MATH 308 101

PROJECT

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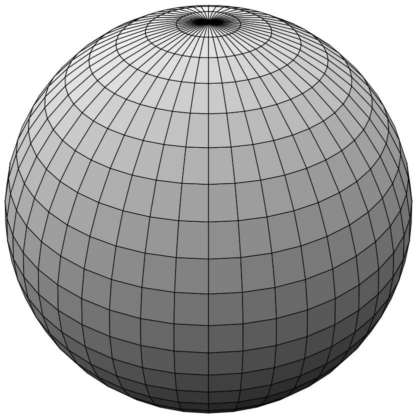
The Shortest Distance

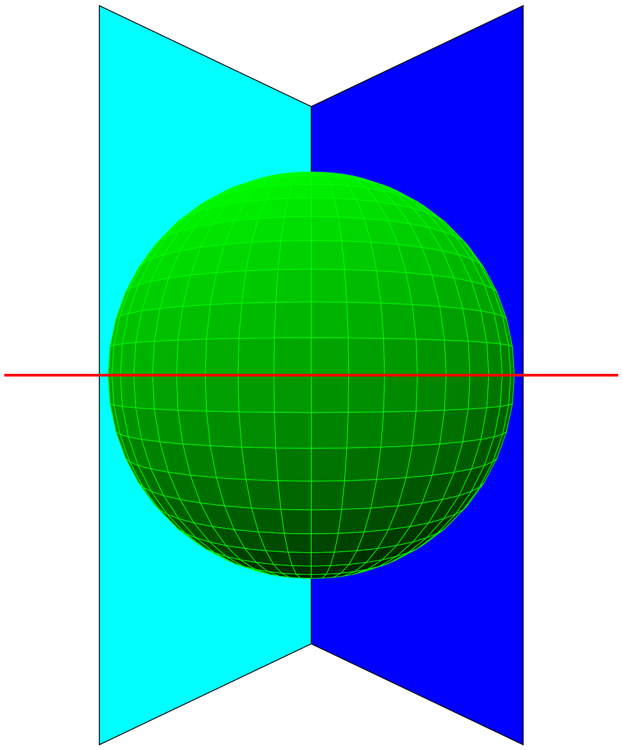
between 2 points on a sphere

is the arc on a great circle -

an arc with the sphere’s radius.



1st, we construct a sperical triangle by slicing a sphere with intersecting planes.

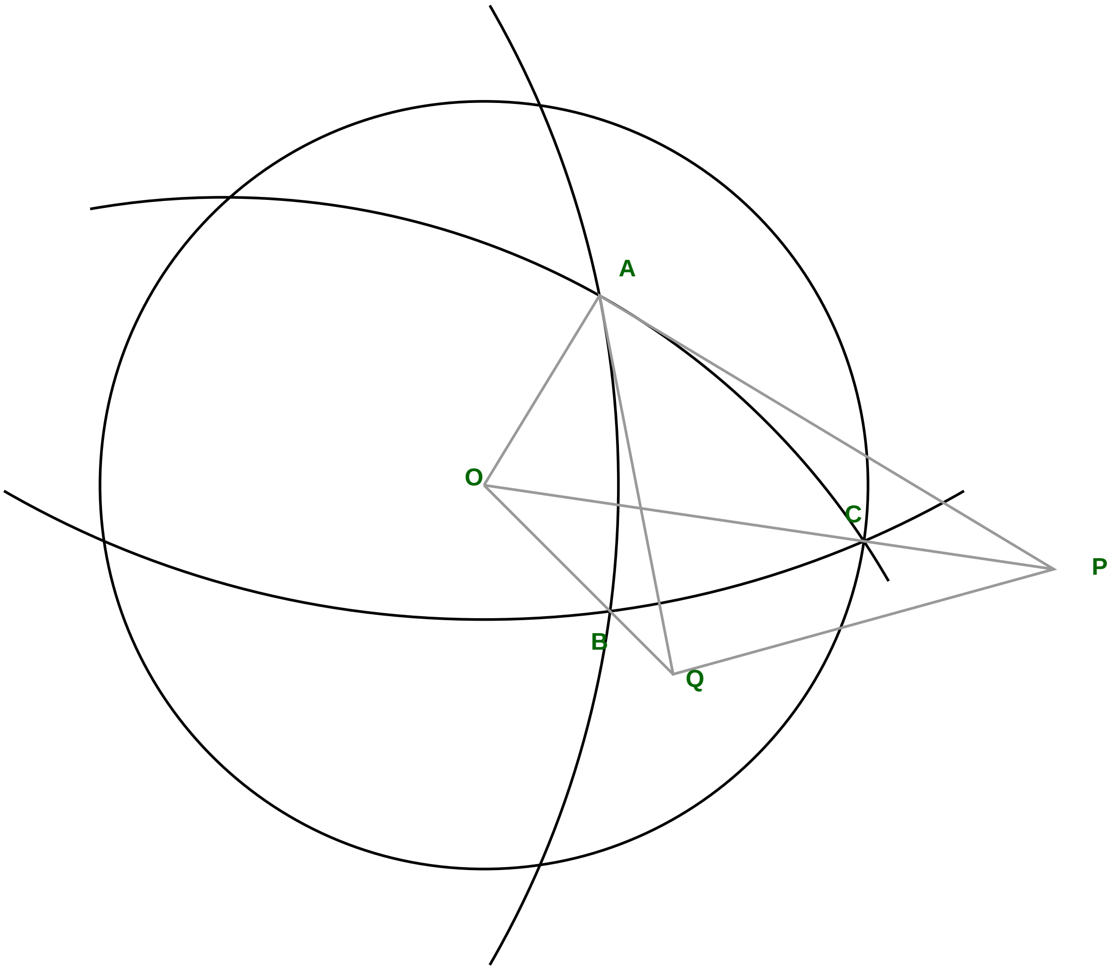


From the spherical triangle,

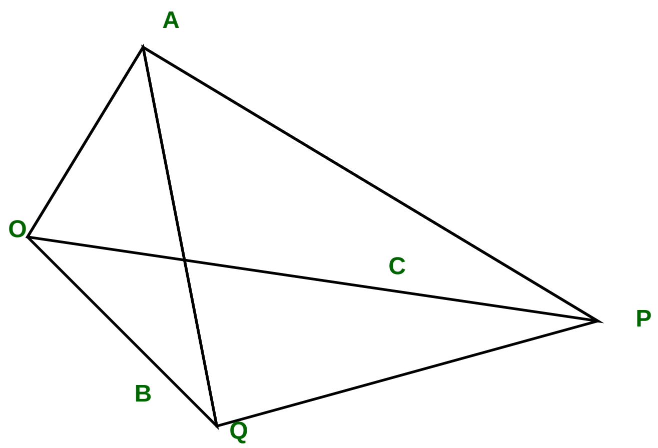
we can calculate a length

on the arc of a great circle

with the cosine rule.



Now we unfold the spherical triangle and apply the cosine law.



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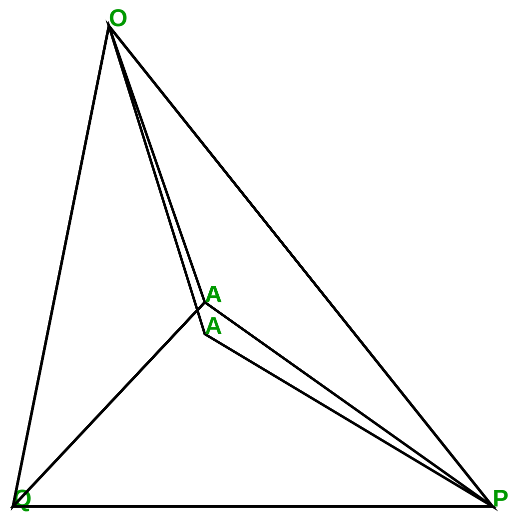

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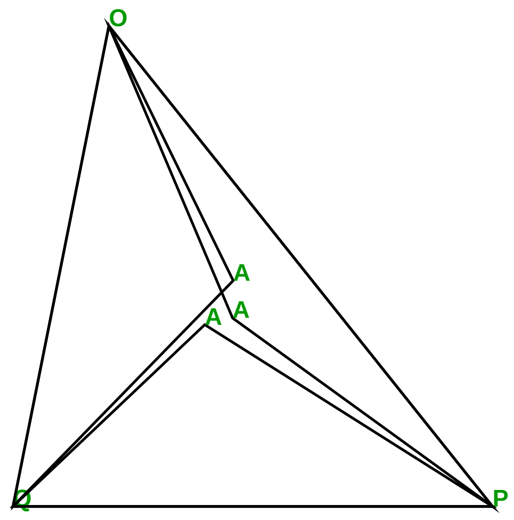

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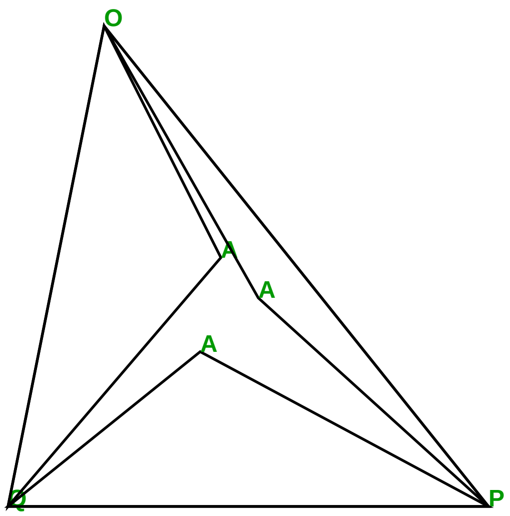

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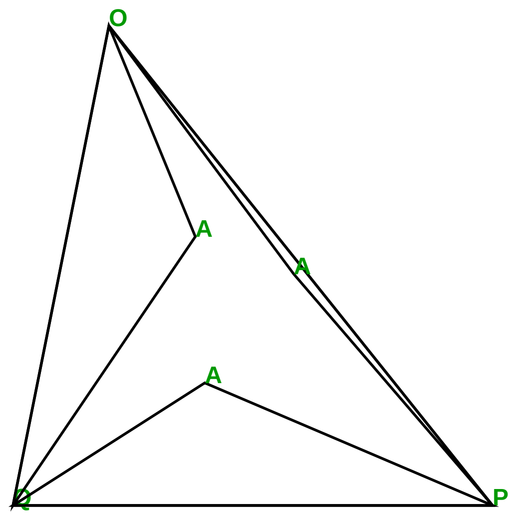

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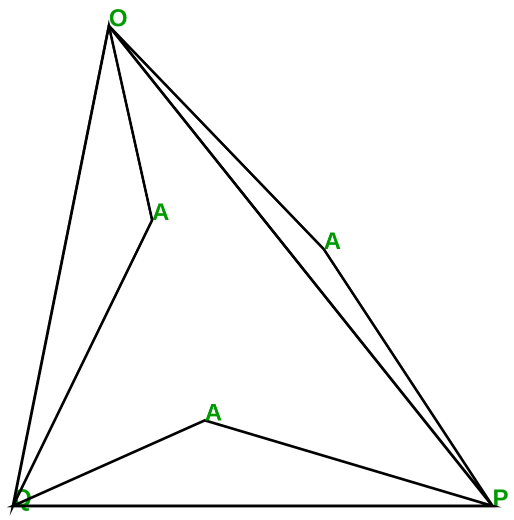

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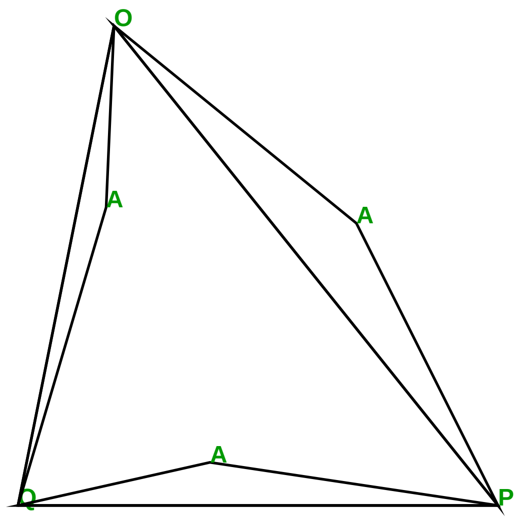

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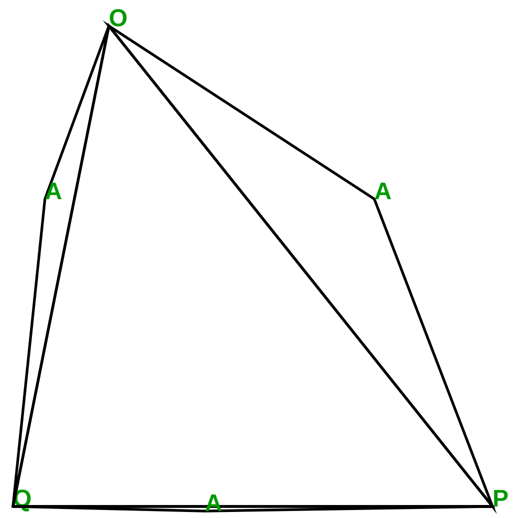

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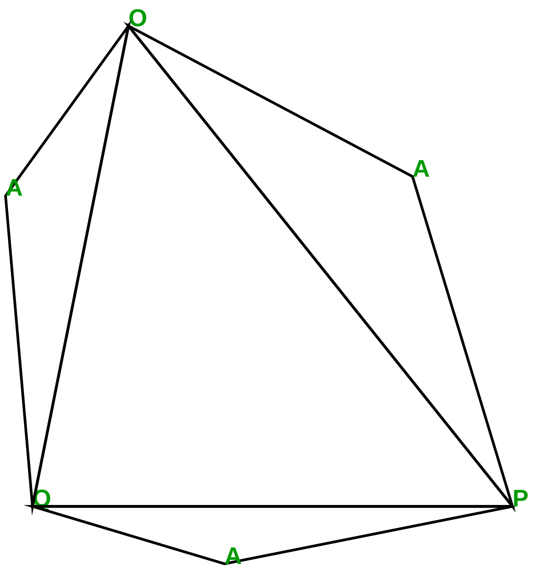

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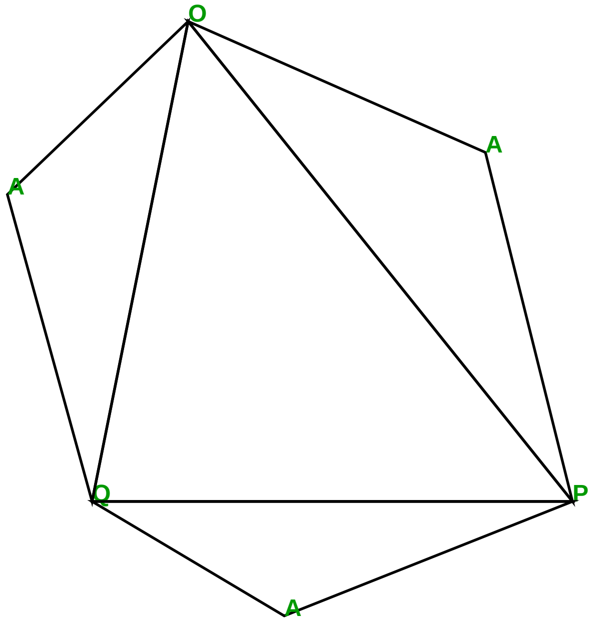

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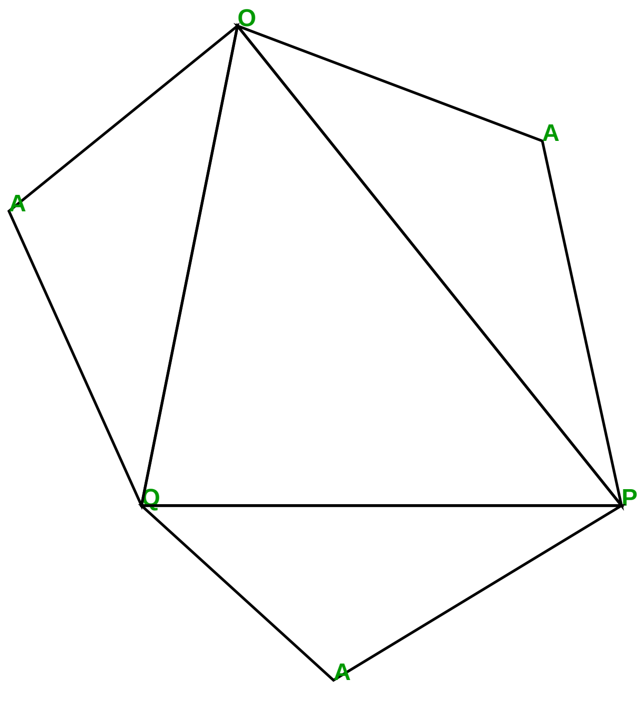

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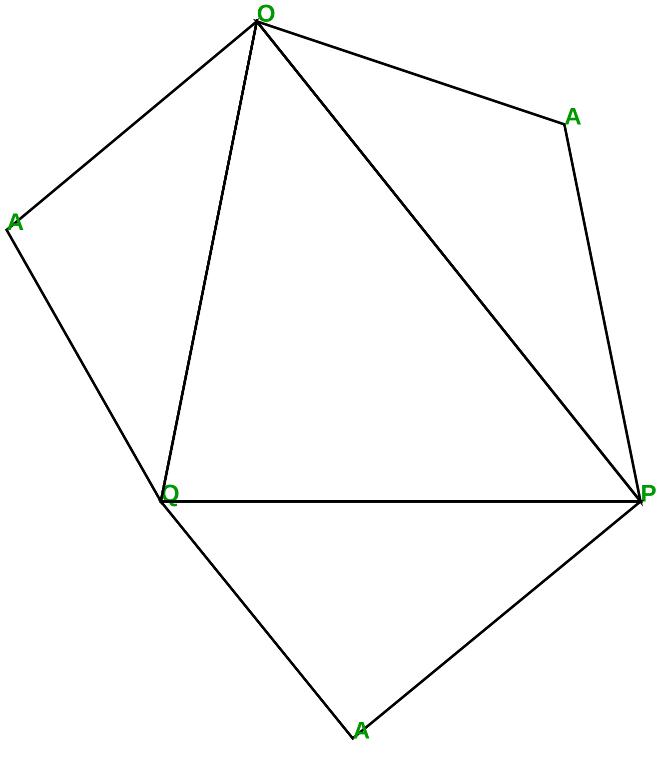

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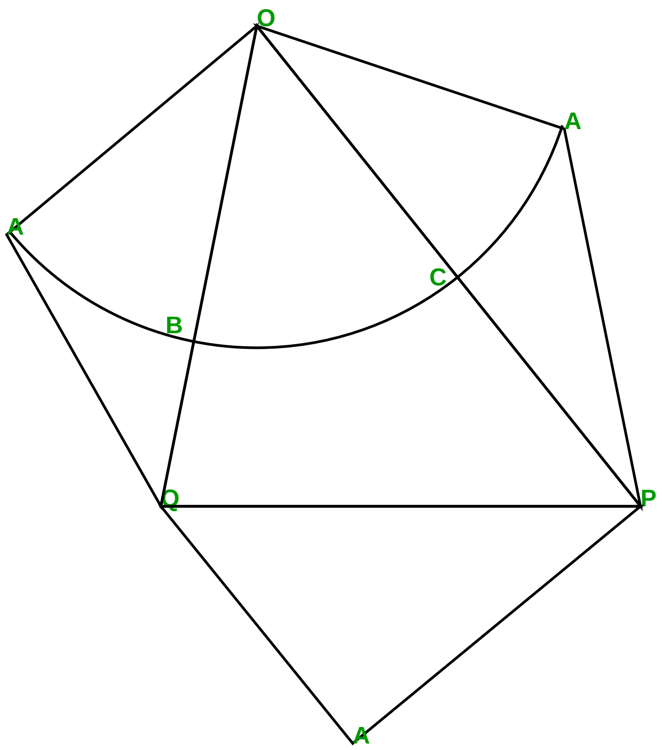
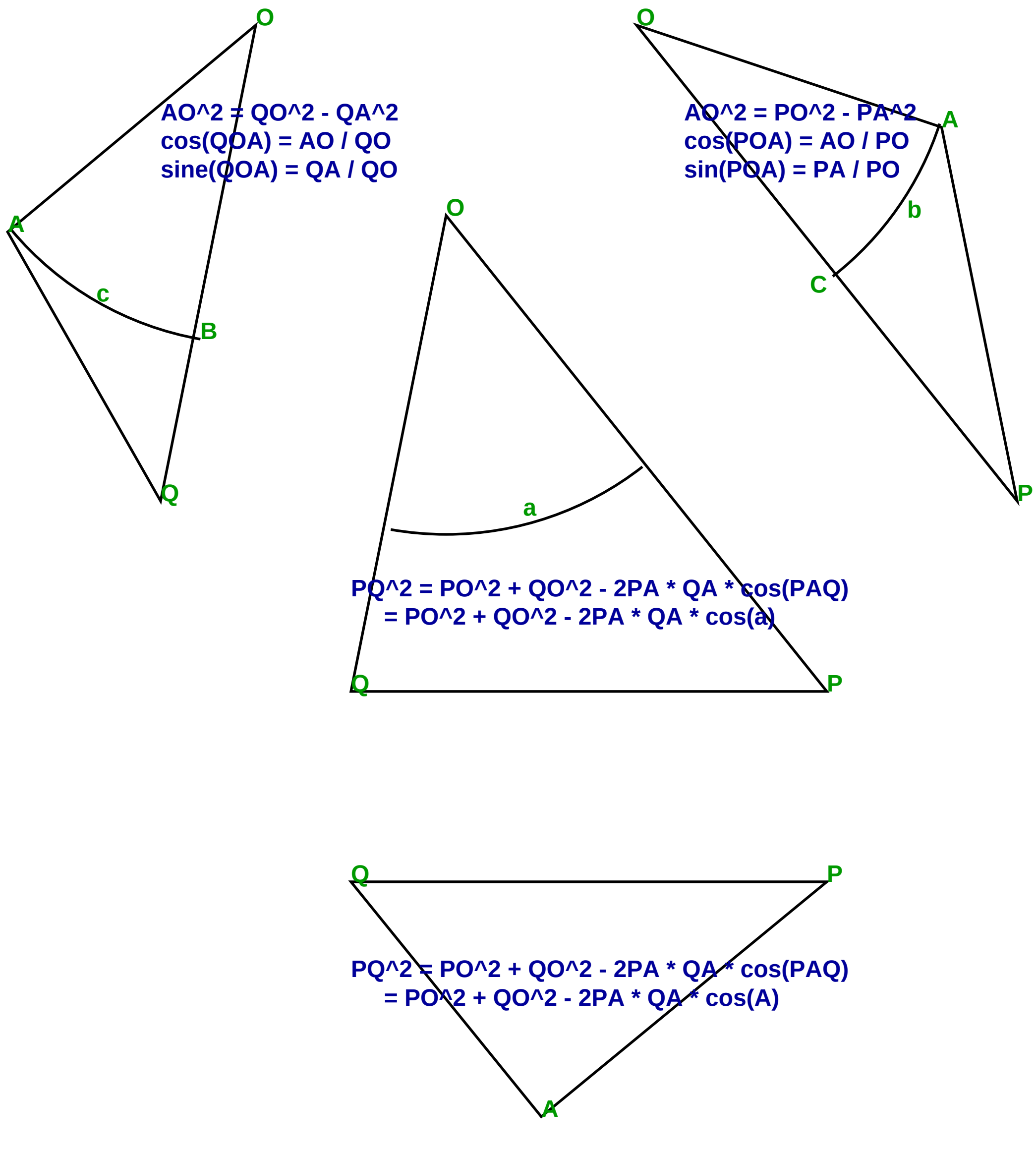


Apply the Cosine Rule and simple Trig...



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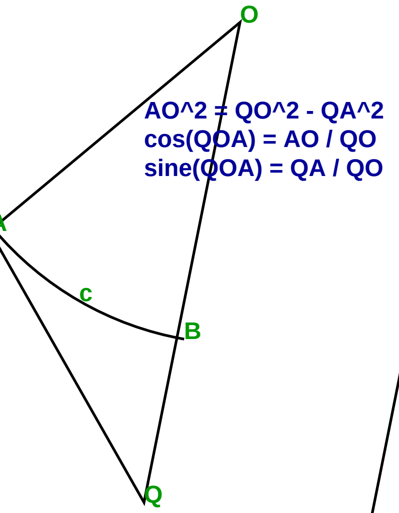

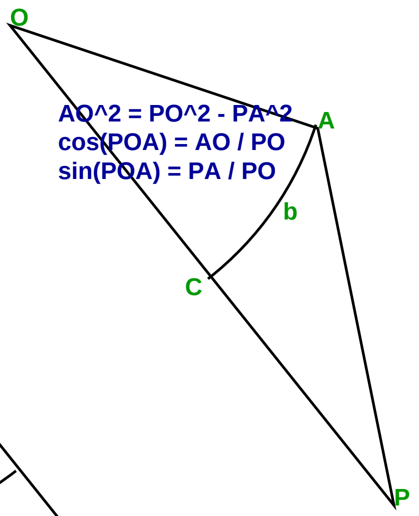

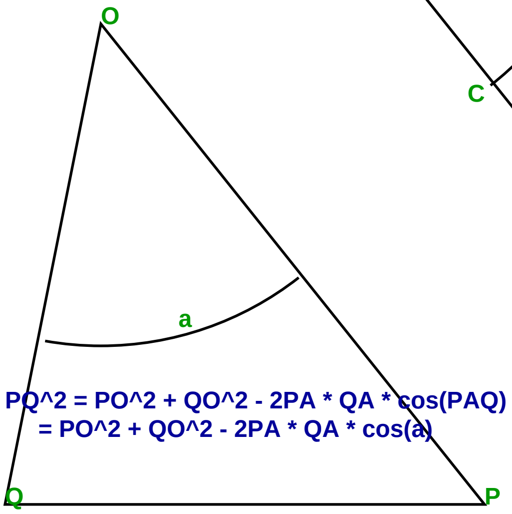

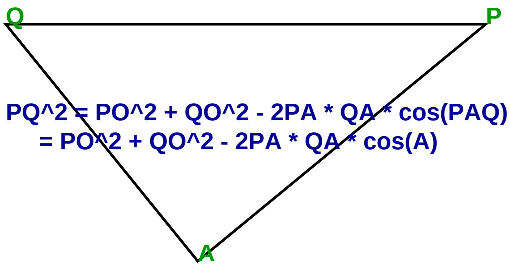

Most important are the 2 equations :
"PQ^2 = PO^2 + QO^2 - 2PA * QA * cos(a)"
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where we can substitue the lenghts of the 2 sides and derive the 3rd side.

PO^2 + QO^2 - 2PA * QA * cos(a)
= PO^2 + QO^2 - 2PA * QA * cos(A)
2PO * QO cos(a) = 2AO^2 + 2PA * QA * cos(A)
cos(a) = (AO / PO)(AO / QO) + (PA / PO)(QA / QO) * cos(A)
= cos(POA) * cos(QOA) + sin(POA) * sin(QOA) * cos(A)
= cos(b) * cos(c) + sin(b) * sin(c) cos(A)
which "...is analogous to, and derived from the flat triangle formula..." : a^2 = b^2 + c^2 -2bc cos(A).

Note the sin law : sin(A) = a*sin(C) / c