Math 308 Project
Euclid’s Element
Book XII Proposition 5

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Book VII Proposition 5

This proposition is developed based on the combinations of the relatively basic propositons XII.3, X.1, XII.4, V.11, V.16, and, of course, further ideas from Euclid.

Related theorems in XII.5

XII.3

The pyramid ABCD is divided into "two pyramids equal to one another, having triangular bases, and similar to the whole pyramid" and into "two equal prisms". Then:

These two prisms are greater than 1/2 of the whole pyramid.

Related theorems in XII.5

X.1

Given two magnitudes AB and C, AB > C.

Related theorems in XII.5

X.1

If we subtracted a magnitude HB, which is greater than half of AB, from AB.

Related theorems in XII.5

X.1

and we subtracted another magnitude KH, which is greater than half of AH, from AH,

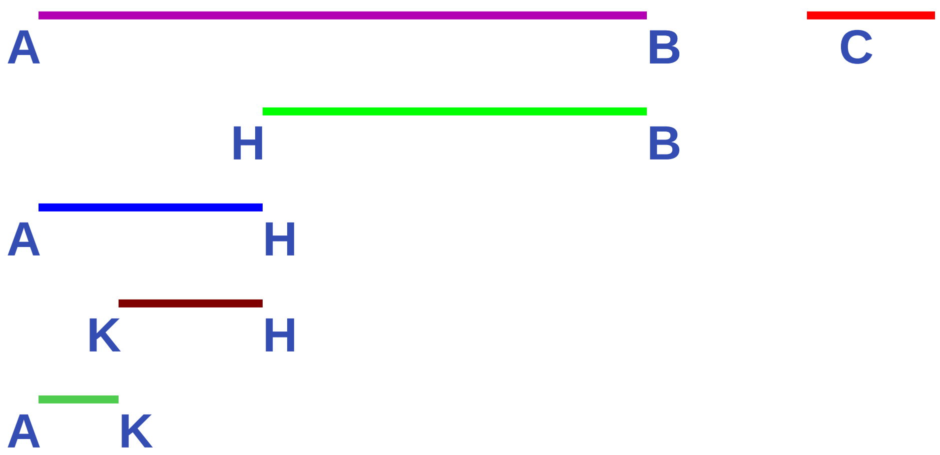
Related theorems in XII.5

X.1

then:

There will be left some magnitude which is less than the magnitude C.

In this case, AK < C.



Related theorems in XII.5

XII.4

Let there be two pyramids of the same height having the triangular bases ABC and DEF, and with the vertices G and H. Let each of these two pyramids be divided into "two smaller pyramids which is equal to one another and similar to the whole pyramid" and into "two equal prisms". Then we say that:

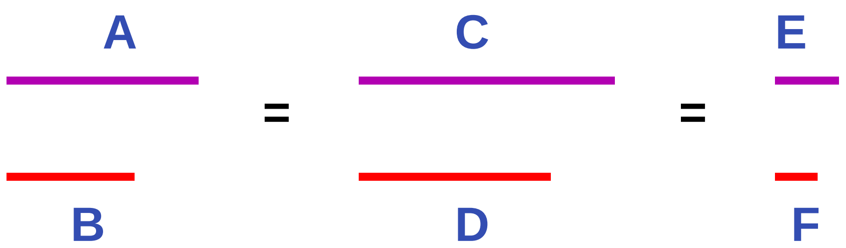
The base ABC:base DEF = all prisms in pyramid ABCG: all prisms in pyramid DEFH

Related theorems in XII.5

V.11

Ratios which are the same with the same ratio are also the same with one another

In other words, Let A:B = C:D and C:D = E:F



Related theorems in XII.5

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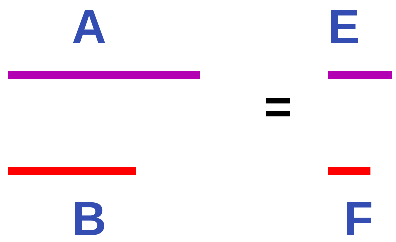
Ratios which are the same with the same ratio are also the same with one another

In other words, Let A:B = C:D and C:D = E:F

Then we say that:

A:B = E:F



Related theorems in XII.5

V.16

If four magnitudes are proportional, then they are also propositonal alternately.
In other words, Let A, B, C, and D be four proportional magnitudes, so that A:B = C:D. Then we say that:

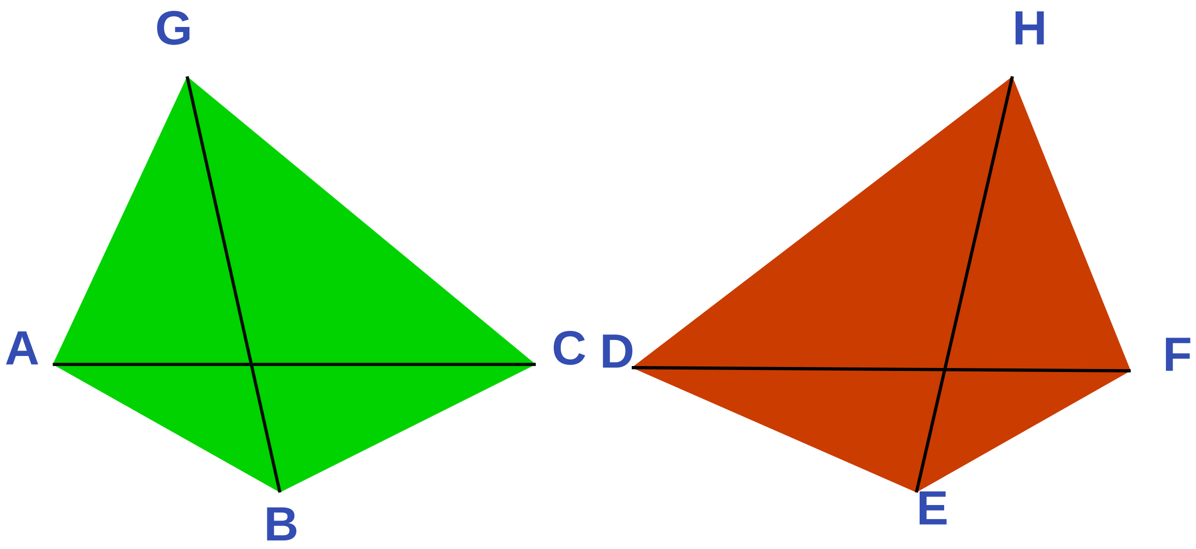
$$A:C = B:D$$

Book XII Proposition 5

XII.5

Pyramids of the same height with triangular bases are to one another as their bases

In other words,



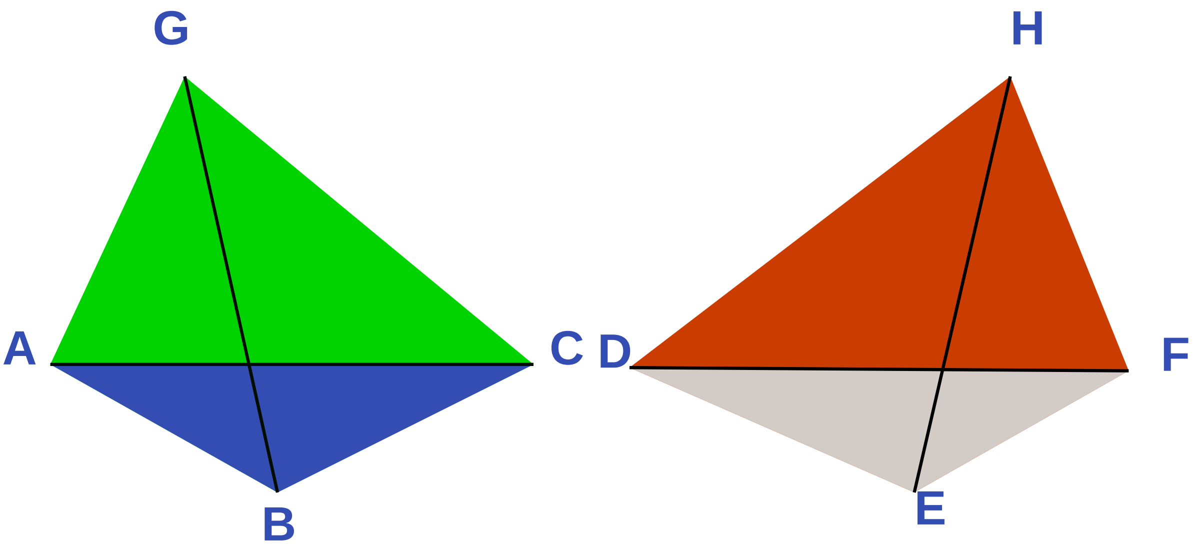
Book XII Proposition 5

XII.5

We say that:

Base ABC : Base DEF =

Pyramid ABCG : Pyramid DEFH



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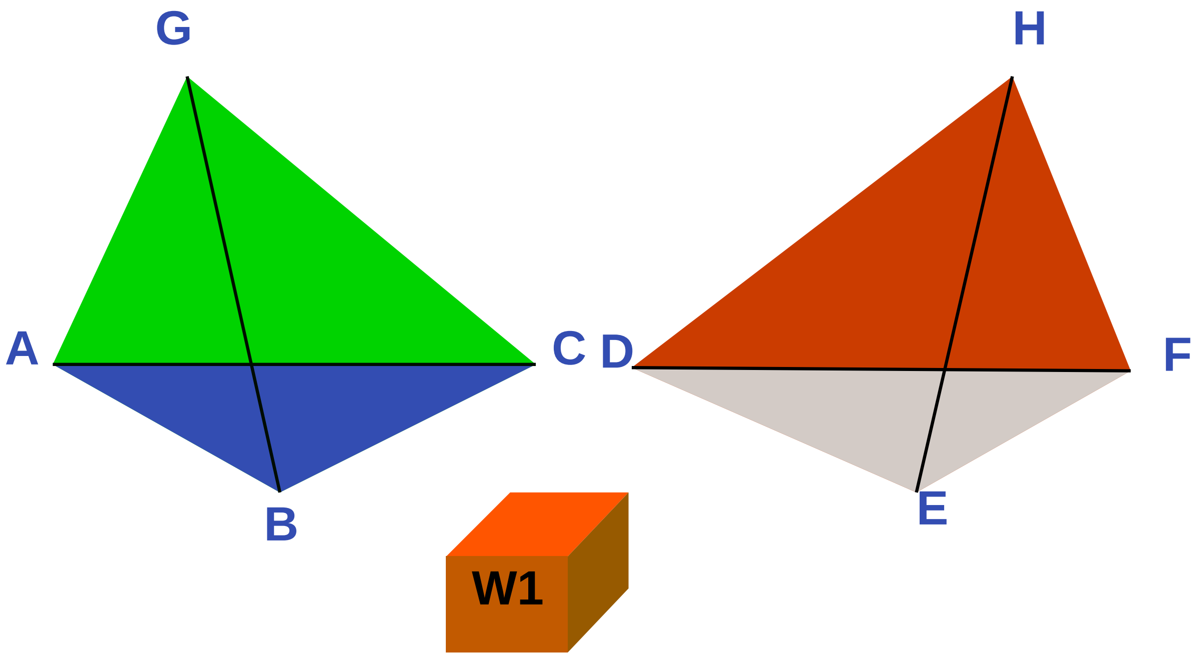
Proof

Assume Base ABC : Base DEF is NOT to
Pyramid ABCG : Pyramid DEFH



then either:

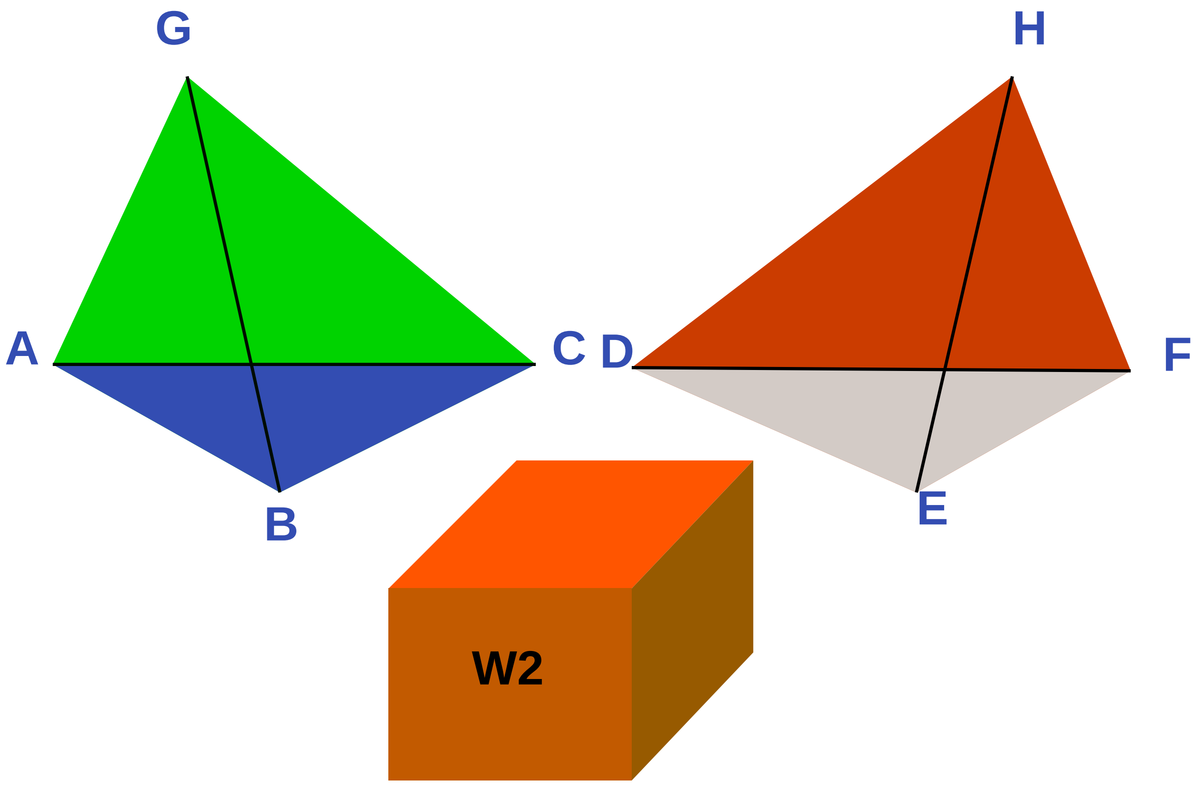
Base ABC:Base DEF = Pyramid ABCG:W1

, W1 is any solid less than Pyramid DEFH.

(called it Case 1)

Book XII Proposition 5

or:
Base ABC:Base DEF = Pyramid AGCG: W2



, W2 is any solid greater than pyramid DEFH
(called it Case 2)

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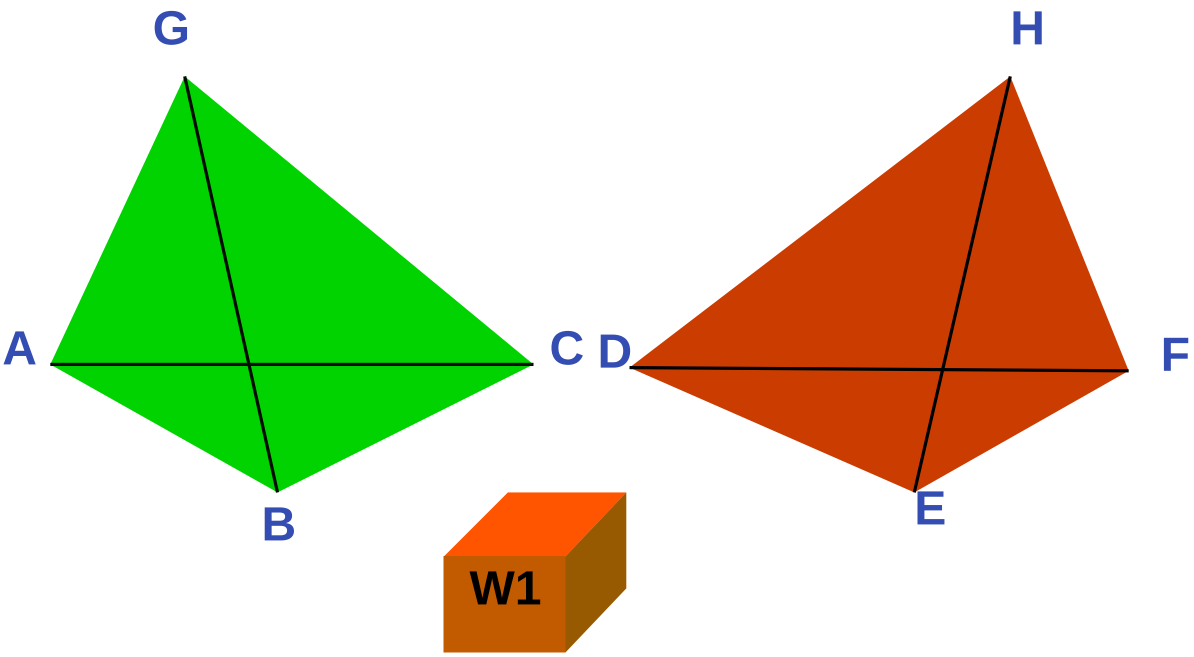
Case 1:

Pyramid ABCG: W1



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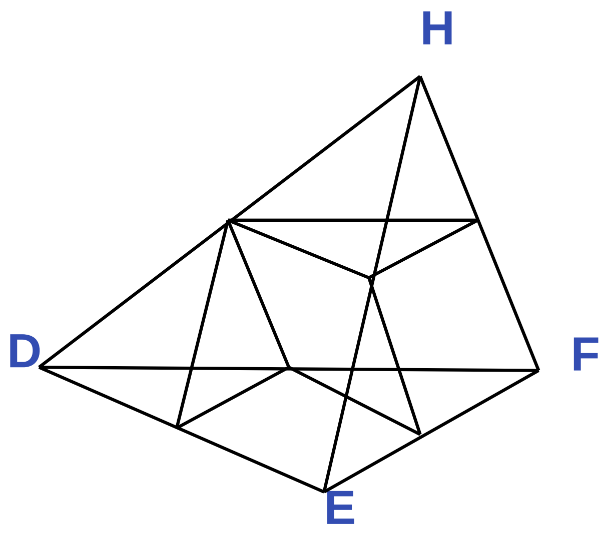
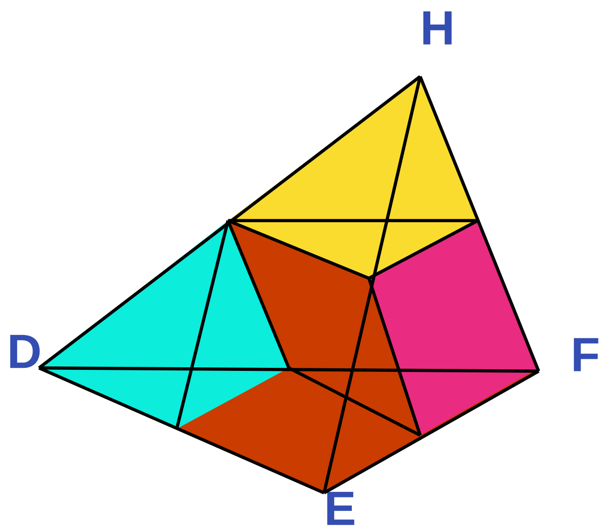
Divide pyramid DEFH



Book XII Proposition 5

into two equal pyramids similar to pyramid DEFH and into two equal prisms.



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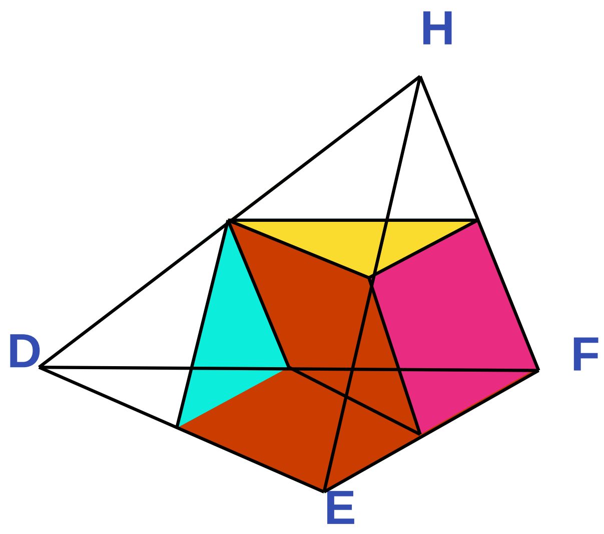
By XII.3, two equal prisms > 1/2 pyramid DEFH



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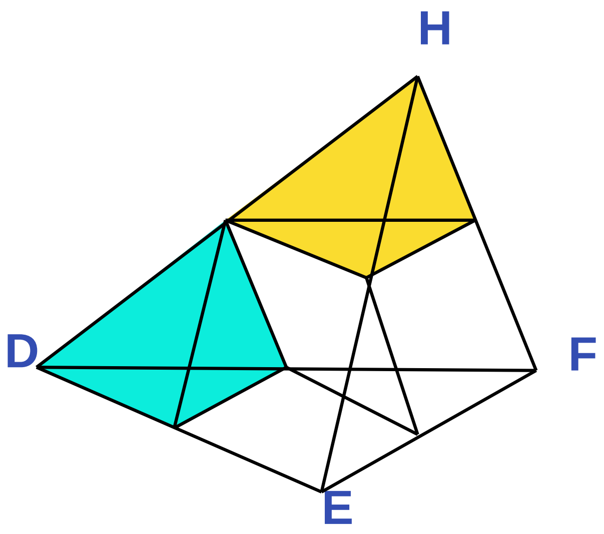
we divide pyramids from the first division similarly,
and repeated again and again..............



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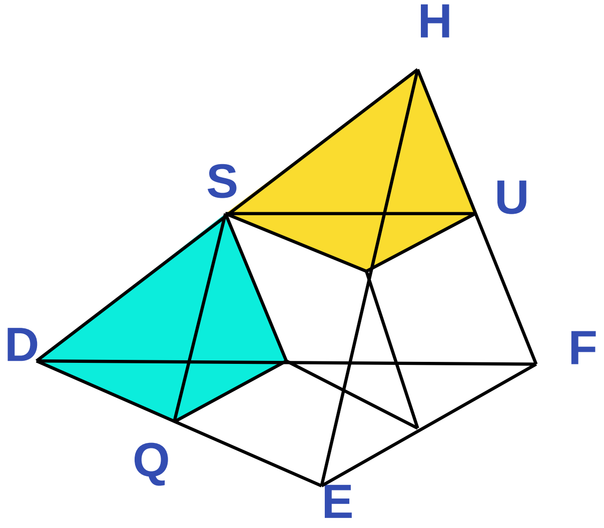
there will be left over pyramid DQRS and
Pyramid STUH.
and by X.1, these two pyramids < W1.



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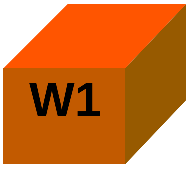
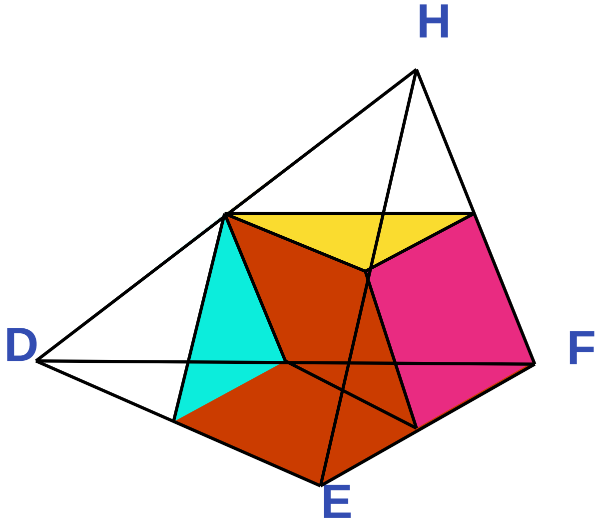
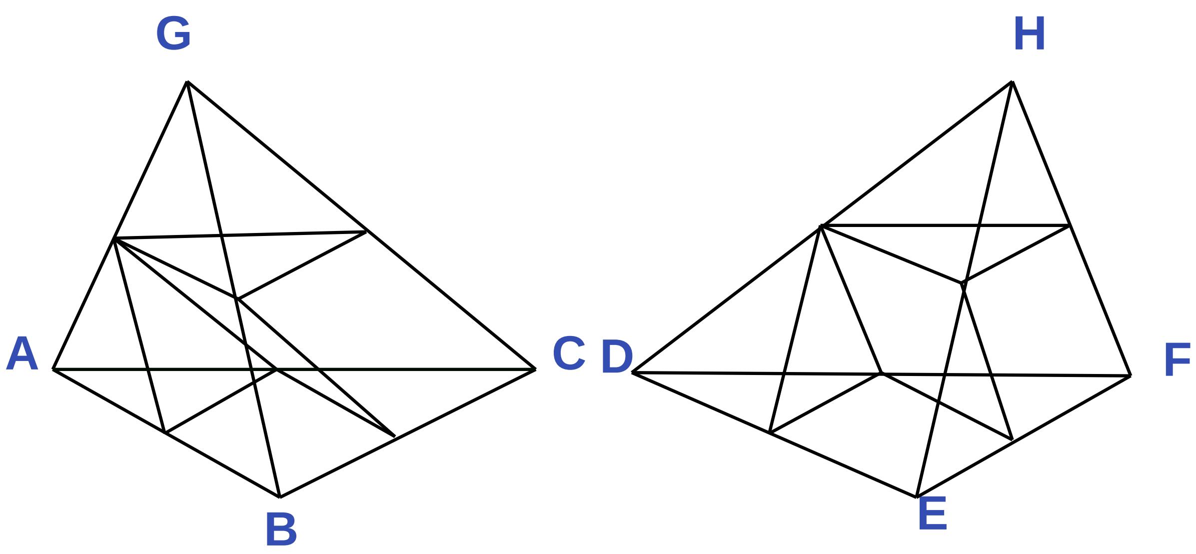
the remaining prisms in pyramid DEFH > W1.





Book XII Proposition 5

we divide pyramid ABCG similarly with pyramid DEFH with the same number of times.



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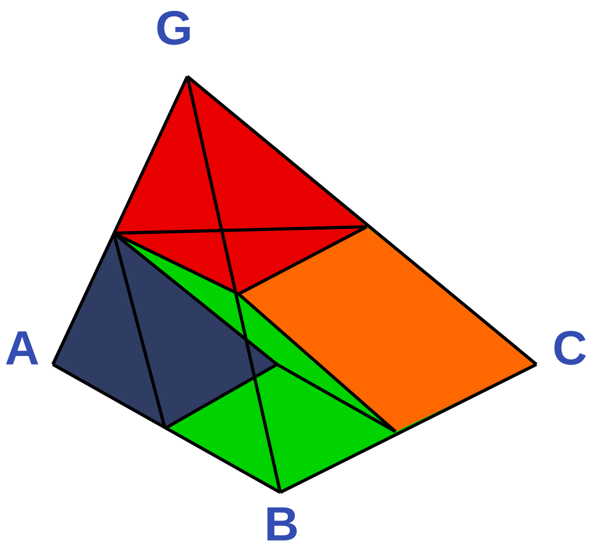
Then,



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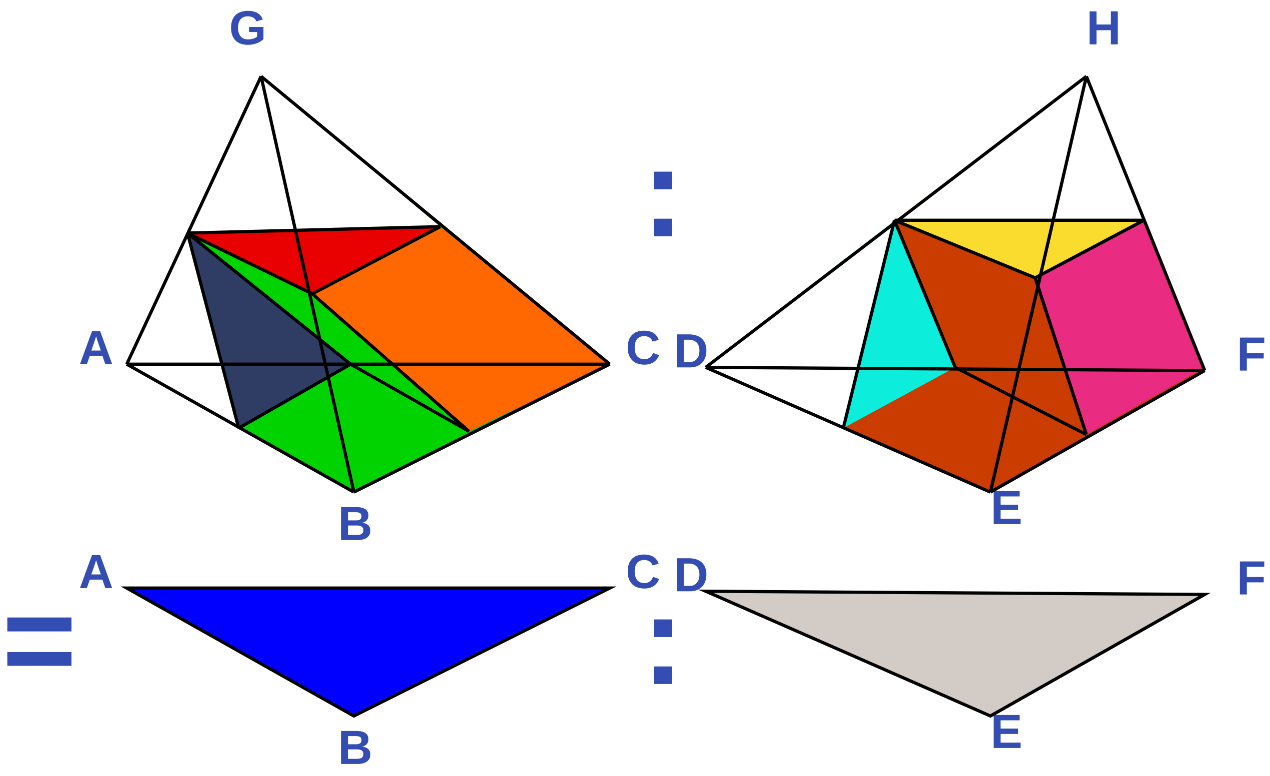
By XII.4,



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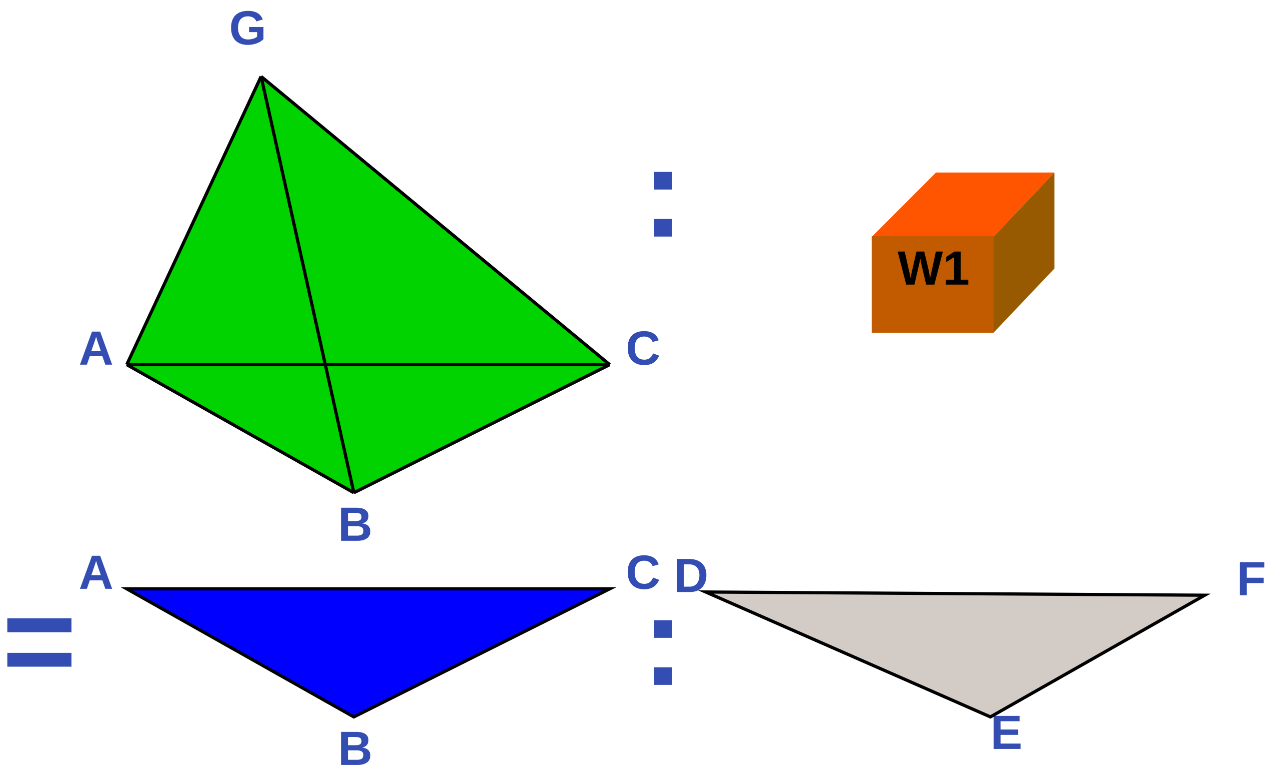
But



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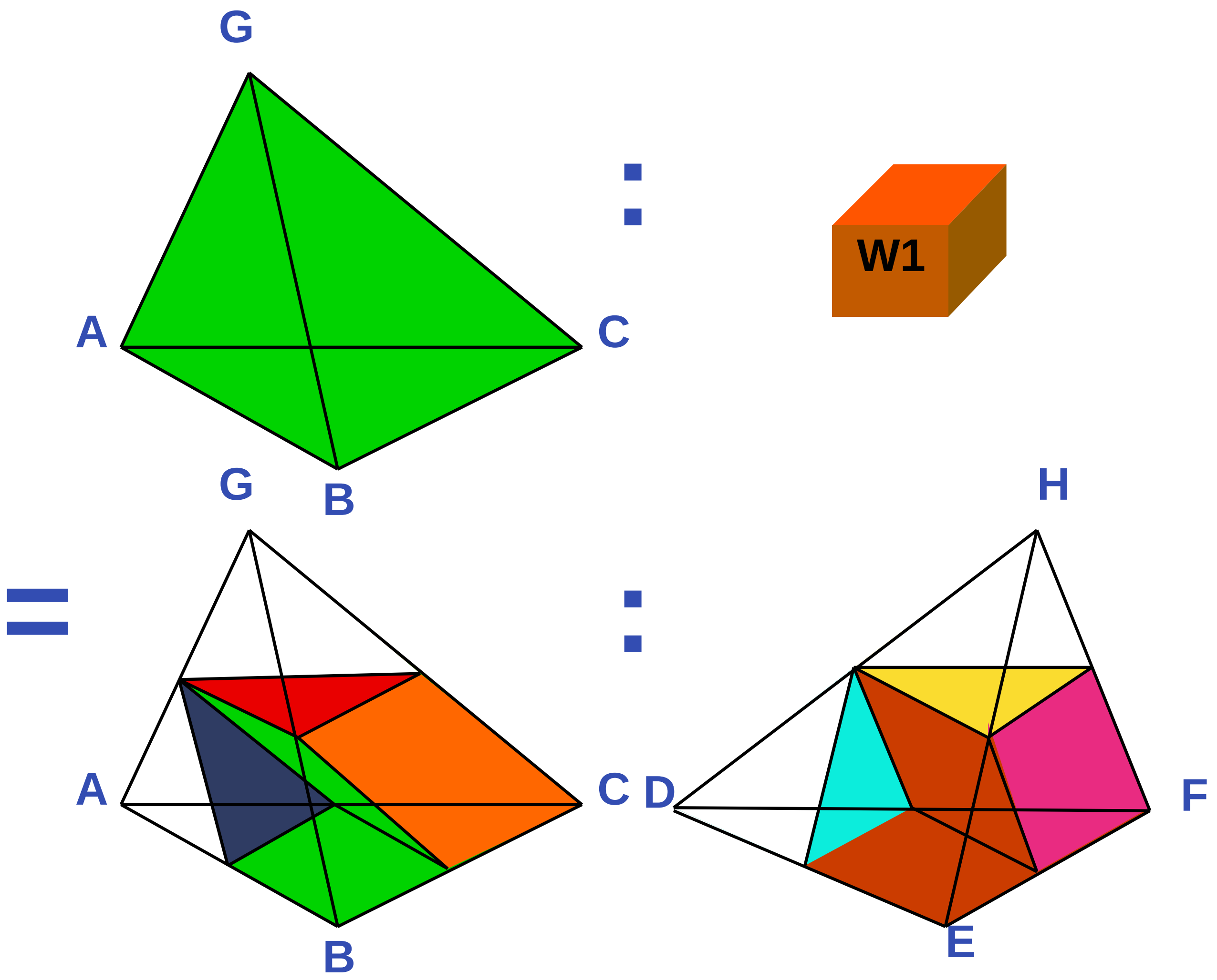
therefore,



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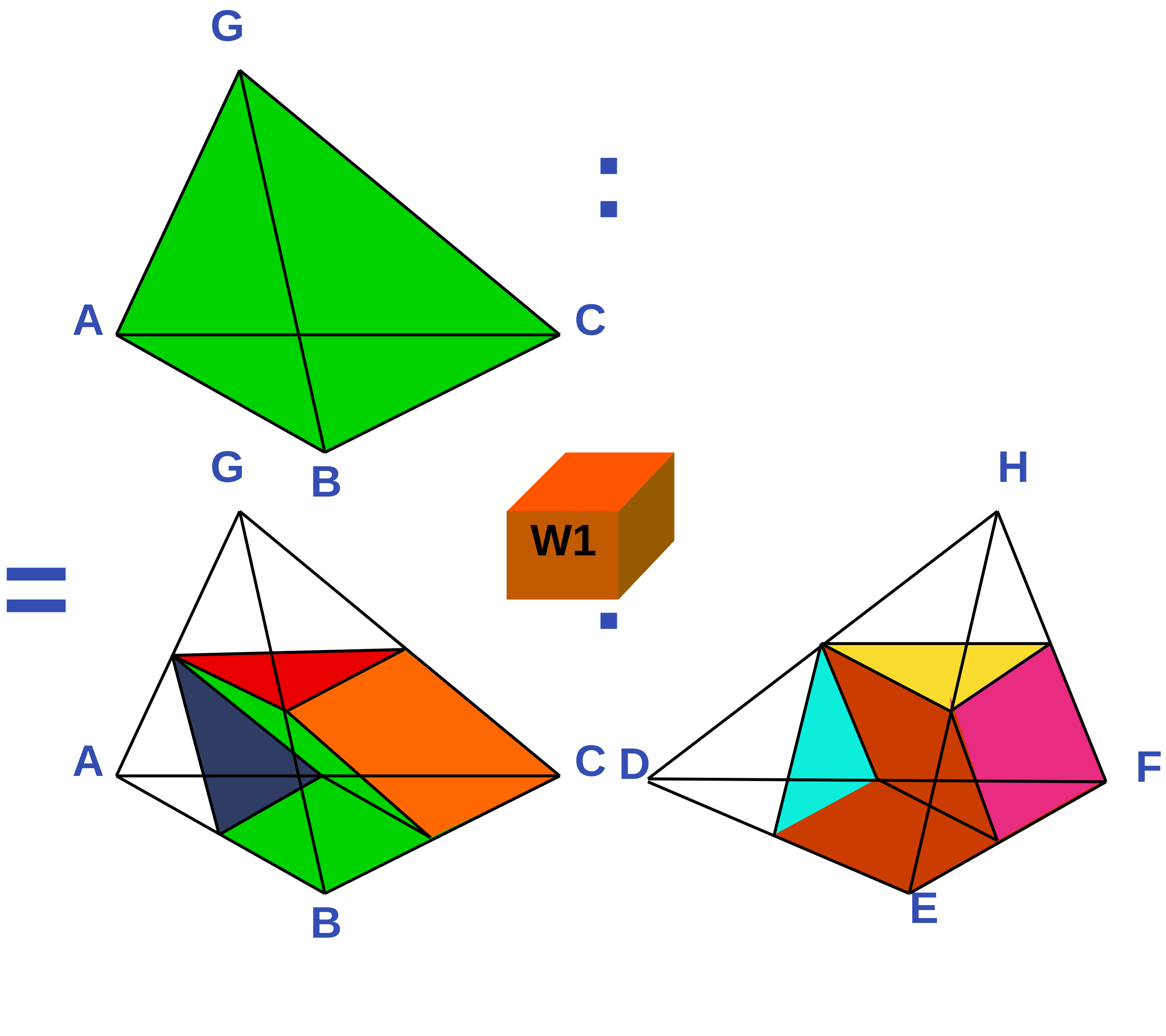
Alternately,



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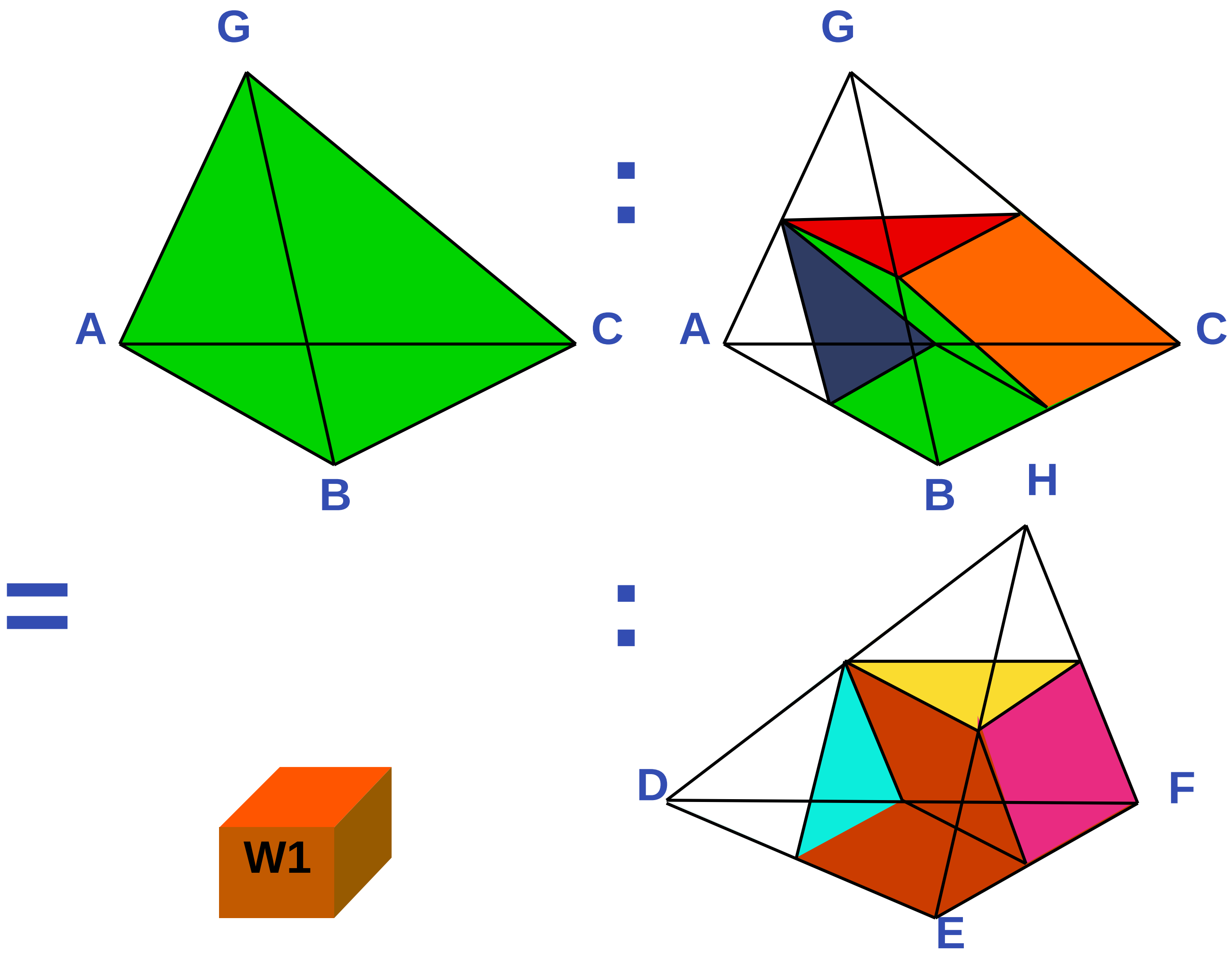
By V.16,



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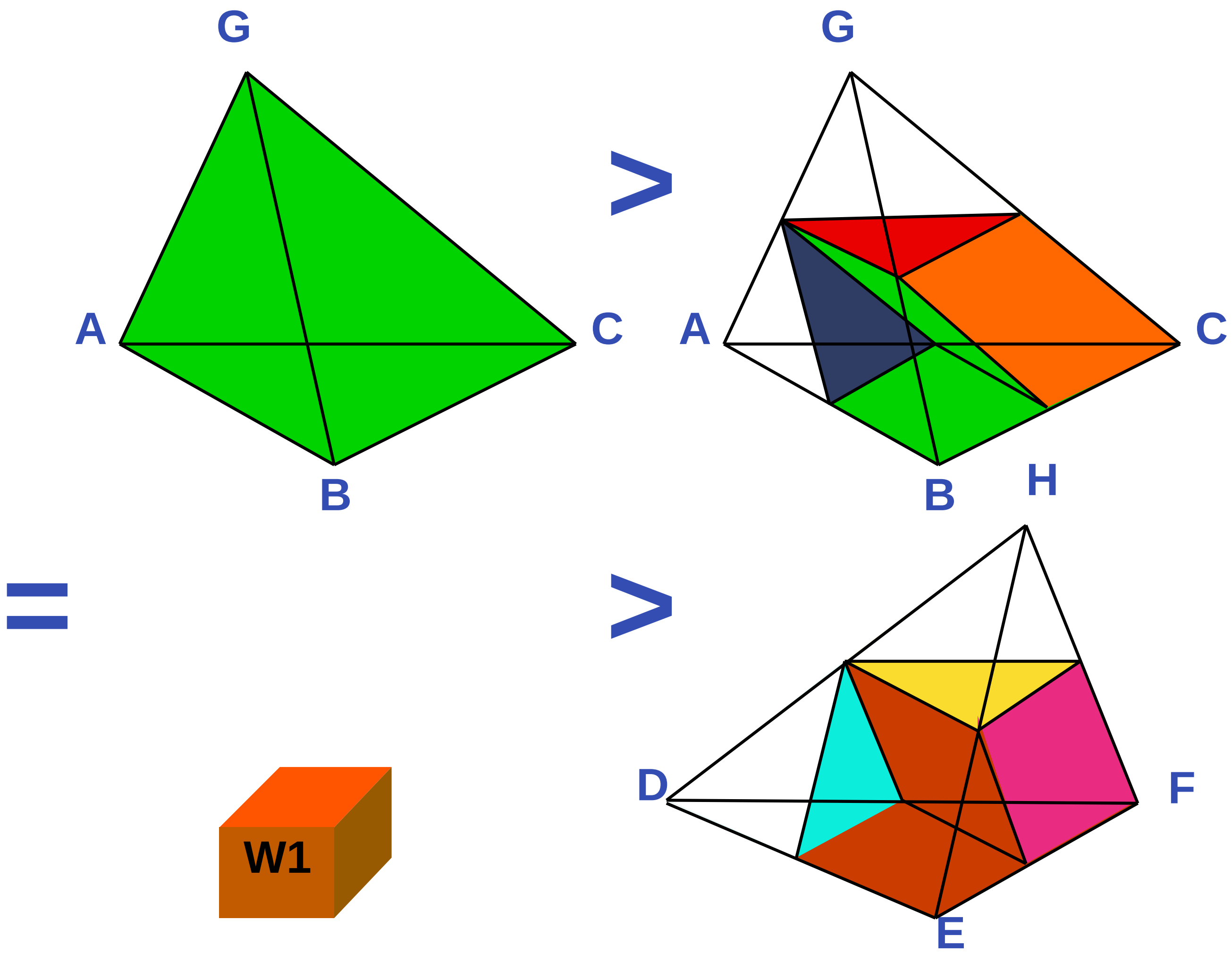
But then,



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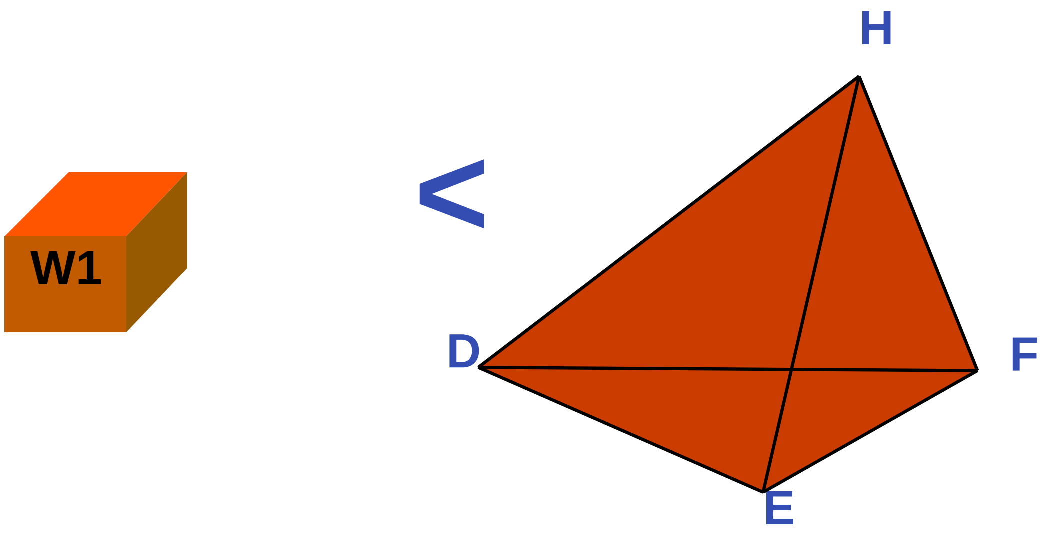
But previously we noted that



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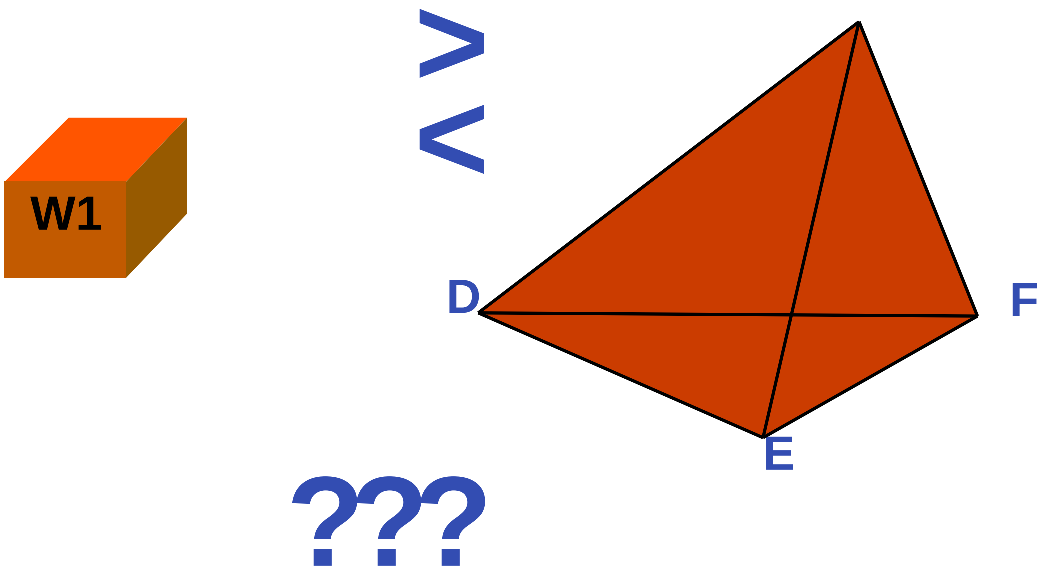
Contradiction!



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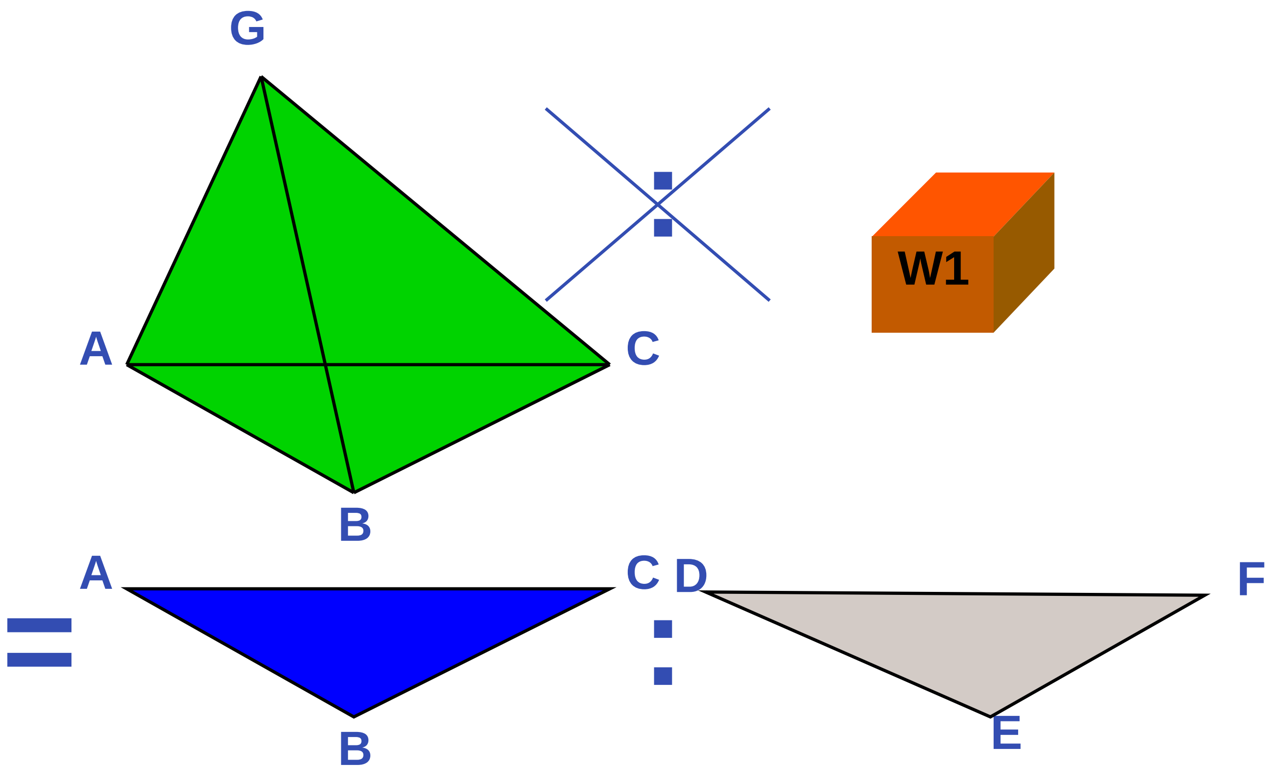
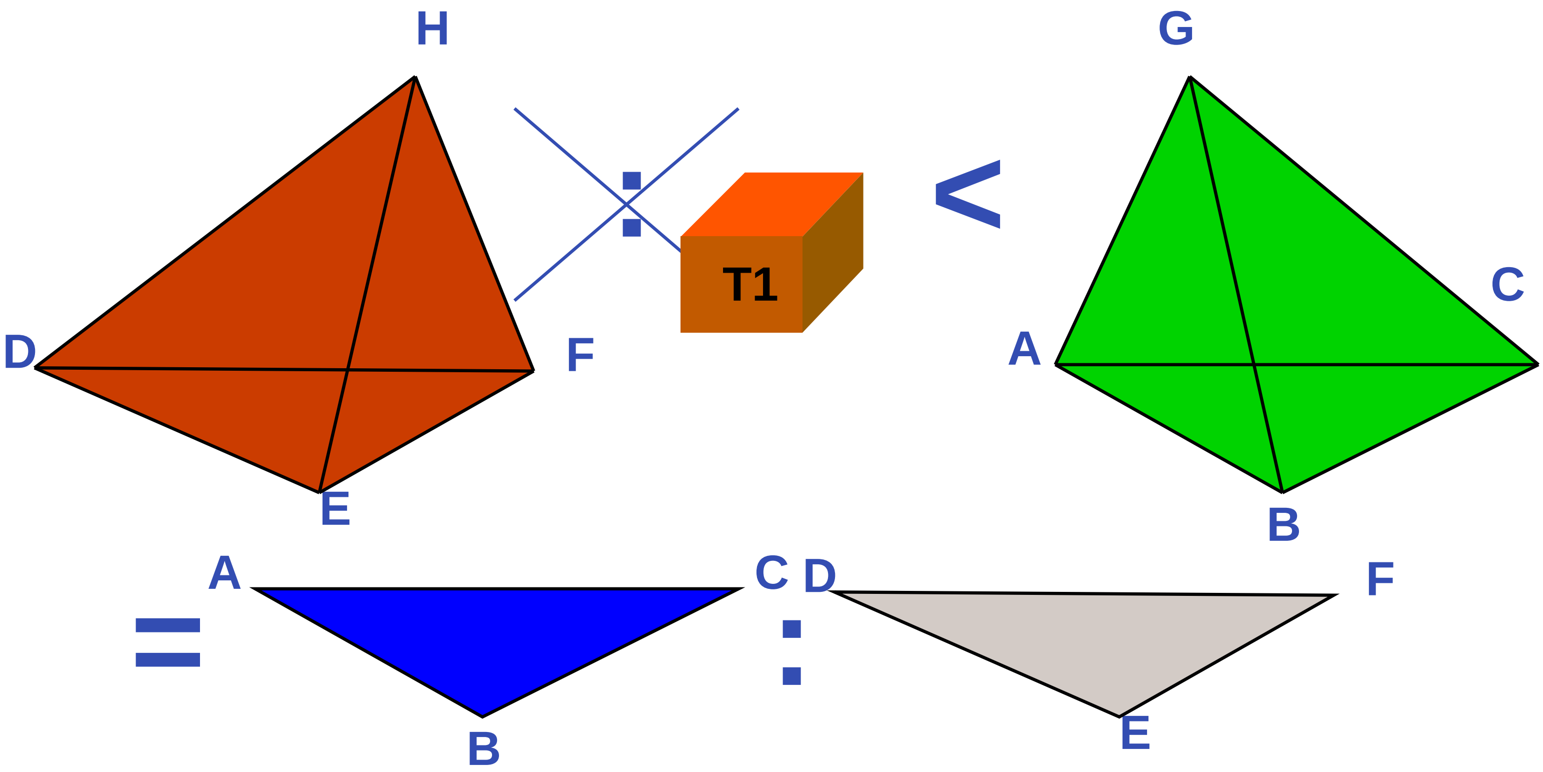
Therefore,



, where W1 is any solid less than pyramid DEFH.
(Case 1 failed)

Book XII Proposition 5

Similarly,

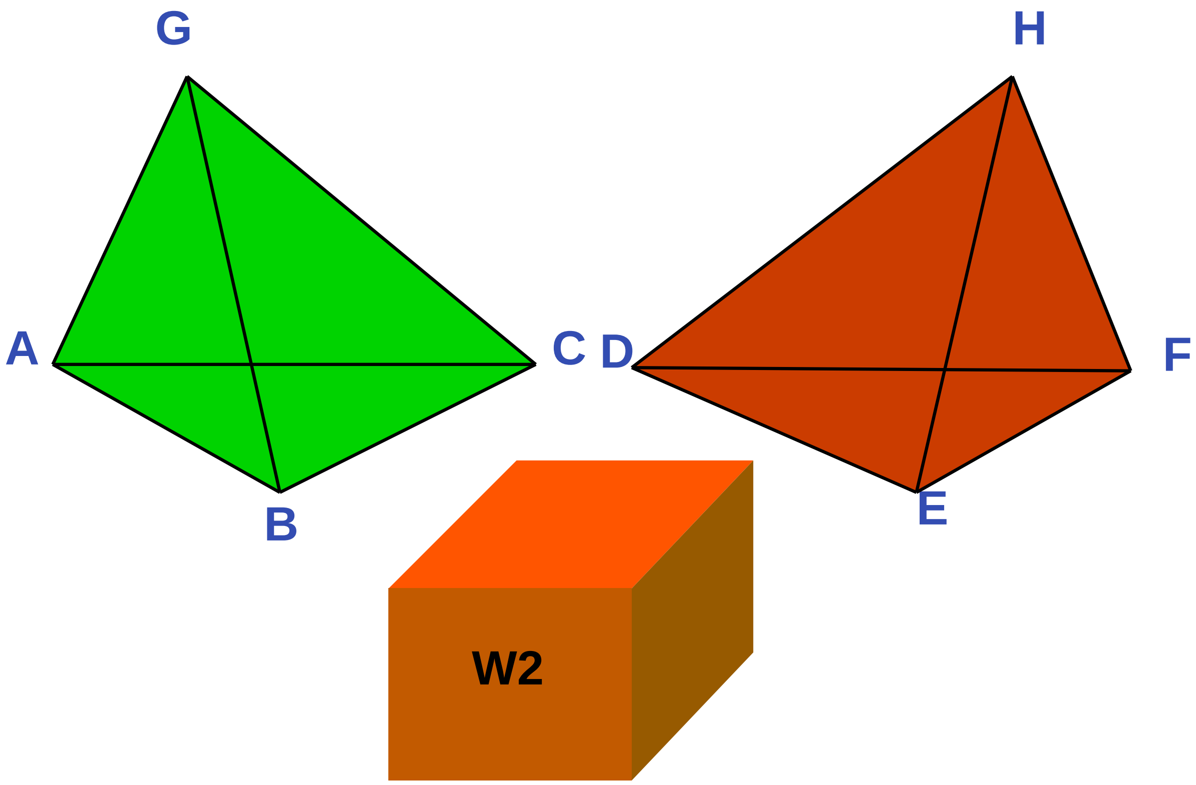


, where T1 is any solid less than pyramid ABCG.

Book XII Proposition 5

Case 2:

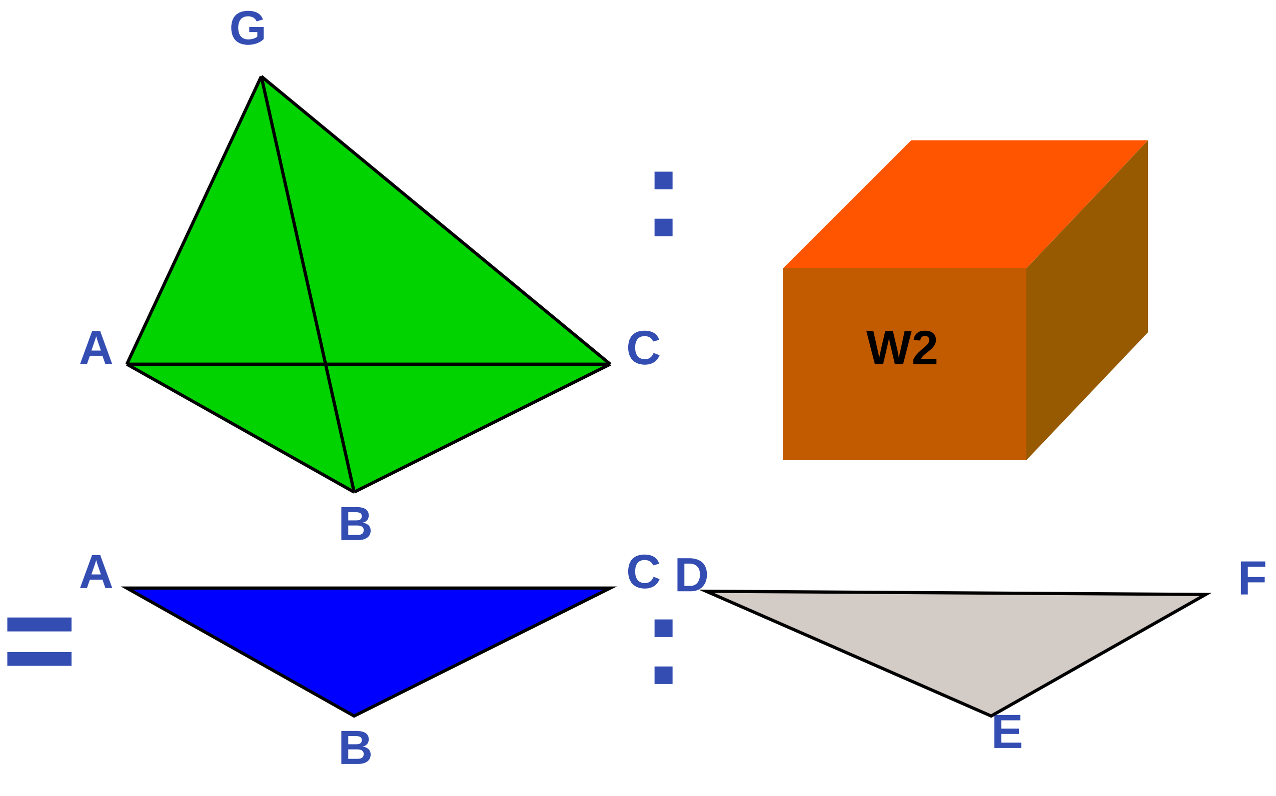
Pyramid ABCG: W2



, W2 is any solid > pyramid DEFH.

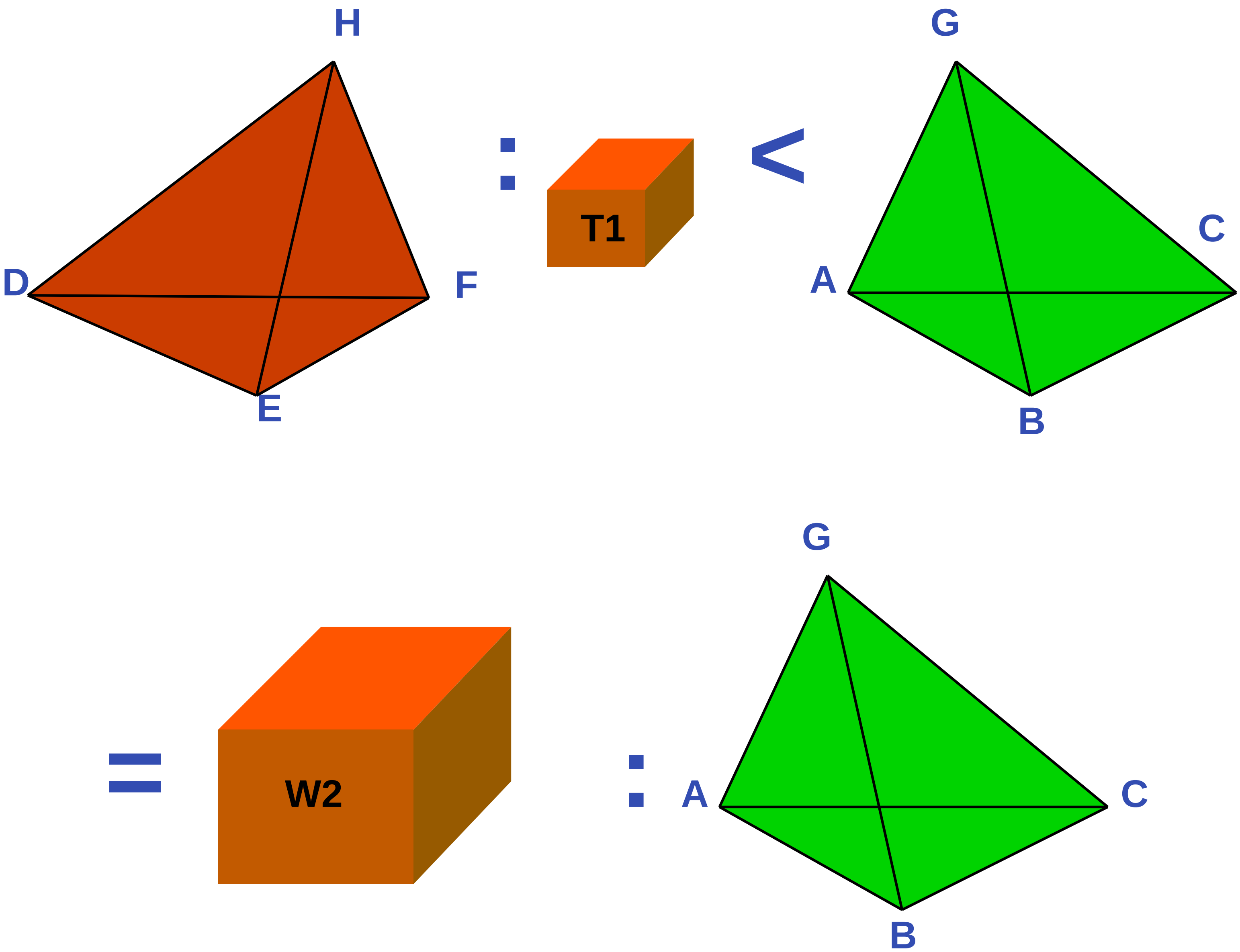
Book XII Proposition 5

Therefore inversely,



Book XII Proposition 5

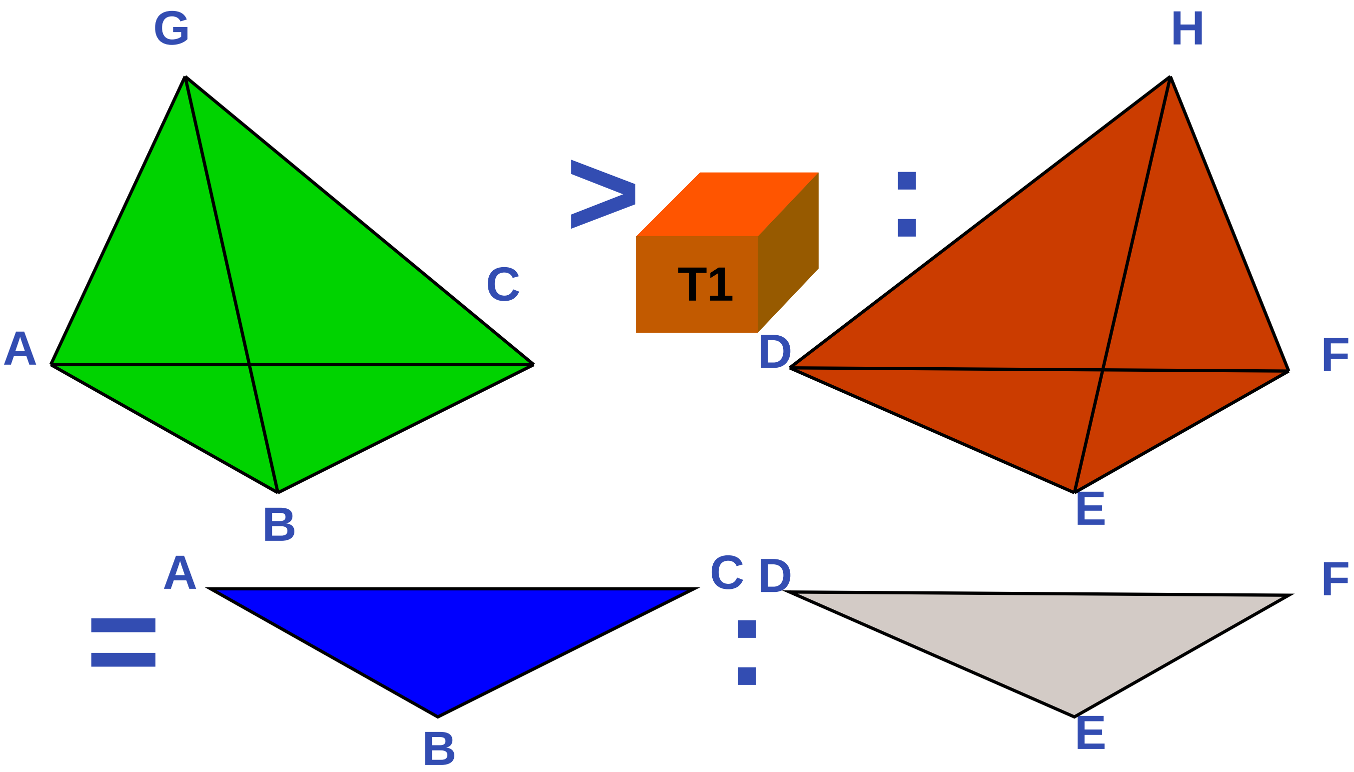
Similarly,



, where T1 is any solid less than pyramid ABCG.

Book XII Proposition 5

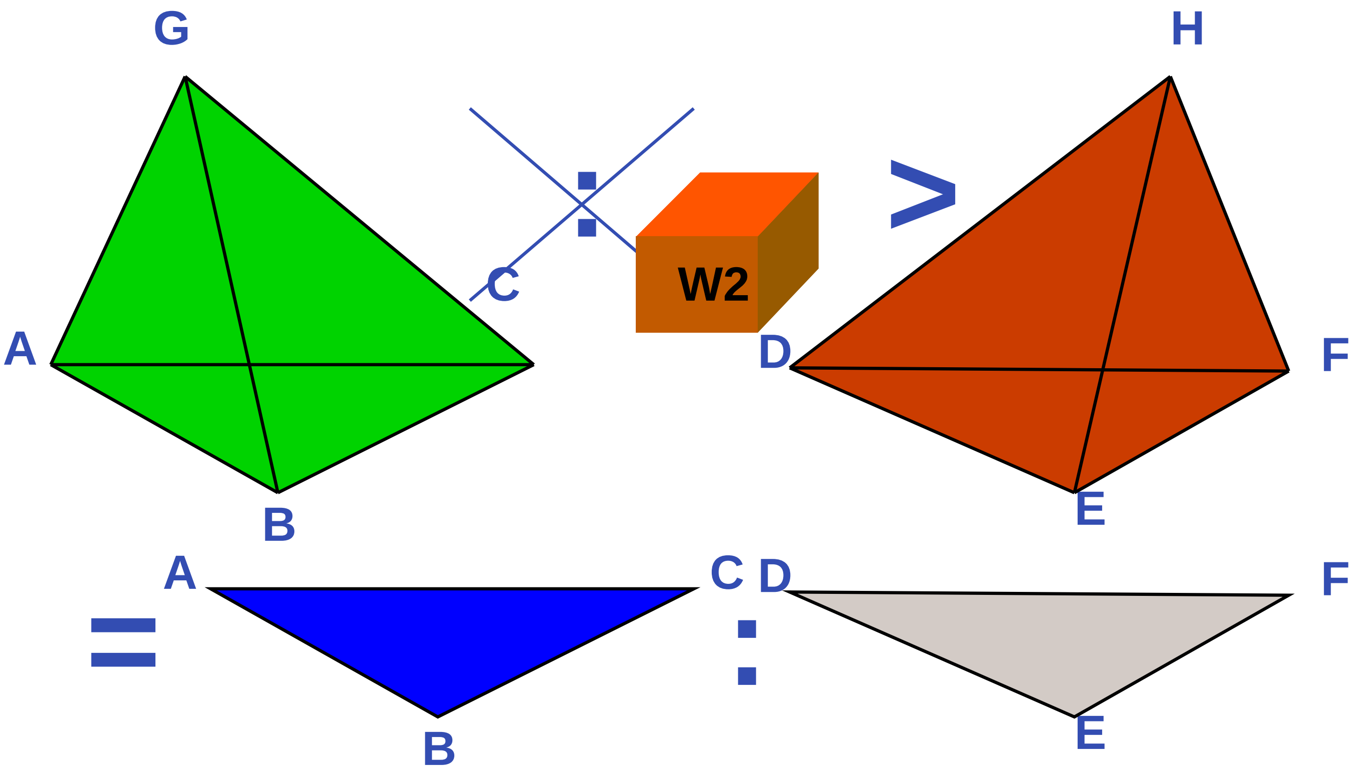
Therefore, by V.11,



, which was proved absurd.

Book XII Proposition 5

Therefore,



(Case 2 failed)

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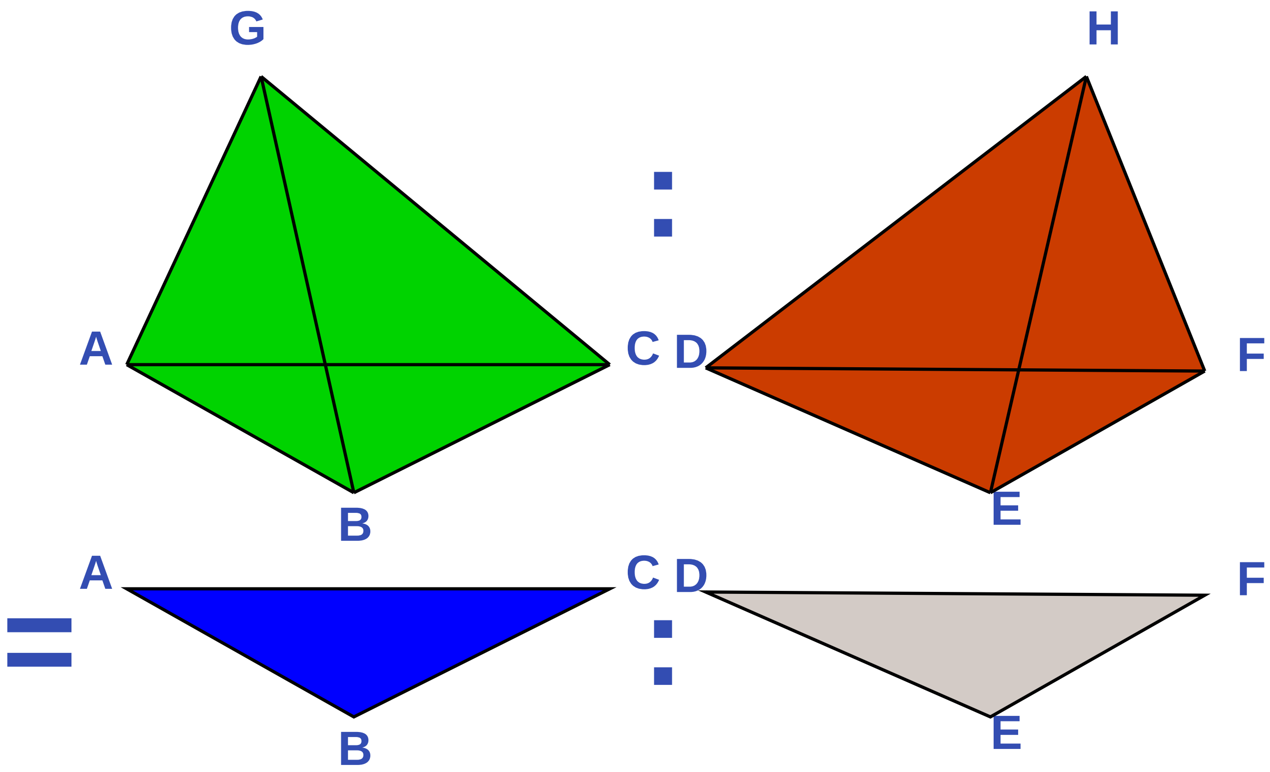
Since both case 1 and 2 failed, we have



Book XII Proposition 5

Therefore, XII.5 is true.
The pyramids of the same height with triangular bases are to one another as their bases.