Project: Proof of Kepler’s second law

As the planet moves in its orbit

a line from the sun to the planet sweeps out equal areas in equal times.

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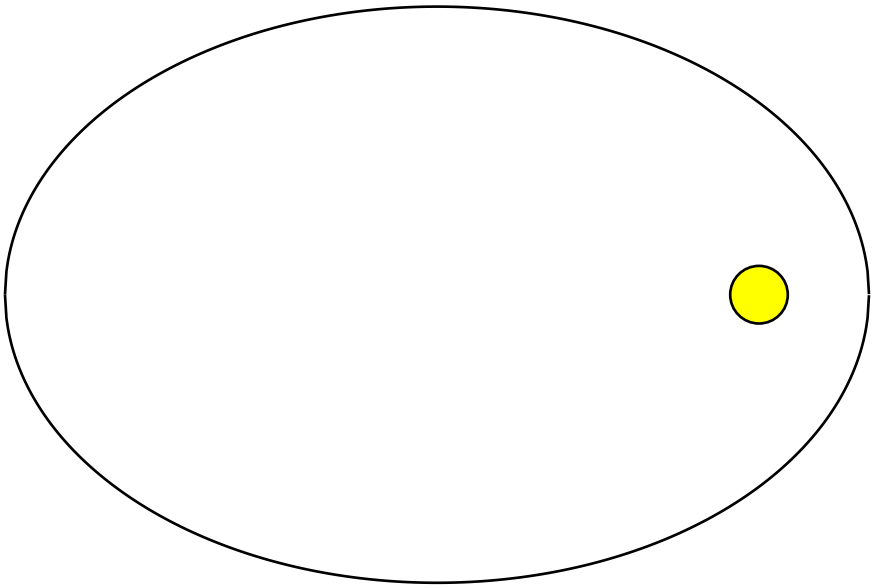
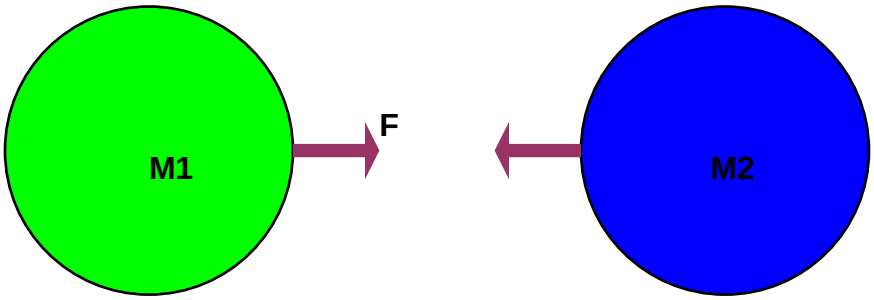
Student Number: 96805015



To prove Kepler's second law, we need following information

Newton's gravitational force

F = G*M1*M2 / (r^2)

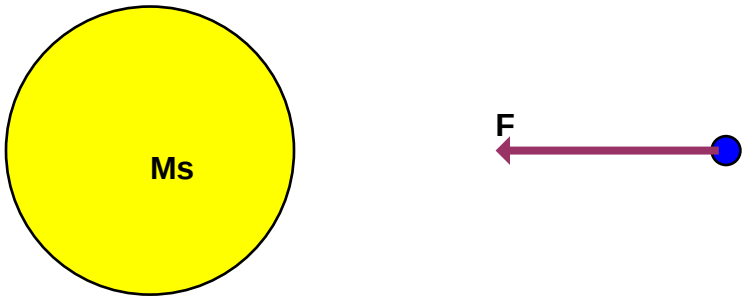


Since we are dealing with planetary motion,

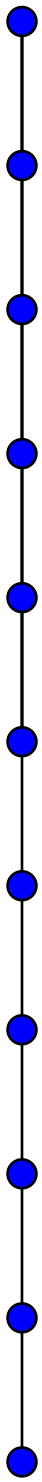
Sun’s mass is much greater than mass of a planet

Therefore, Sun does not move by gravitational force

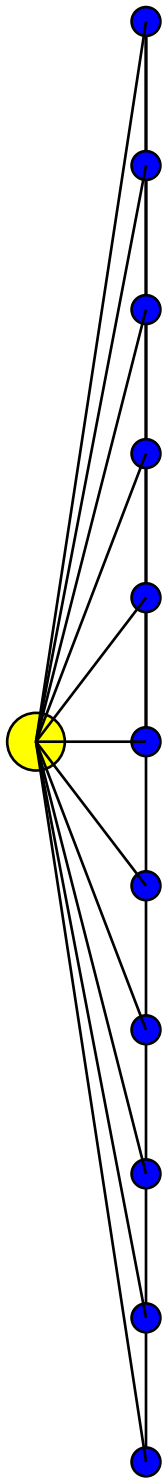
F = G*Ms / (r^2)



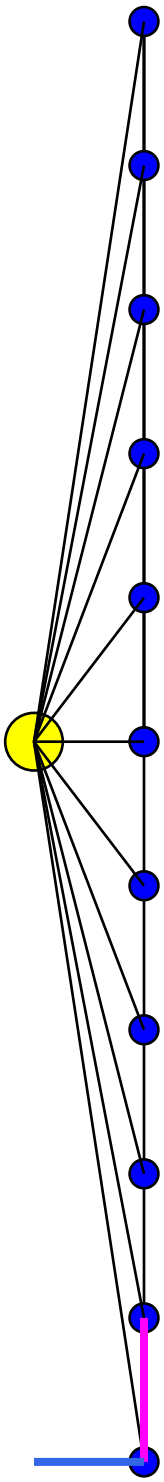
Now, Suppose a planet moves in a straigh line with certain velocity V



Sun’s position is fixed, and it is exerting a gravitational force



Note that each triangle area has same size of base and height



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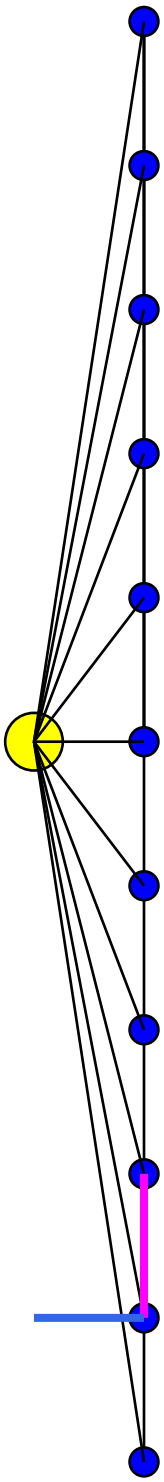

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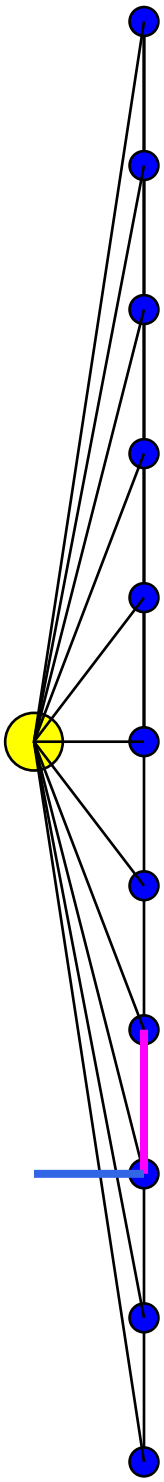

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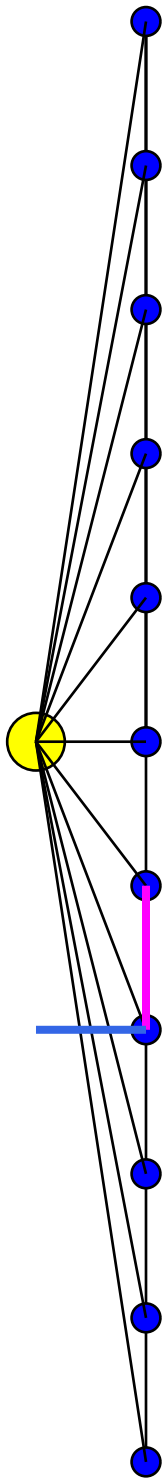

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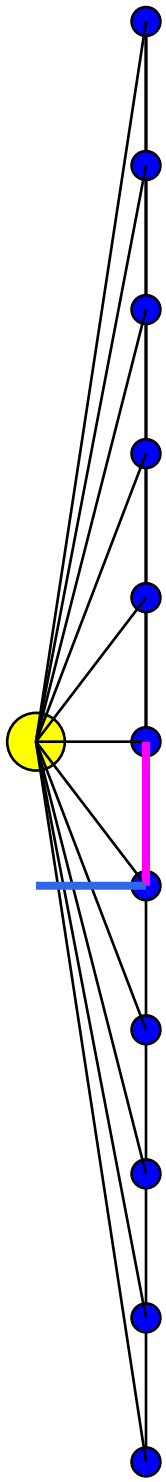

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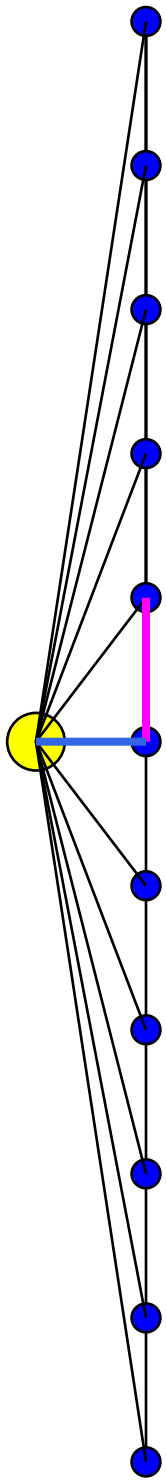

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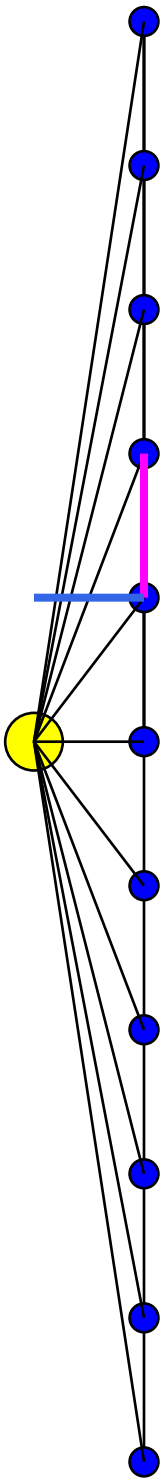

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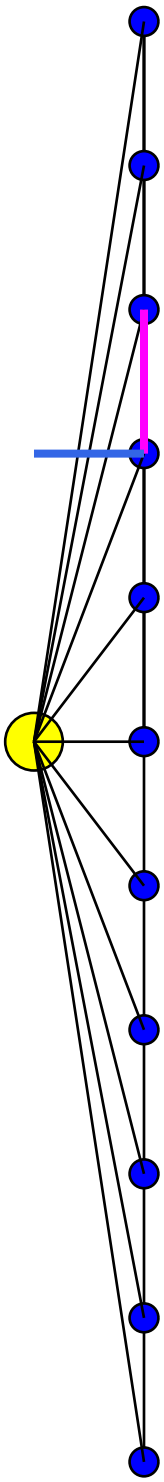

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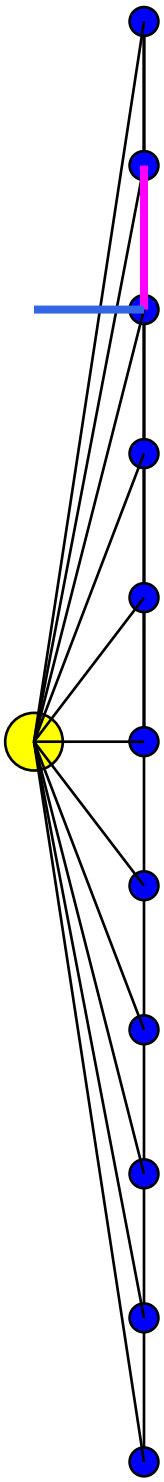

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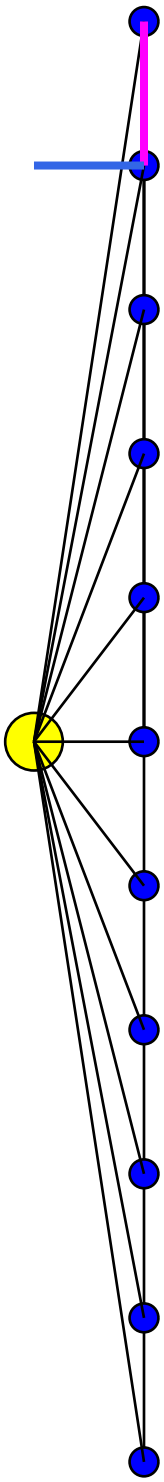
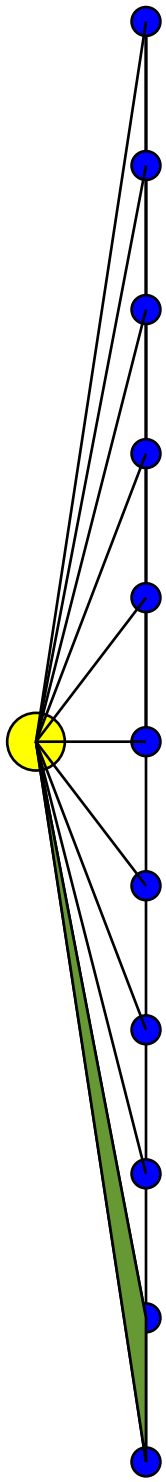


By Euclid’s proposition I.38, these triangles have same area



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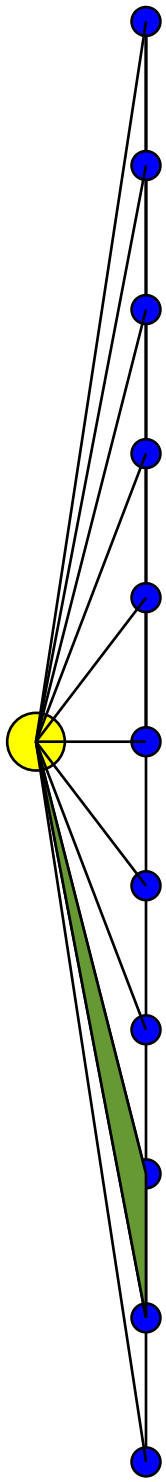

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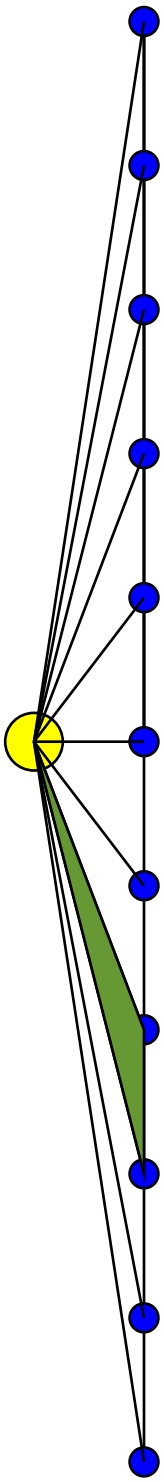

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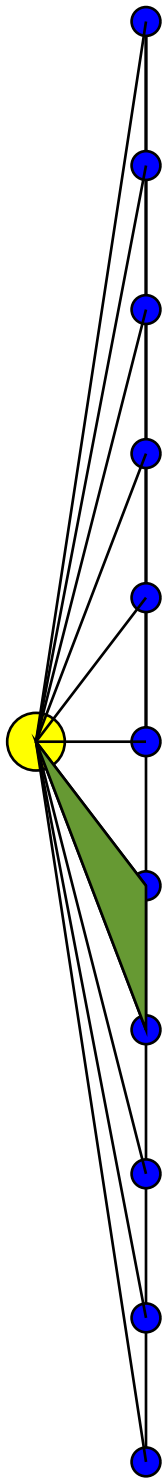

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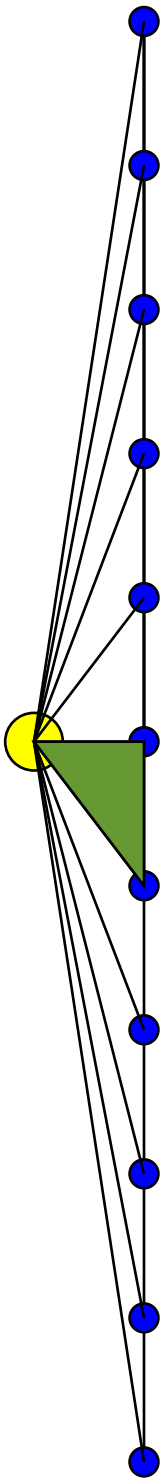

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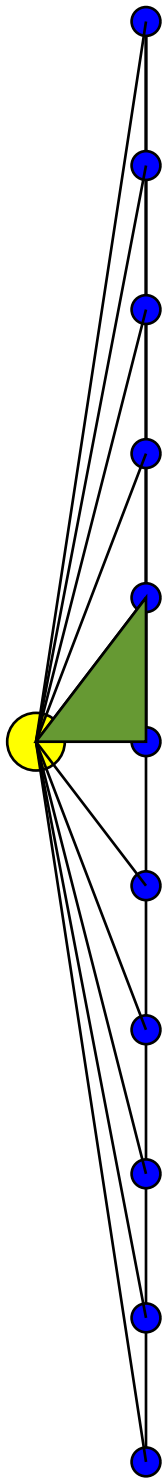

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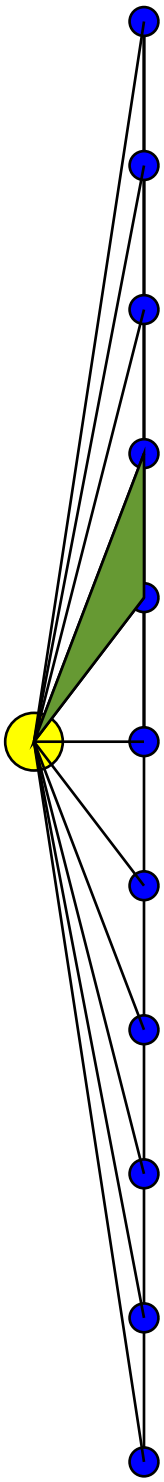

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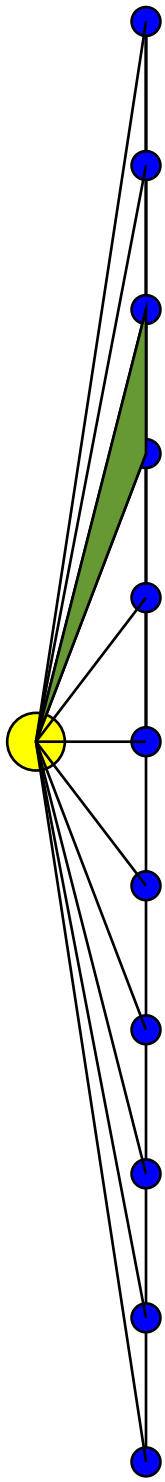

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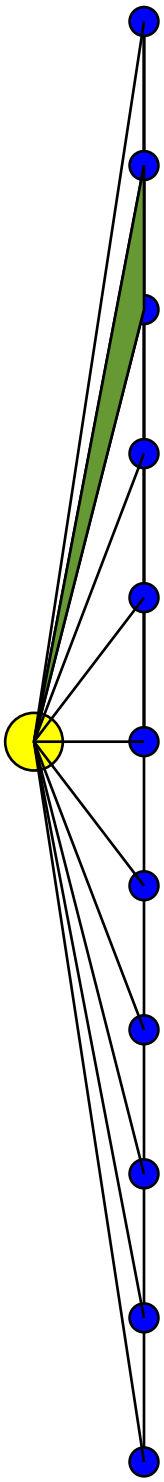

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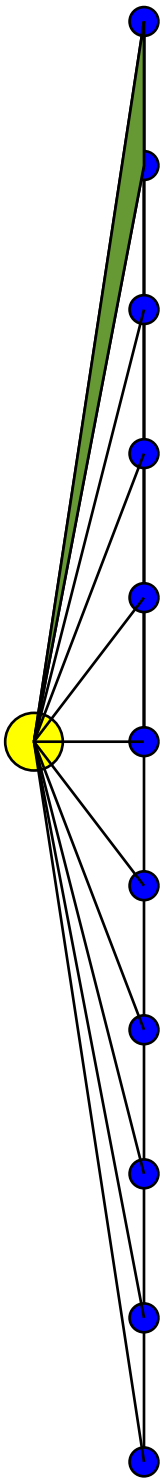
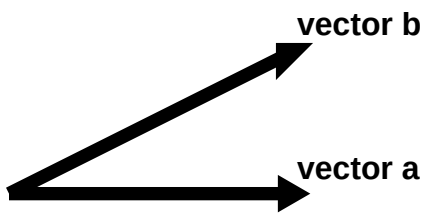


|a X b| / 2 = area of triangle



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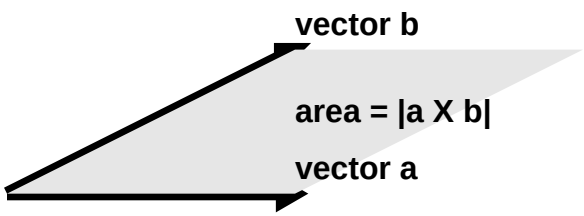
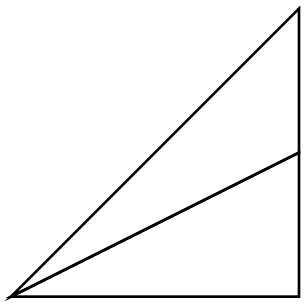


Note that angular momentum: L = m*v X r

where m is mass of an object, v is a tangential velocity vector, r is a distance vector

Now let's consider following 2 triangles that have same area



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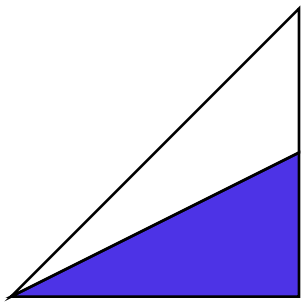

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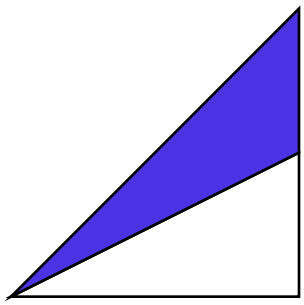

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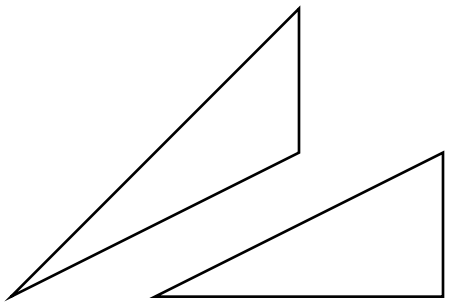

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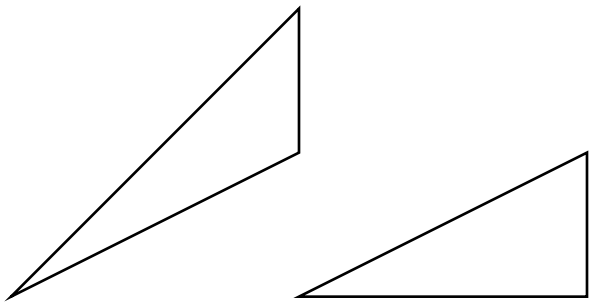

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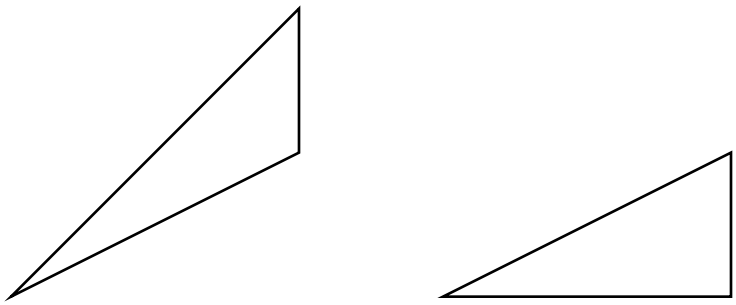

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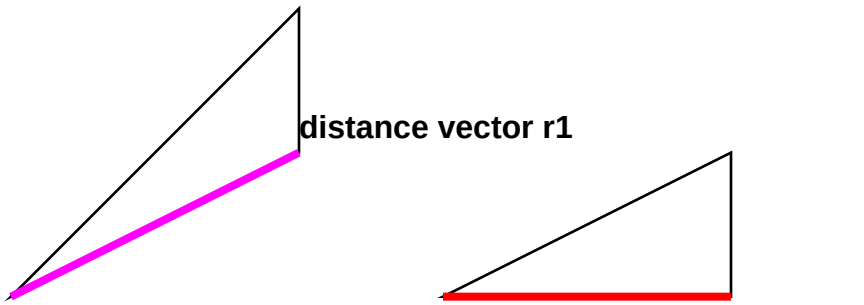

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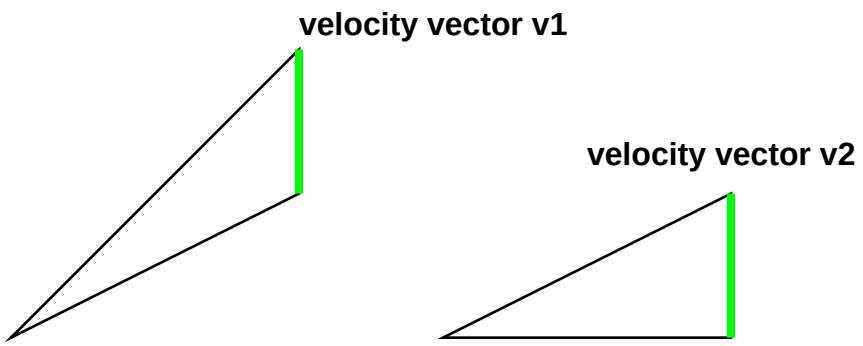

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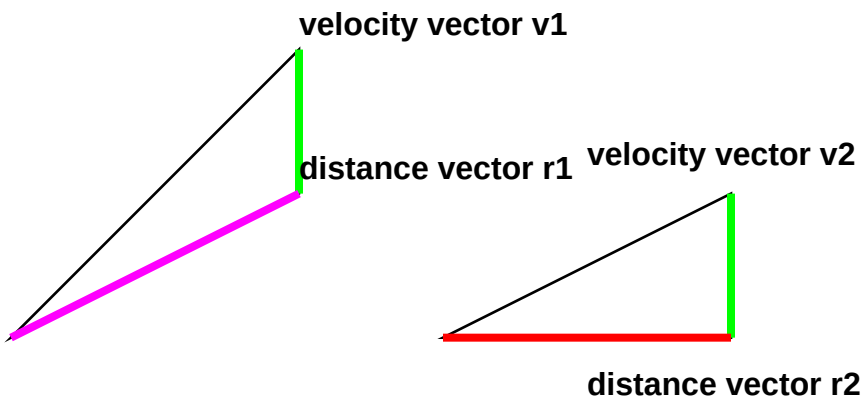
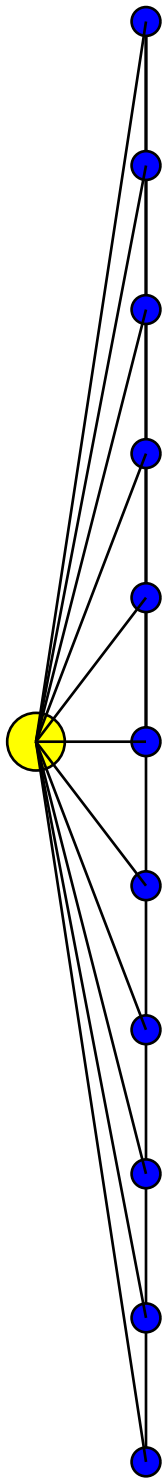
Since their area is same, Vn X Rn is same for all n.

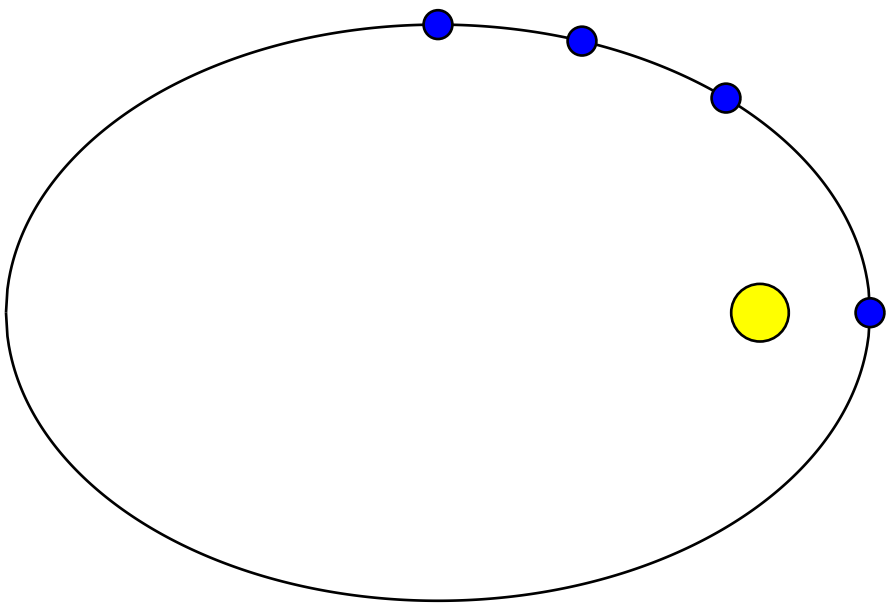
This is conservation of angular momentum



Therefore, if a planet moves in a straight line, Kepler's second law holds.



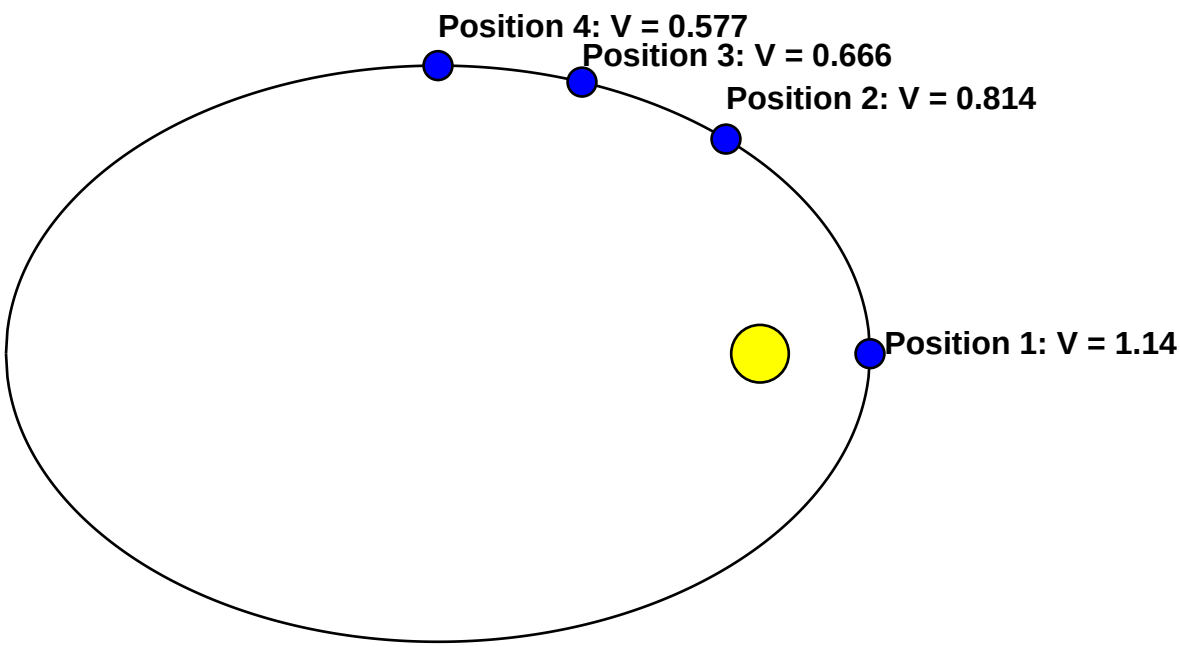
However, a planet moves in an elliptical orbit



Find tangential velocity at each point.

F = G * Ms / (r^2) , Centripetal Acceleration = V/(r^2)

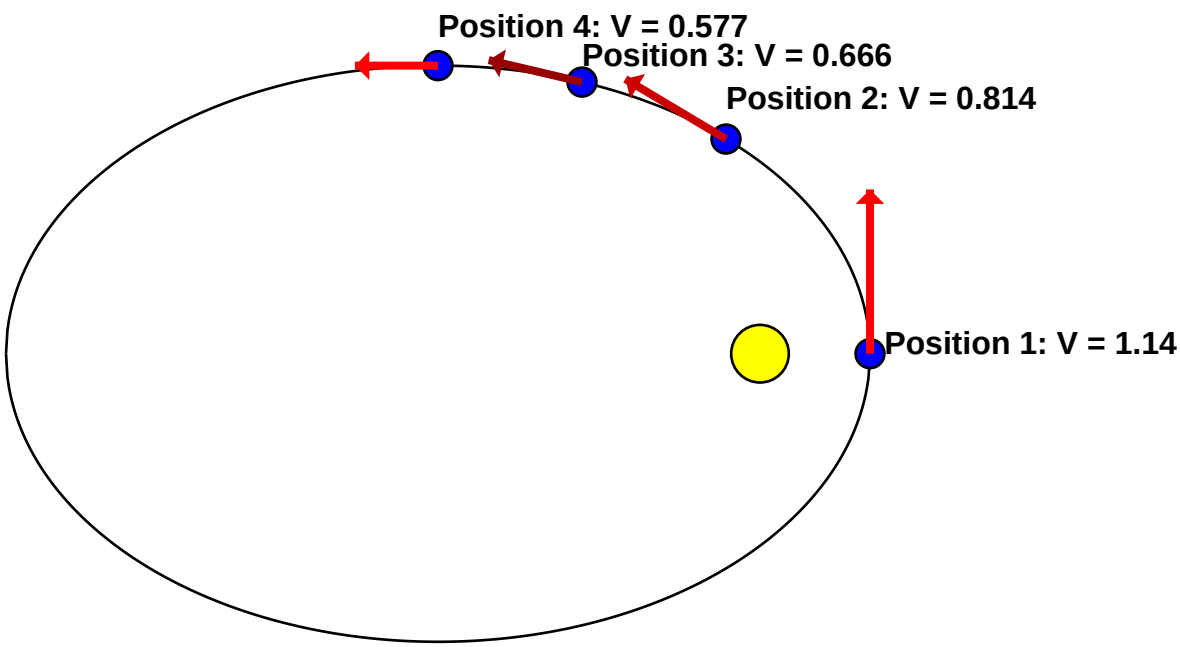
==> G * Ms / r = V^2 ==> k/r = V^2 ==> V = sqrt(k/r), We let k = 1 for convention



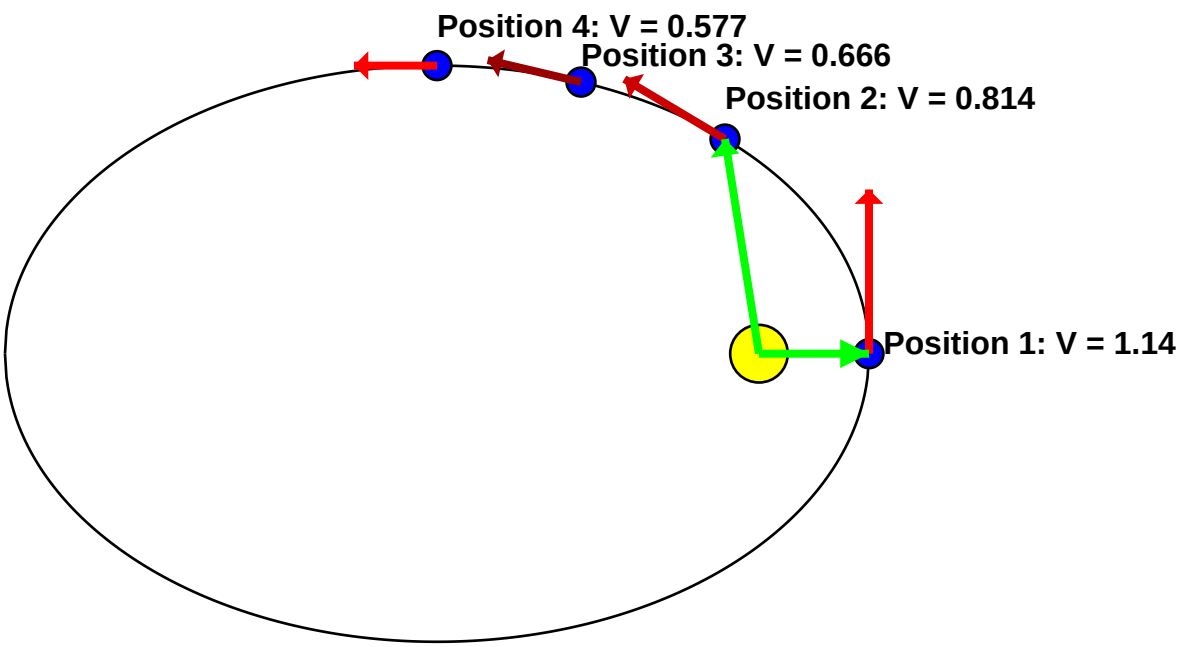
Find direction of velocity at each point.

Implicit diffentiation: dy/dx = -4x/9y

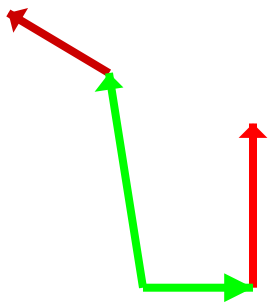
==> Gives us a slope of tangent velocity ==> Direction



Add direction vector for each point.



Let’s consider Position 1 and Position 2



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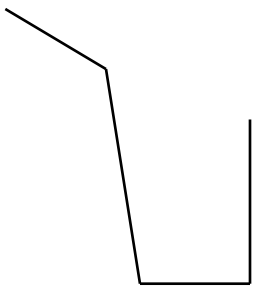

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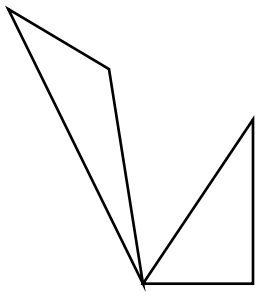

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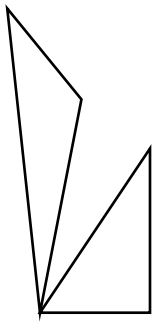

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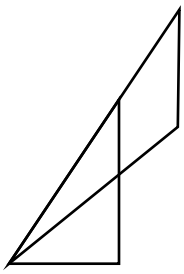

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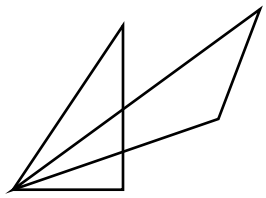

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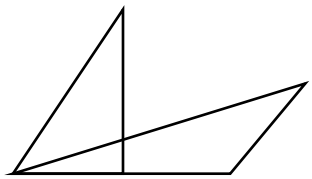

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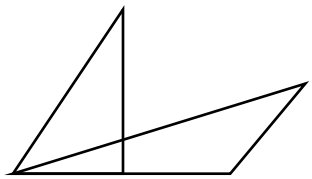

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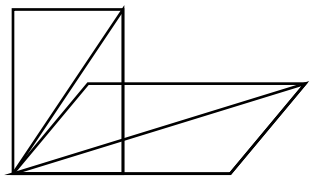
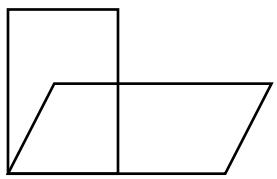


Let’s consider Position 1 and Position 2



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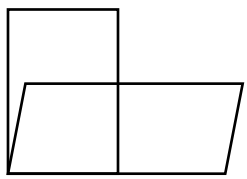

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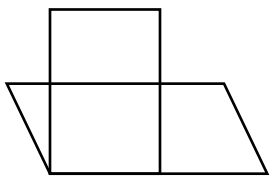

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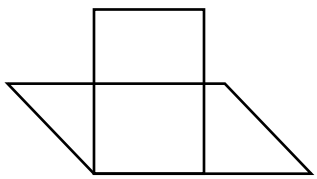

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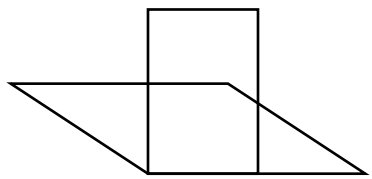

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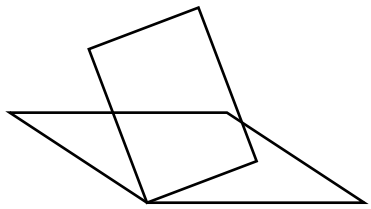

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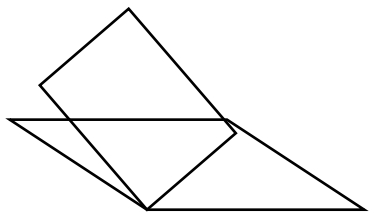

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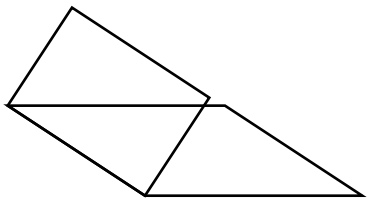
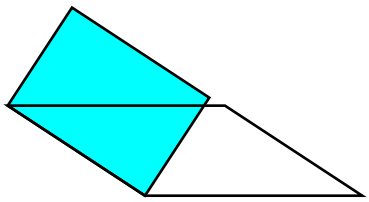


By Euclid’s Proposition, the area of two rectangle is same



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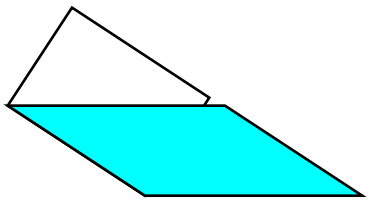
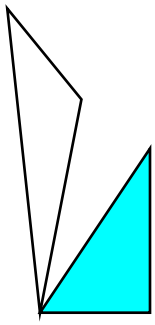


Therefore, these two original triangles have same area



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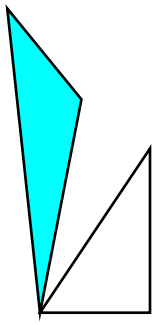
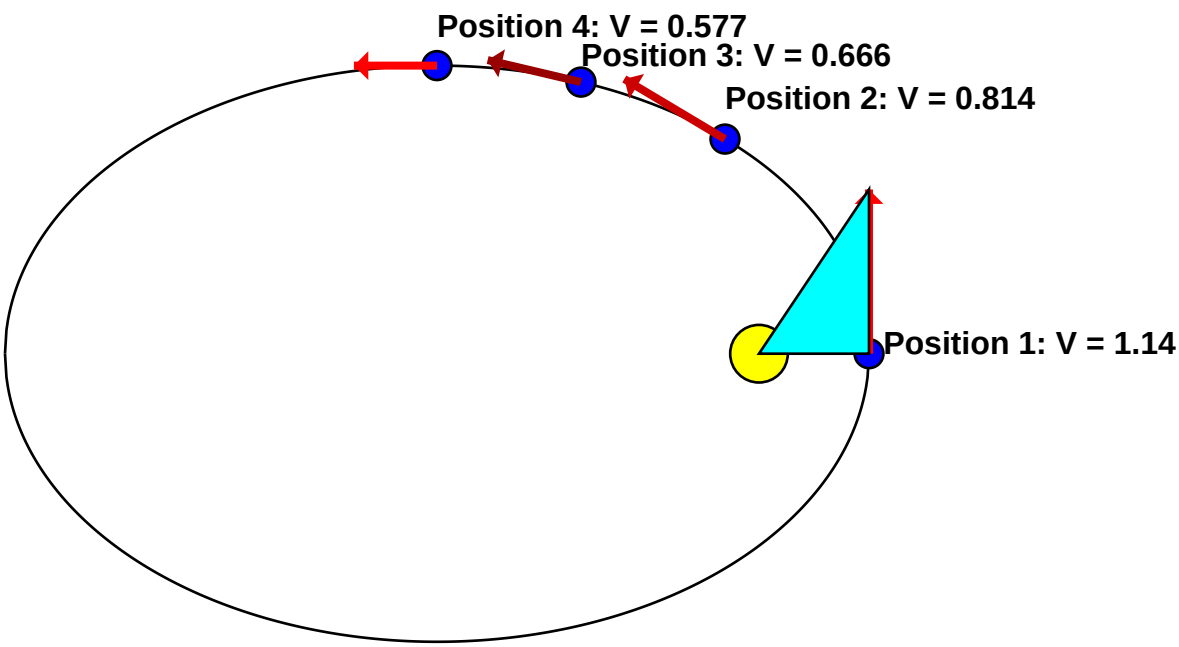


Therefore, in an orbital motion, Kepler's second law holds.



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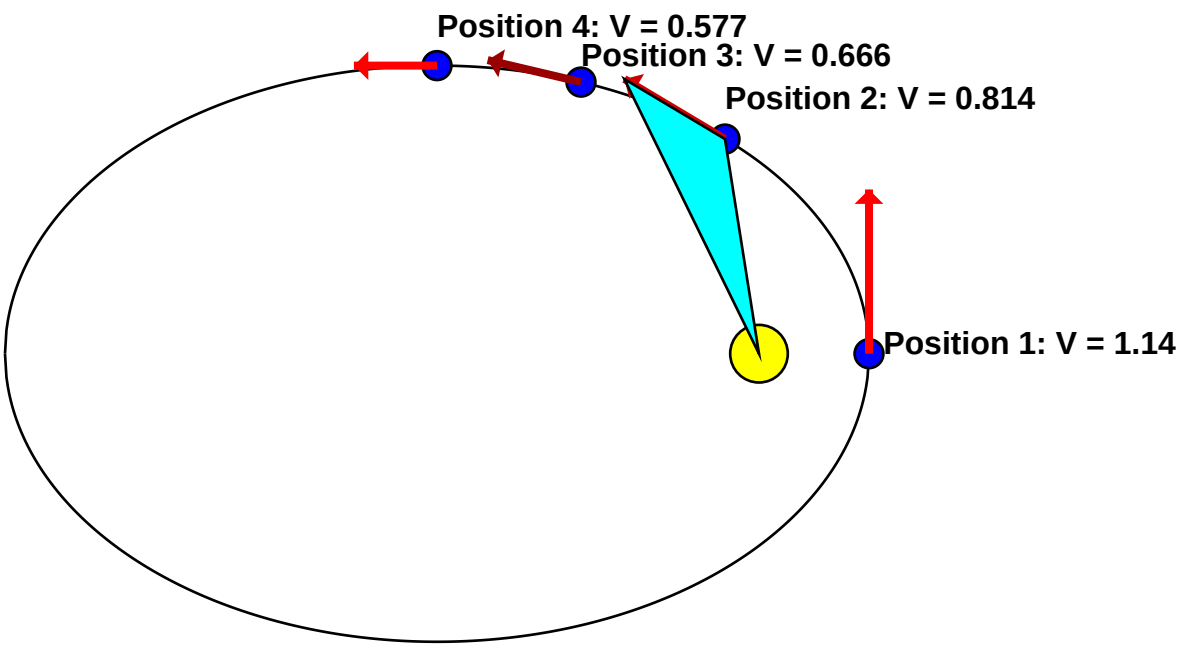

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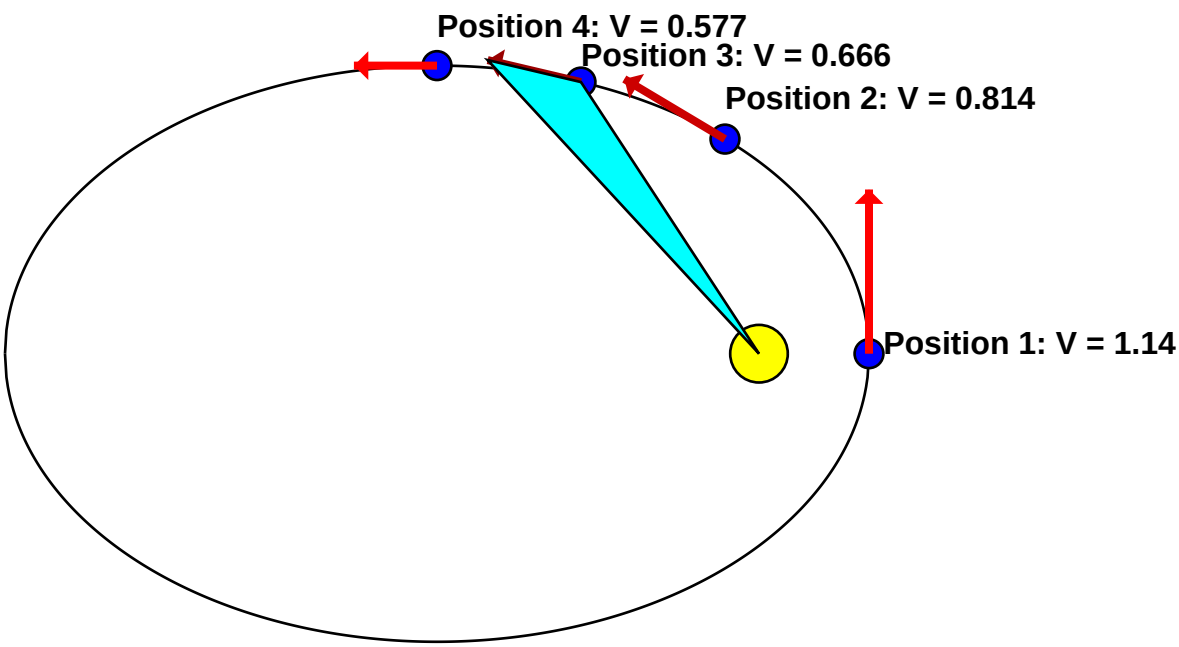

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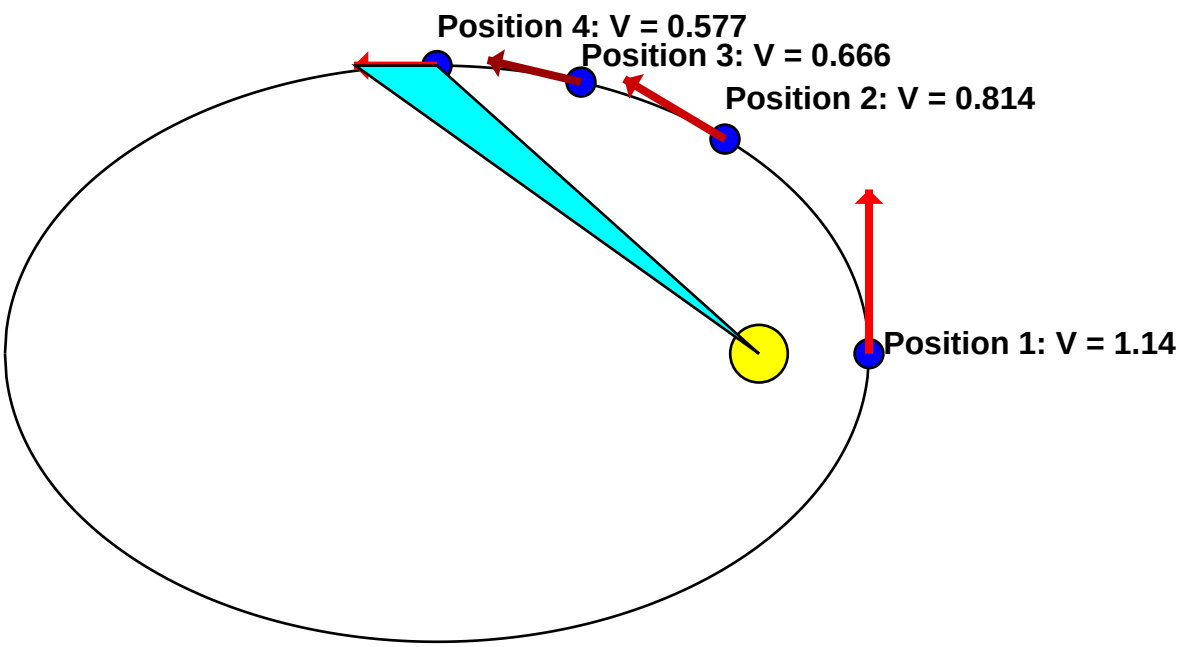

THE END

Special Thanks to: Dr. Bill Casselman

Thanks to : Issac Newton, Johannes Kepler

Reference: Dr.Casselman's notes about Kepler's law

Reference: http://www.physics.nmt.edu/~raymond/classes/ph13xbook/node126.html

Reference: http://dept.physics.upenn.edu/courses/gladney/mathphys/java/sect5/subsubsection5_1

P.S: Every magnitude and direction of all vectors was calculated

i.e. all vectors are scaled for real demonstration