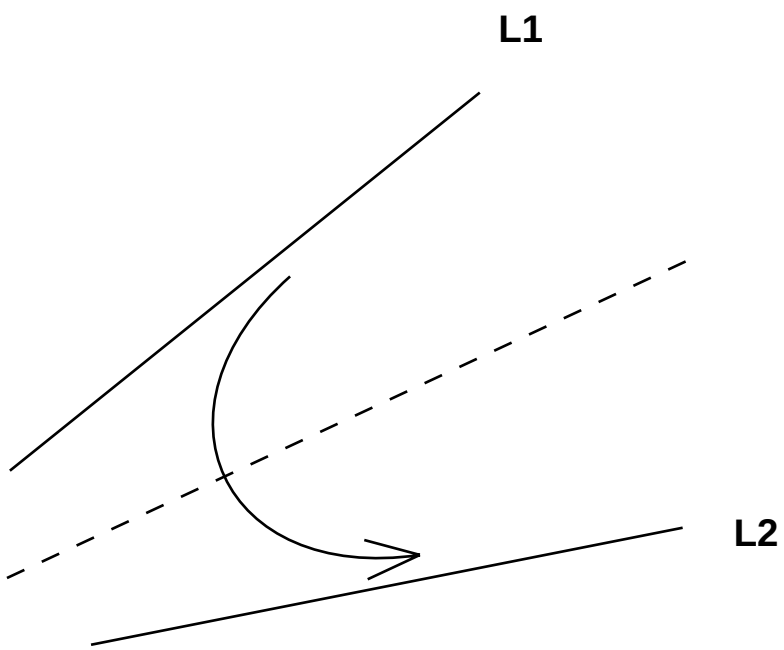
Given 2 points p1 and p2, we can fold p1 onto p2.

AXIOM III

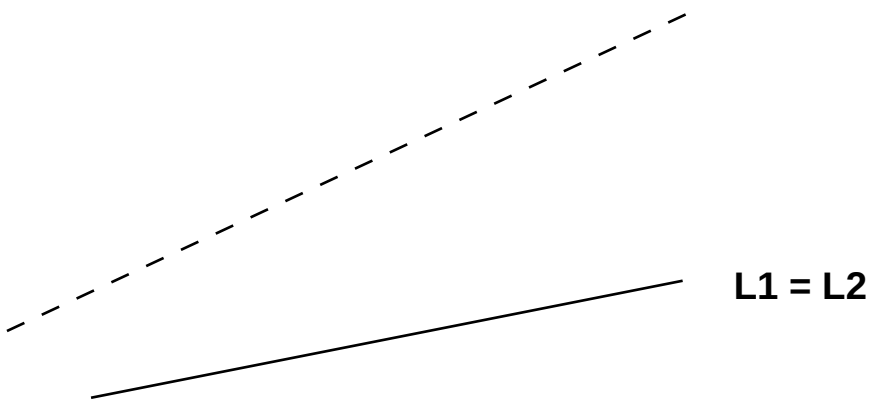
Given 2 lines L1 and L2, we can fold L1 onto L2.



Given 2 lines L1 and L2, we can fold L1 onto L2.



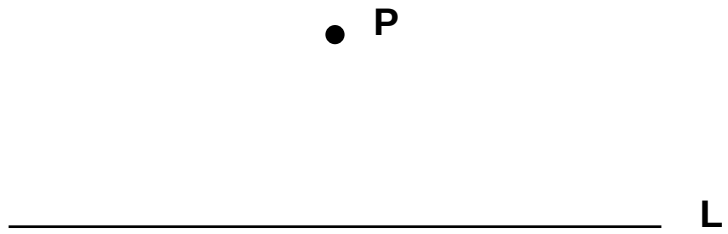
Given 2 lines L1 and L2, we can fold L1 onto L2.



Given 2 lines L1 and L2, we can fold L1 onto L2.

AXIOM IV

Given a point p and a line L,
we can make a fold perpendicular to
L passing through p.



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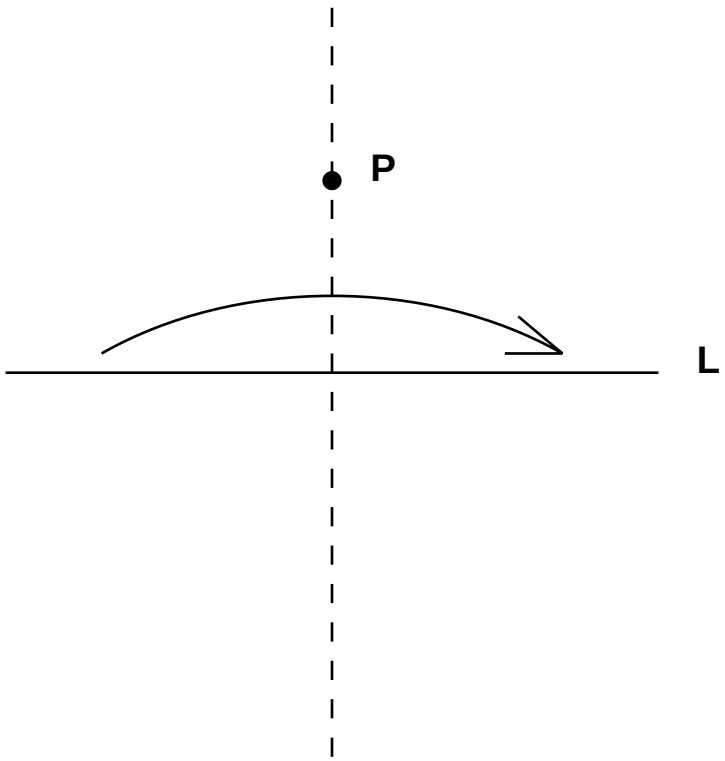

Given a point p and a line L,
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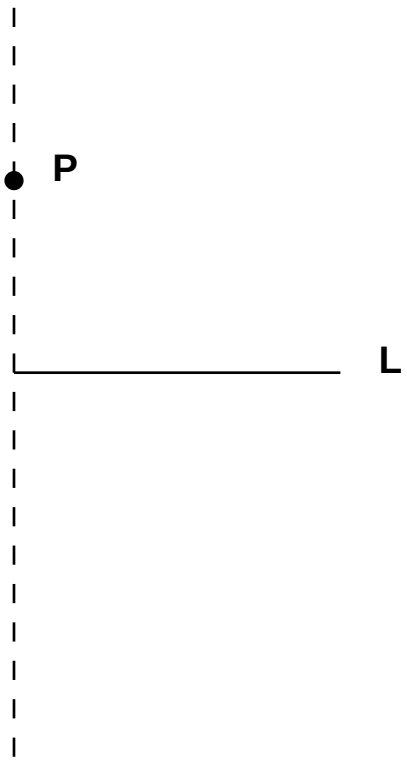
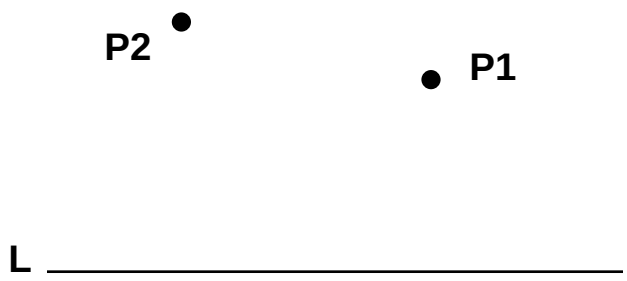


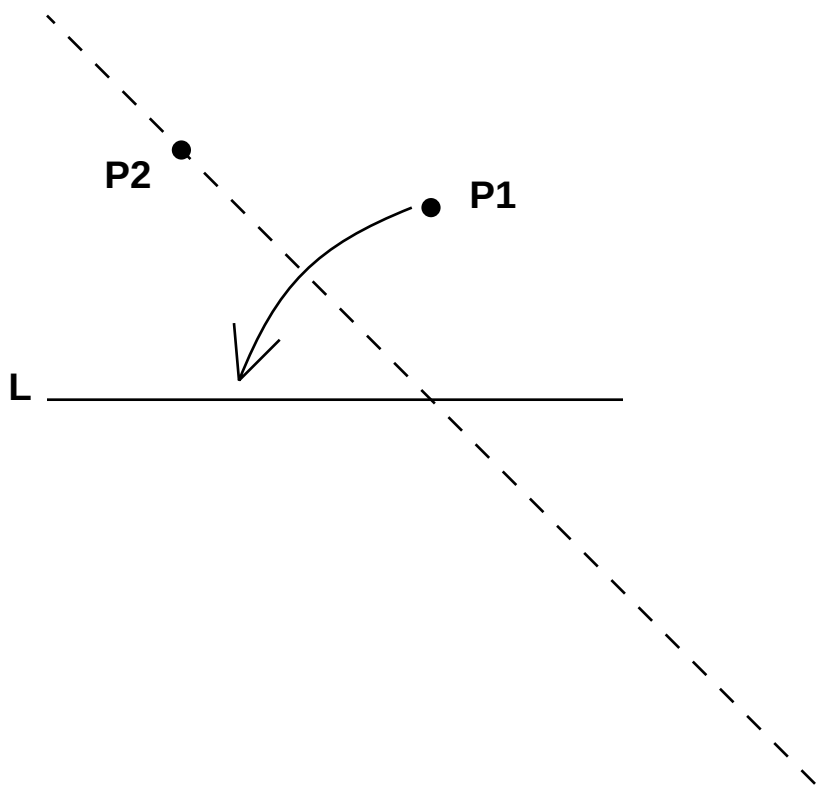
Given a point p and a line L,
we can make a fold perpendicular to
L passing through p.

AXIOM V

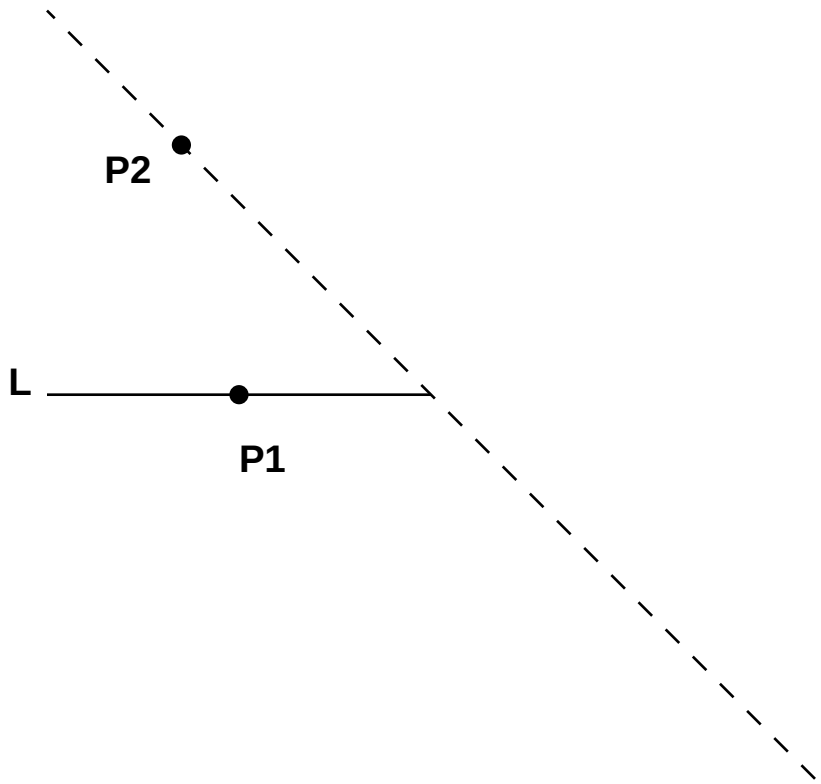
Given 2 points p1 and p2 and a line L, we can make a fold that places p1 onto L and passes through p2.



Given 2 points p1 and p2 and a line L, we can make a fold that places p1 onto L and passes through p2.



Given 2 points p1 and p2 and a line L, we can make a fold that places p1 onto L and passes through p2.

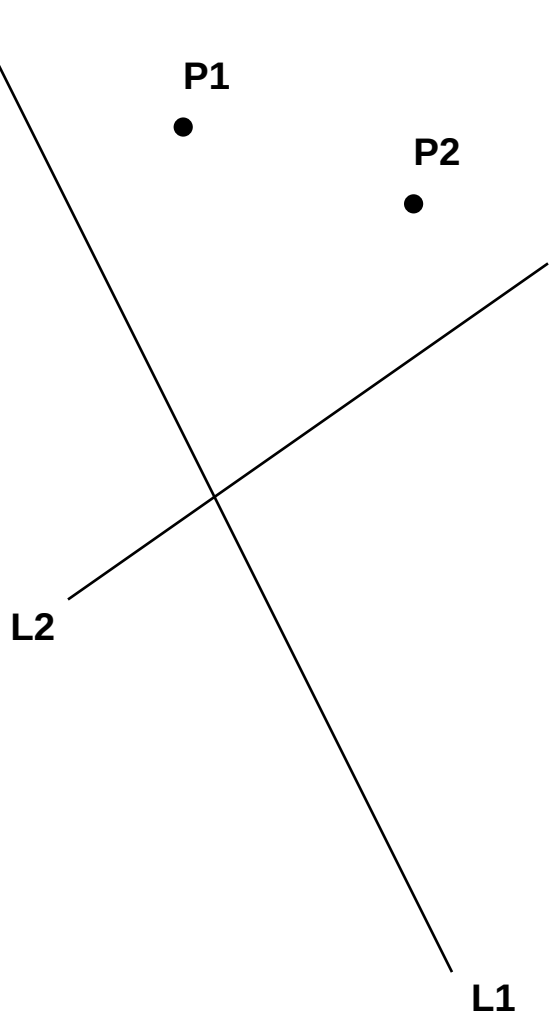


Given 2 points p1 and p2 and a line L, we can make a fold that places p1 onto L and passes through p2.

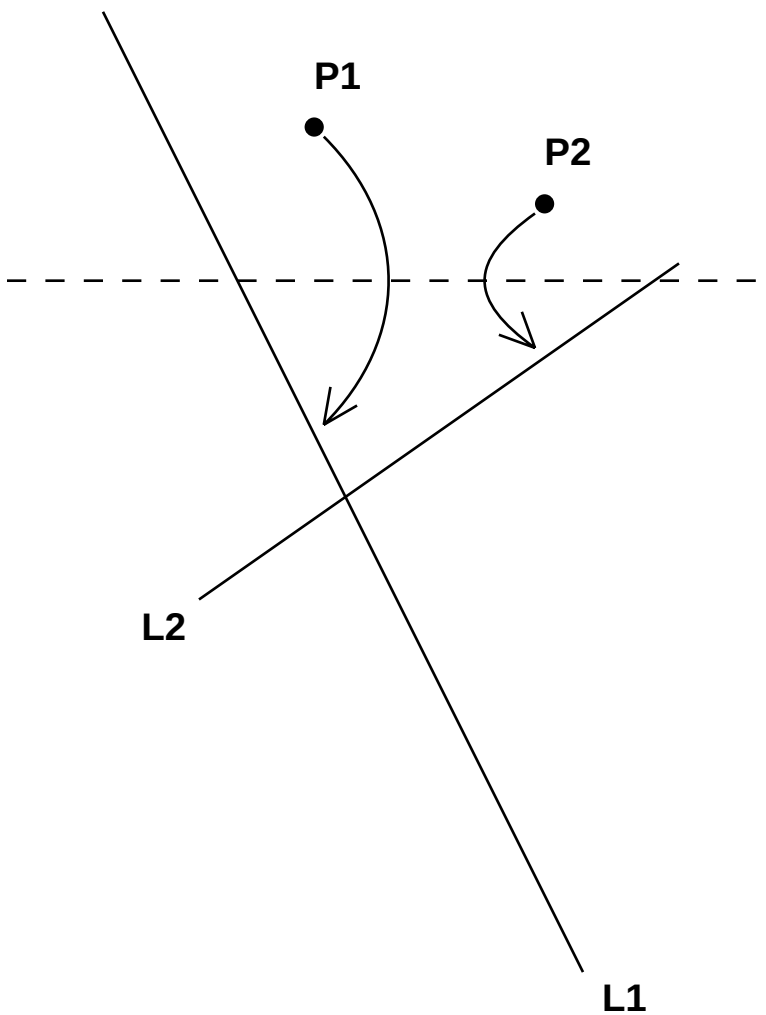
AXIOM VI

Given 2 points p1 and p2 and 2 lines L1 and L2, we can make a fold that places p1 onto L1 and p2 onto L2.

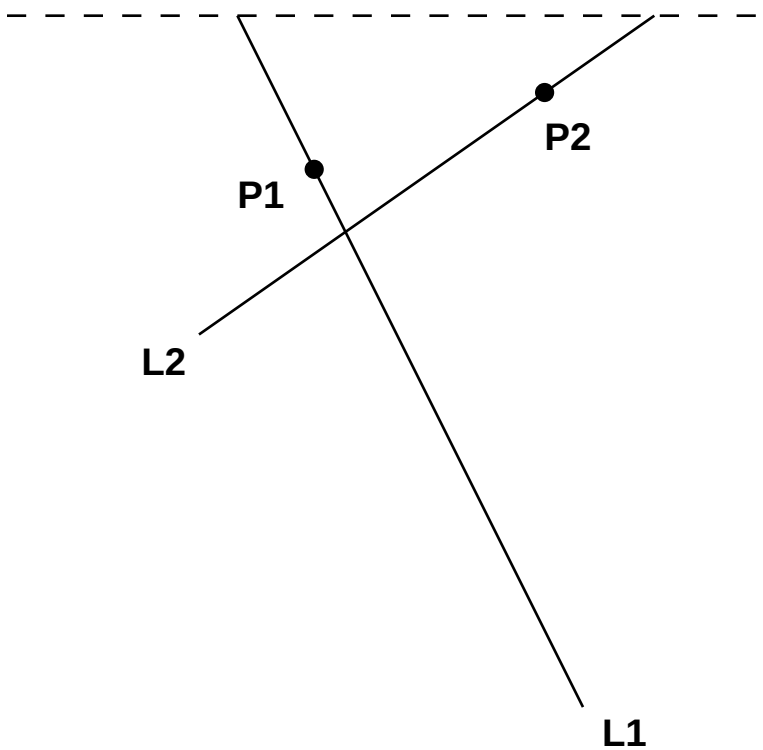
Given 2 points p1 and p2 and 2 lines L1 and L2, we can make a fold that places p1 onto L1 and p2 onto L2.



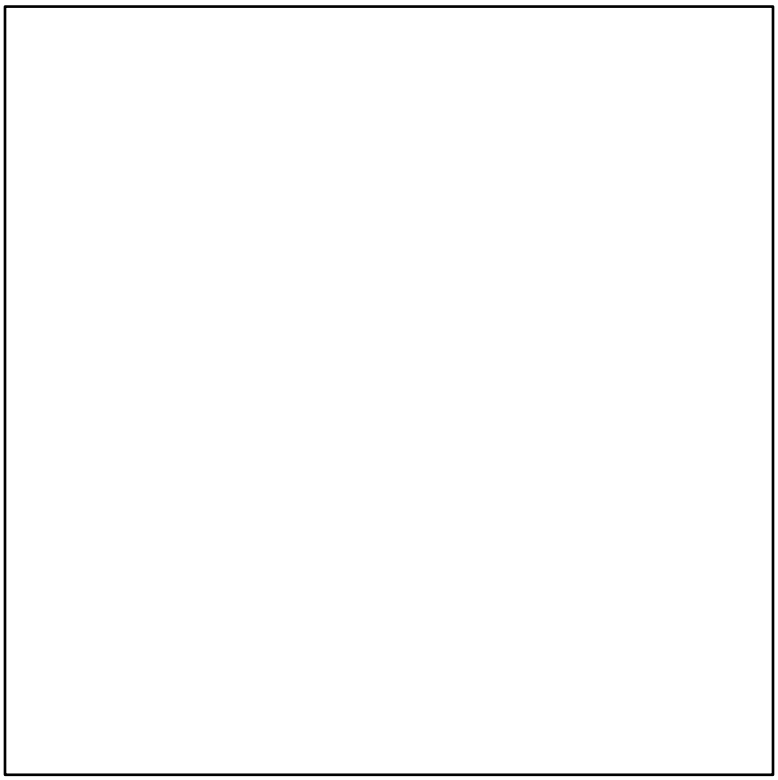
Given 2 points p1 and p2 and 2 lines L1 and L2, we can make a fold that places p1 onto L1 and p2 onto L2.



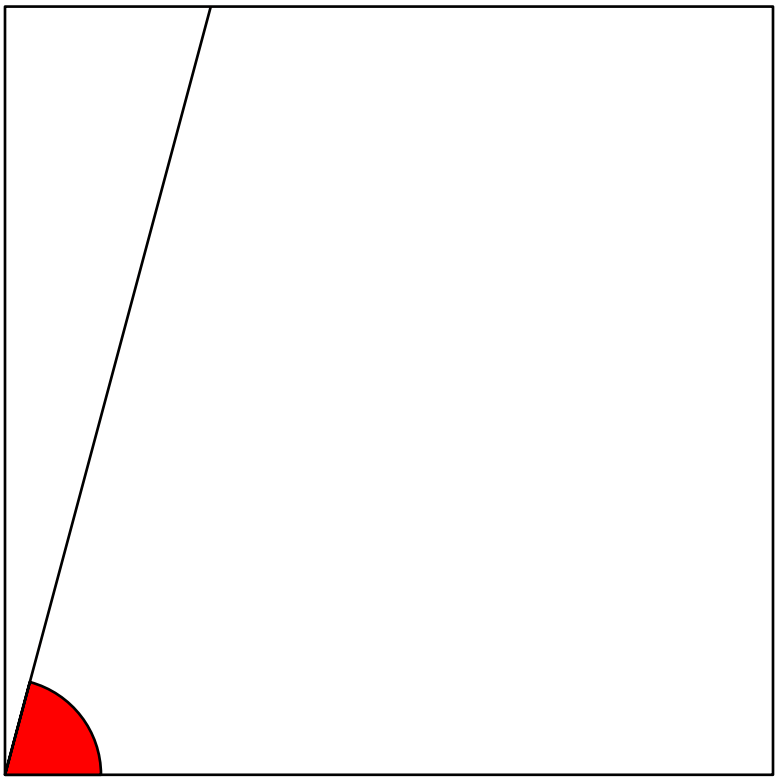
Given 2 points p1 and p2 and 2 lines L1 and L2, we can make a fold that places p1 onto L1 and p2 onto L2.



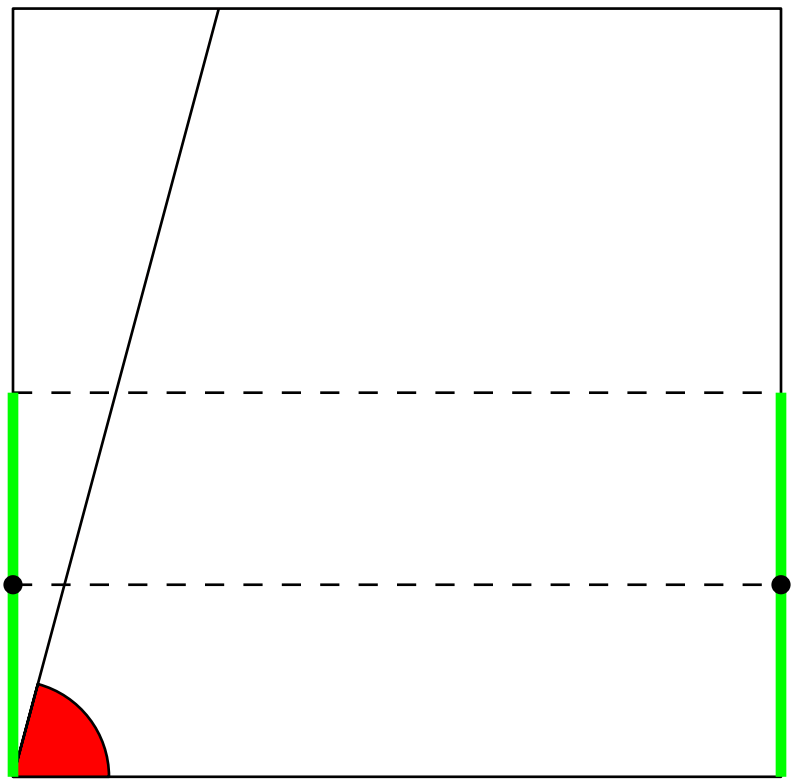
Having seen the 6 axioms, we now proceed to see how an arbitrary angle can be trisected using these axioms.



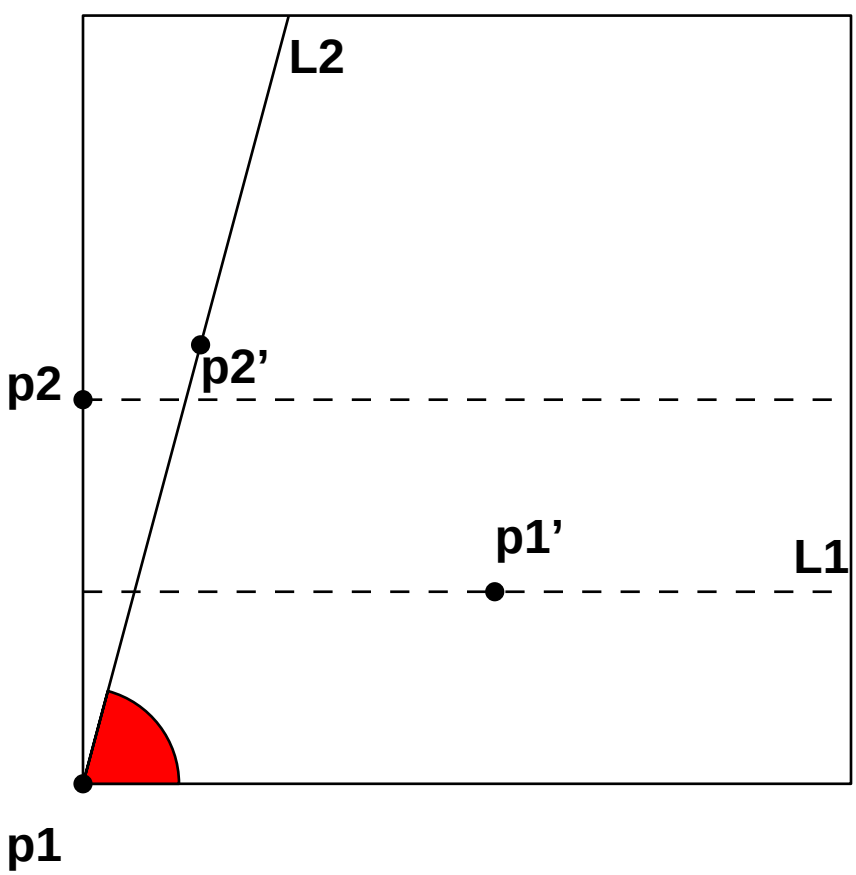
We start with a square piece of paper.



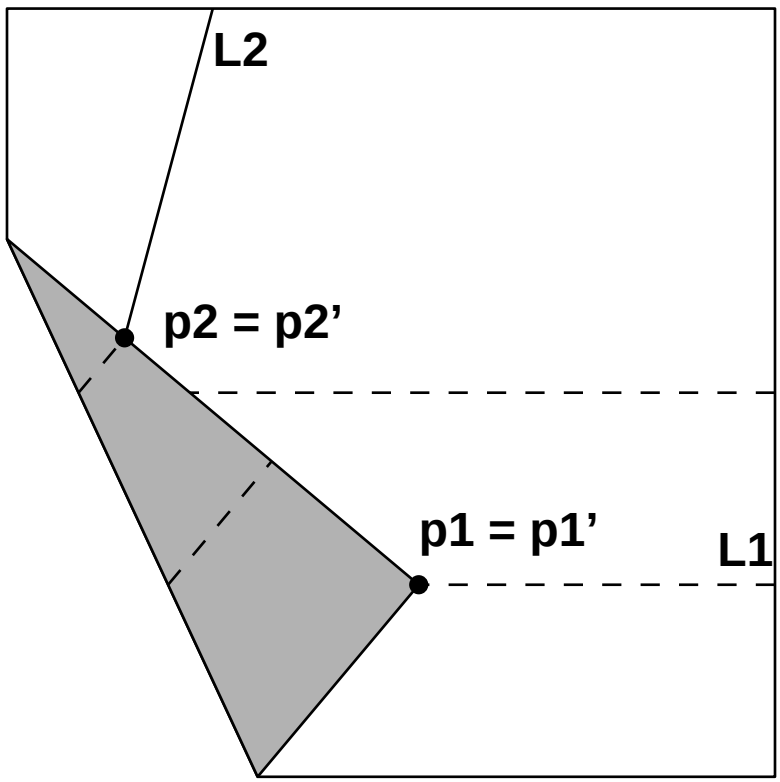
Select an arbitrary angle to be trisected.



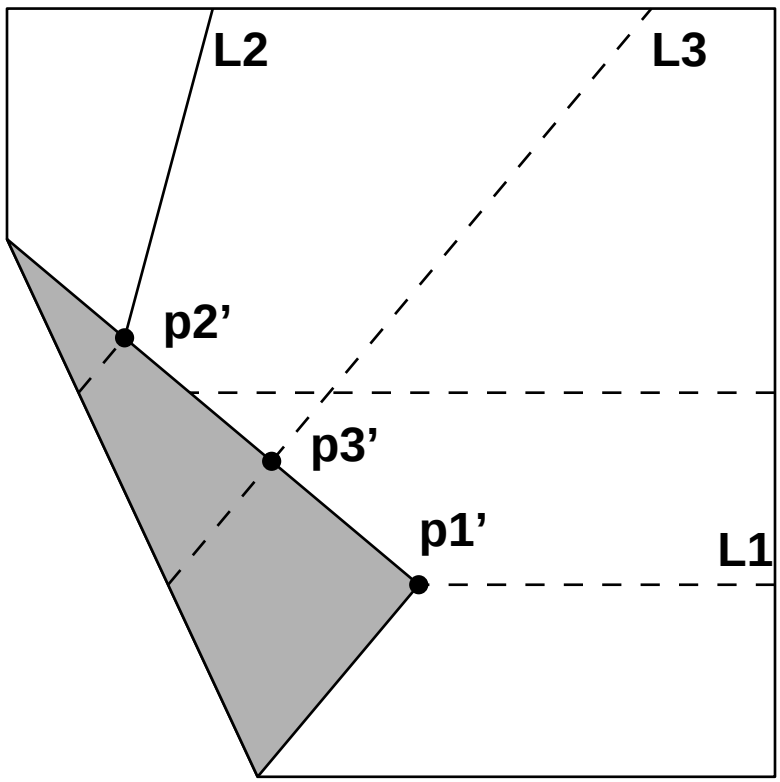
By Axiom III, fold 2 parallel creases that spaced evenly aparted.



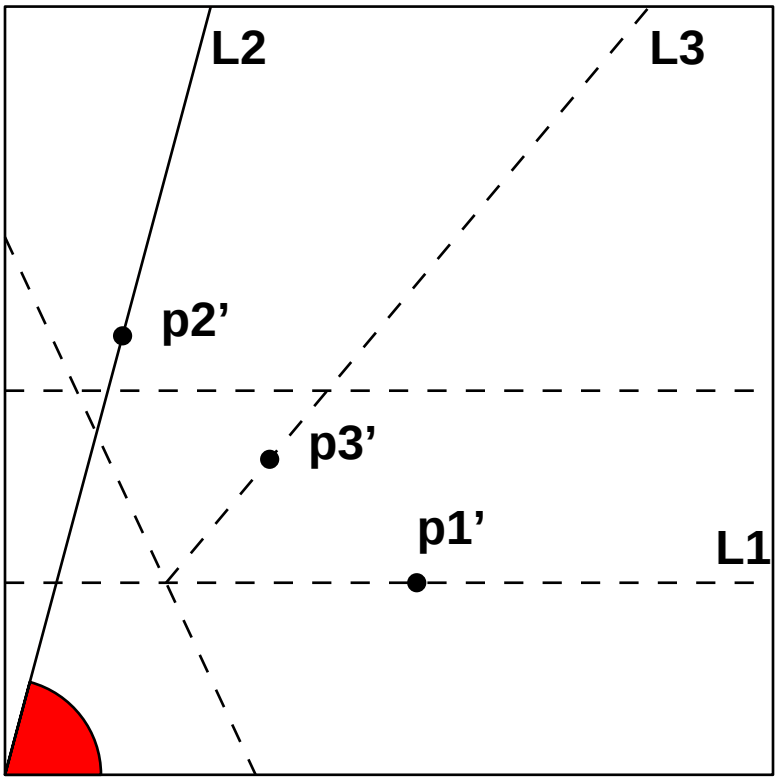
Want to fold p1 onto p1', and p2 onto p2'.



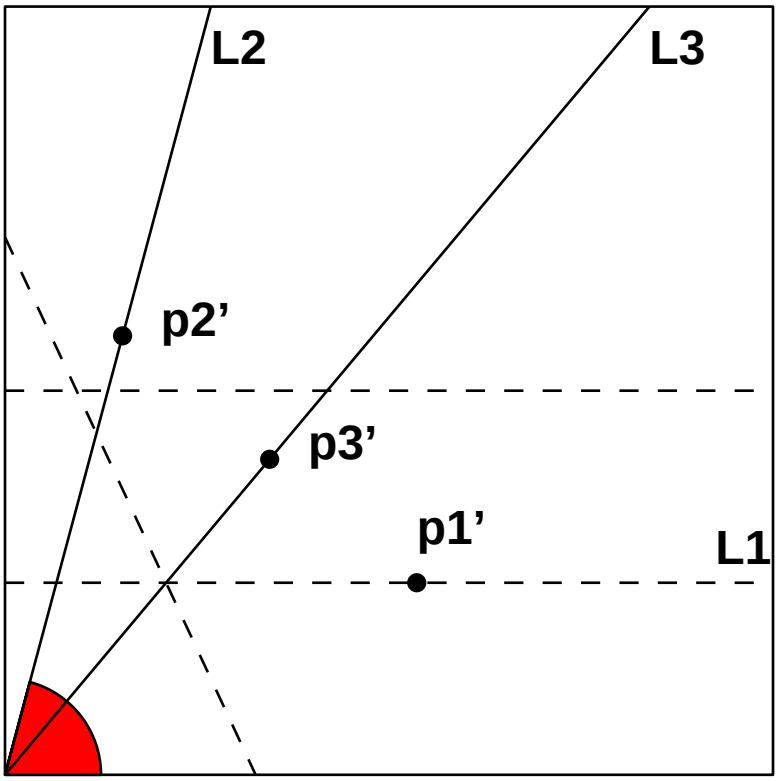
Axiom VI allows this fold.



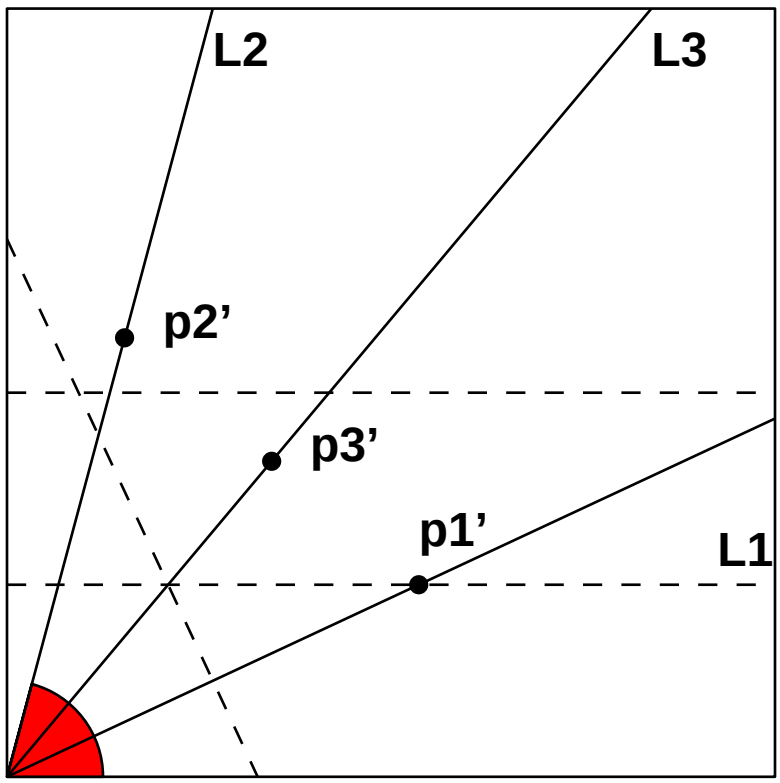
Extend the crease L1 through p3’ to meet the boundary. This is a new crease L3.



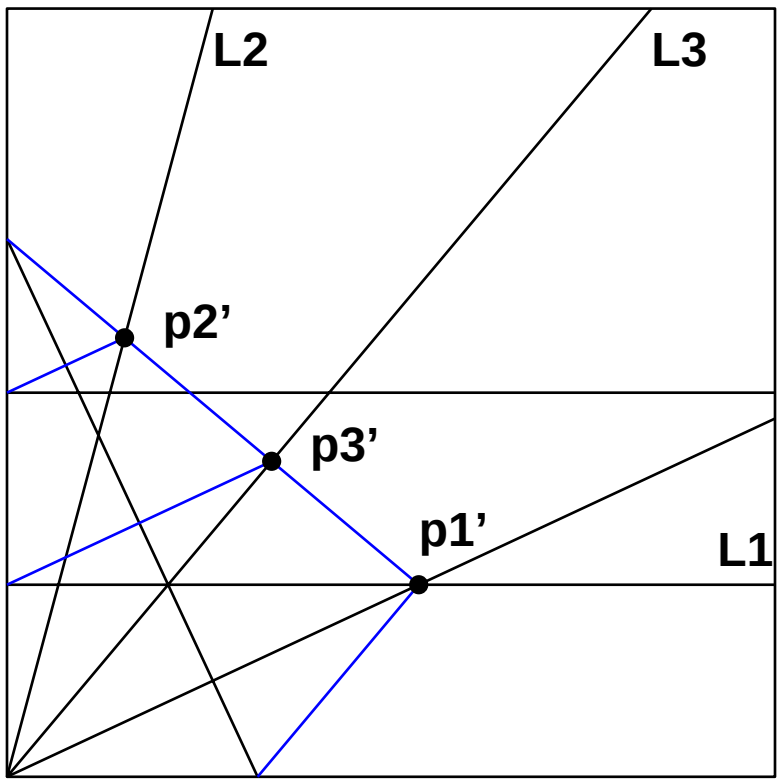
Unfold the paper.



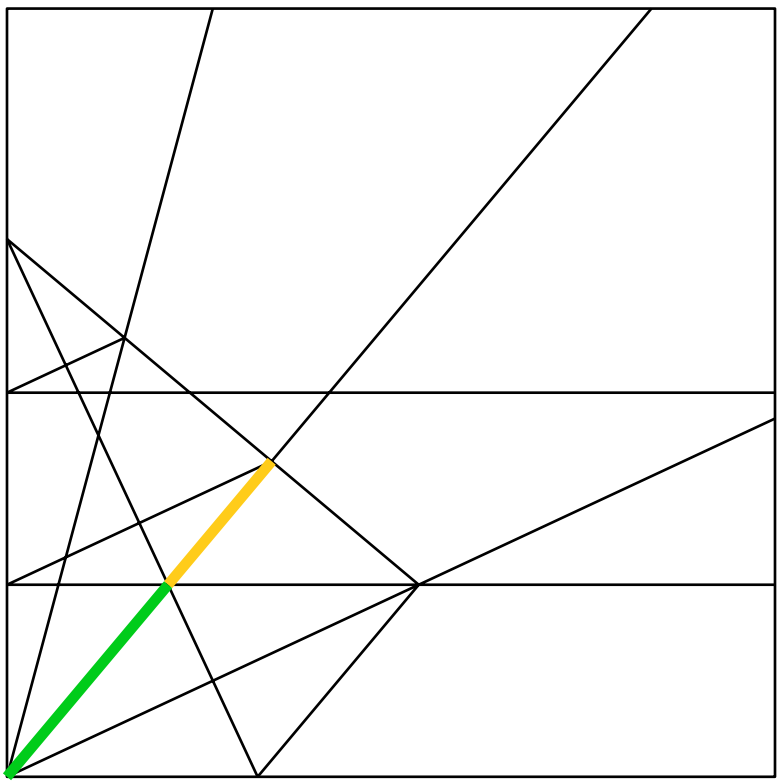
Draw L3 in a solid line.



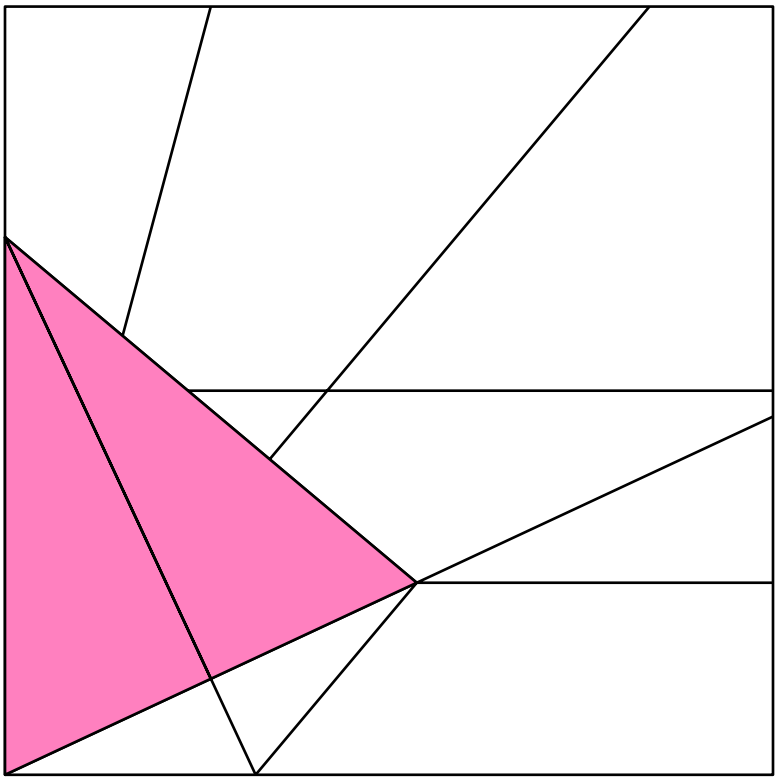
By Axiom III, fold the lower side to meet L3. A new crease created passes through p1’.
We trisected the angle!



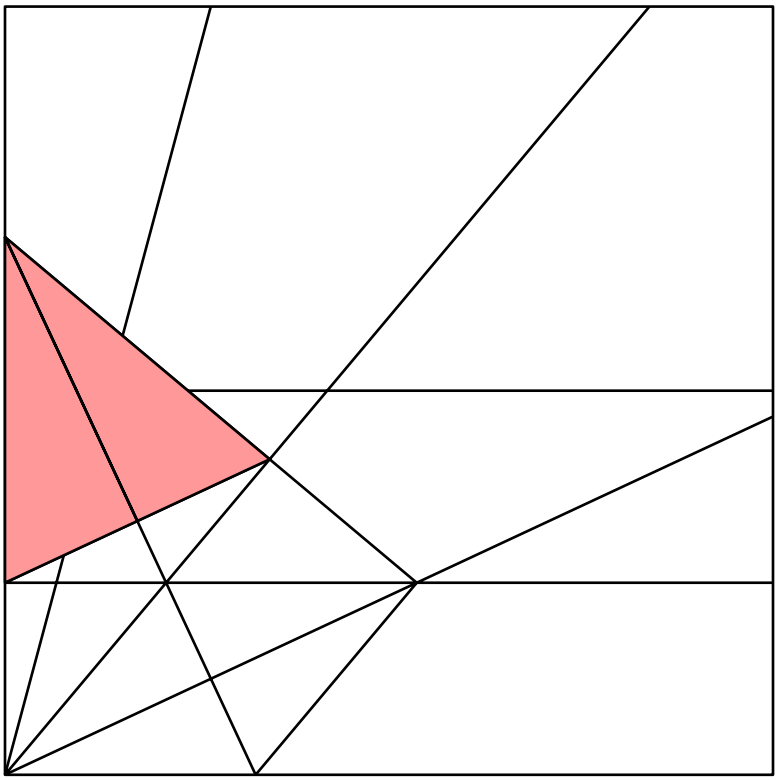
To see why, let’s add the blue lines and draw the creases out.



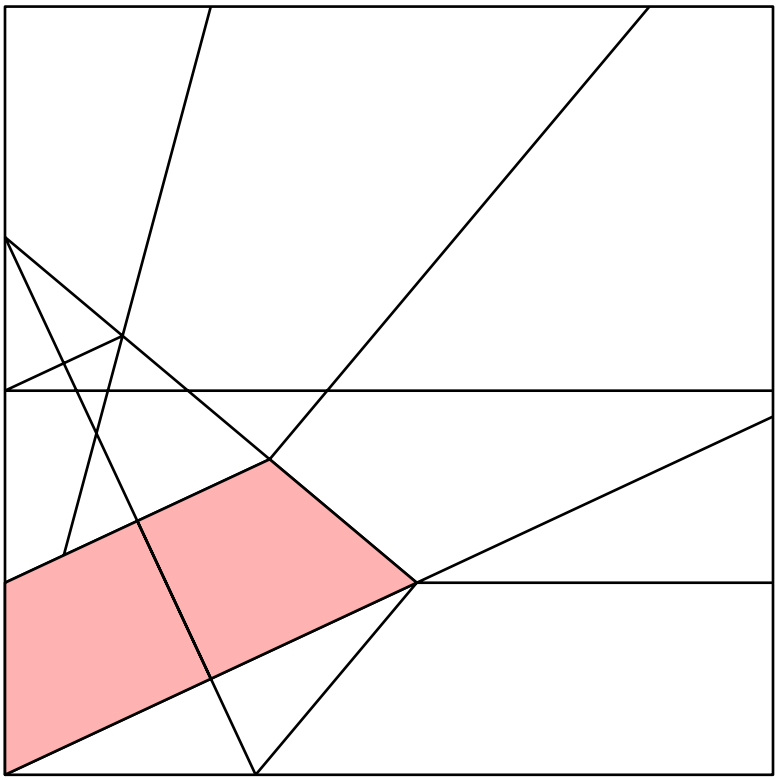
First, we have to show these 2 lines are colinear.



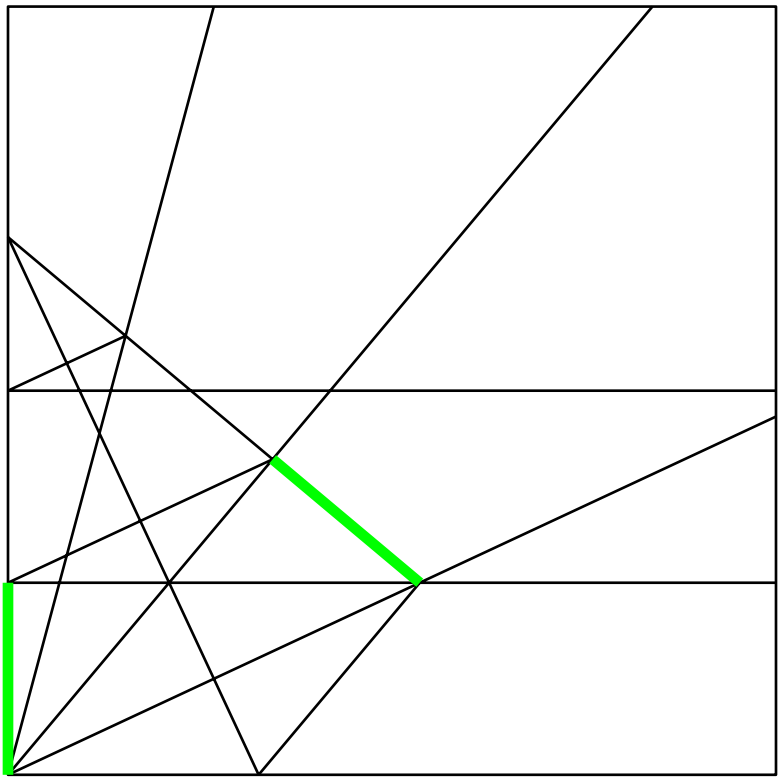
These 2 triangles are congruent because of the fold.



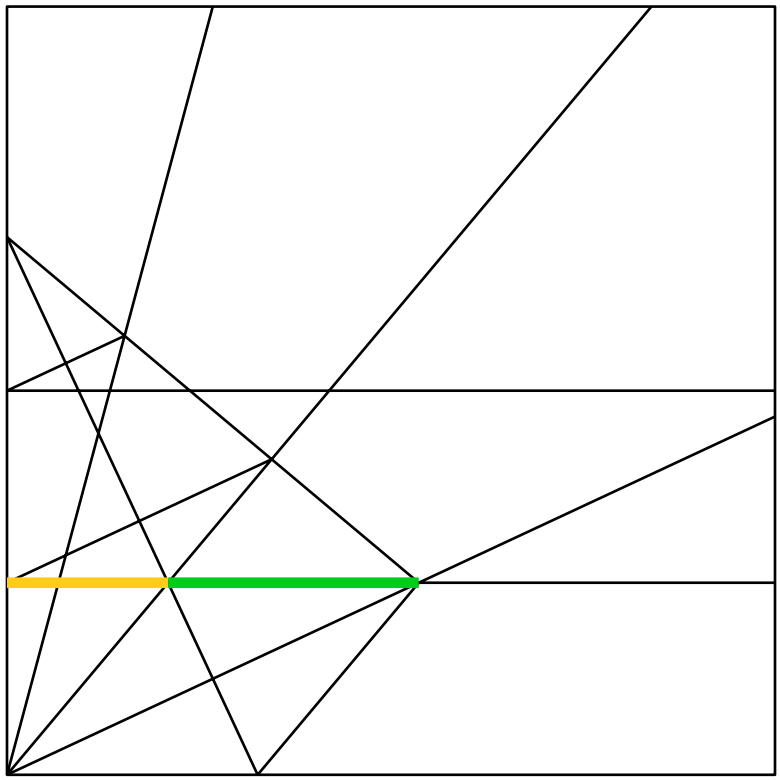
So as these two triangles.



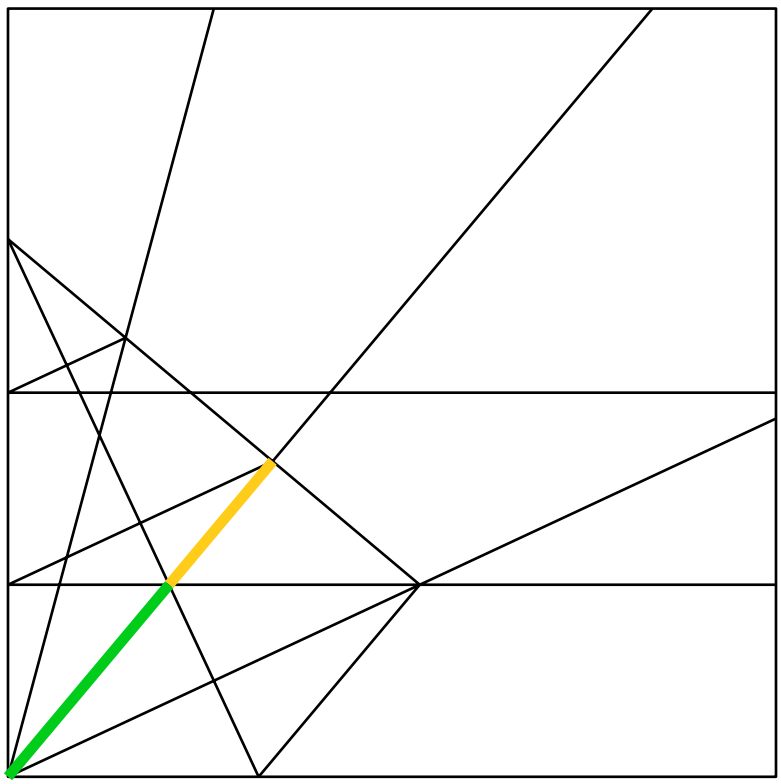
So as these two trapezoids.



These two lines are equal.

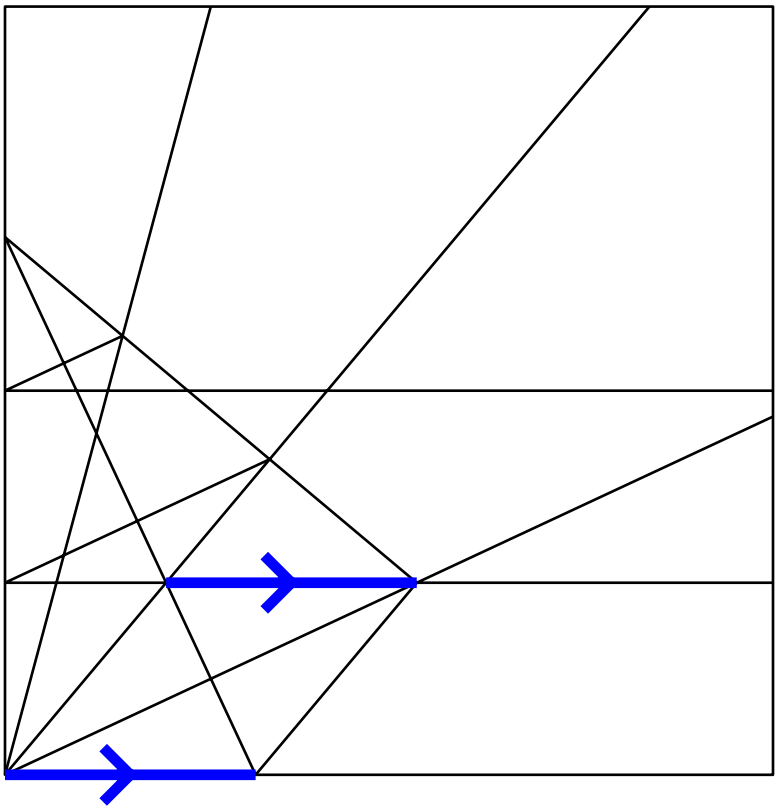


We know these 2 lines are colinear since both are on L1.

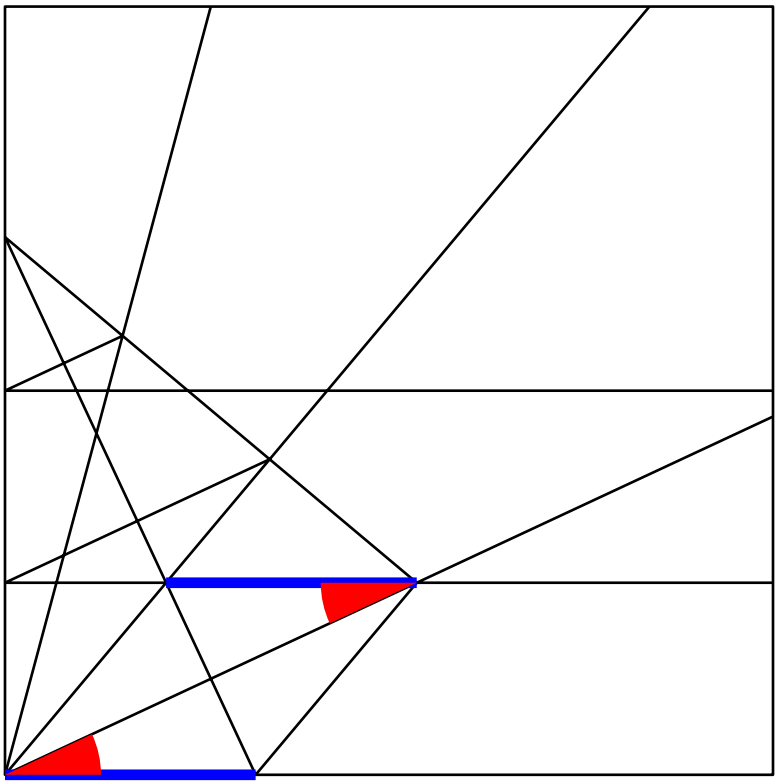


Diagonals of symmetric trapezoid are equal. Therefore these 2 lines are colinear.

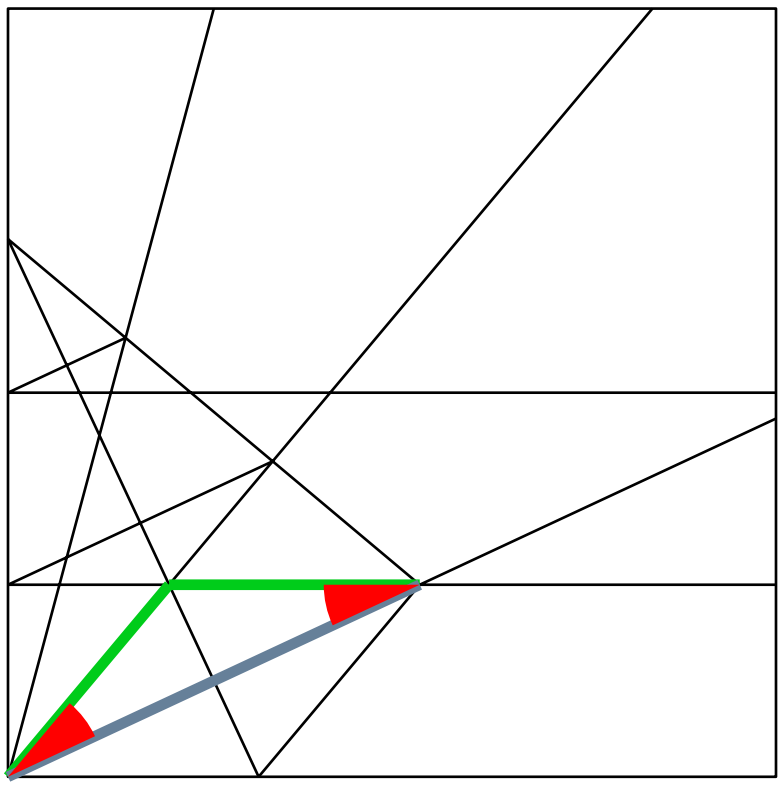
Now we have to show the angle is trisected.



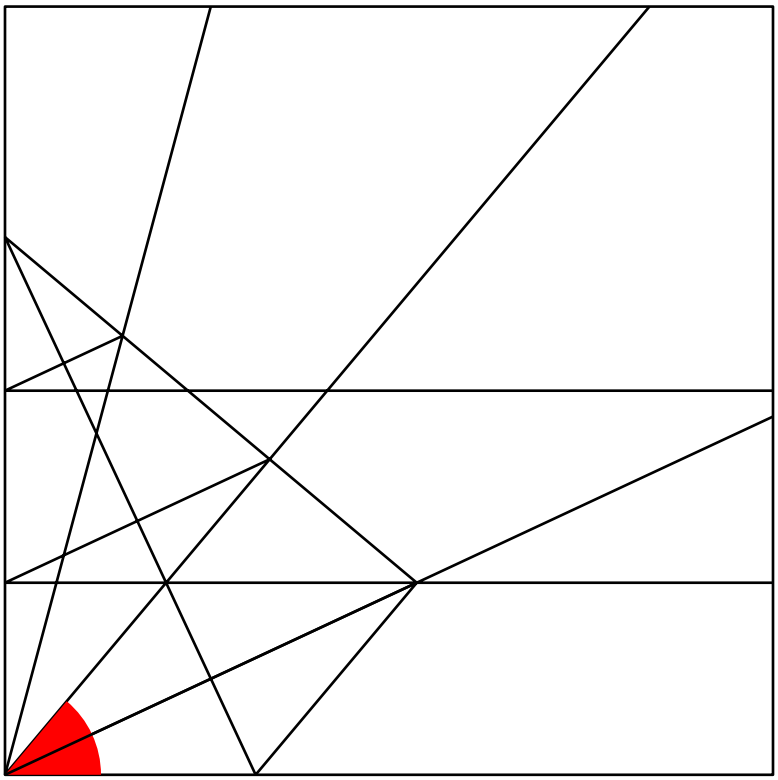
First, notice that these 2 lines are parallel.



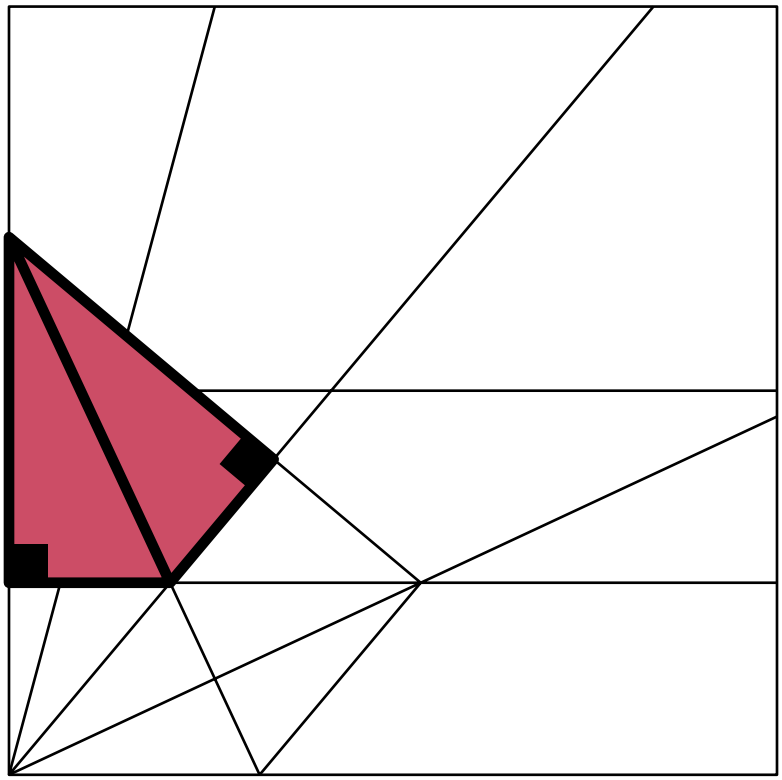
Alternate interior angles of parallel lines are equal.



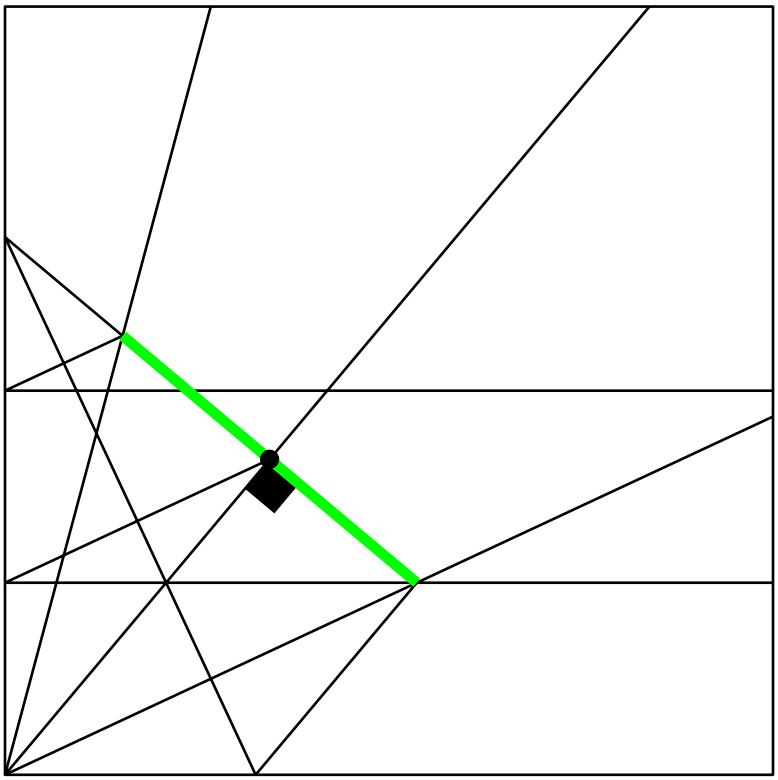
Also, this is an isosceles triangle.
So its base angles are equal.



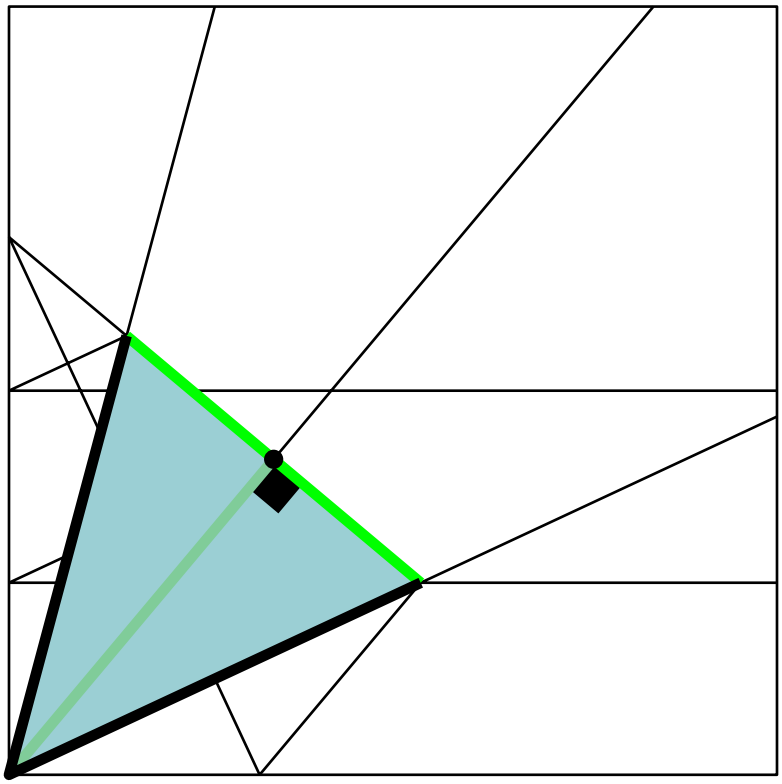
So these 2 angles are equal.



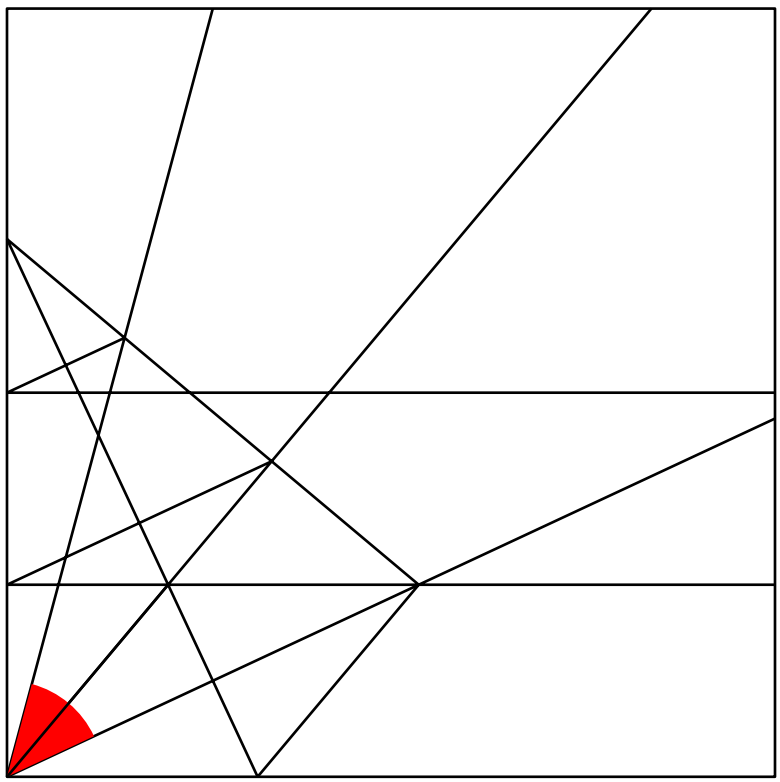
Moreover, these 2 right-angle triangles are congruent because of the fold.



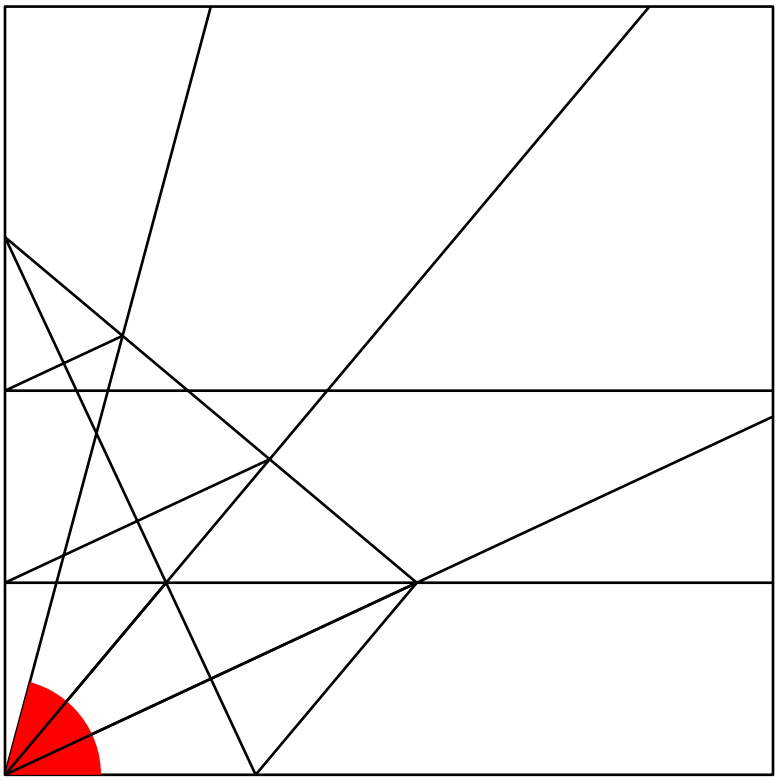
These 2 line segments are equal because the creases are evenly spaced.



These 2 triangles are both right-angled, have equal bases, and share a common height; they are congruent.

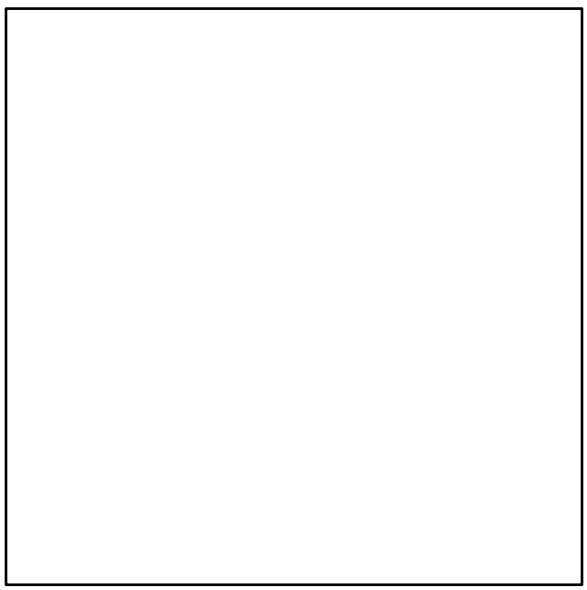


So these 2 angles are equal.

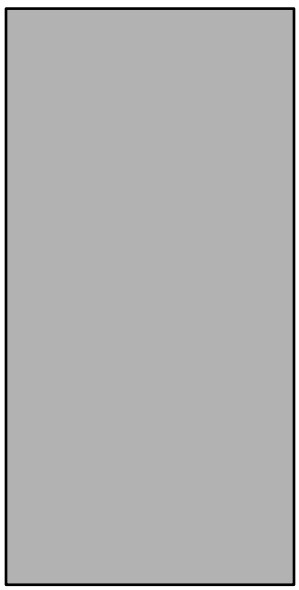


In the end, all three angles are equal.
The original angle is trisected!!!

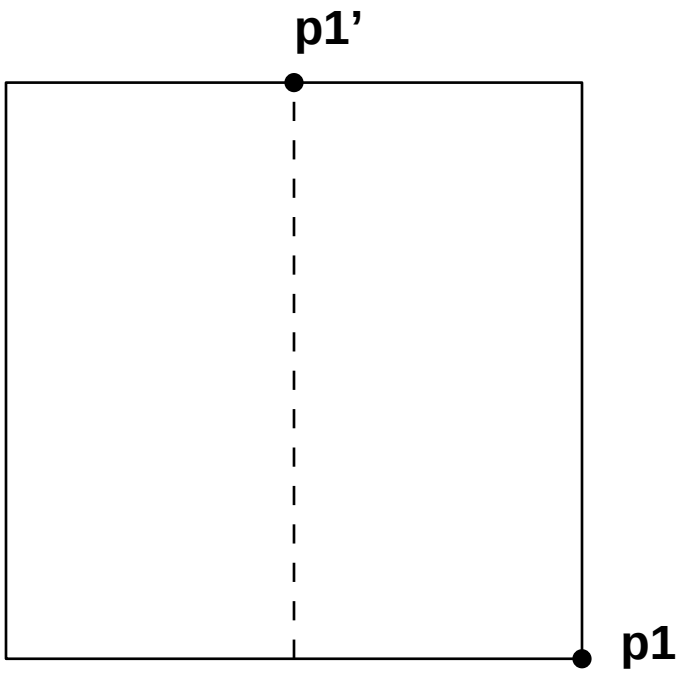
Besides trisecting an angle, we can also divide a sheet of paper into thirds using only origami.



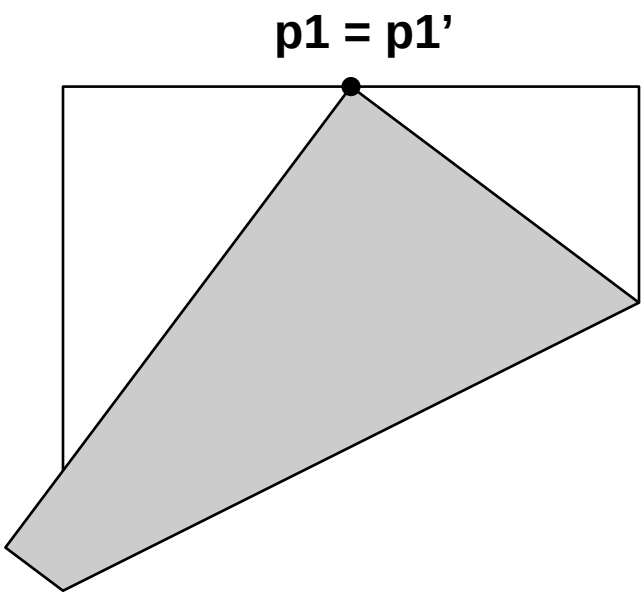
We start with a square piece of paper.



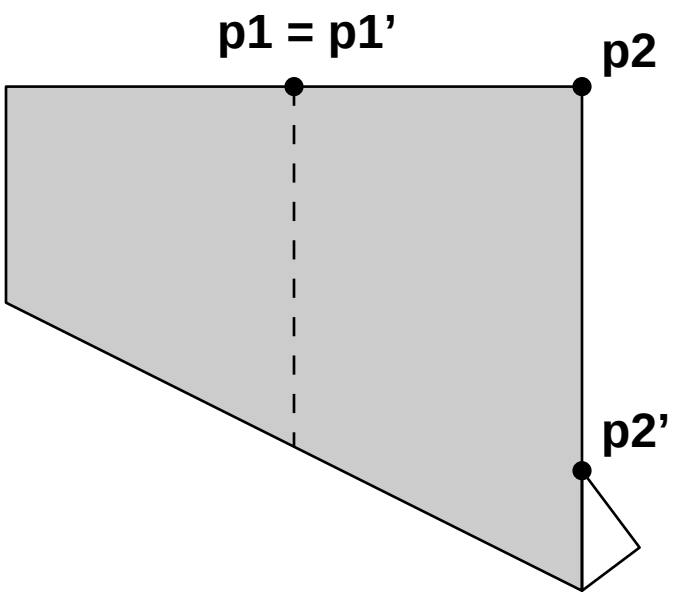
By Axiom III, we can fold it in half.



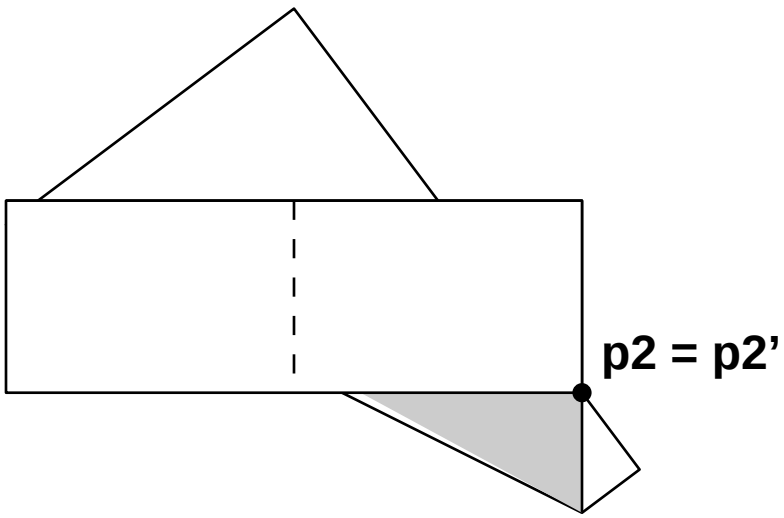
Unfold it with a crease left.
Want to place p1 onto p1’.



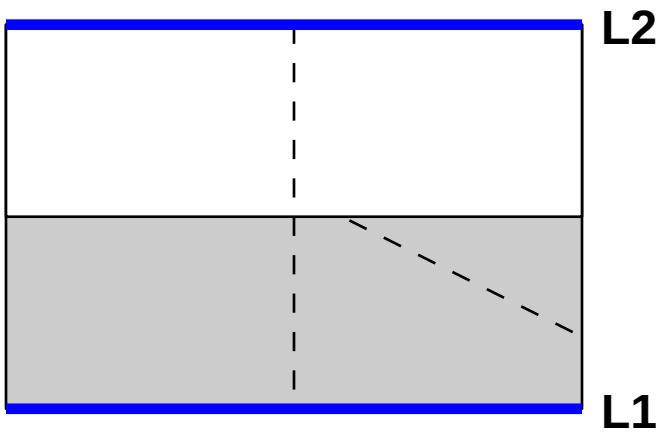
Axiom II allows this move.



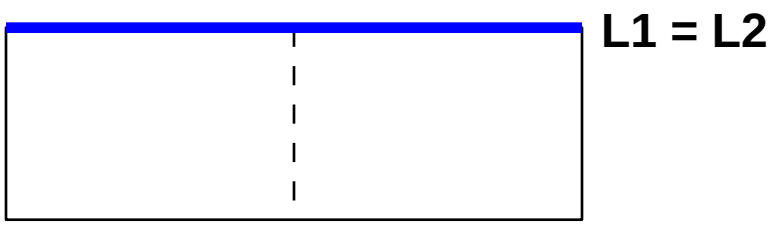
Turn the paper over.
Want to place p2 onto p2’.



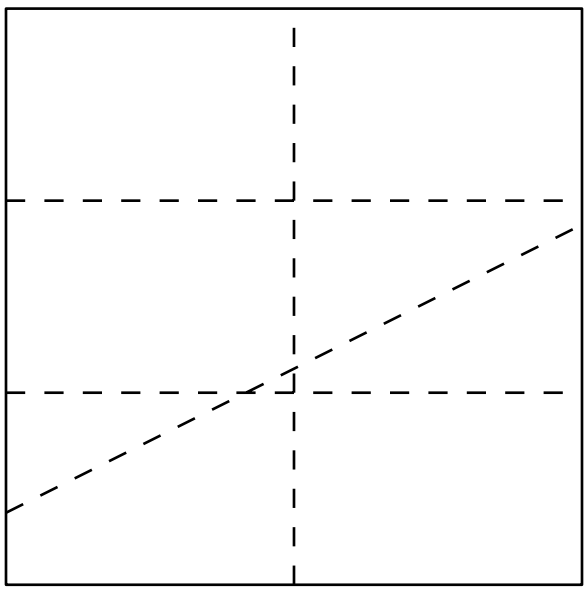
Again, axiom II allows this move.



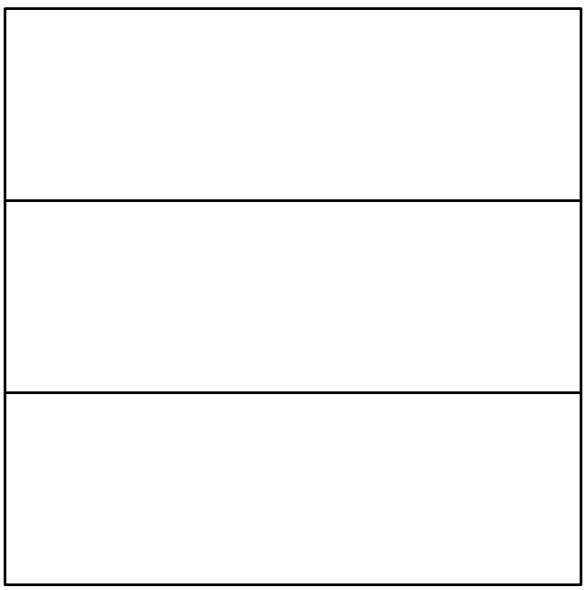
Unfold our first slanted fold at the back.
Want to fold L1 onto L2.



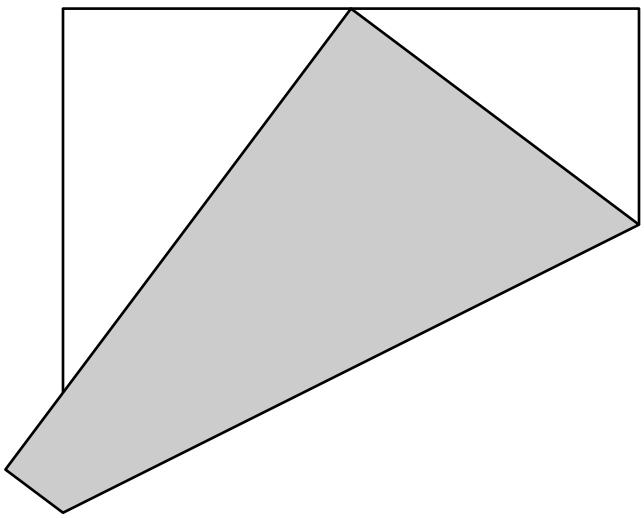
This time it’s axiom III that allows us to do that.



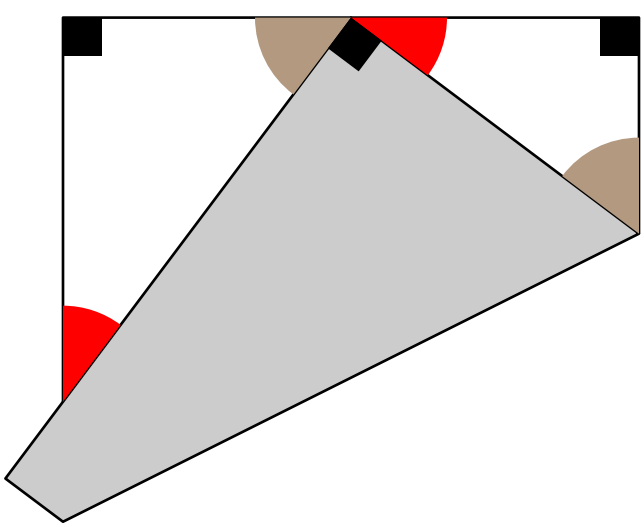
Turn the paper over, and unfold all.



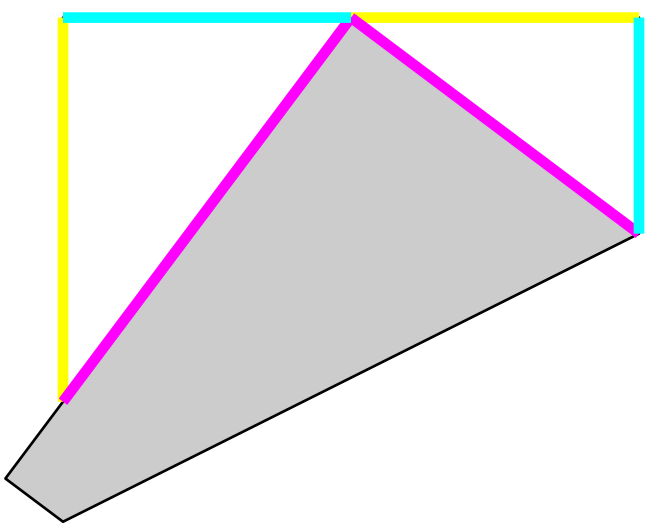
The paper is now divided into thirds!



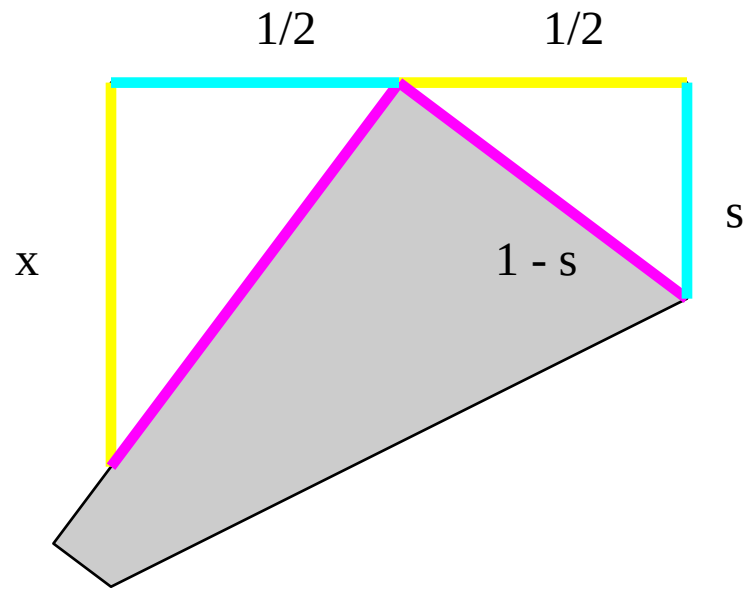
To see why, let’s look at the paper after the second fold.



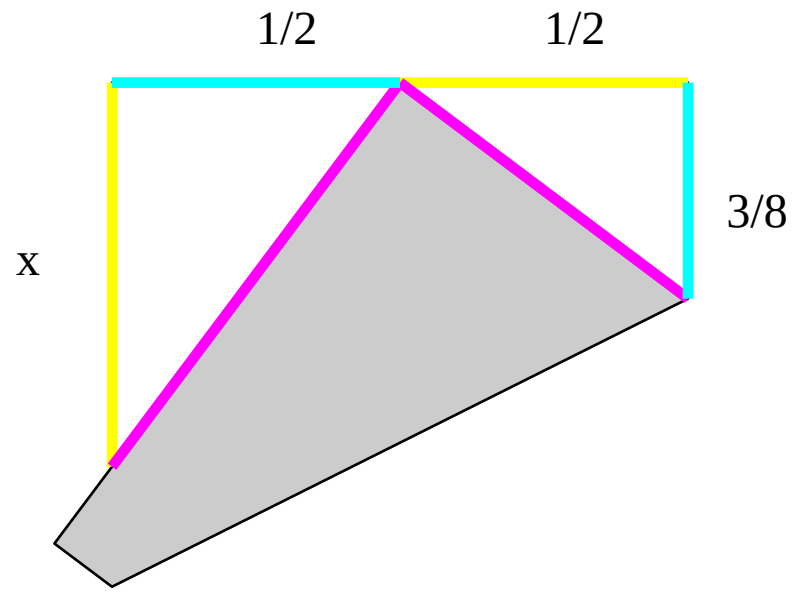
The angles of the 2 triangles are equal.
So the triangles are similar.



Since the triangles are similar,
the sides of the same colour are in the same ratio.



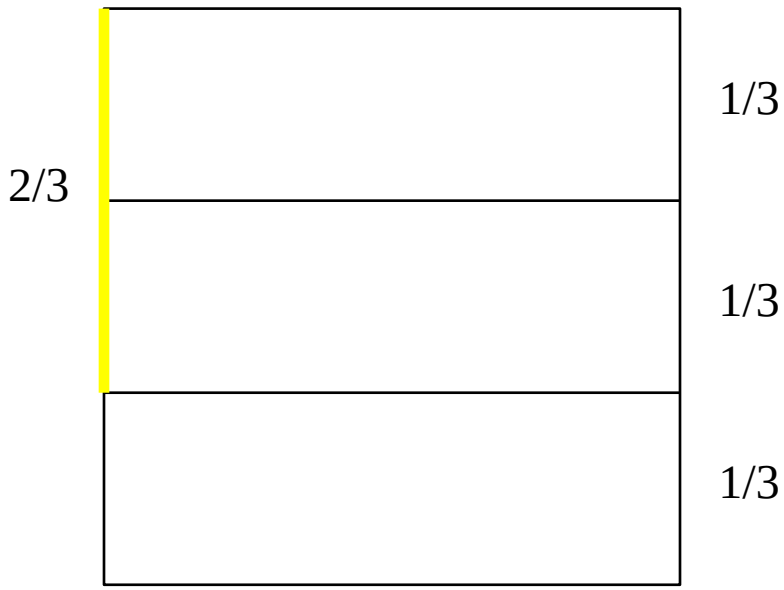
Solve for s by Pythaogus' theorem,
s = 3/8.



By similar triangles,

$$\frac{x}{1/2} = \frac{1/2}{3/8}$$

$$x = 2/3$$



It’s clear the paper is divided into thirds!

Origami brings 2D geometry to another level. It is used to construct many wonderful objects, including polyhedra, tessellations, and many more. Some mathematicians also attempt to use origami to prove other geometric theorems, including construction of higher degree polynomials, topology and etc.

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Project:

The Power of Origami

THE END