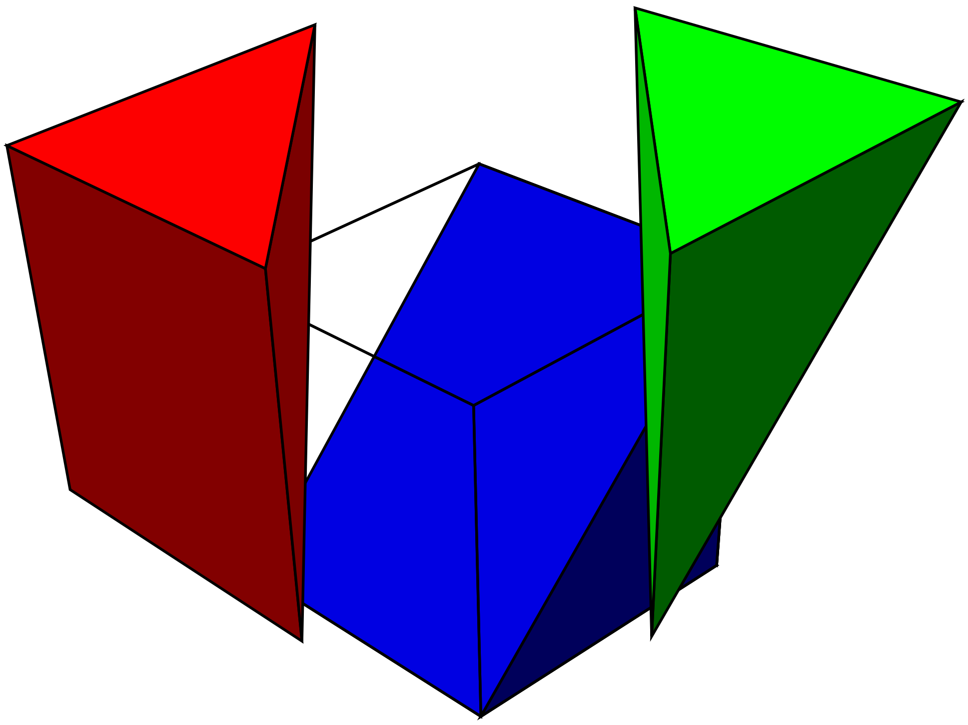
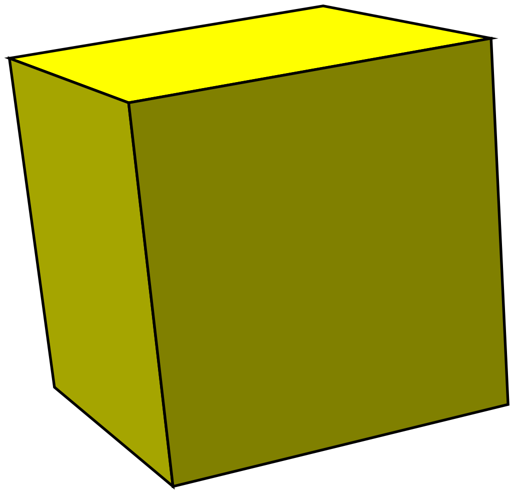
Lui Hui's Method of Proving Formulas for Volume



Math 308, section 101
TermProject
Jonathan Pilley
Std # 23110877

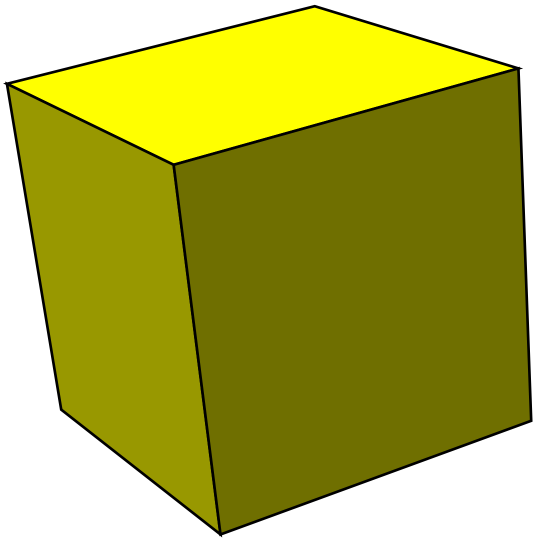
In the year 263AD a Chinese mathematician, named Liu Hui, wrote a book called The Sea Island Mathematical Manual. This book is a commentary on The Nine Chapters of Mathematical Art, known in Chinese as Chiu-Chang Suan-Shu. Chapter five of The Nine Chapters, Consultations on Engineering Works, states the formulas for the volume of many different solids. Liu Hui used a geometric approach to proving these formulas in the Sea Island Manual. He started by dissecting a unit cube into 3 objects for which he determined the volume. This gave him four objects which he used in his method to prove the stated volume formulas. This project does not attempt to examine all of the work Liu Hui did in this area, but rather to introduce the four basic objects used by Liu Hui and then show how he used them with an example.

The first object Liu Hui considered is a unit cube, which acted as a building block for the other 3 basic objects. He worked on the idea that a cube or box solid has volume equal to the product of the length, width, and height. In addition he knew that the volume of a whole solid equals the volume of all the parts of the solid.



Unit Cube Volume = Base x Width x Height

Let's have a closer look at this object.



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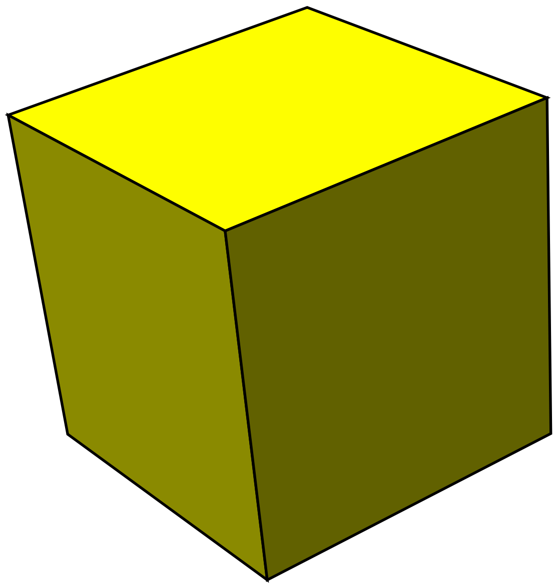

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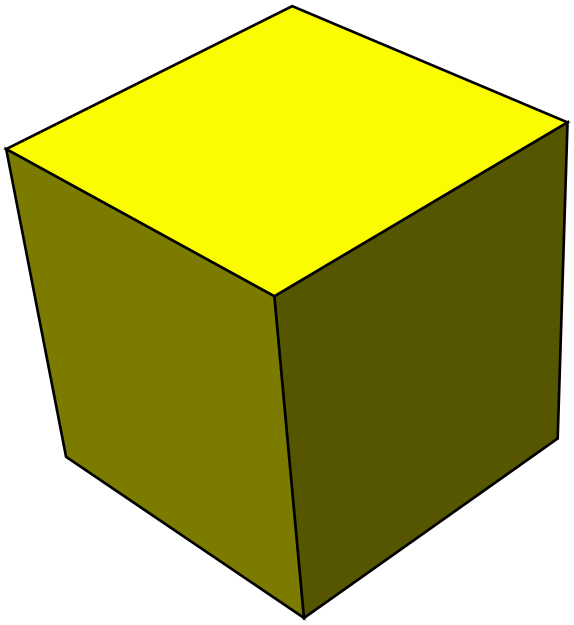

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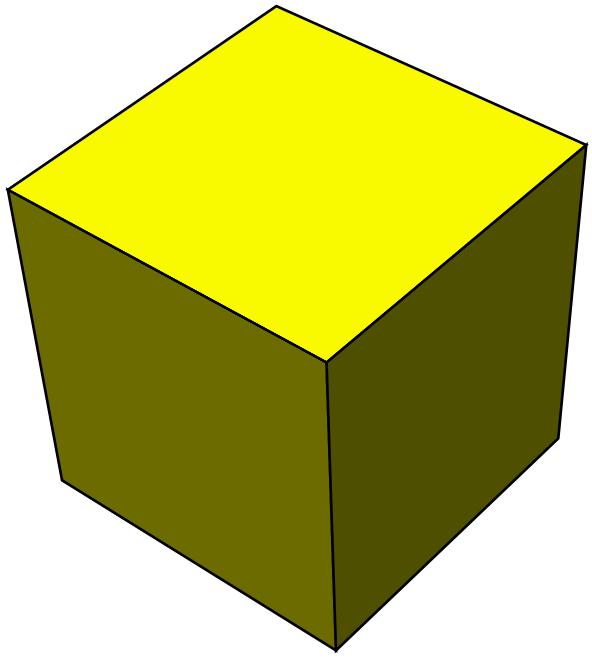

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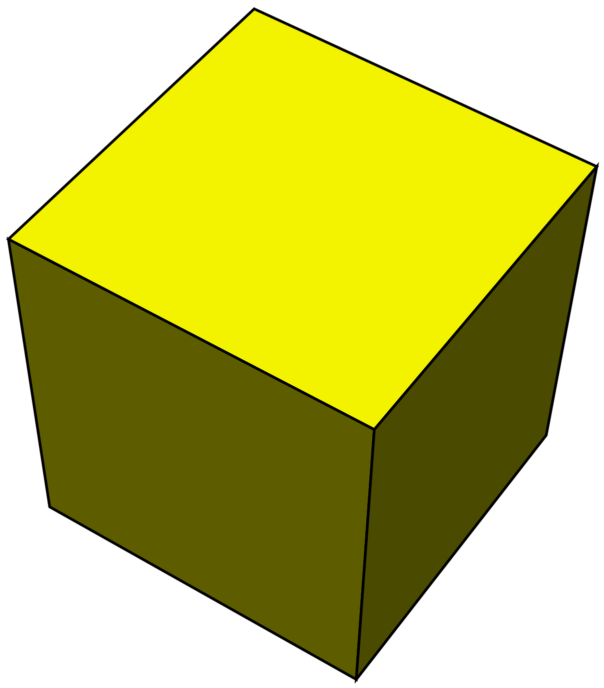

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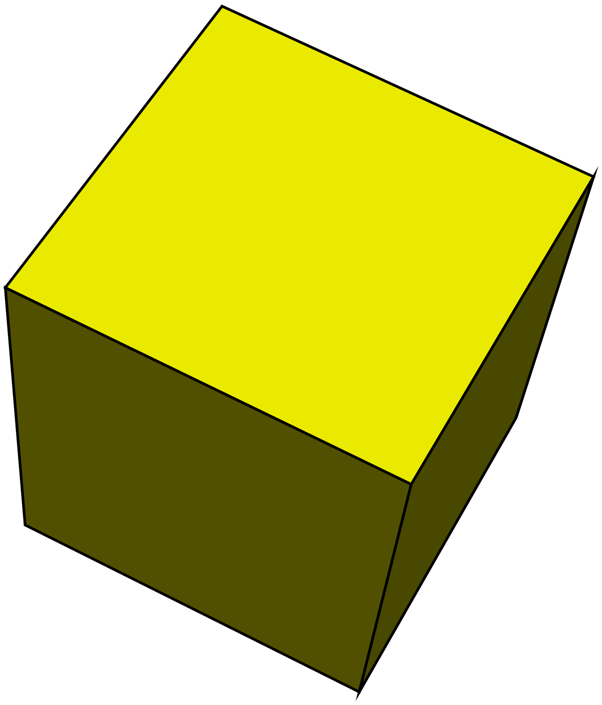

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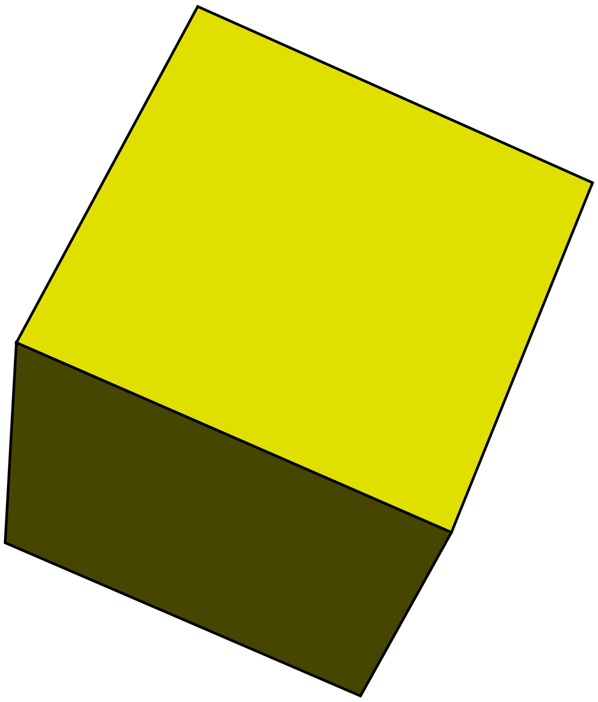

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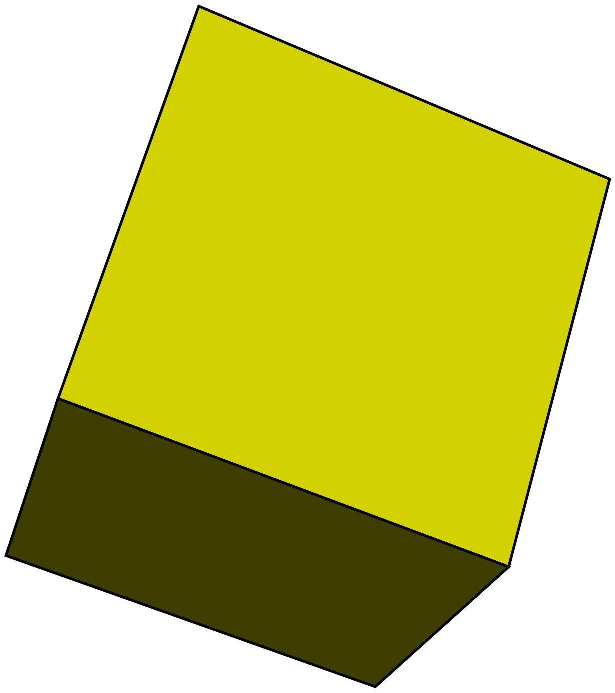

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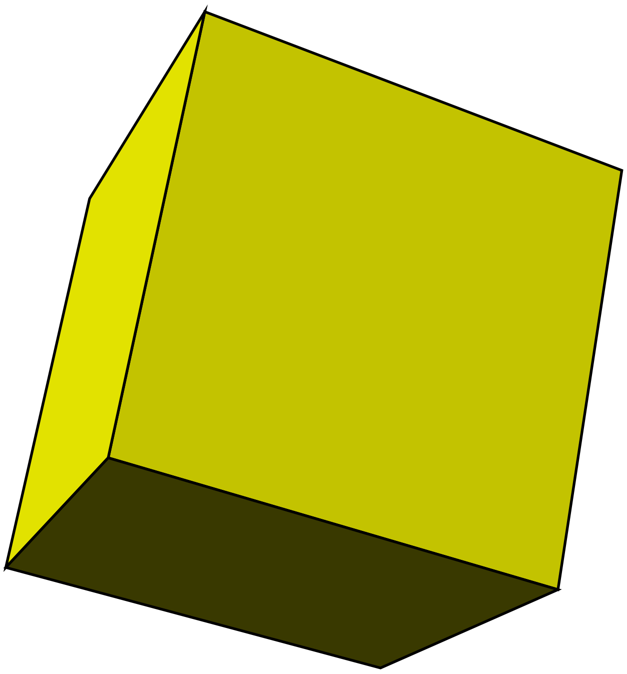

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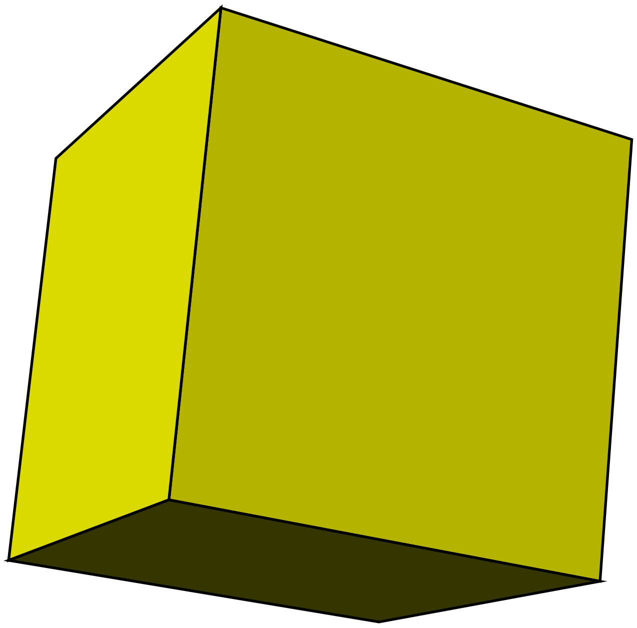

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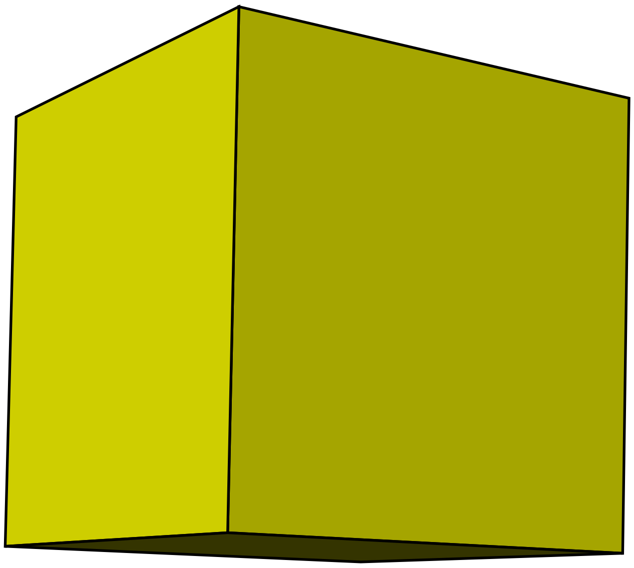

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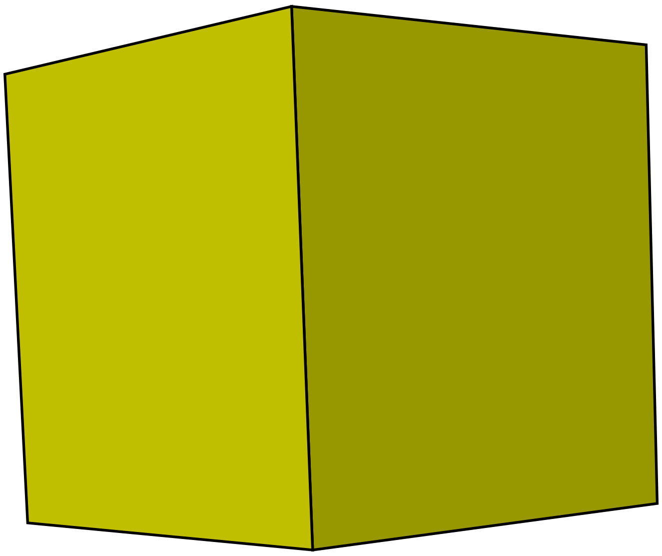

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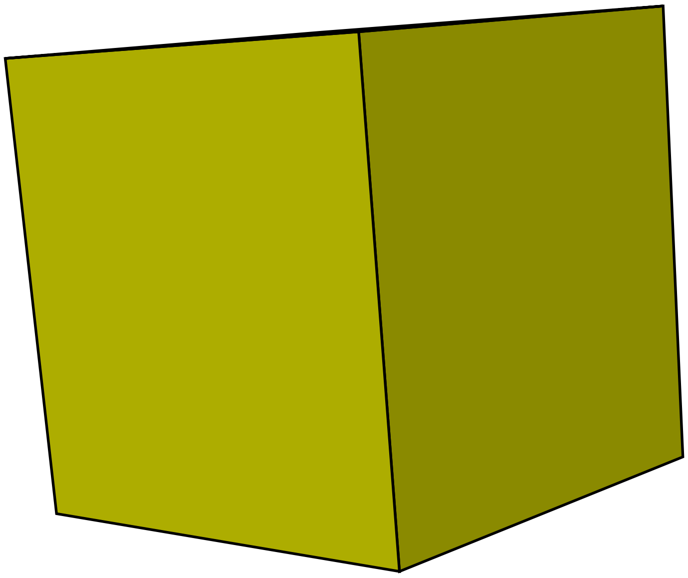

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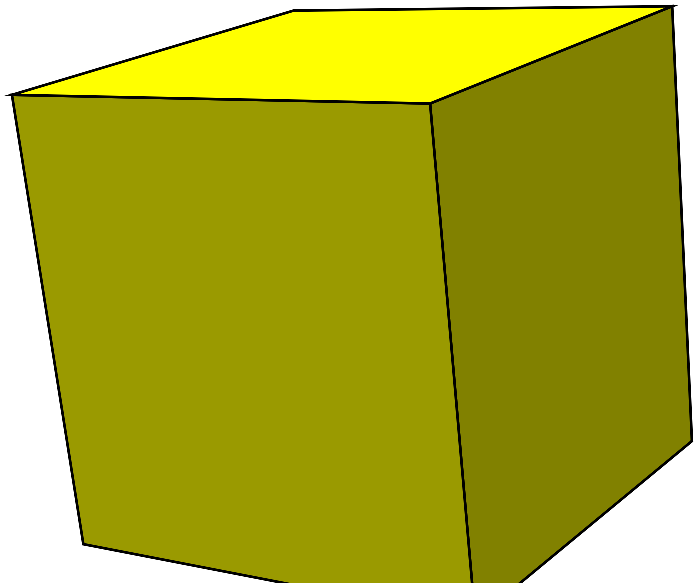

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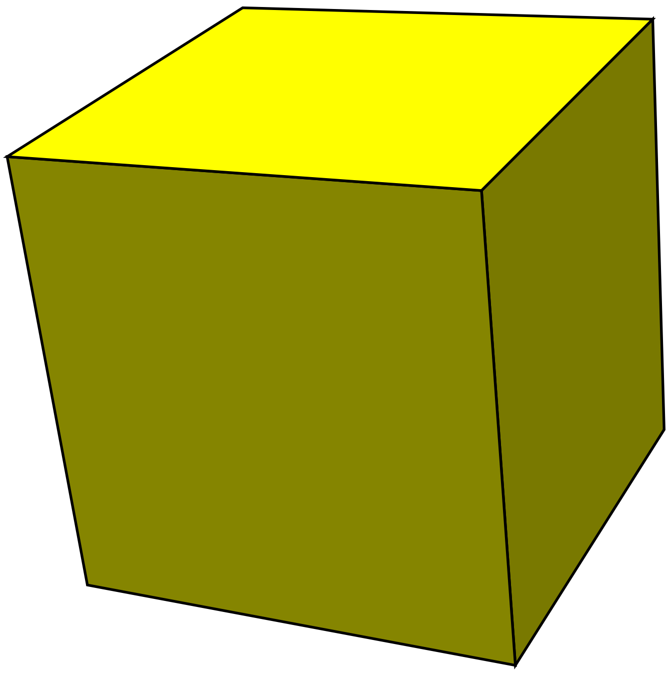

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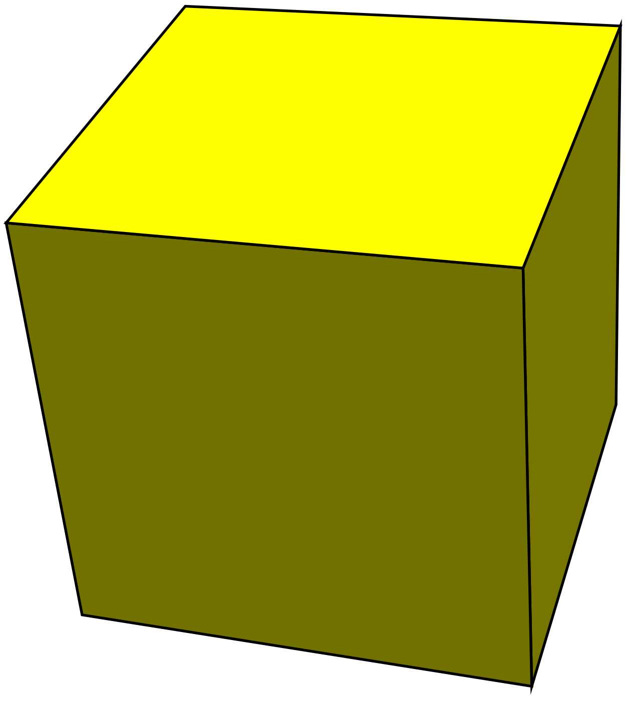

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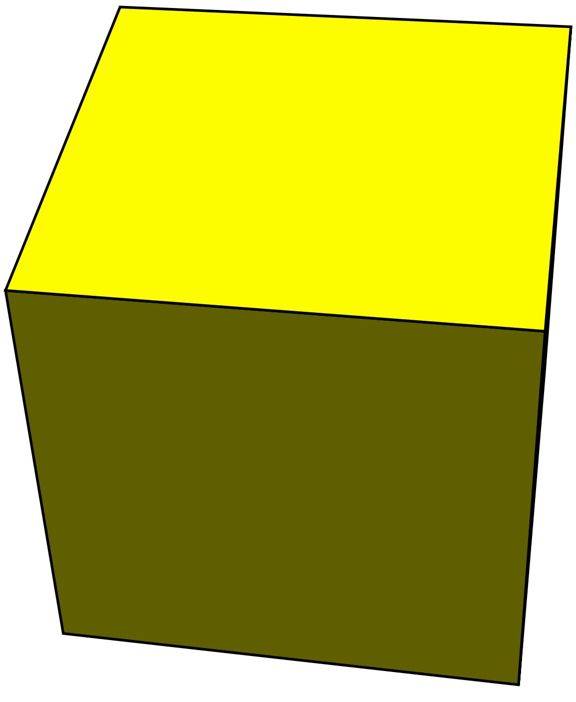

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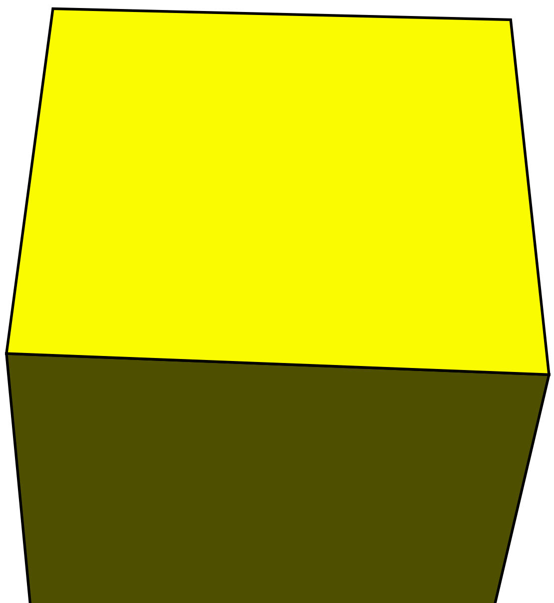

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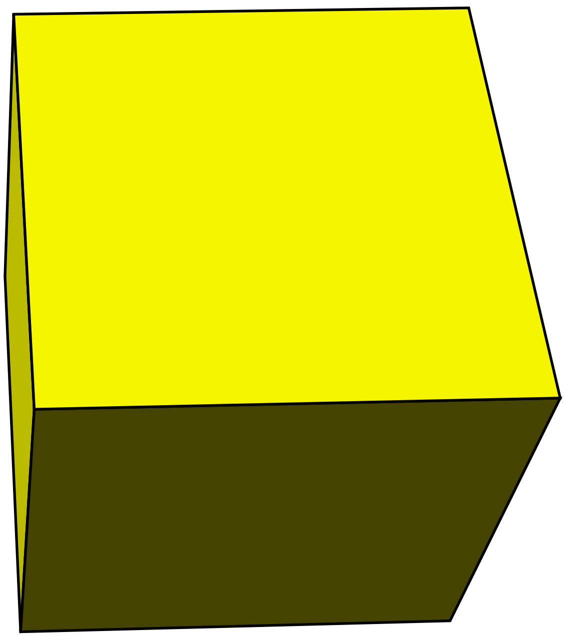

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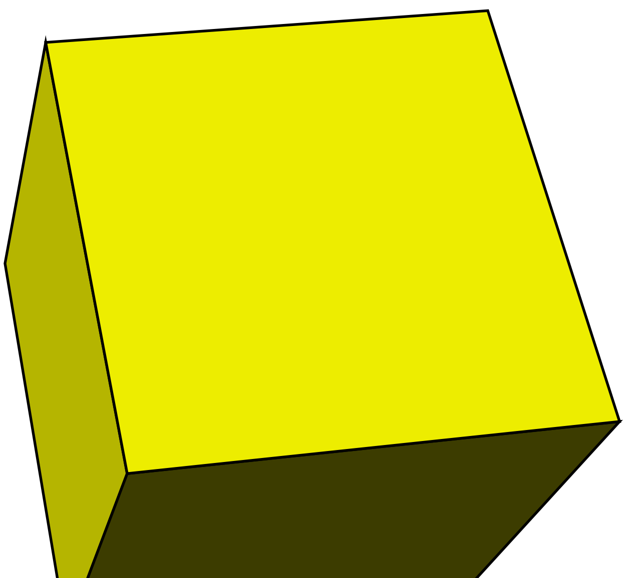

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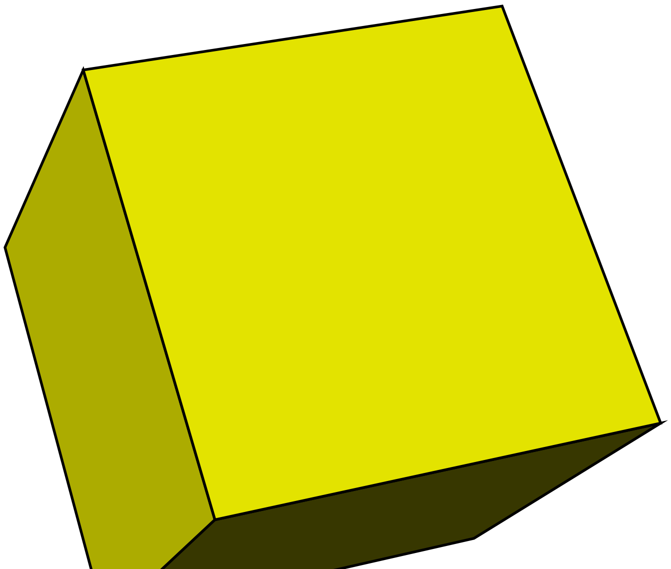

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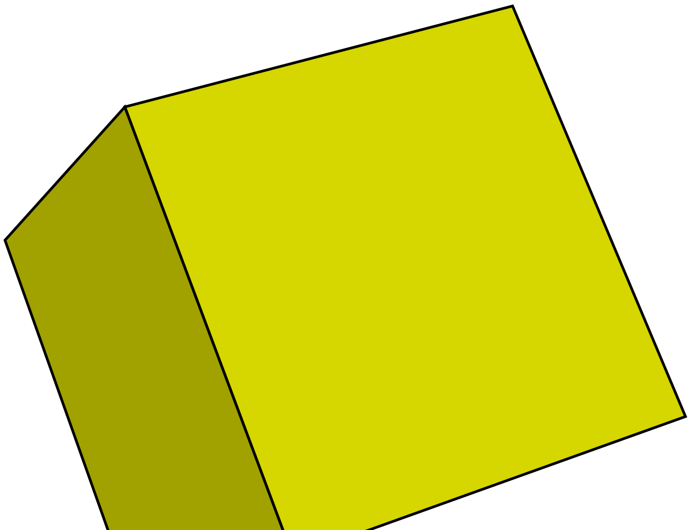

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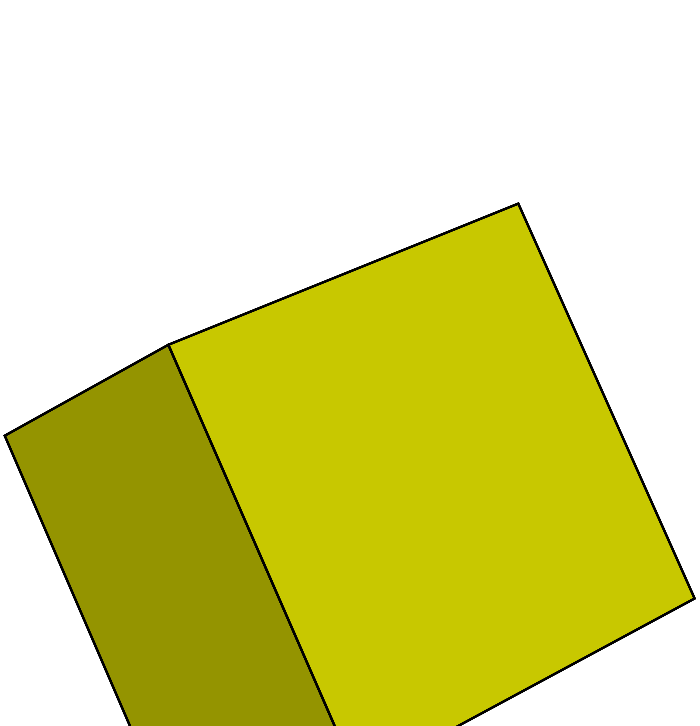

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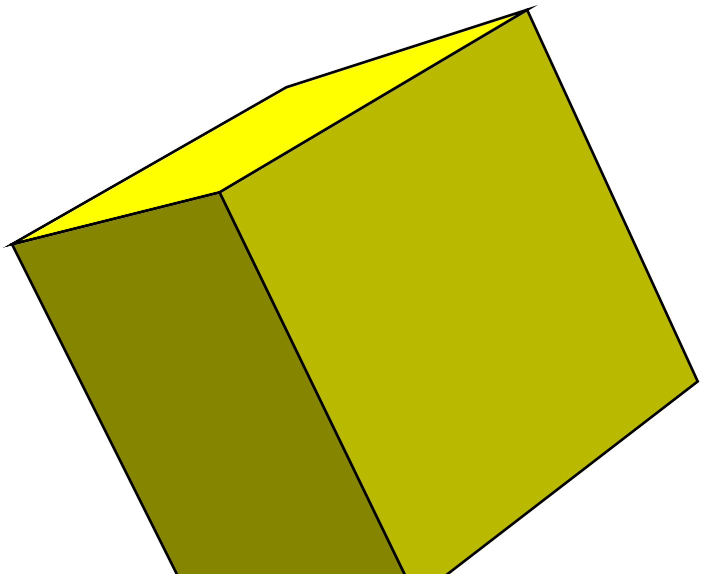

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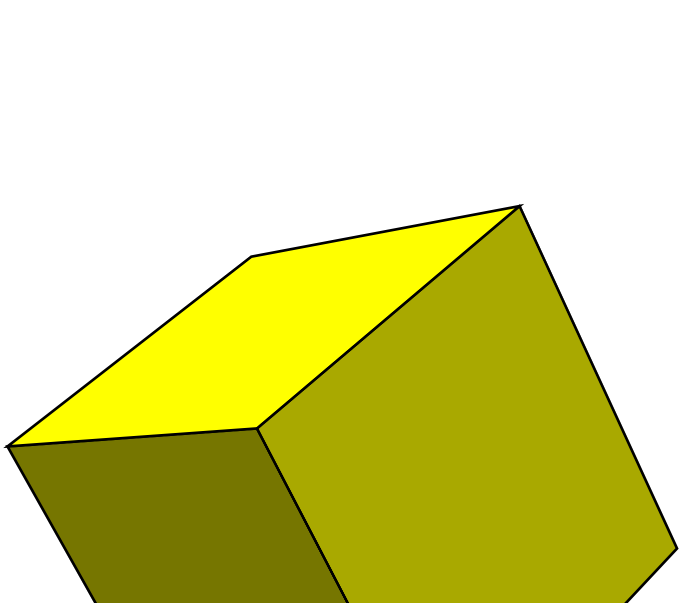

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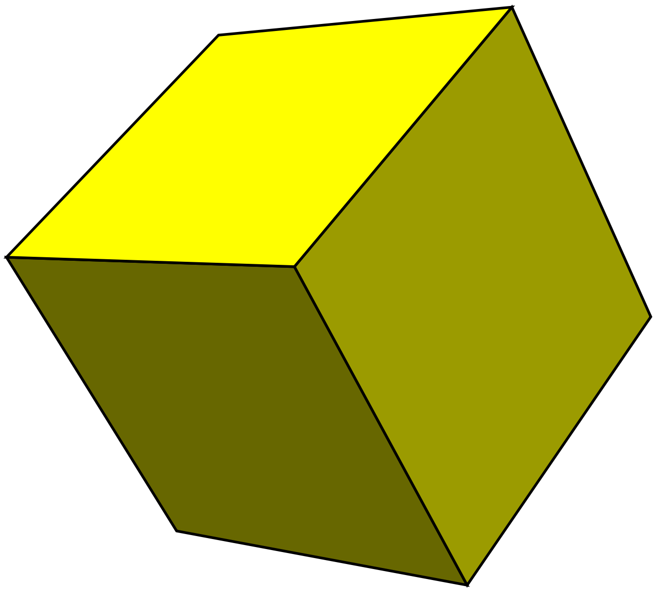

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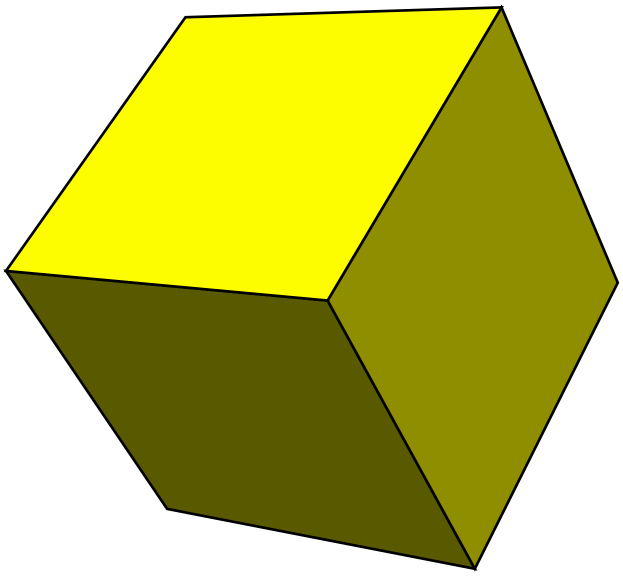

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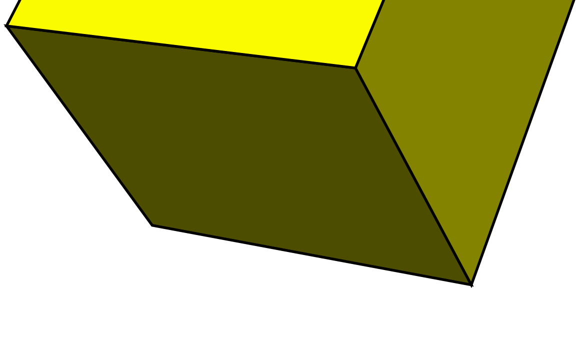

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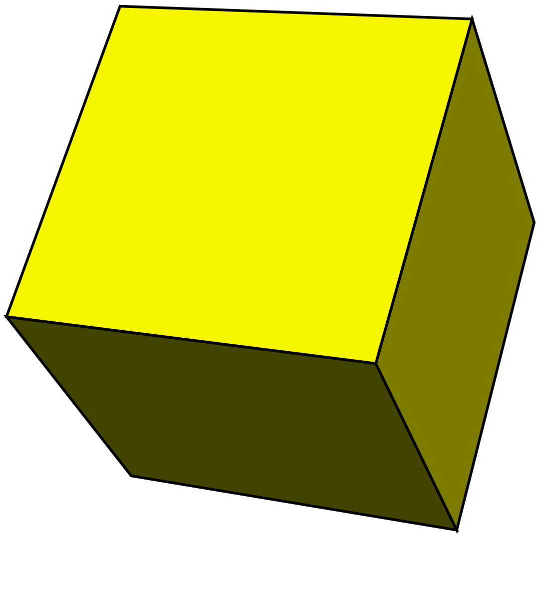

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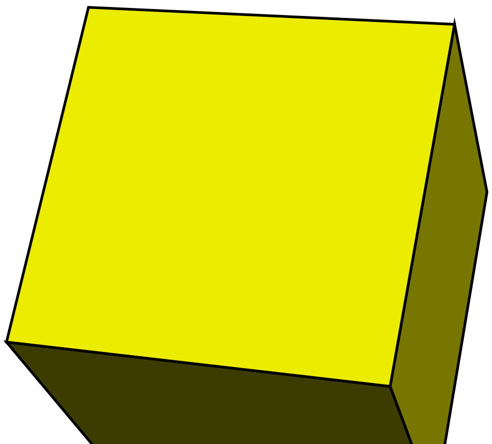

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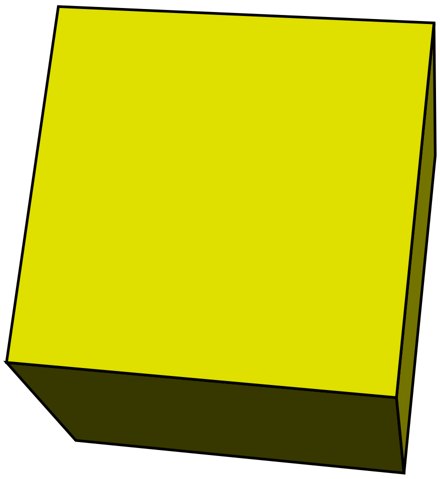

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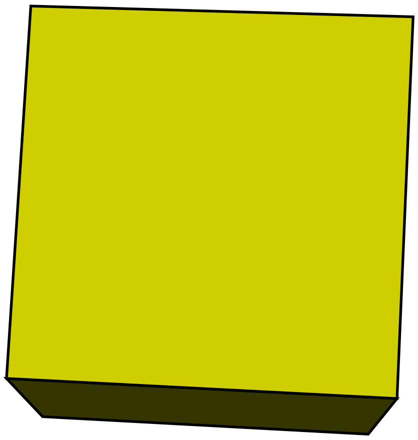

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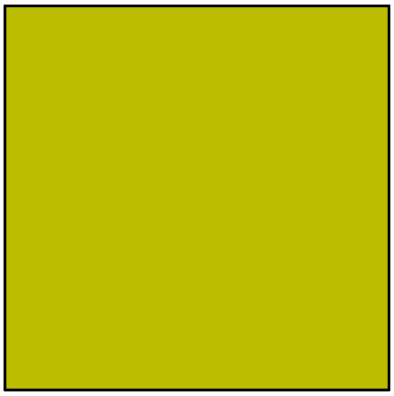

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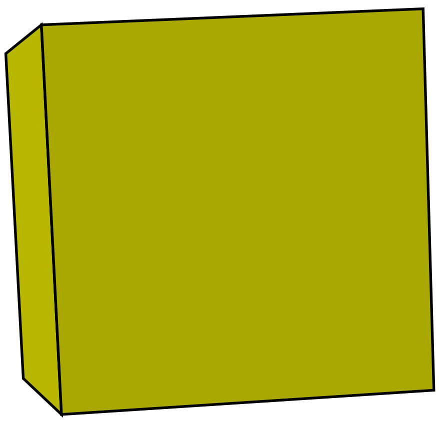
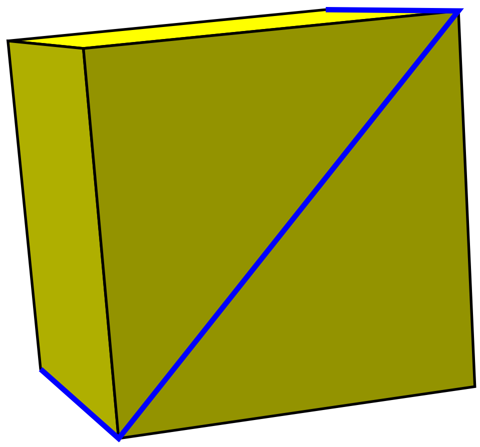


Unit Cube Volume = Base x Width x Height

Let's have a closer look at this object.

The initial solid Liu Hui generated from a unit cube is made by cutting a cube along two edges that are diagonally opposite to each other. The two objects created are identical and have volumes equal to one half of the cube's volume each. The name given to this object was Ch'ien-tu, which literally translates to moat-wall.



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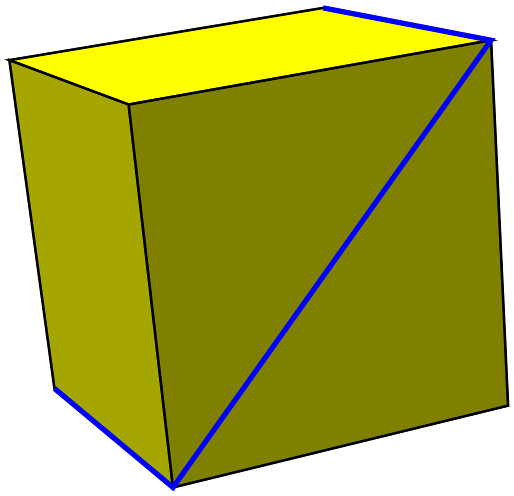
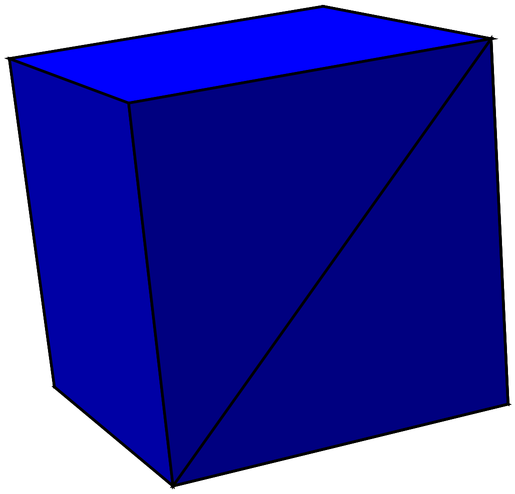
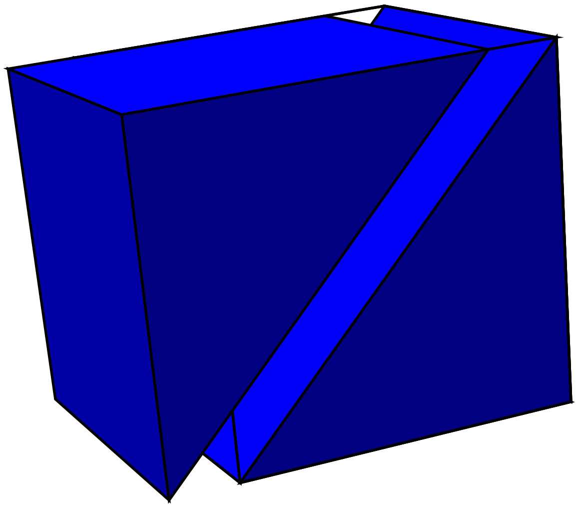


We can see that the unit cube can be constructed using two of these objects. This means that Liu Hui determined that a Ch’ien-tu has volume equal to one half of the base times the width times the height.



Two Ch’ien-tu can be combined to form a unit cube.

Thus a unit cube has twice the volume of a Ch’ien-tu.



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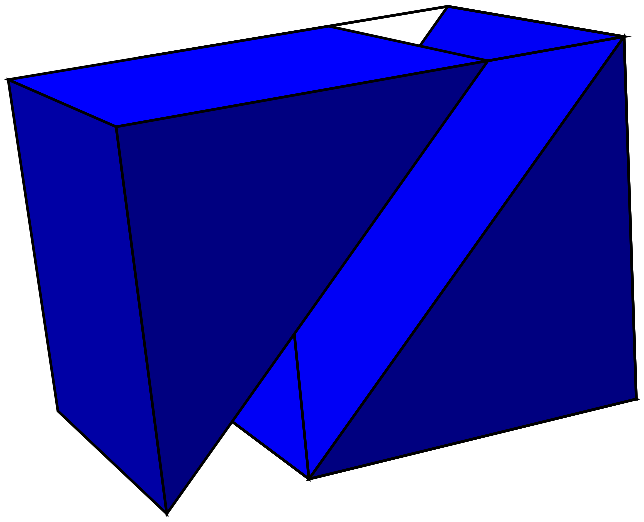

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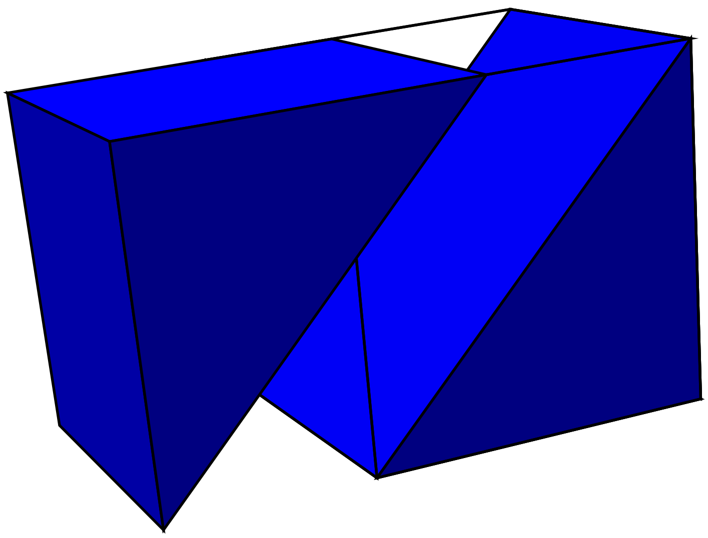

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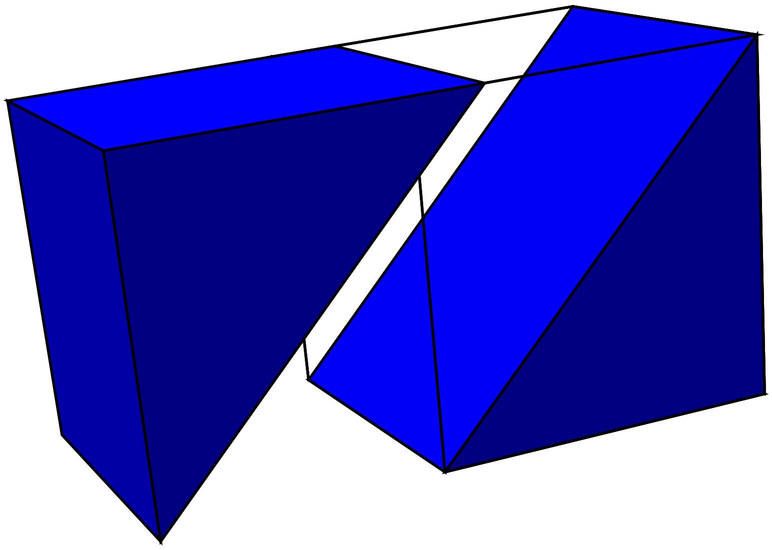

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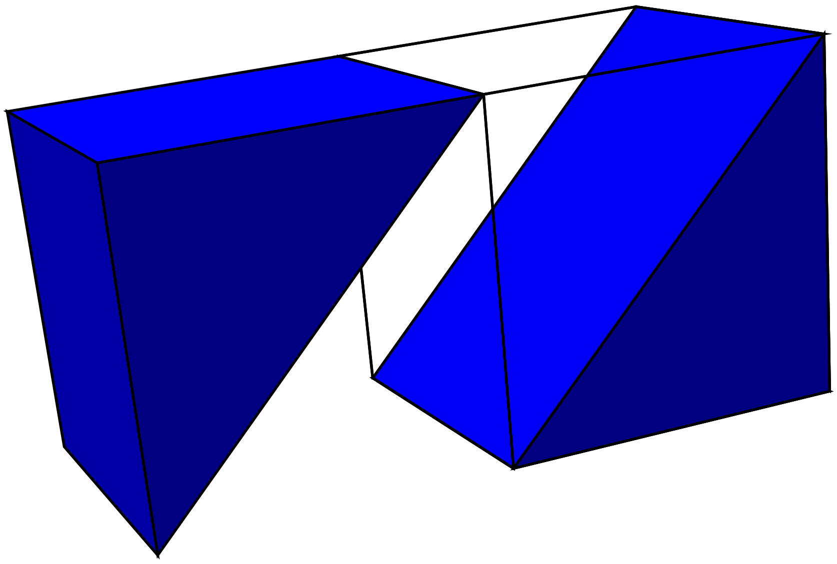

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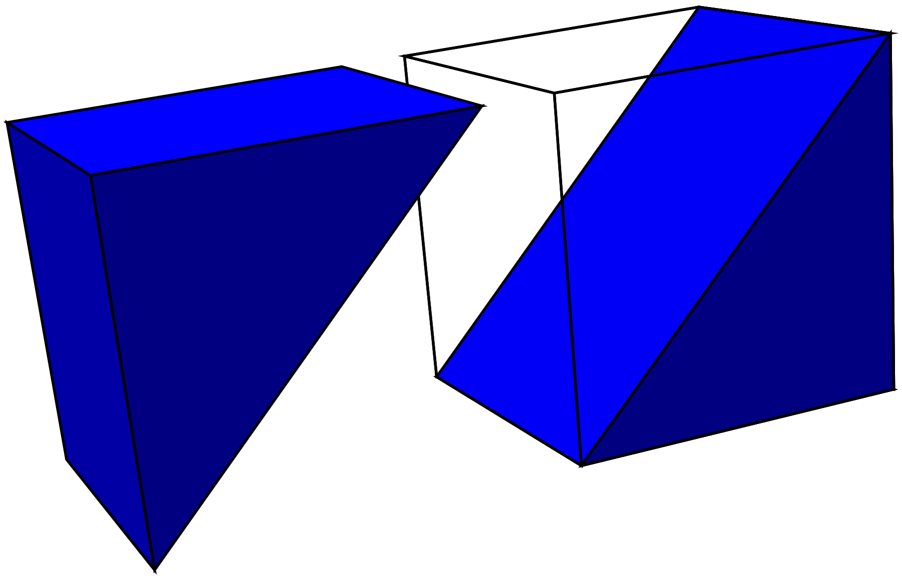

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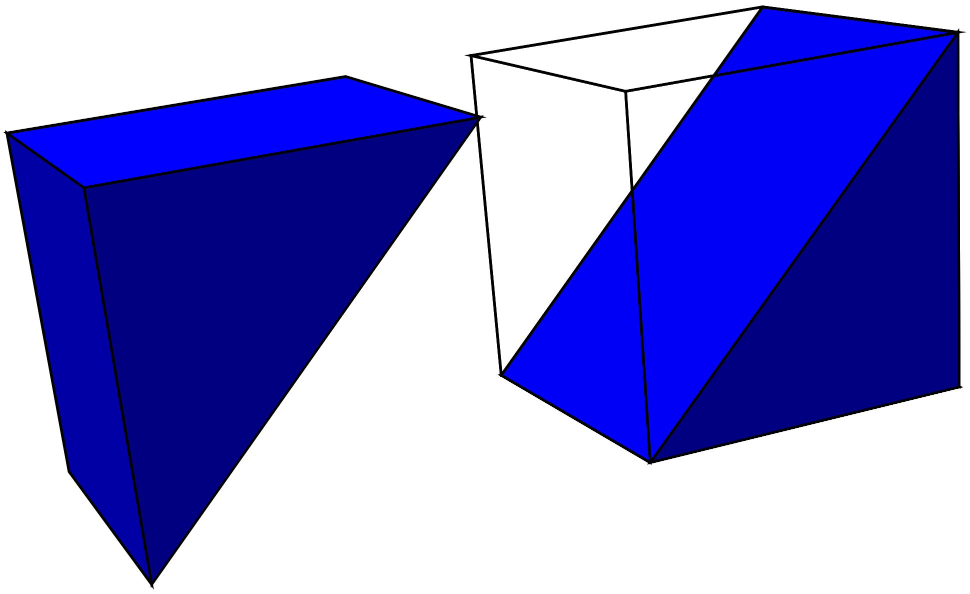

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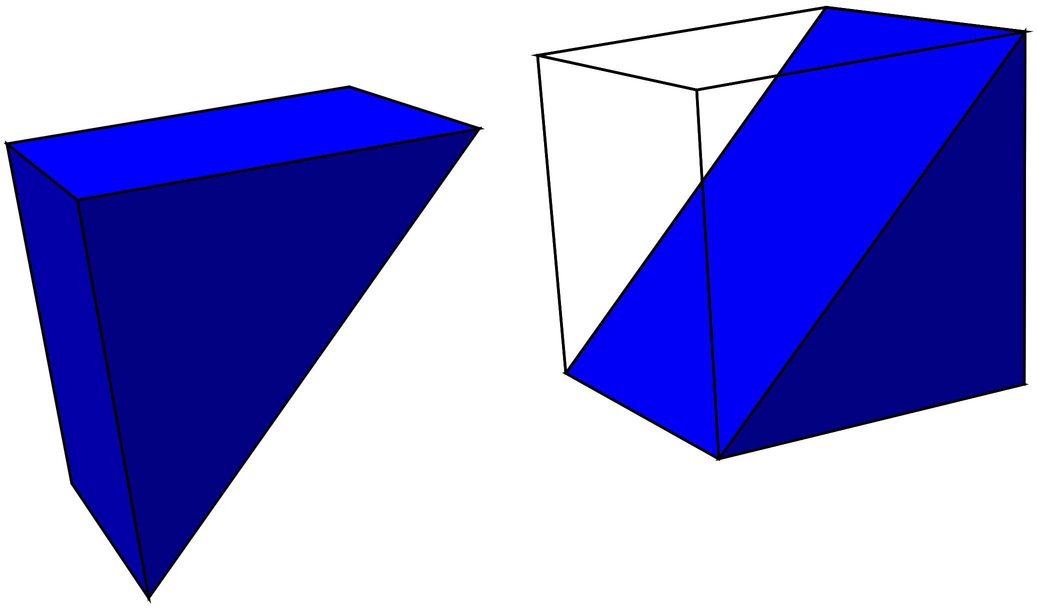

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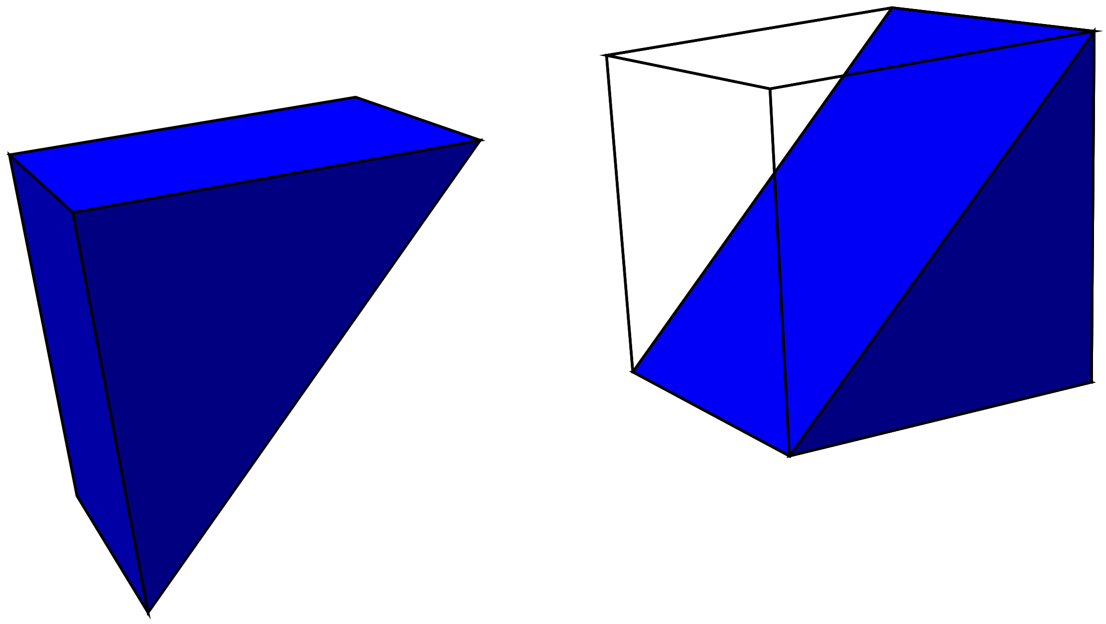

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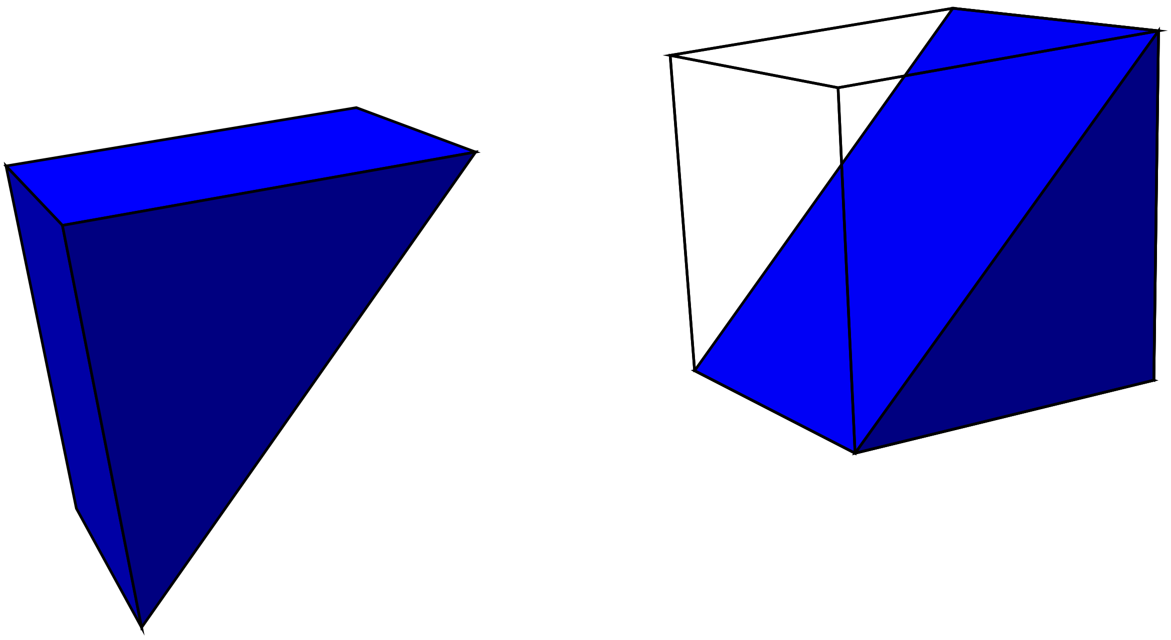

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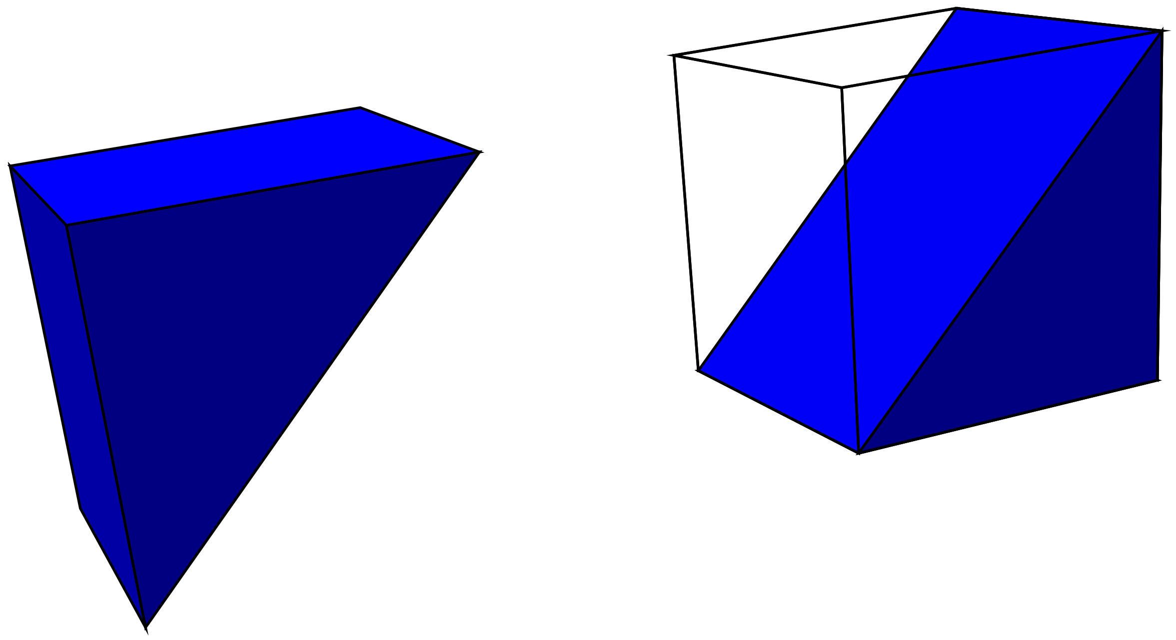

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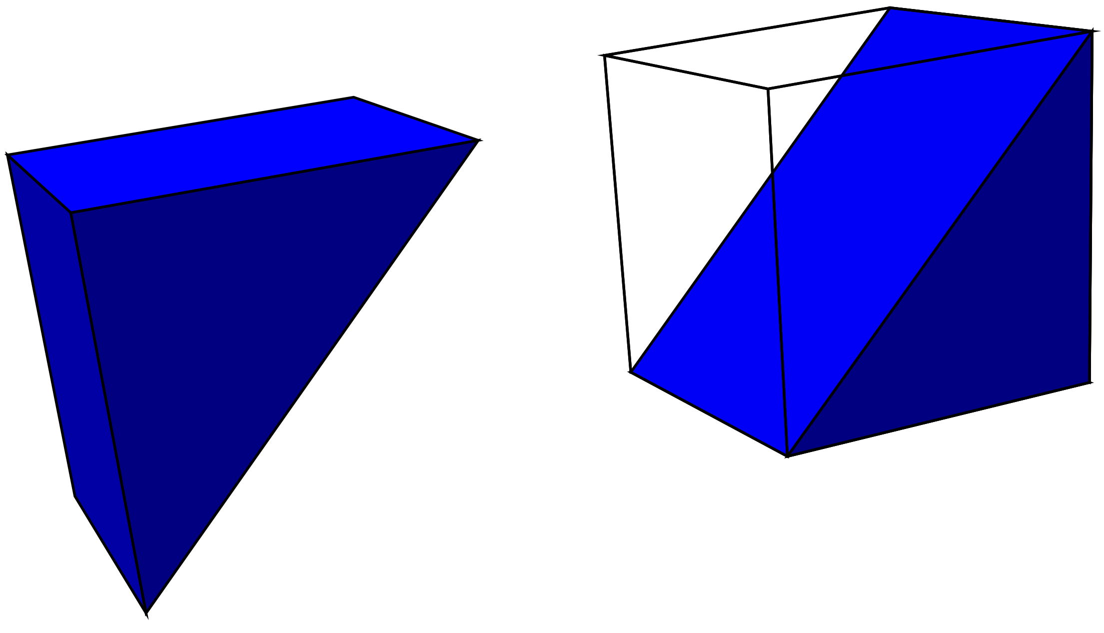

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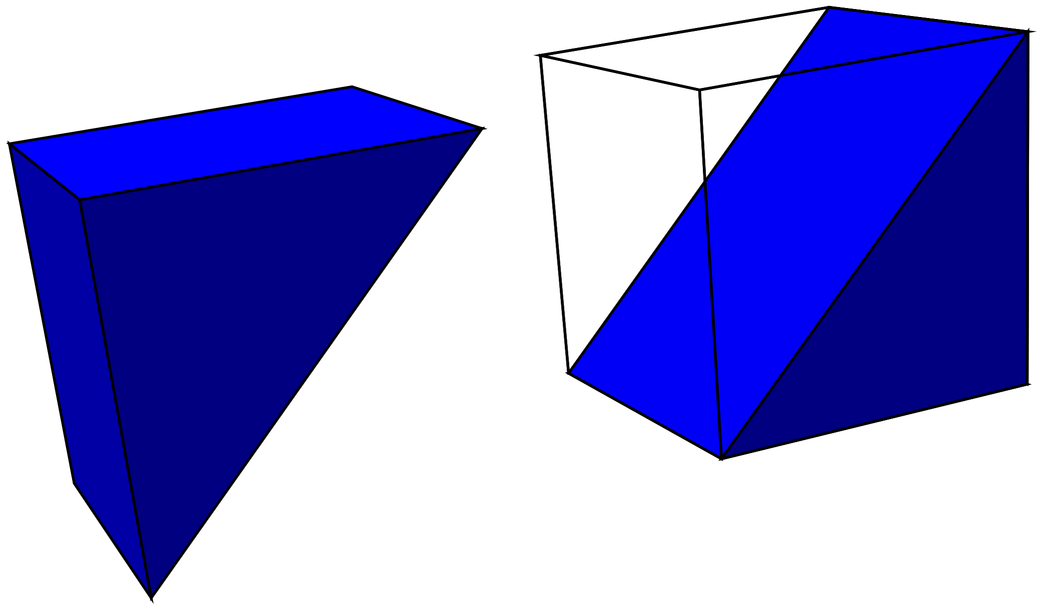

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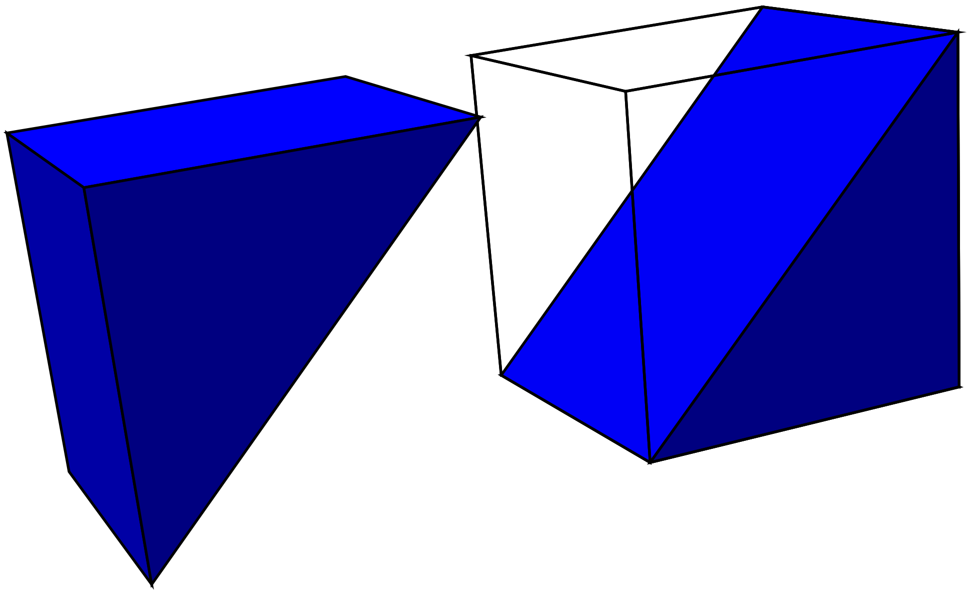

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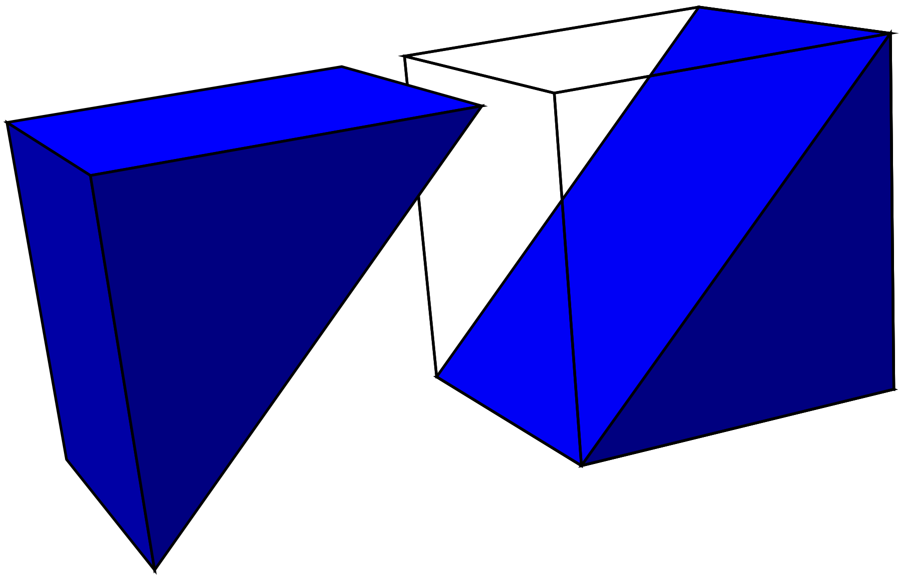

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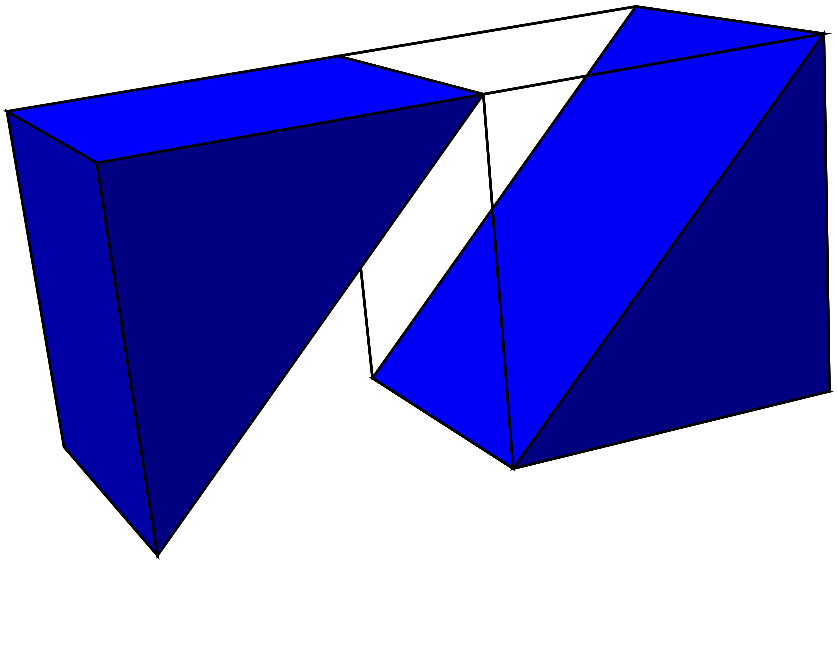

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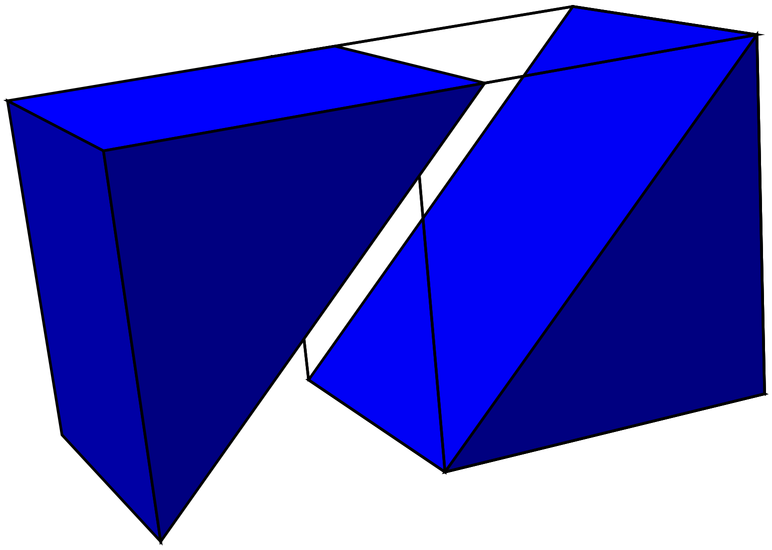

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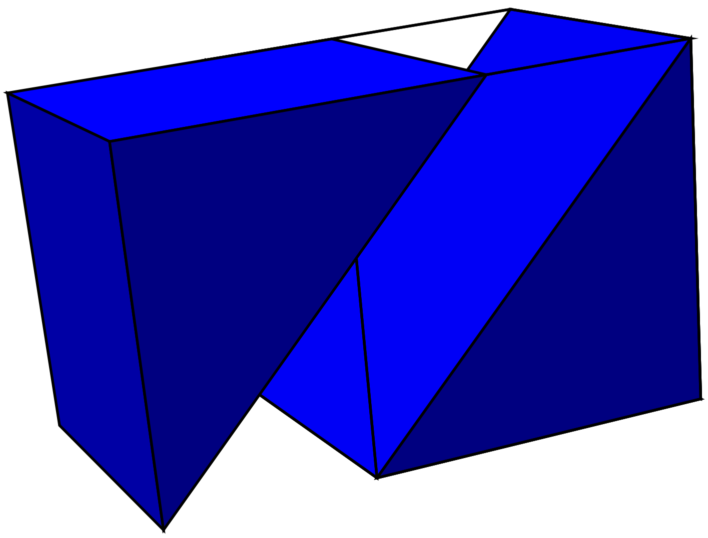

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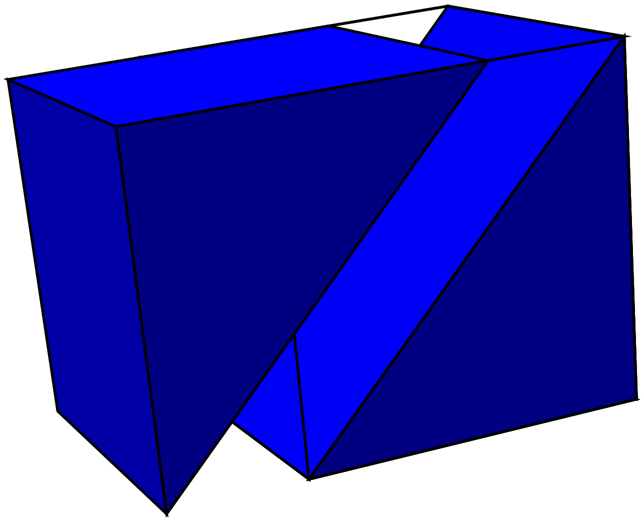

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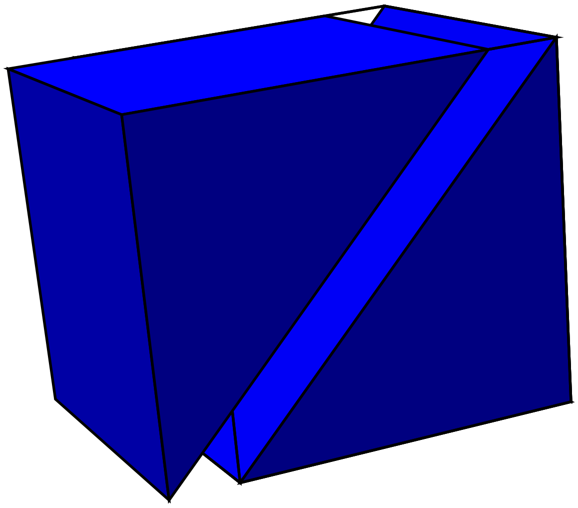
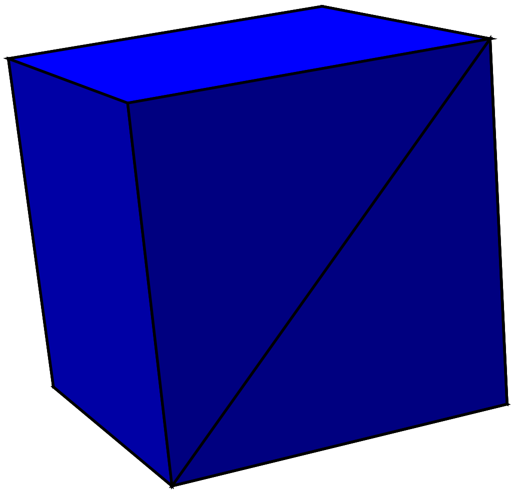


Two Ch’ien-tu can be combined to form a unit cube.

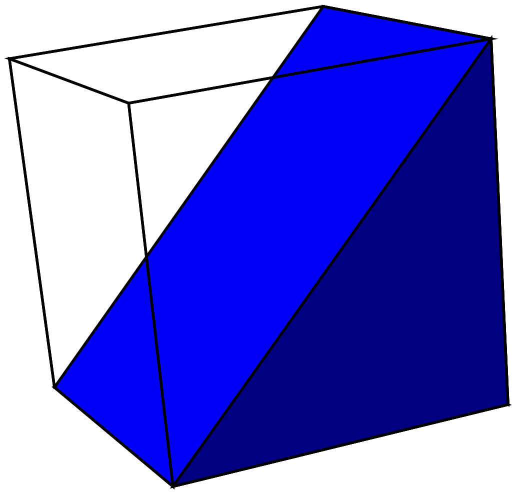
Thus a unit cube has twice the volume of a Ch’ien-tu.



Two Ch’ien-tu can be combined to form a unit cube.

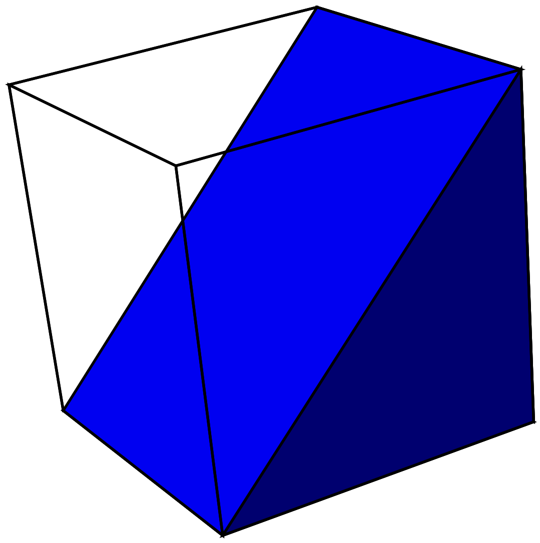
Thus a unit cube has twice the volume of a Ch’ien-tu.

A Ch'ien-tu is constructed by cutting a unit cube along two edges that are diagonally opposite to each other. The resulting solid is a right angled prism with a triangular base. The object is often called a wedge.



Ch'ien-tu Volume = 1/2 x Base x Width x Height

Let's have a closer look at this object.



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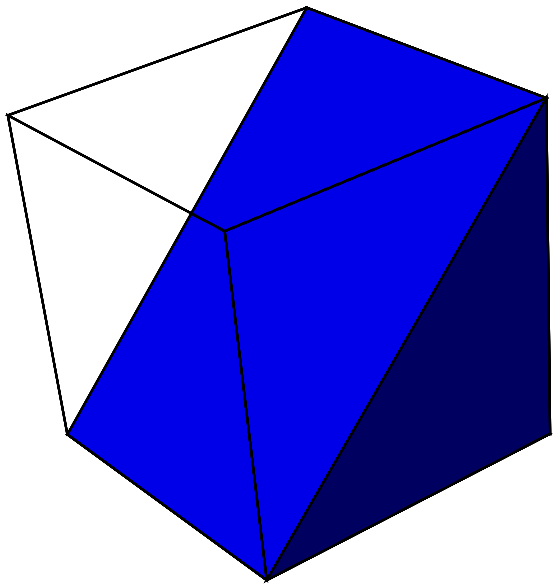

Ch'ien-tu Volume = 1/2 x Base x Width x Height

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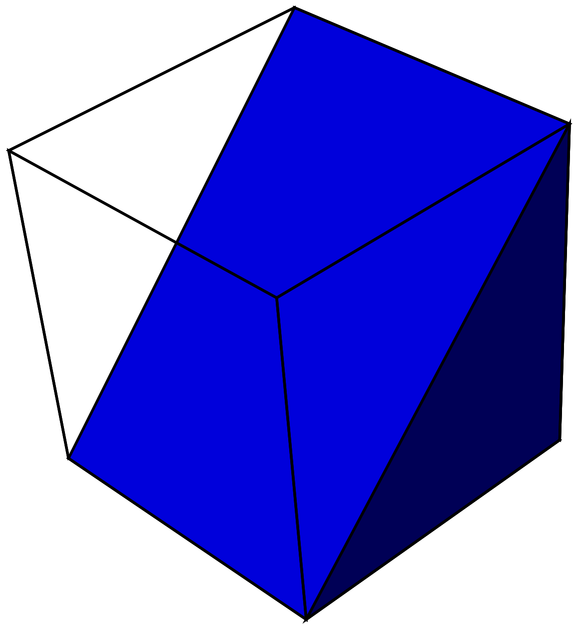

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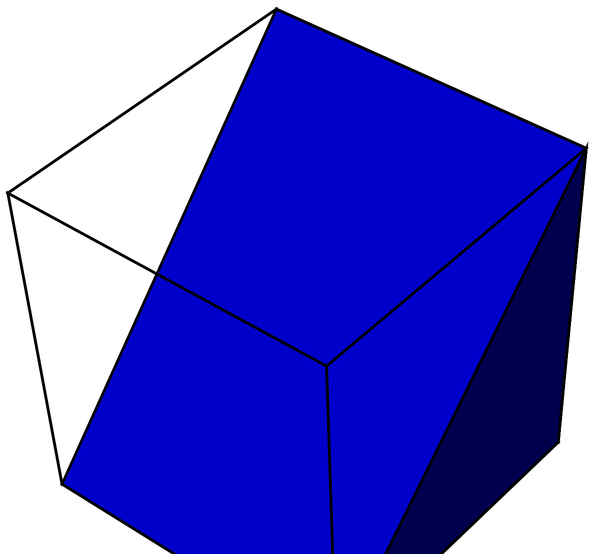

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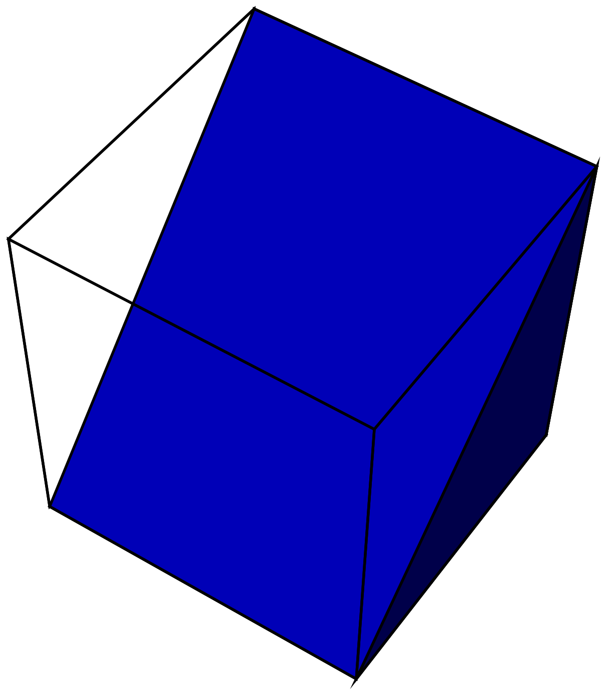

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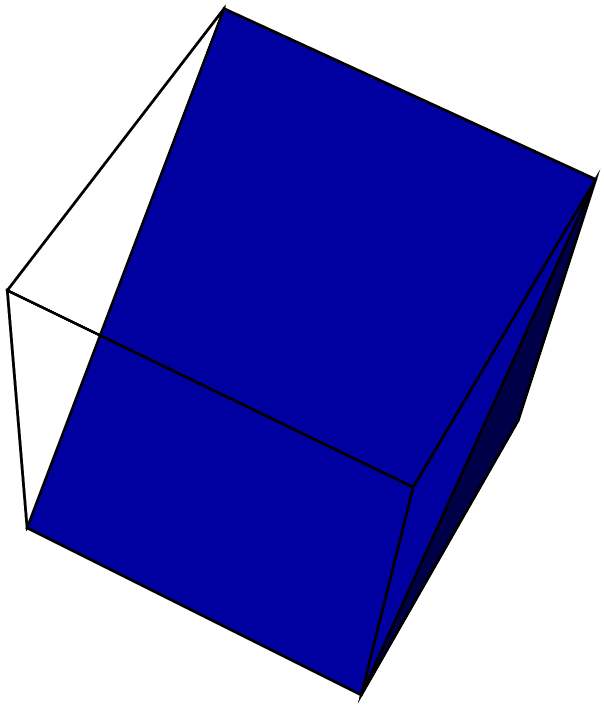

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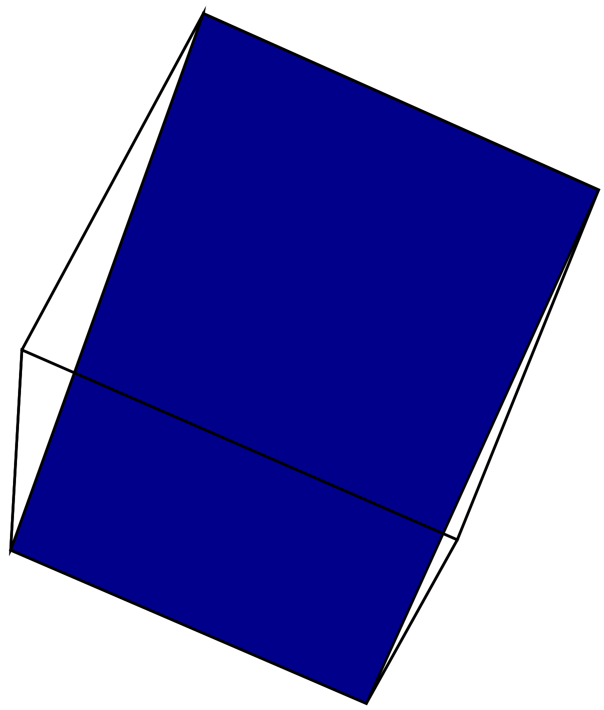

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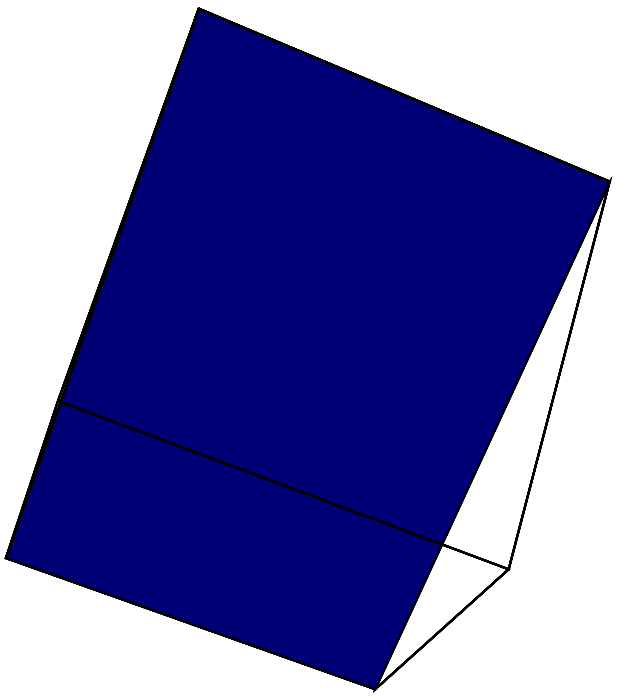

Ch’ien-tu Volume = 1/2 x Base x Width x Height

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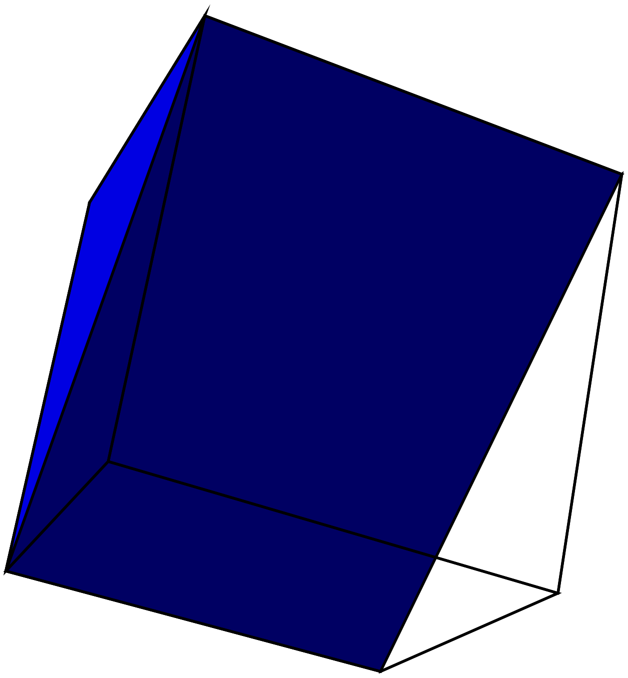

Ch’ien-tu Volume = 1/2 x Base x Width x Height

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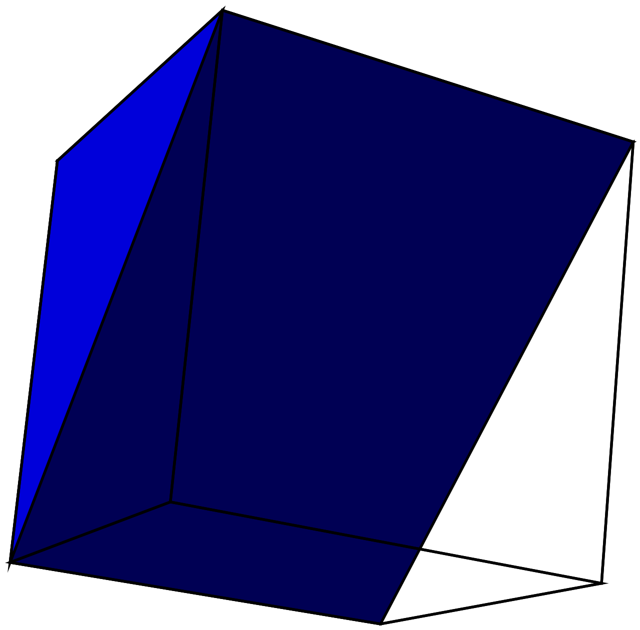

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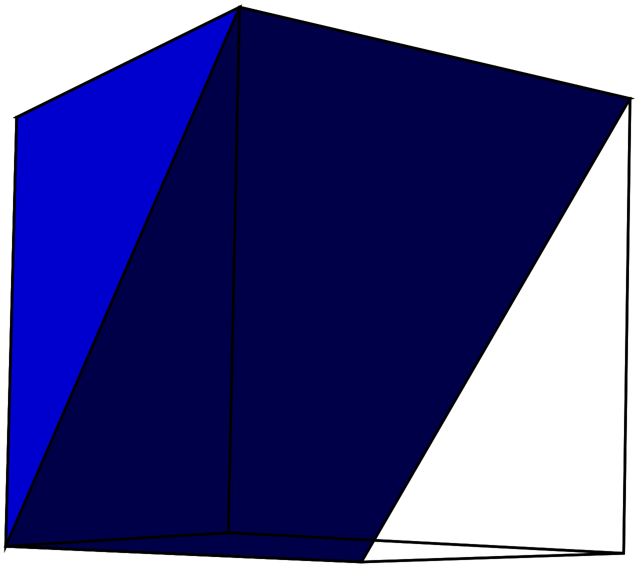

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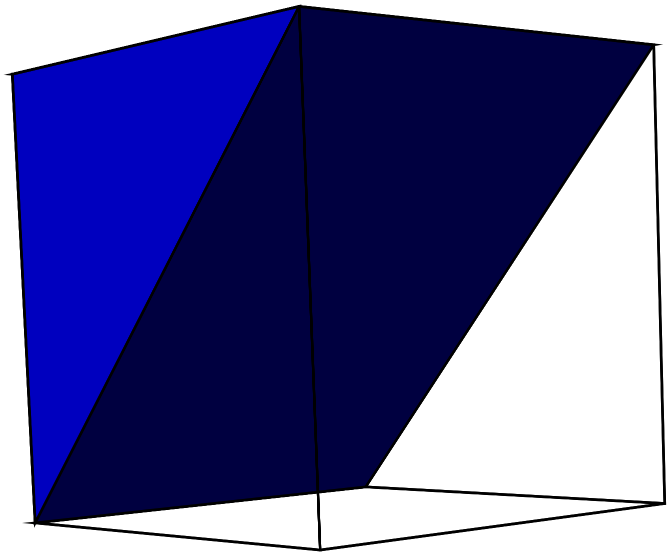

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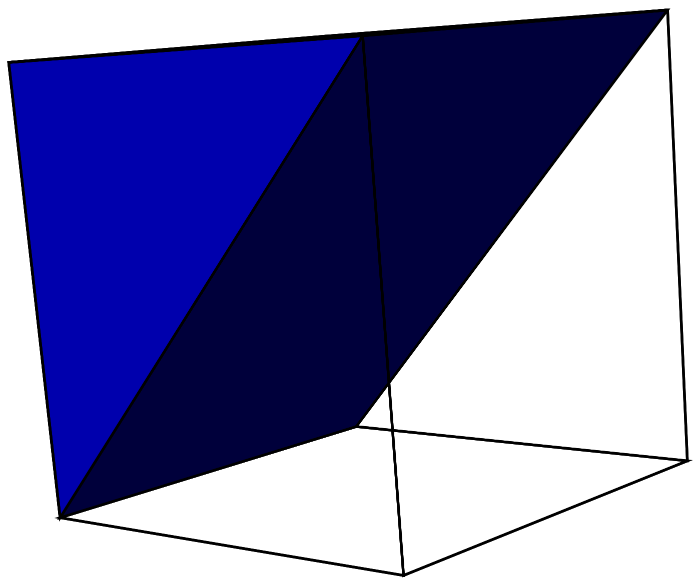

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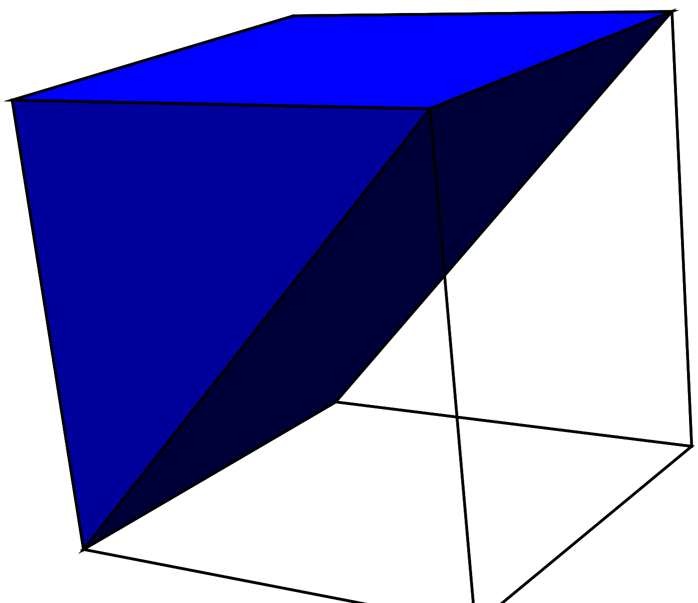

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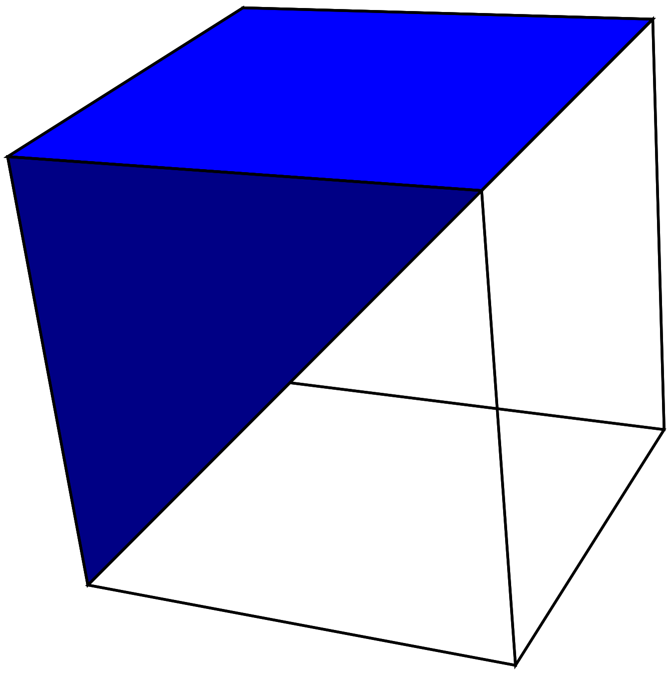

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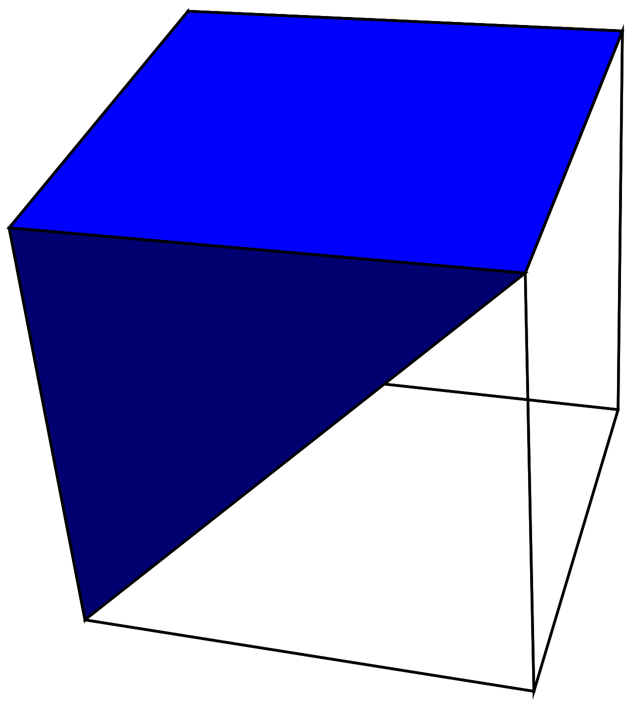

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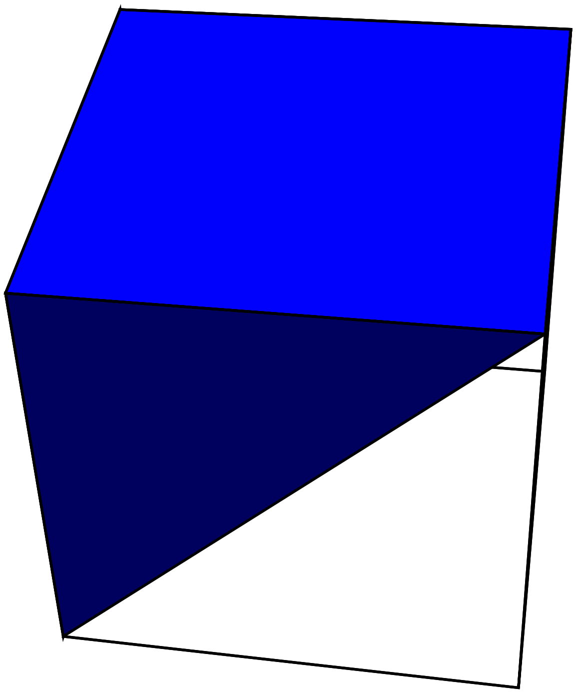

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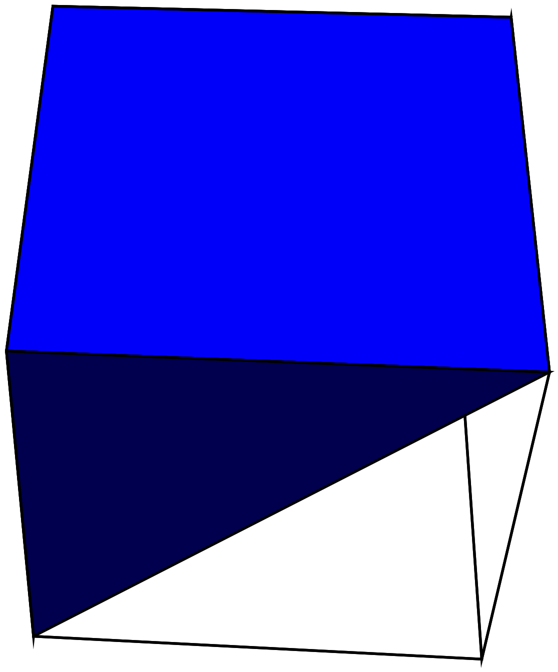

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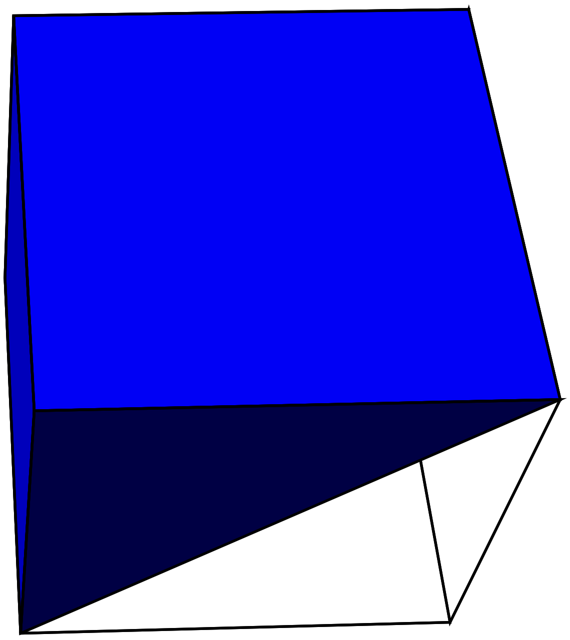

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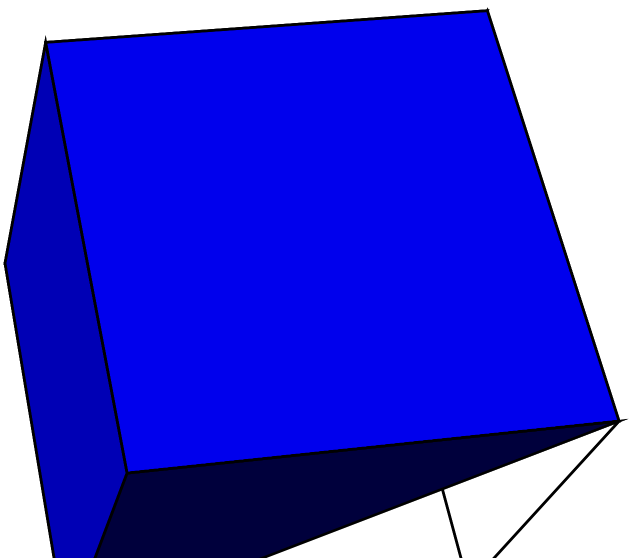

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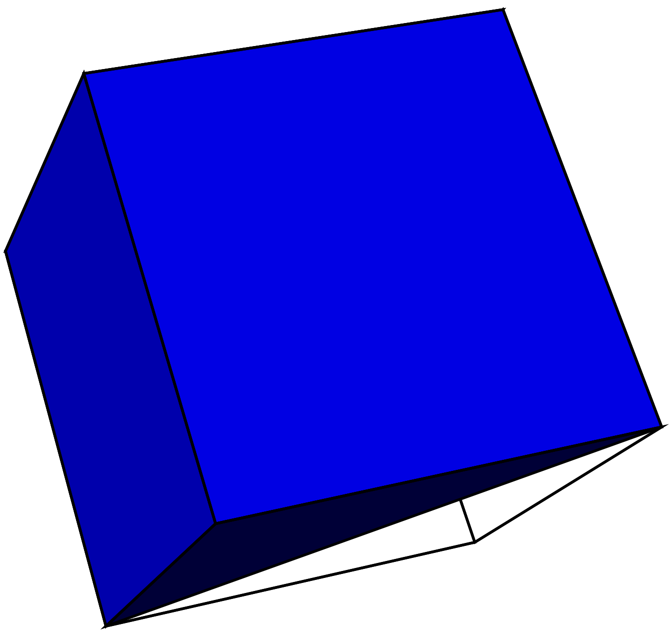

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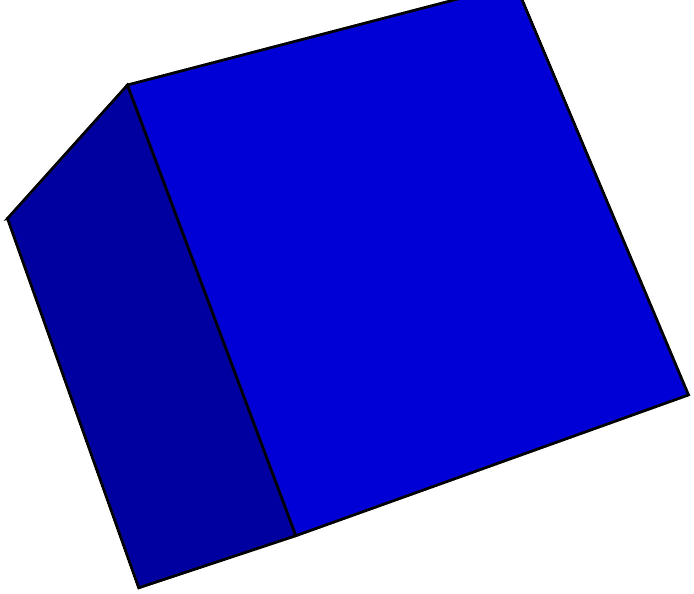

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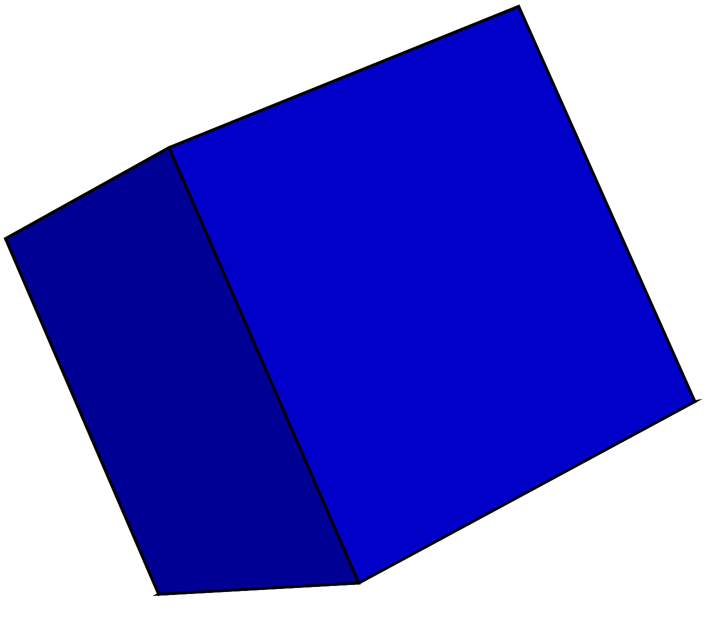

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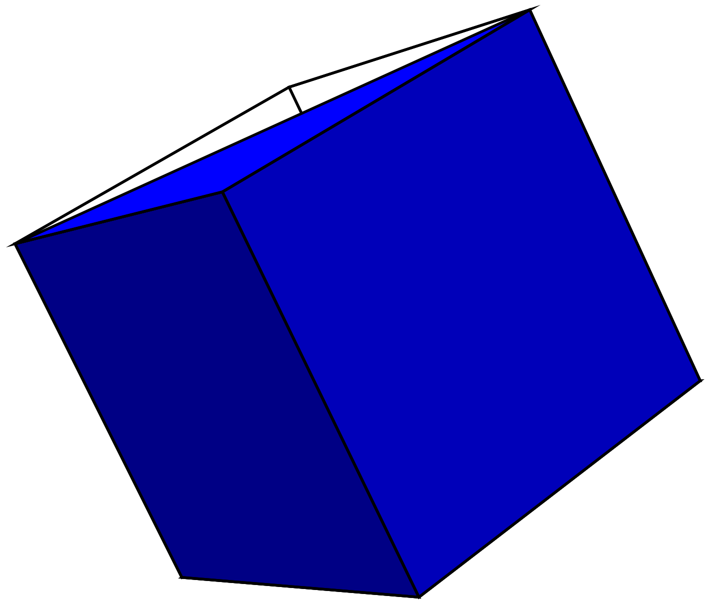

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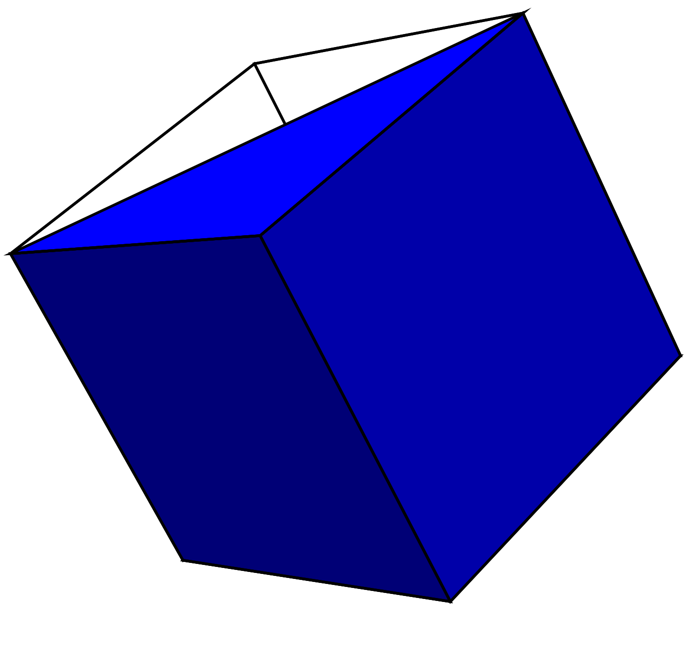

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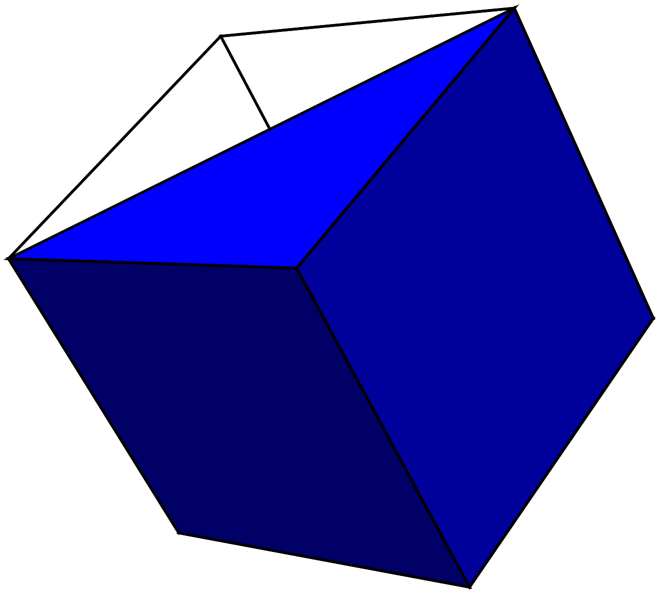

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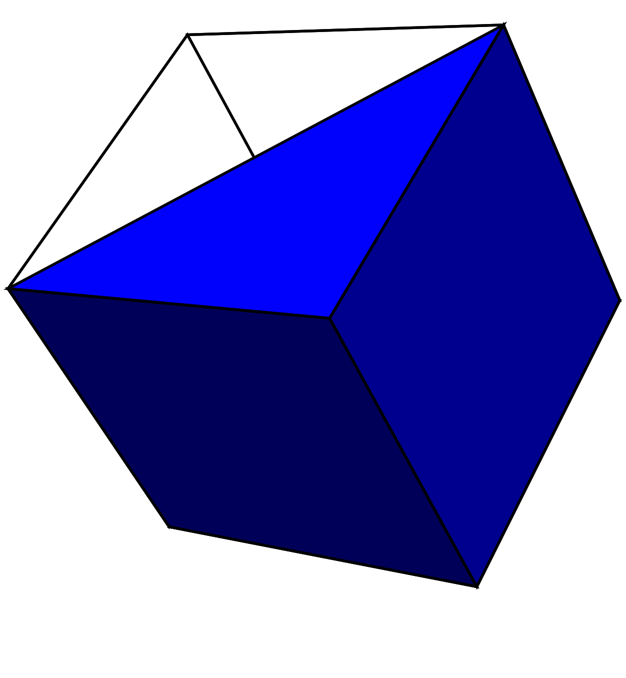

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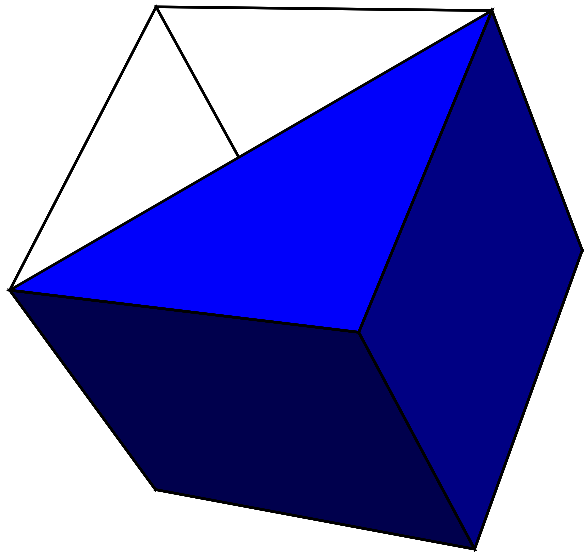

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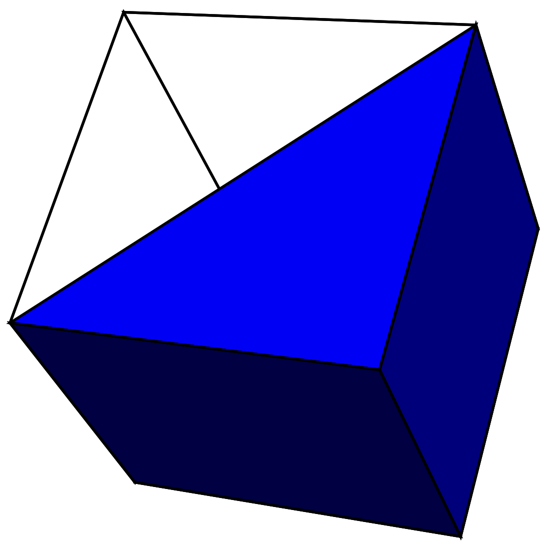

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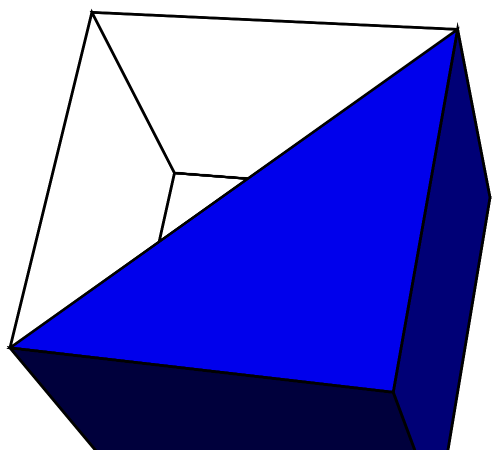

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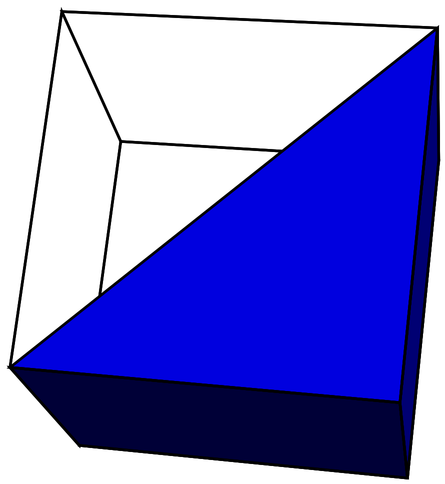

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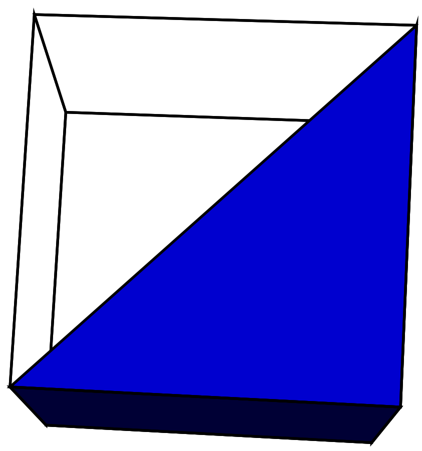

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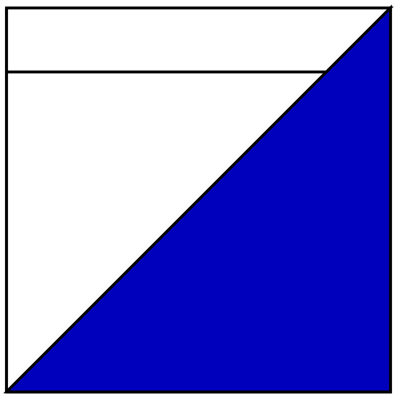

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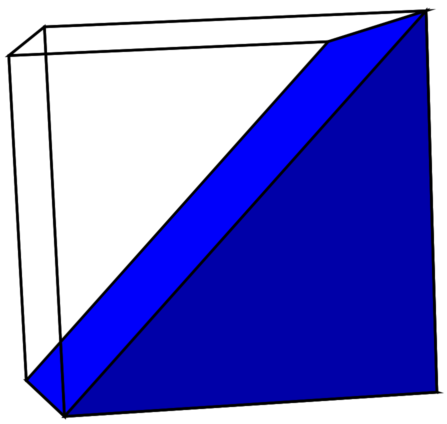
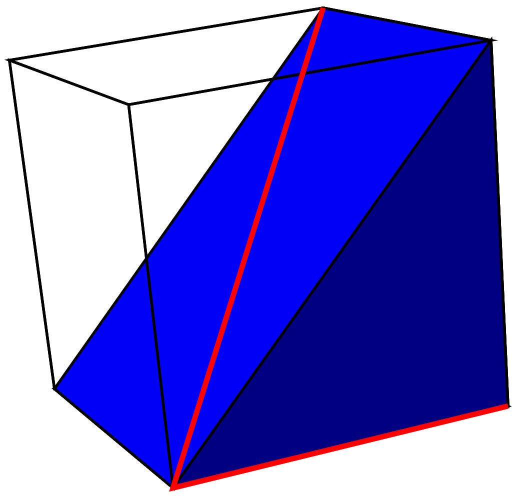


Ch’ien-tu Volume = 1/2 x Base x Width x Height

Let’s have a closer look at this object.

The next basic object was constructed by cutting a Ch’ien-tu diagonally across the face of the wedge shape. The result is two different objects which do not have the same volume. The smaller object on the right is called a Pieh-nao, which literally means Turtle’s Shoulder Joint.



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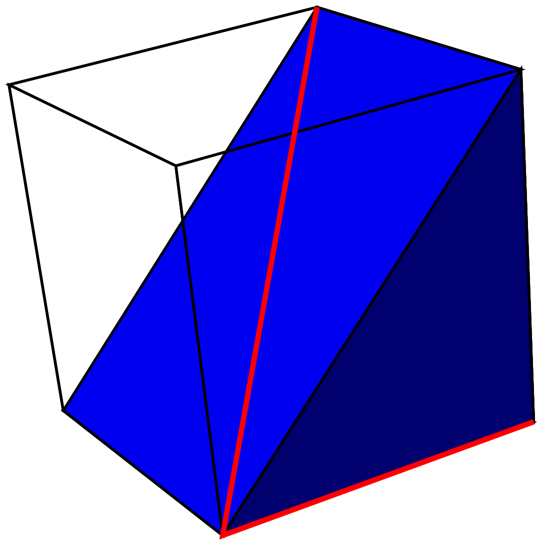

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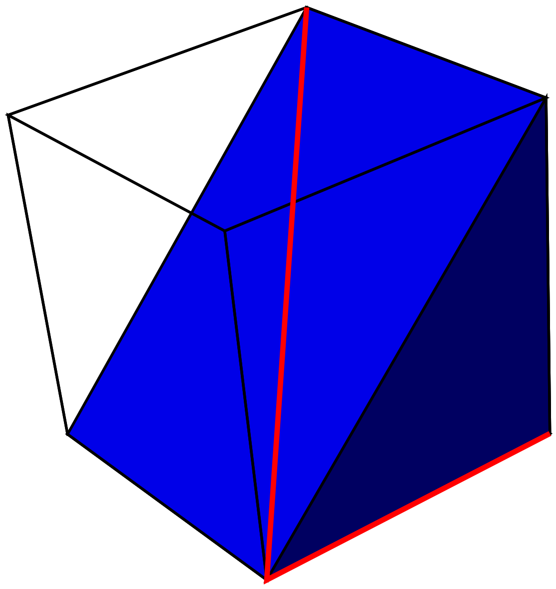
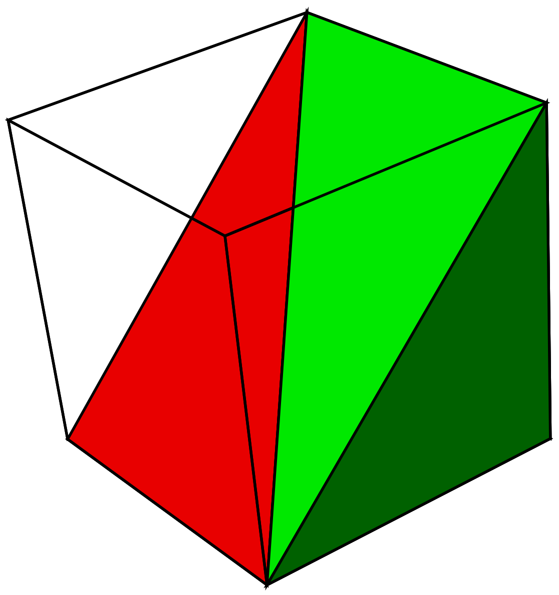
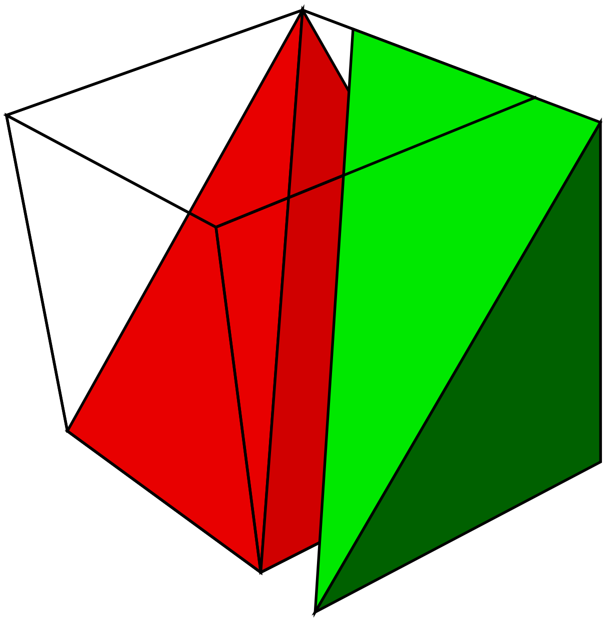


The volume of a Pieh-nao was found by Liu Hui to be equal to one sixth of the volume of the unit cube from which it was created. We will see shortly just how Liu Hui came to this conclusion. The Pieh-nao is the third of the basic objects used in proving the volume formulas.



A Ch’ien-tu can be cut up to form a Pied-nao and another different object. Let’s see how this division works.



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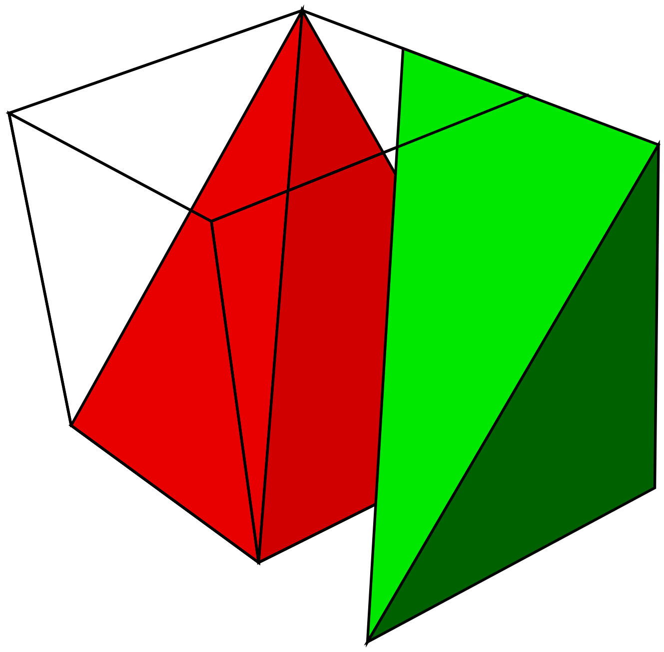

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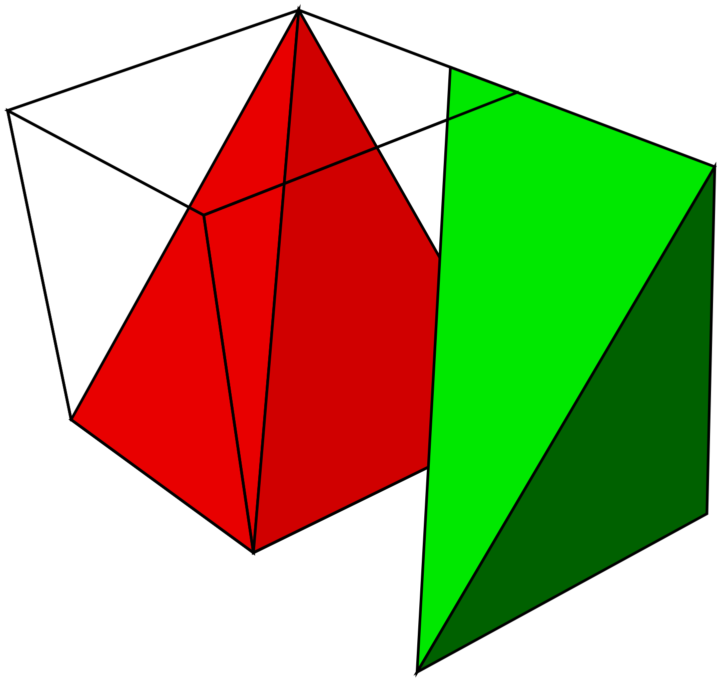

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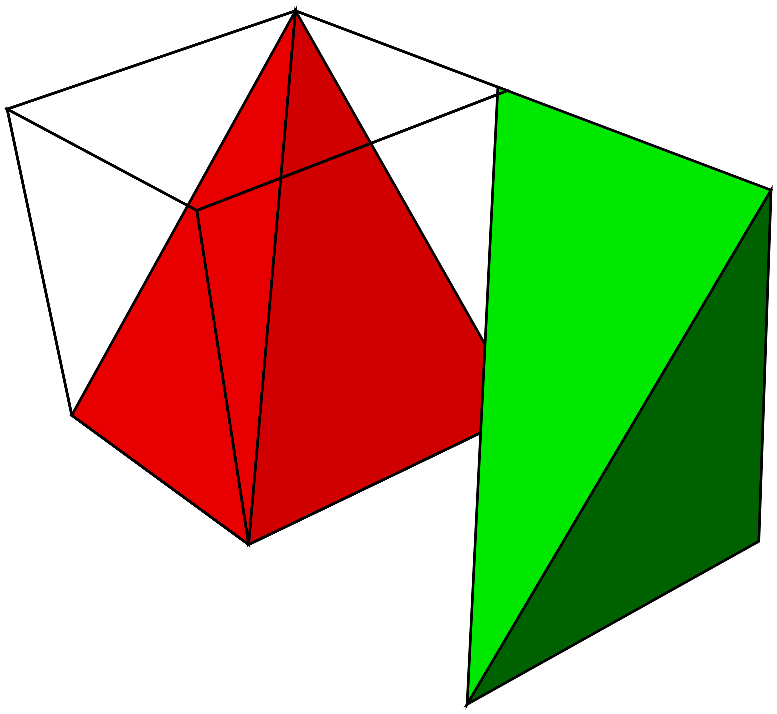

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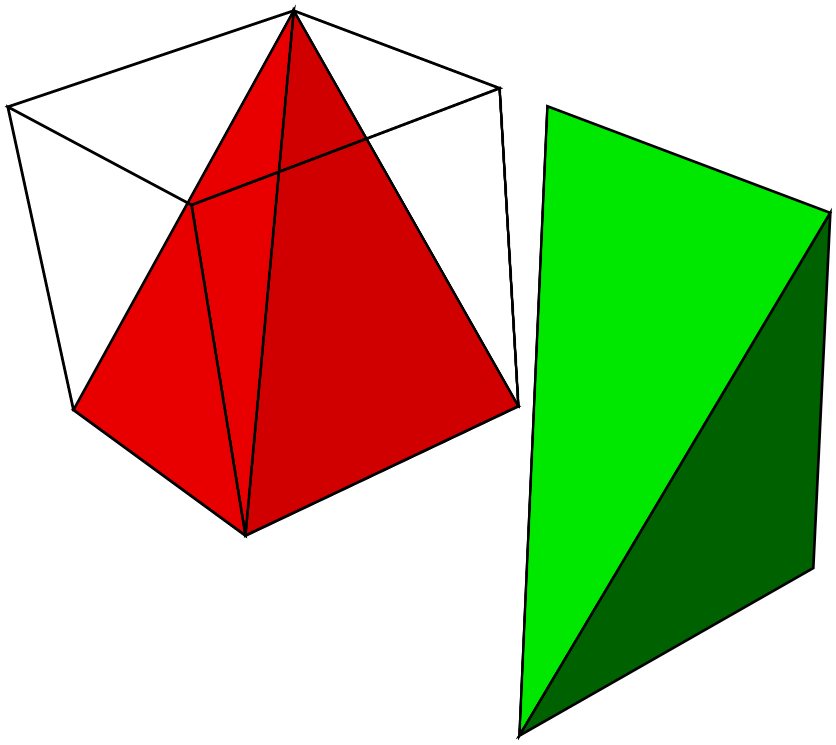

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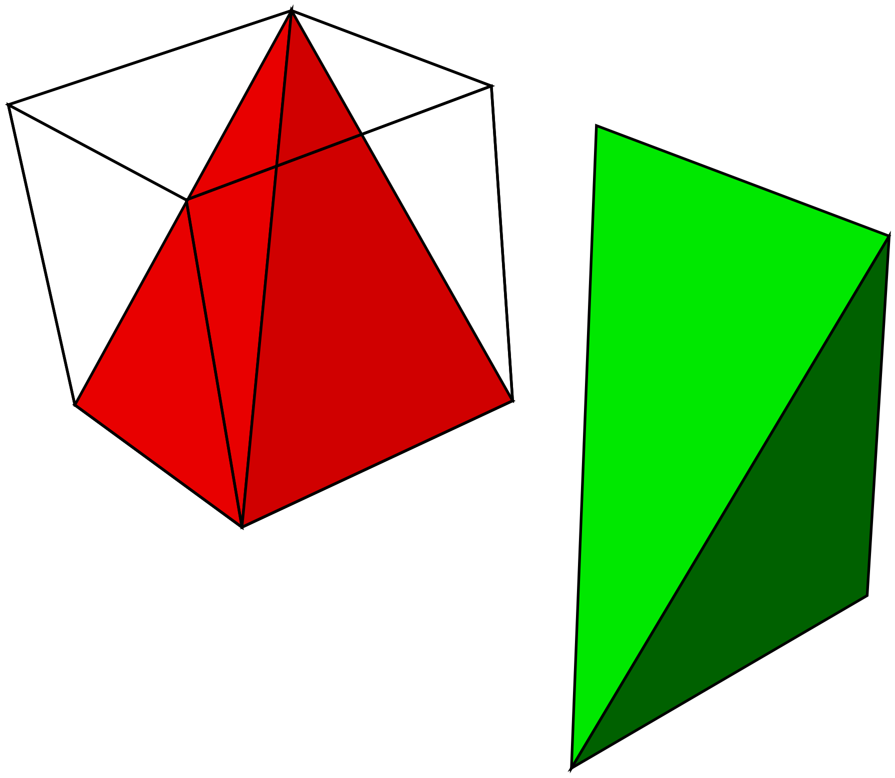

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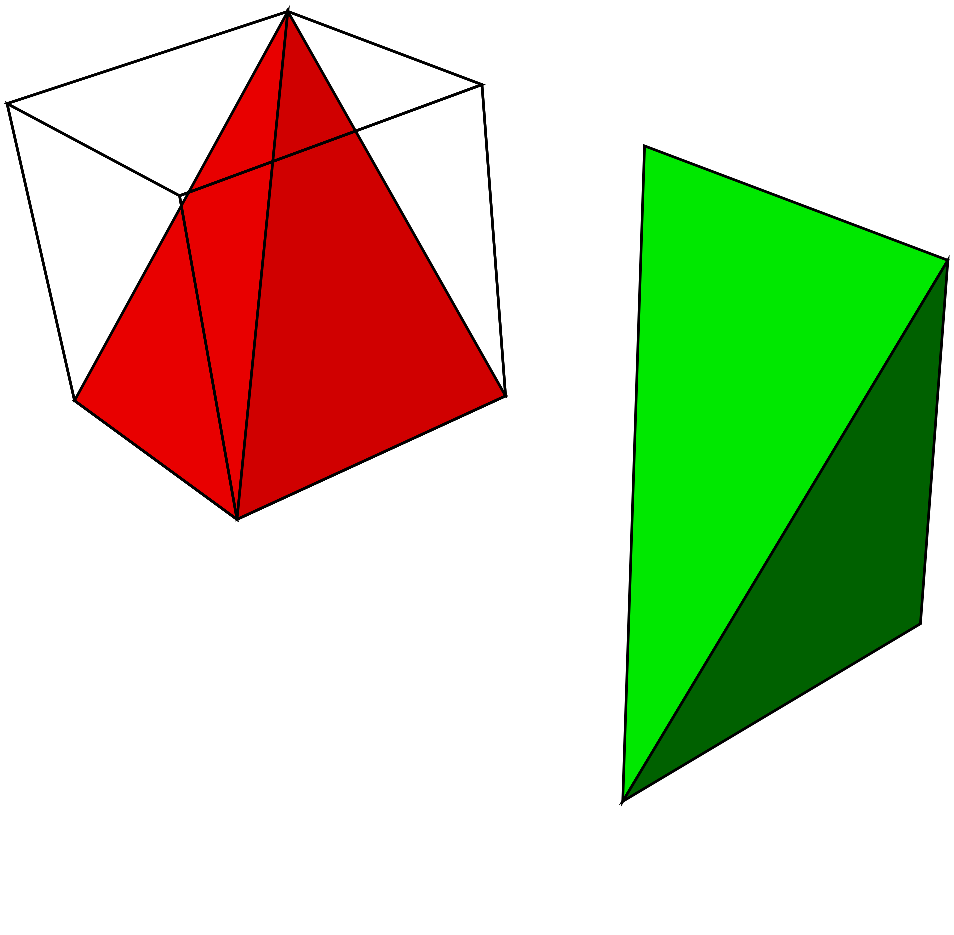

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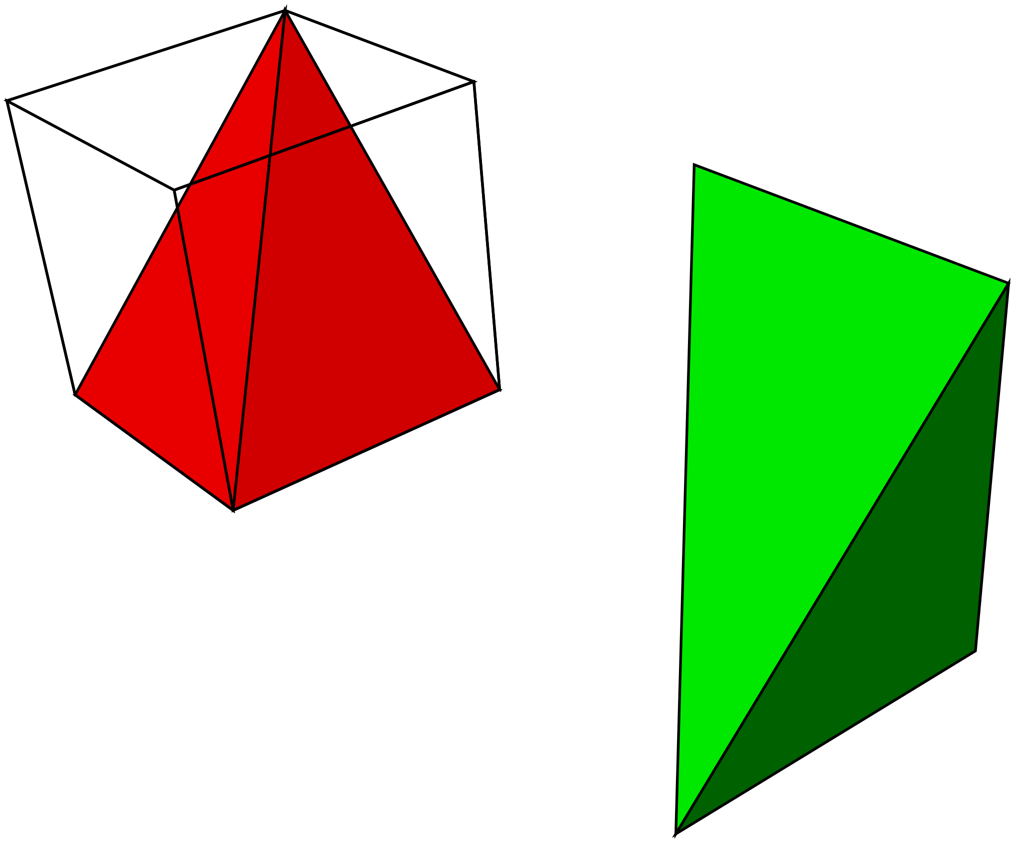

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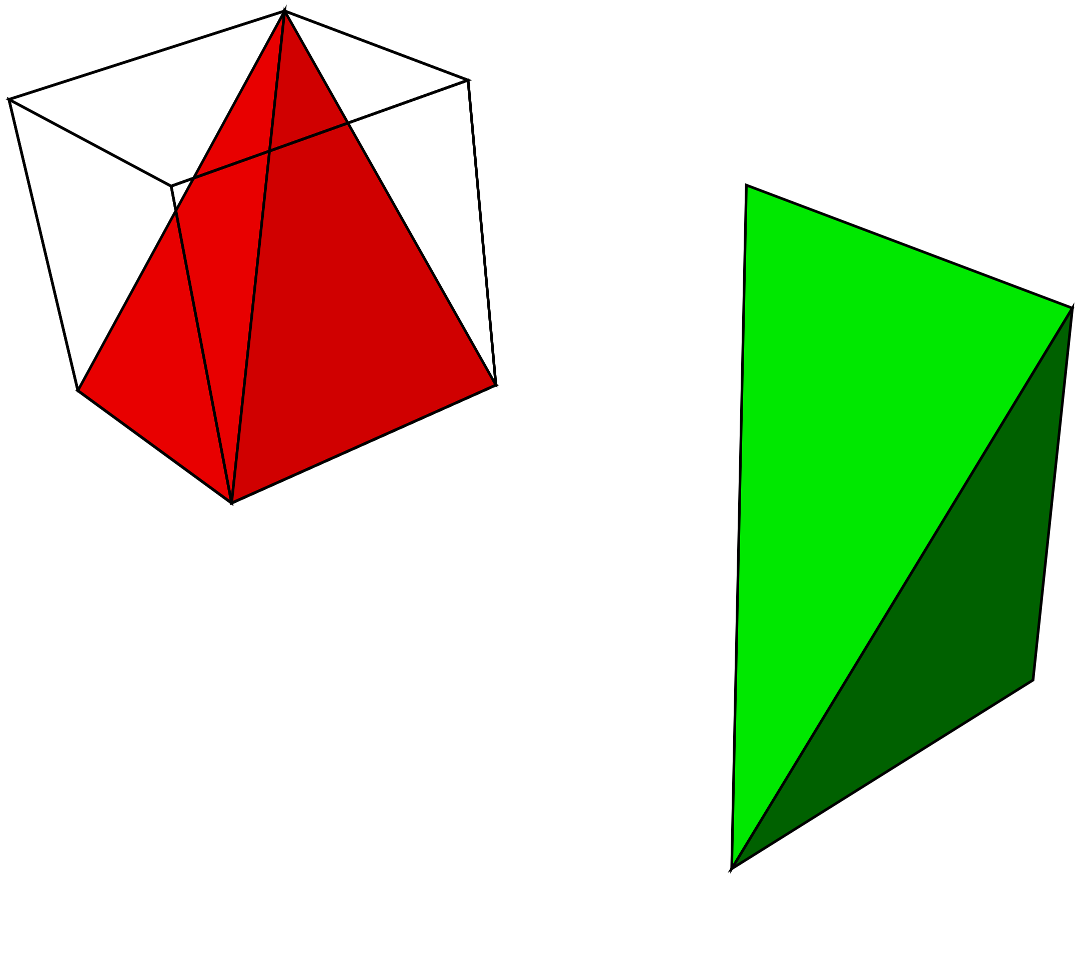

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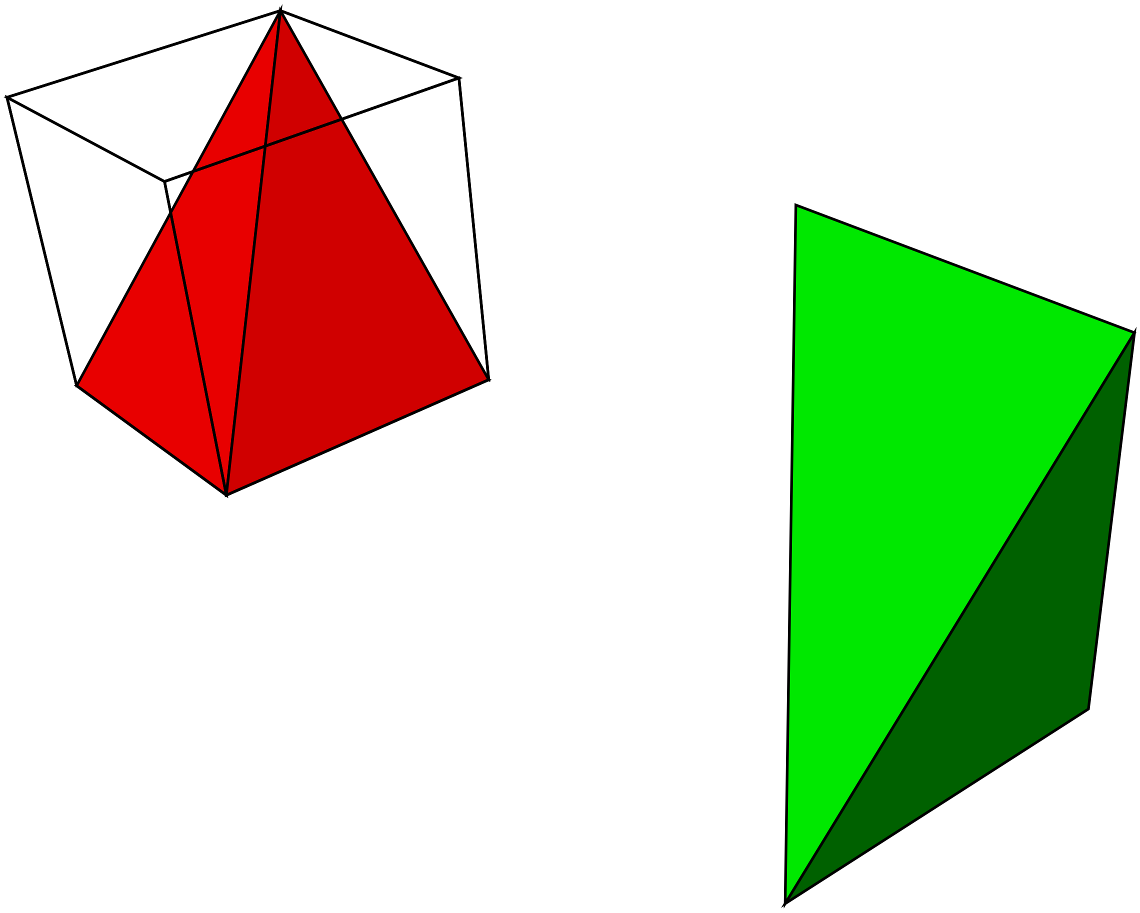

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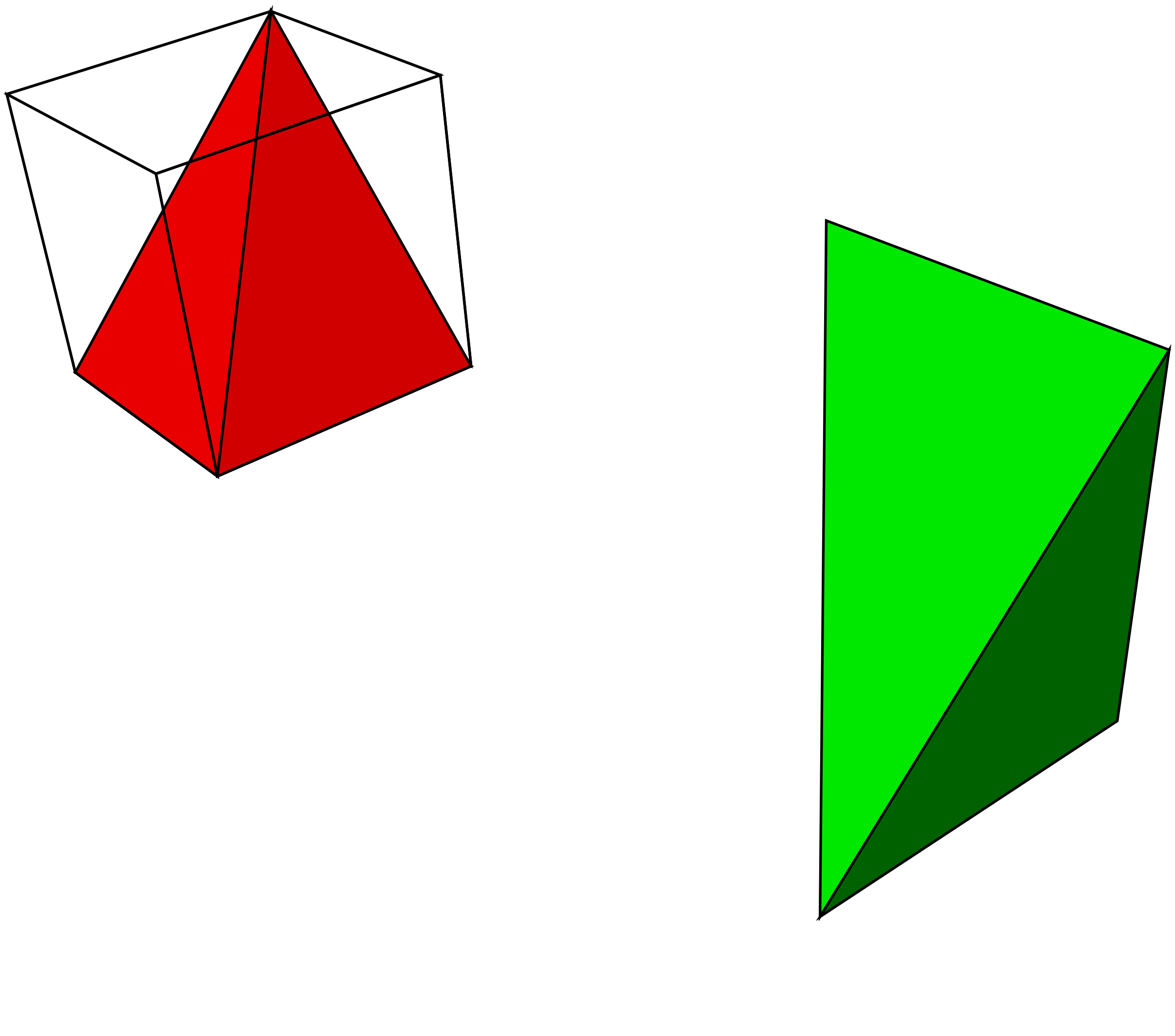

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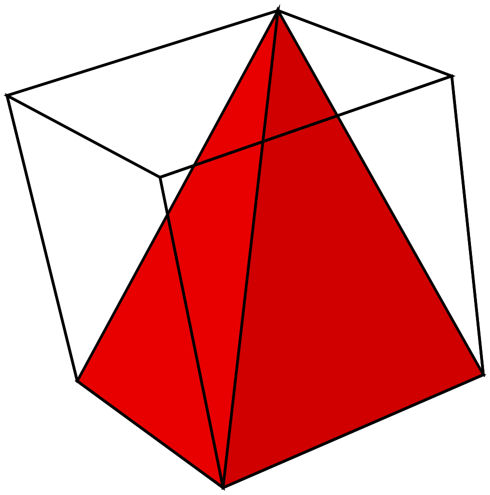

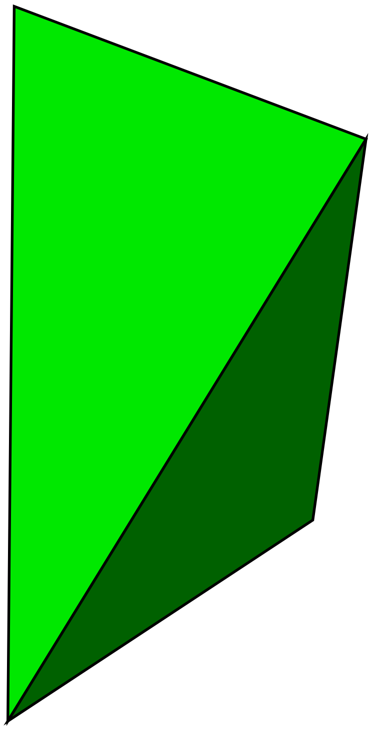

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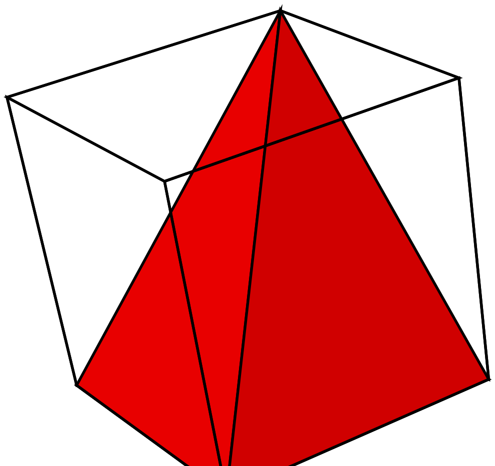

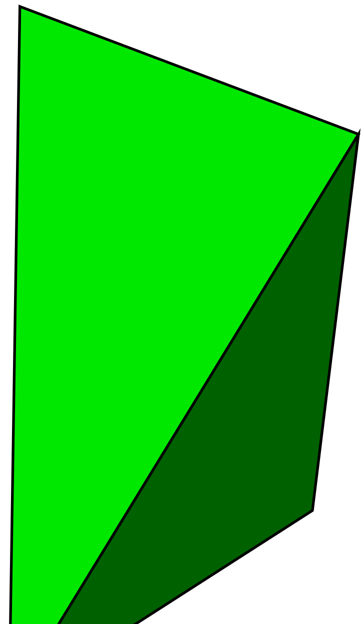

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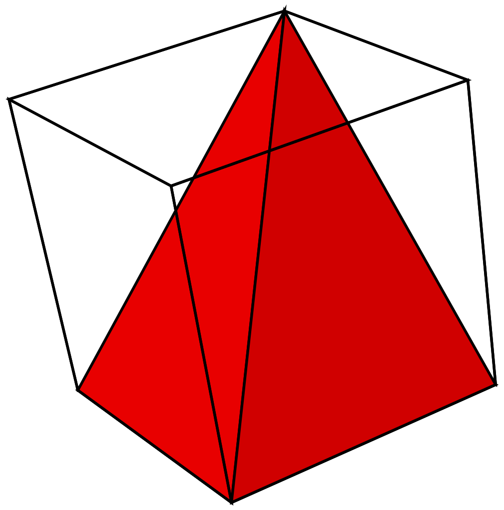

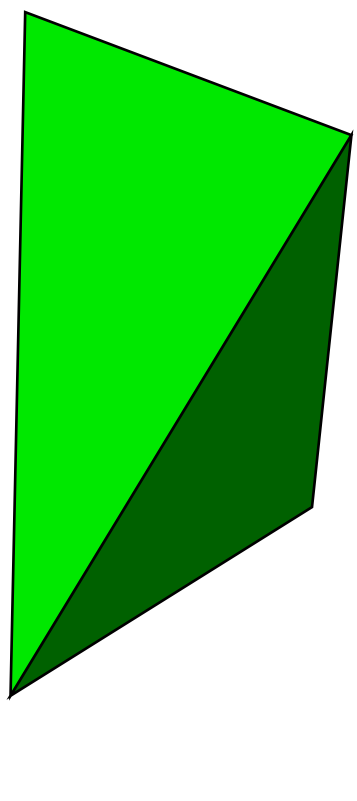

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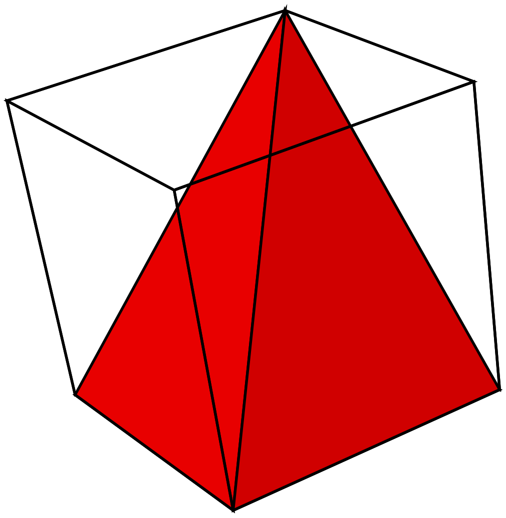
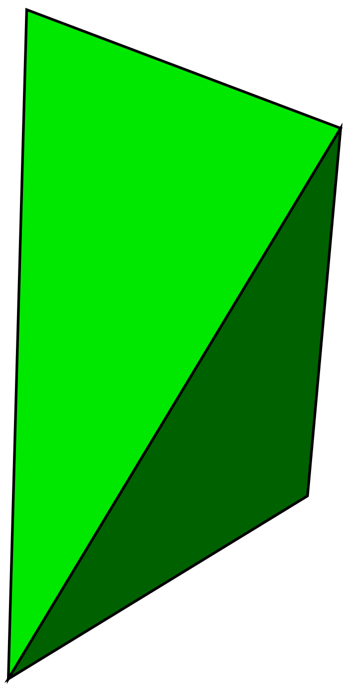
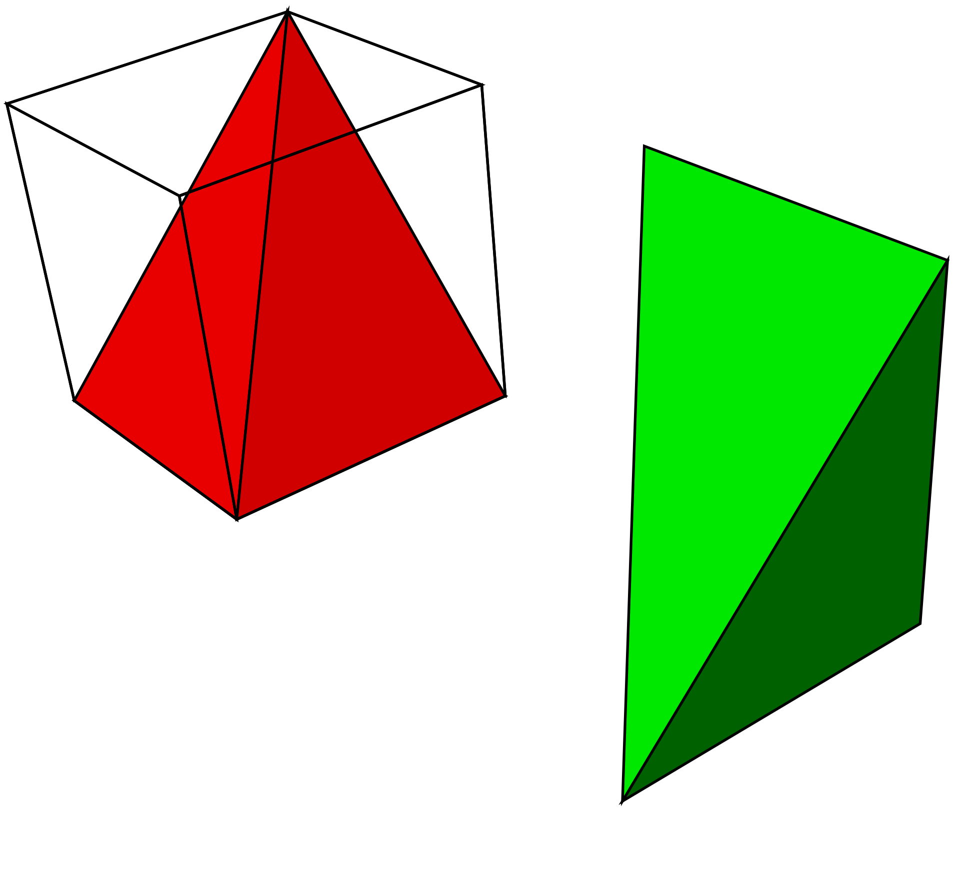
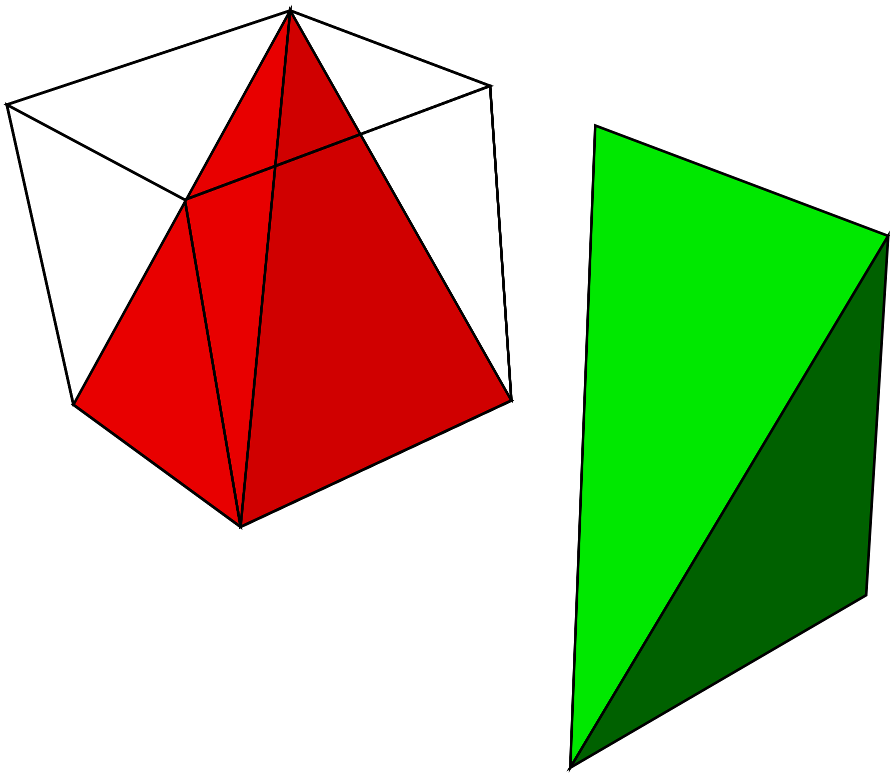




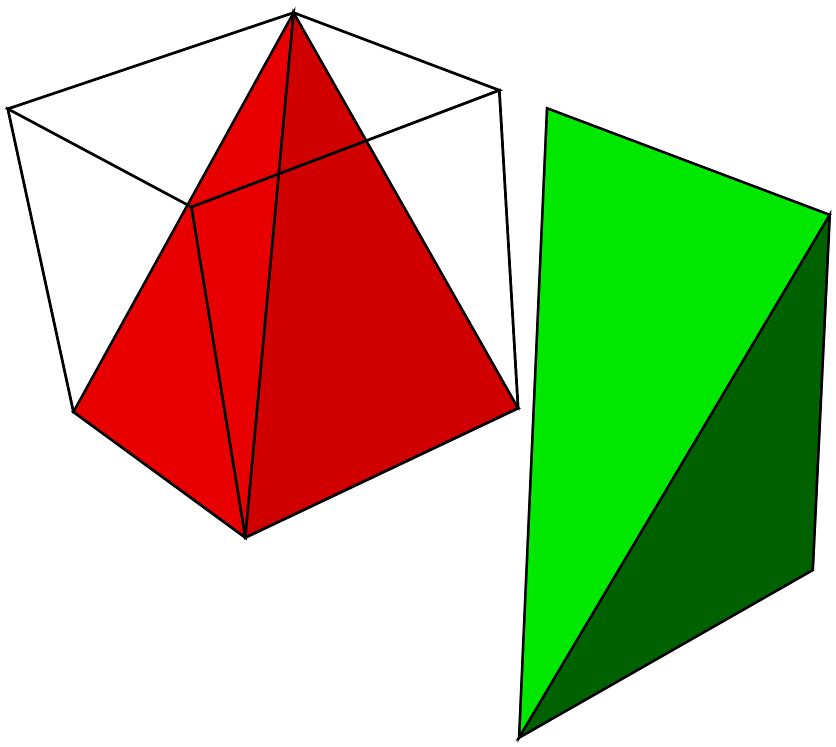
A Ch’ien-tu can be cut up to form a Pied-nao and another different object. Let’s see how this division works.



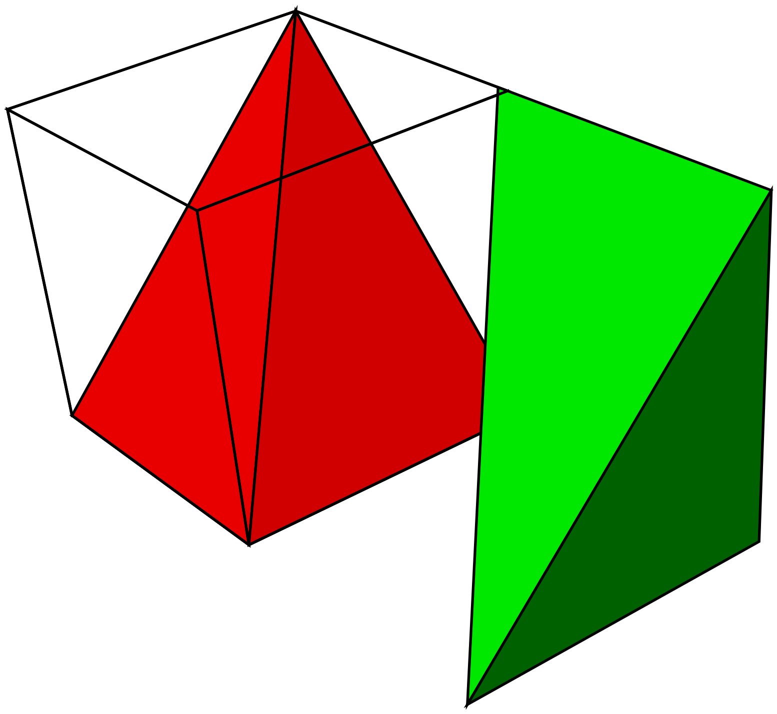
A Ch’ien-tu can be cut up to form a Pied-nao and another different object. Let’s see how this division works.



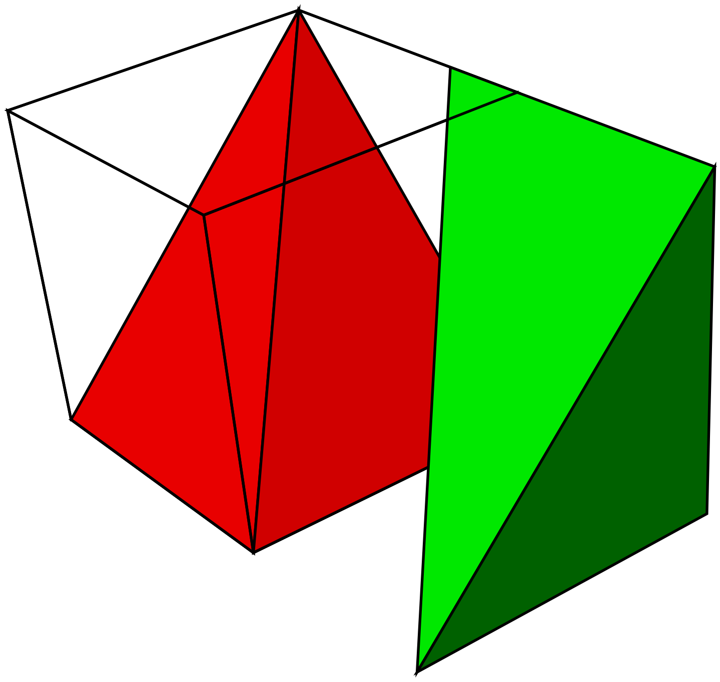
A Ch'ien-tu can be cut up to form a Pied-nao and another different object. Let's see how this division works.



A Ch’ien-tu can be cut up to form a Pied-nao and another different object. Let’s see how this division works.



A Ch’ien-tu can be cut up to form a Pied-nao and another different object. Let’s see how this division works.



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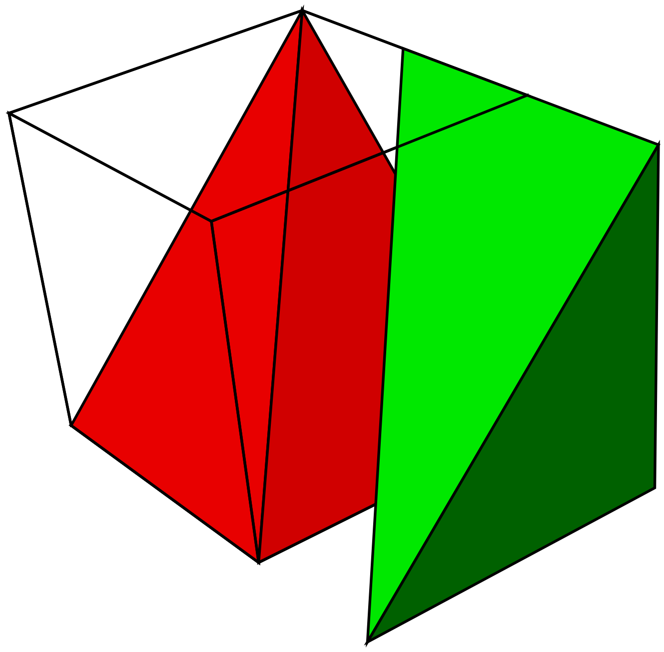

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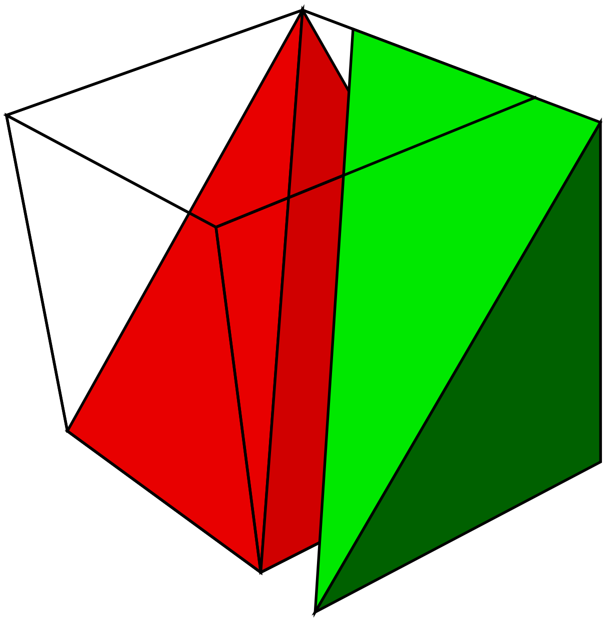

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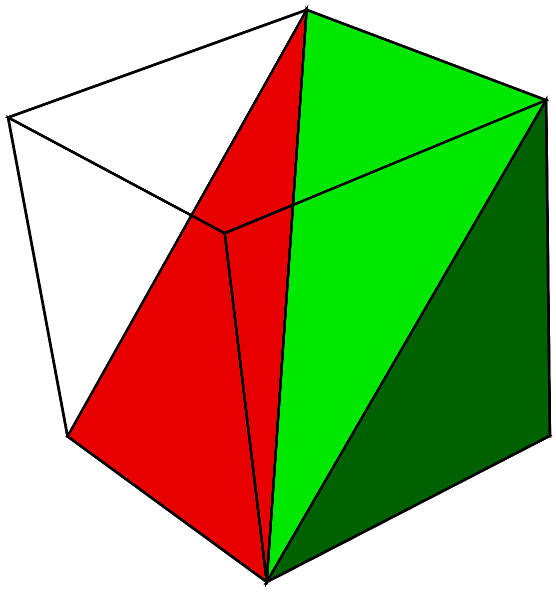
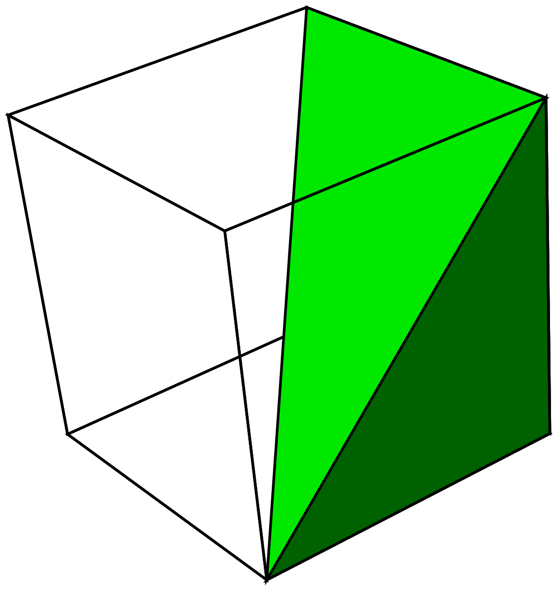


A Ch’ien-tu can be cut up to form a Pied-nao and another different object. Let’s see how this division works.

The Pieh-nao is a tetrahedron with a right triangle base.

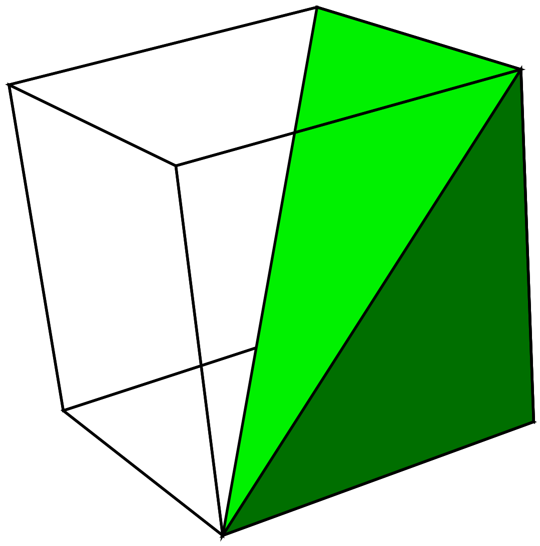
One lateral edge is perpendicular to the base and is not

at the right angled vertex of the tetrahedron base.



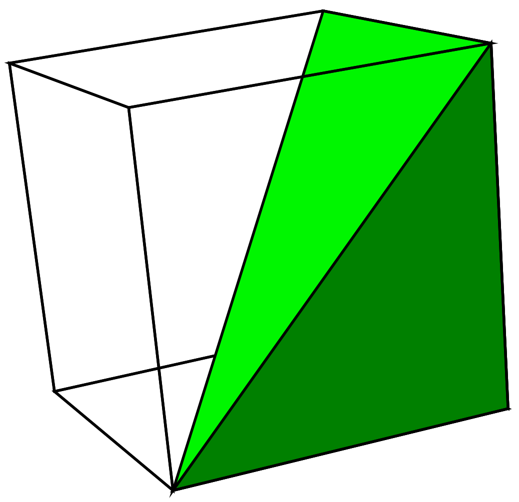
Pieh-nao Volume = 1/6 x Base x Width x Height

Let's have a closer look at the Pieh-nao.



Pieh-nao Volume = 1/6 x Base x Width x Height

Let’s have a closer look at the Pieh-nao.



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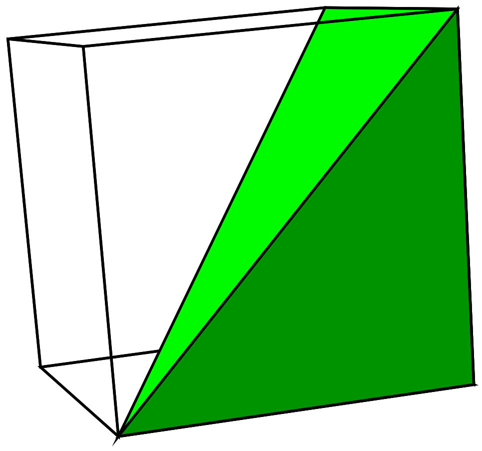

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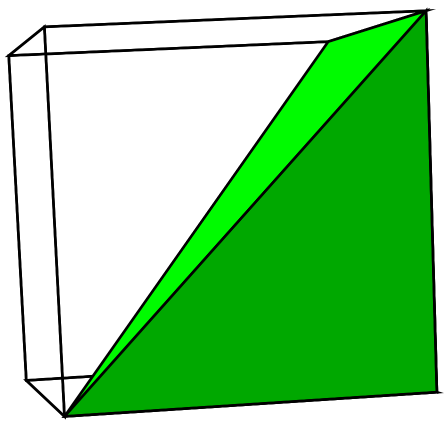

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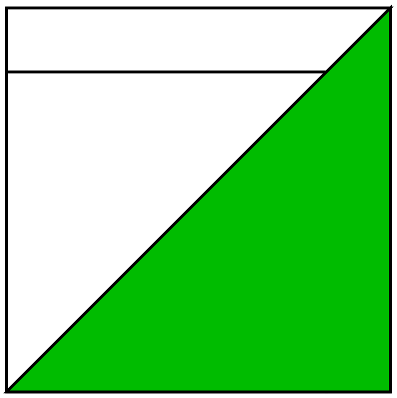

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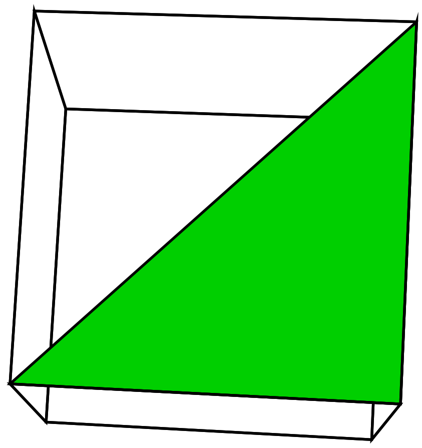

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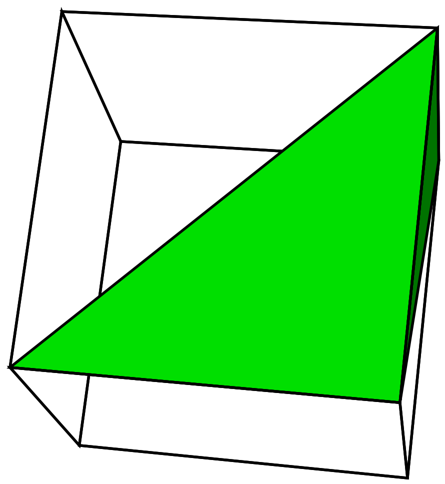

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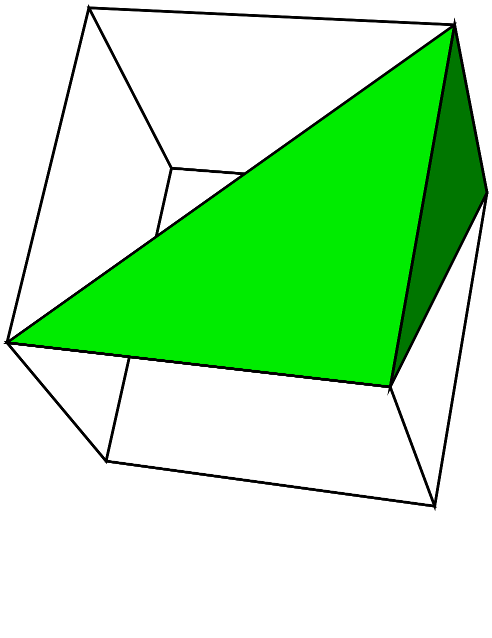

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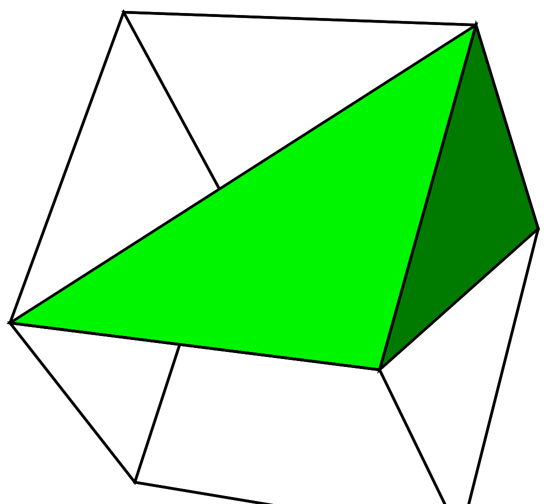

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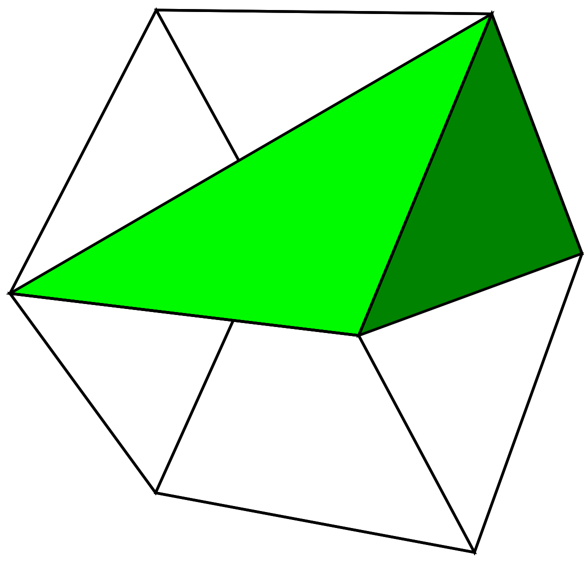

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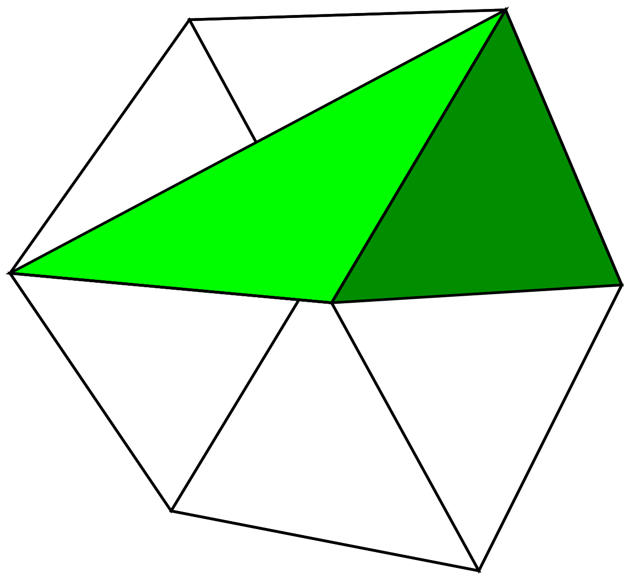

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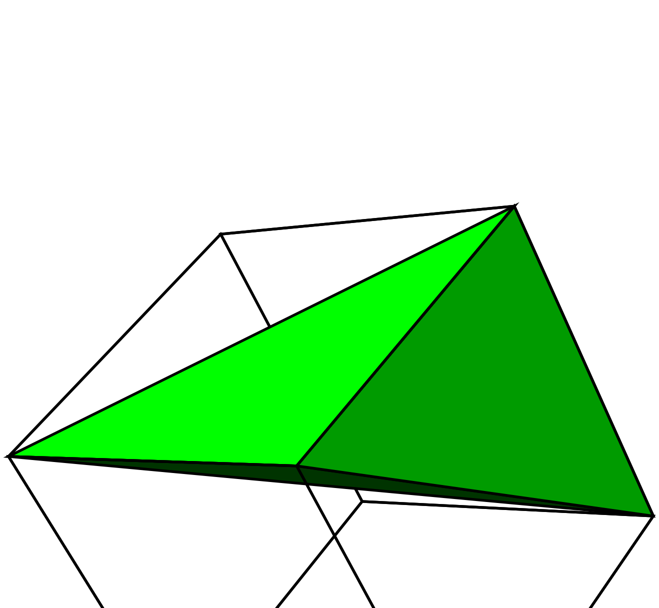

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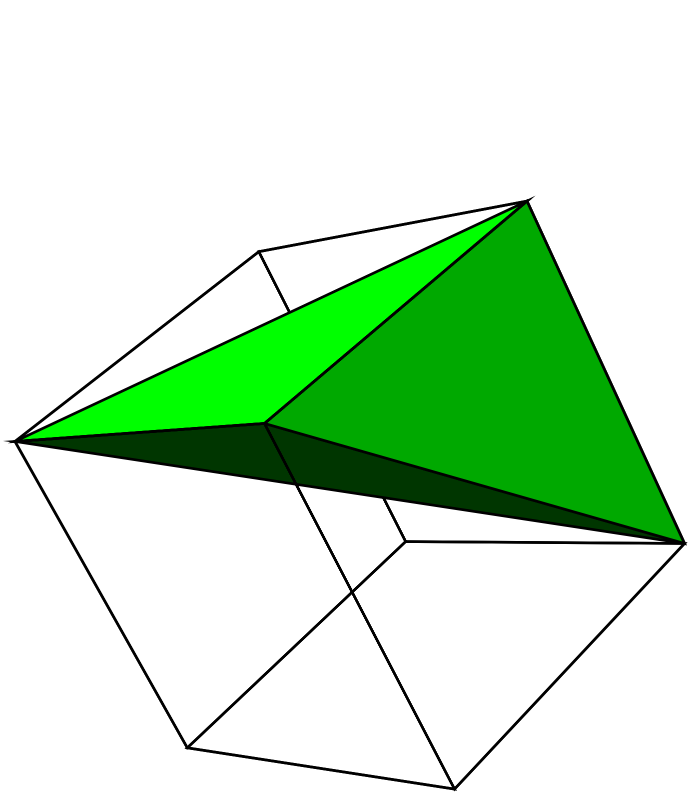

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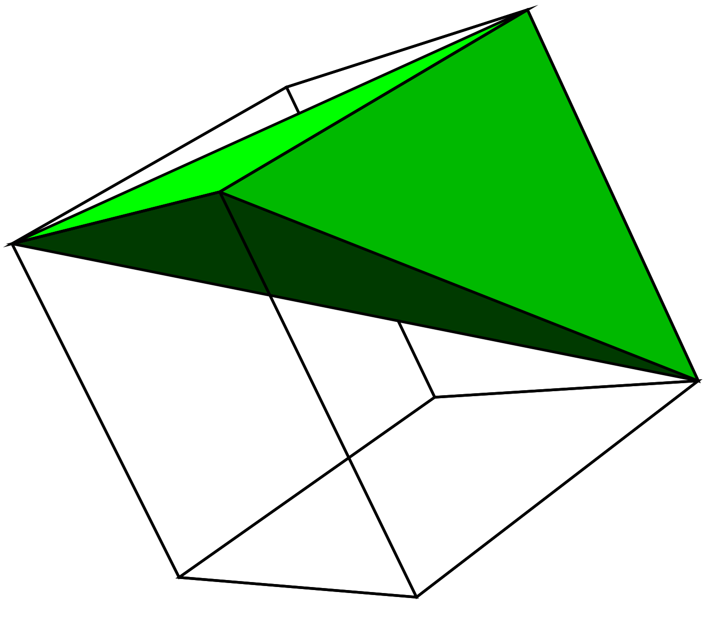

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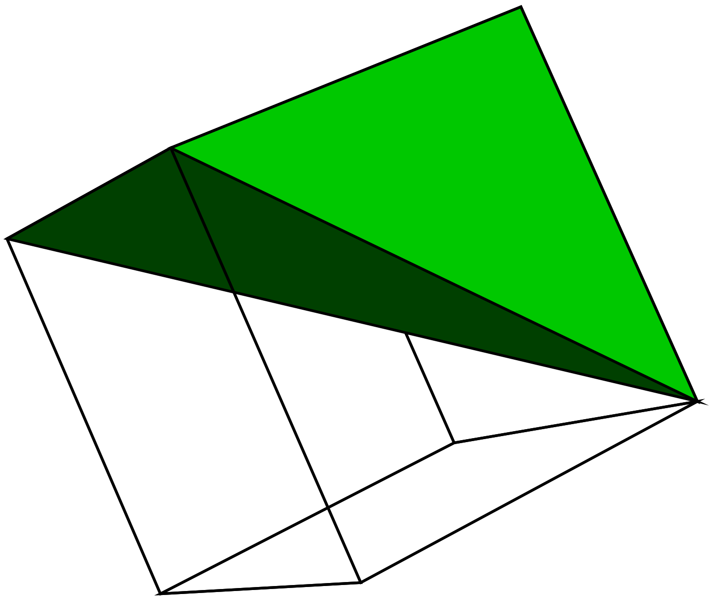

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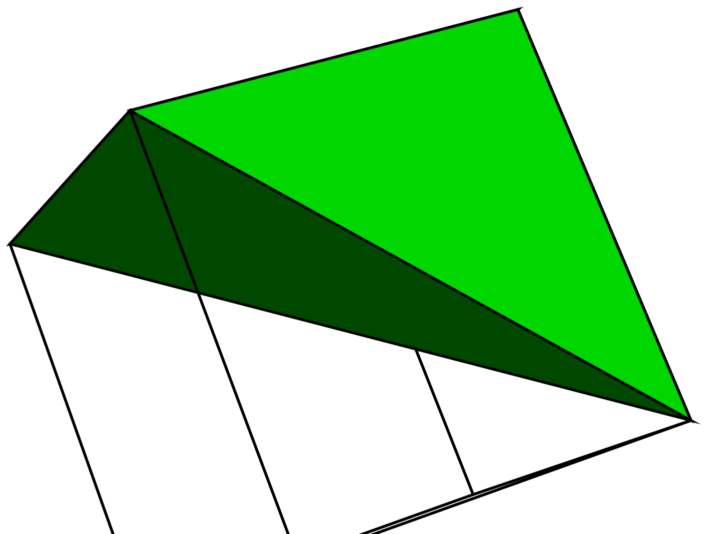

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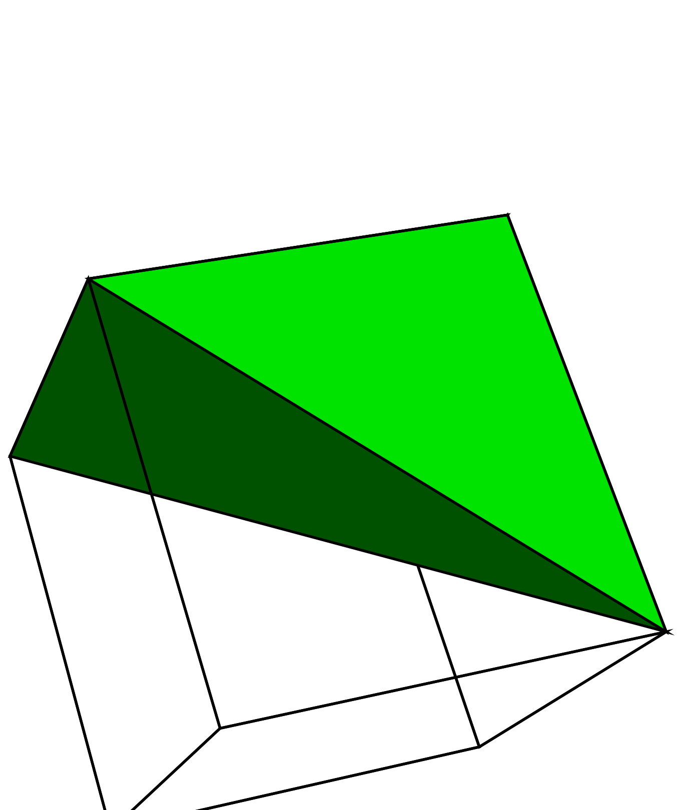

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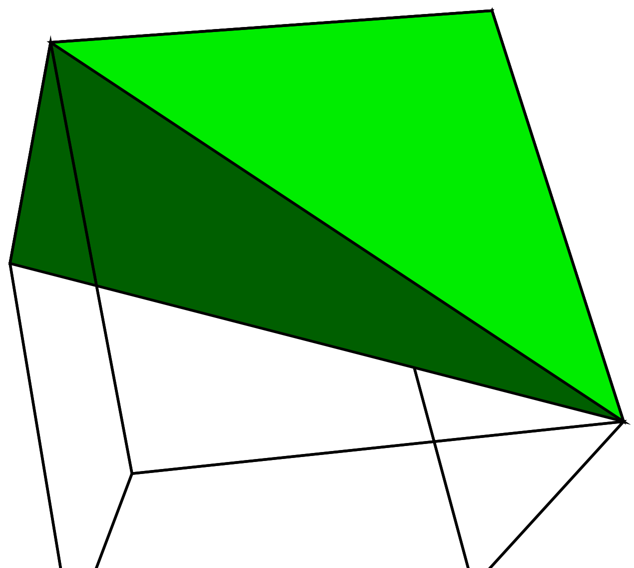

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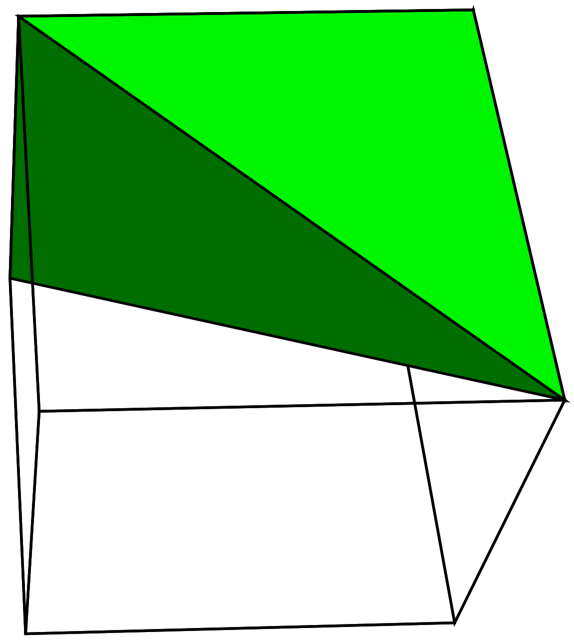

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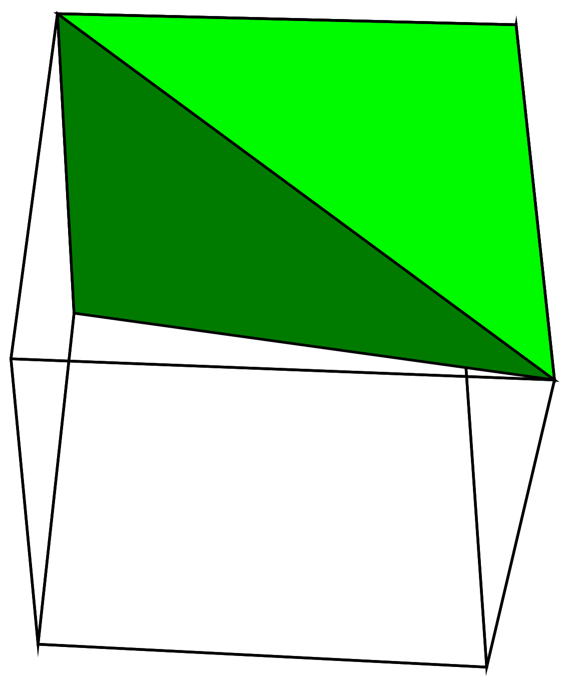

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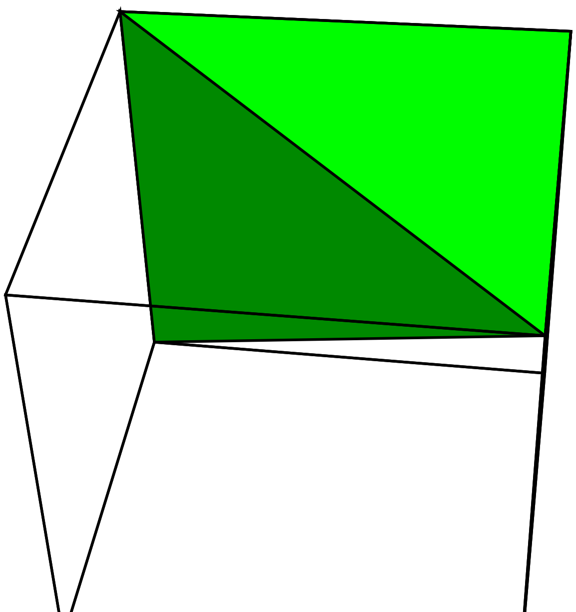

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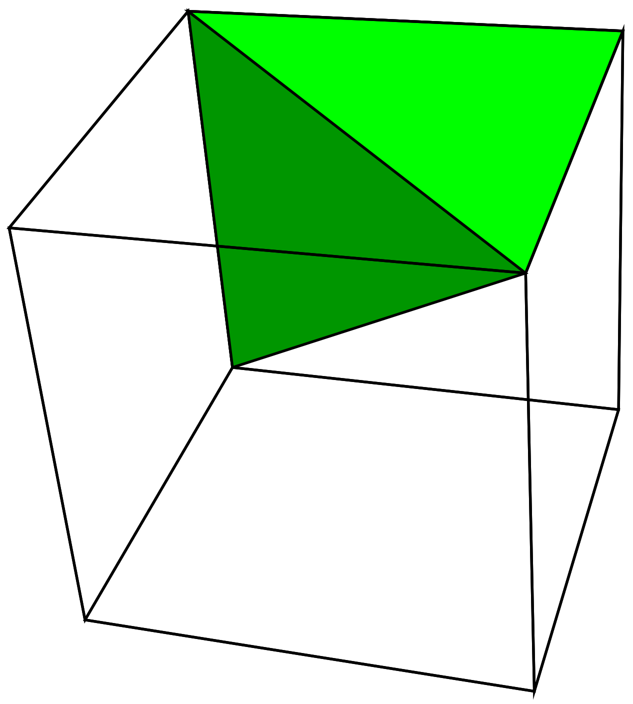

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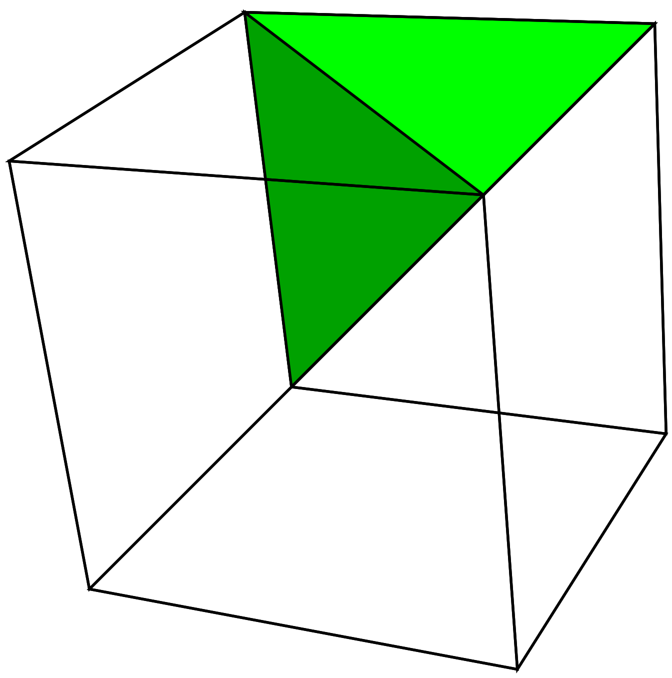

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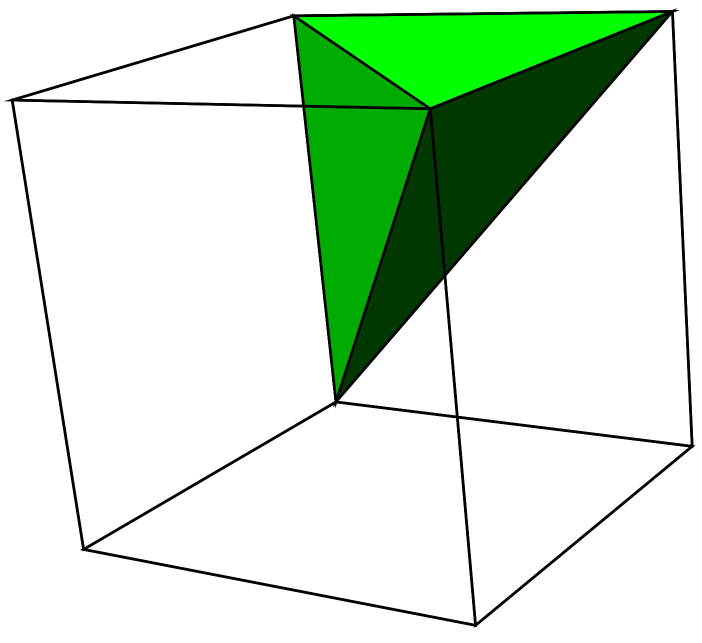

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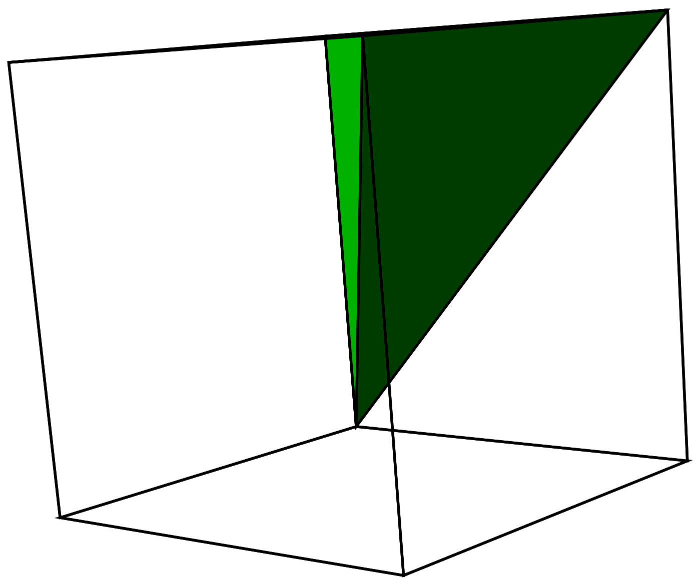

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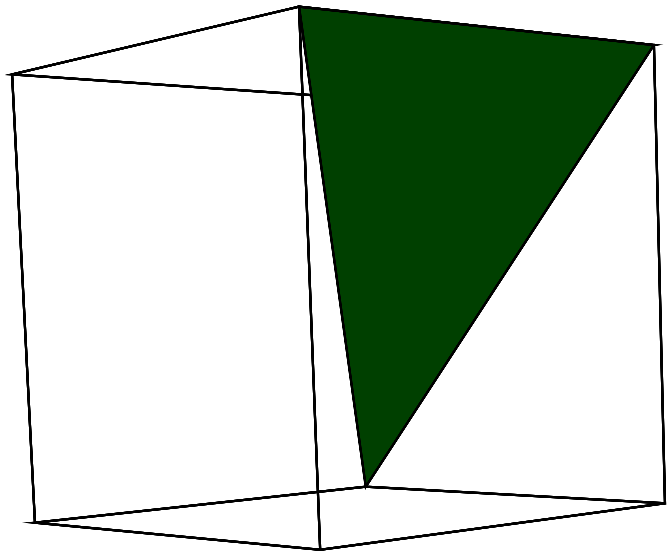

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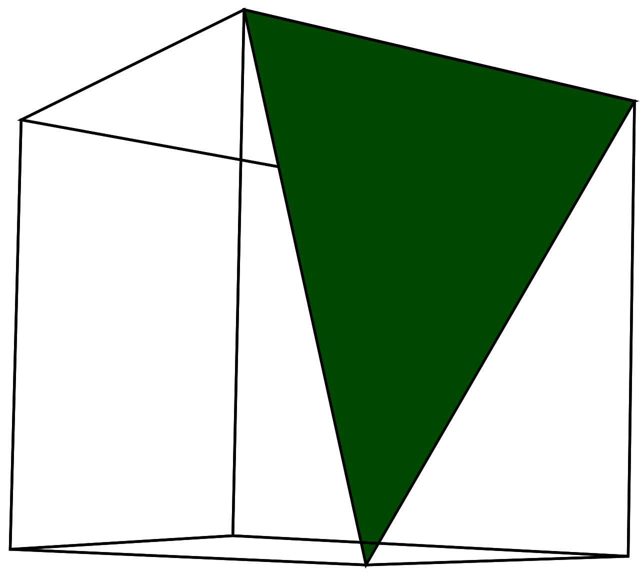

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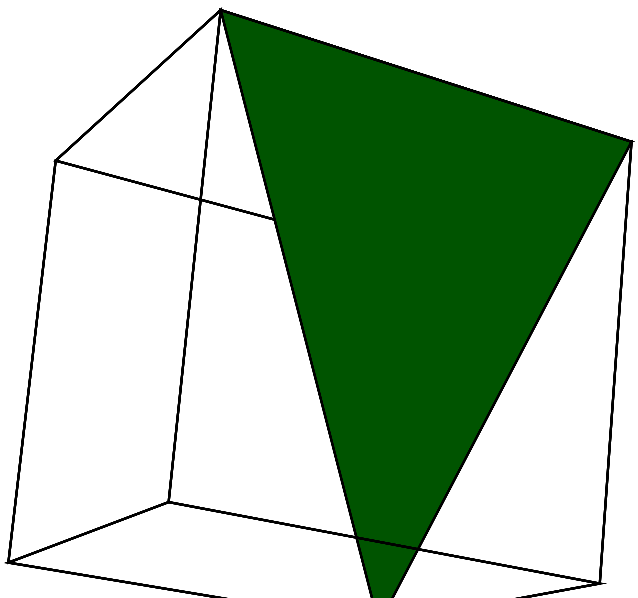

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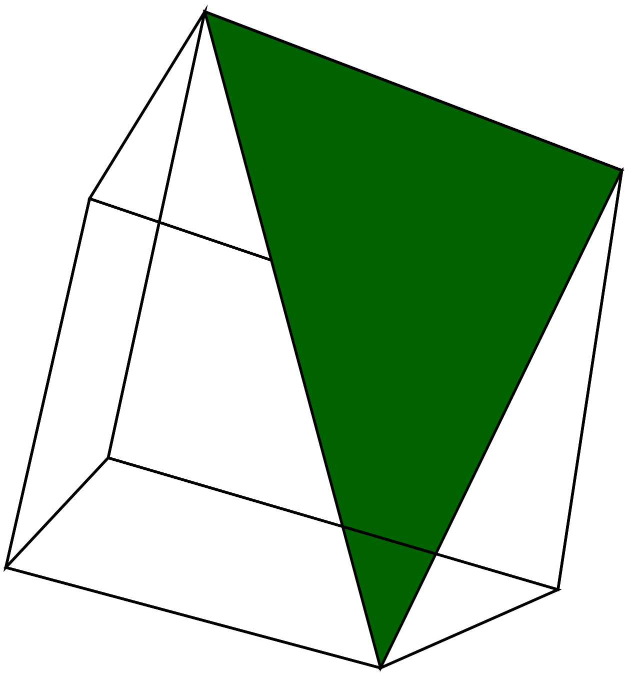

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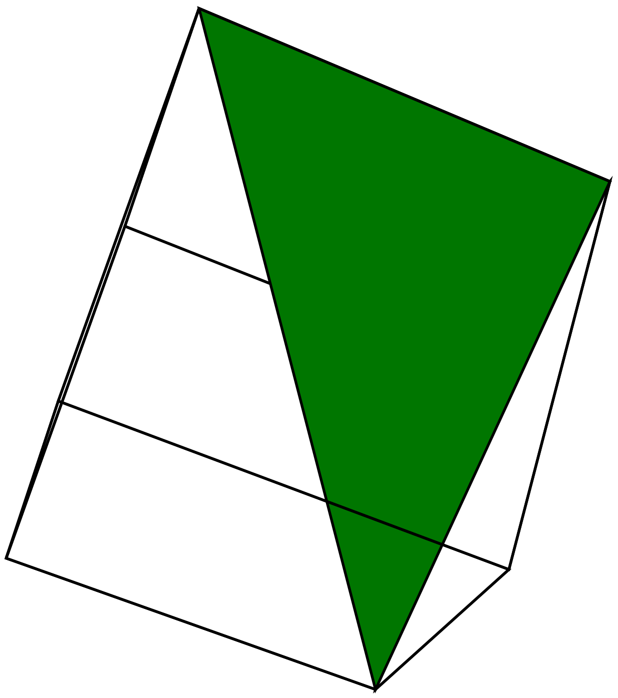

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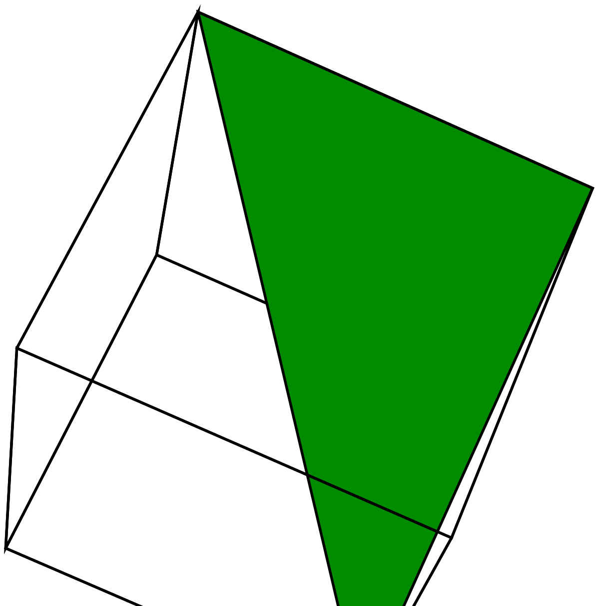

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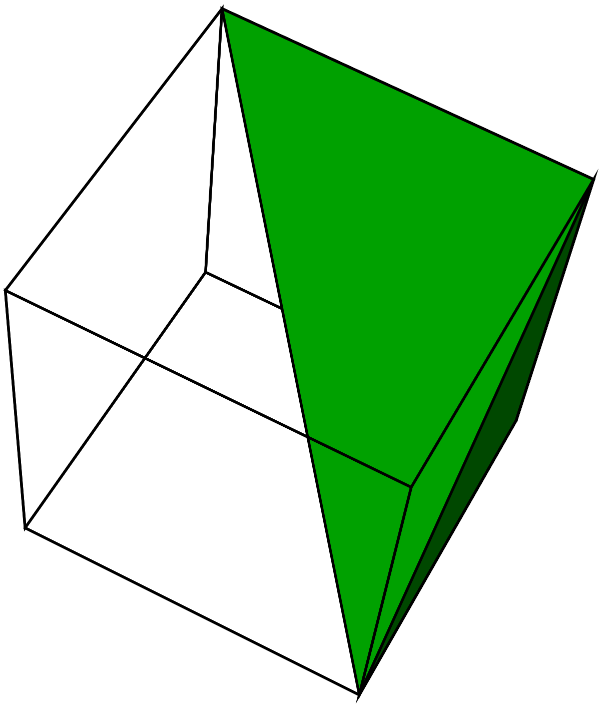

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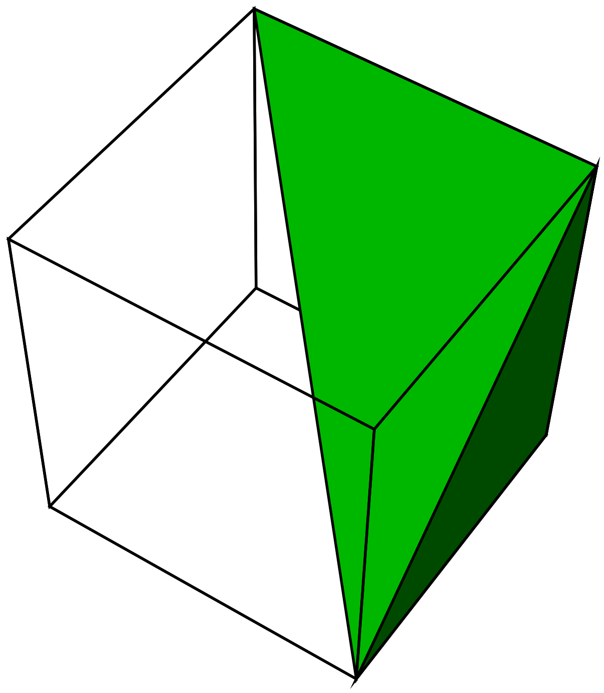

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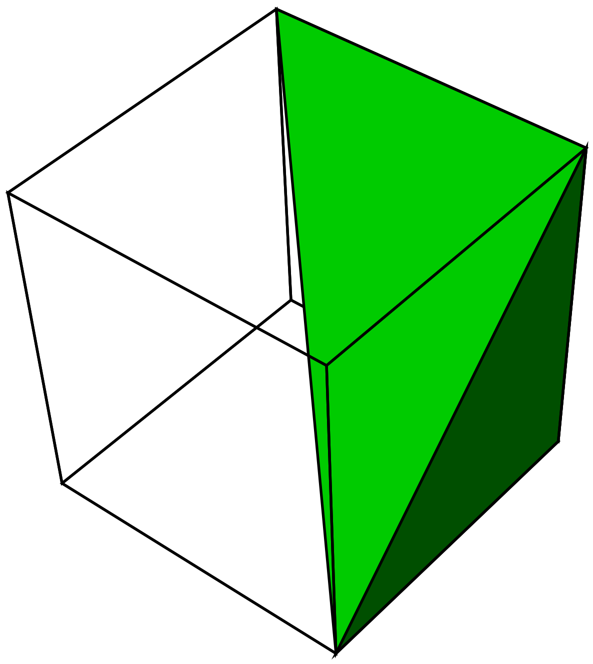

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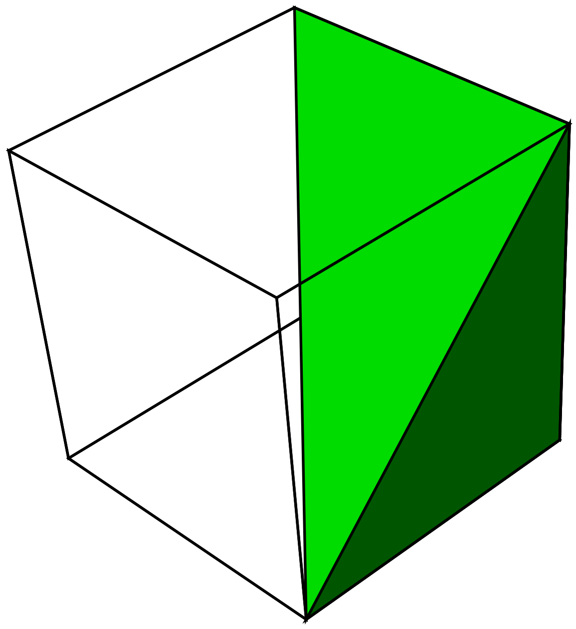

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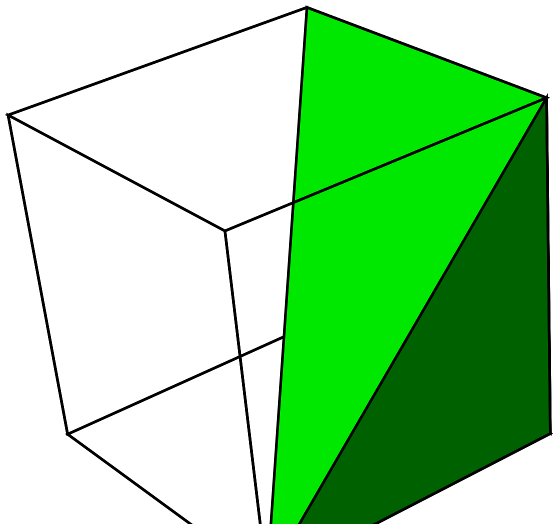
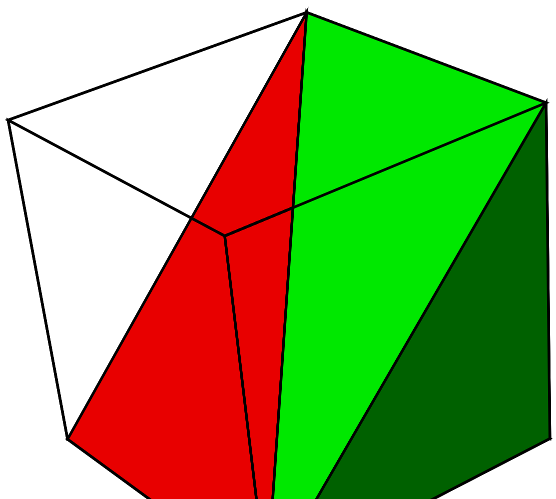


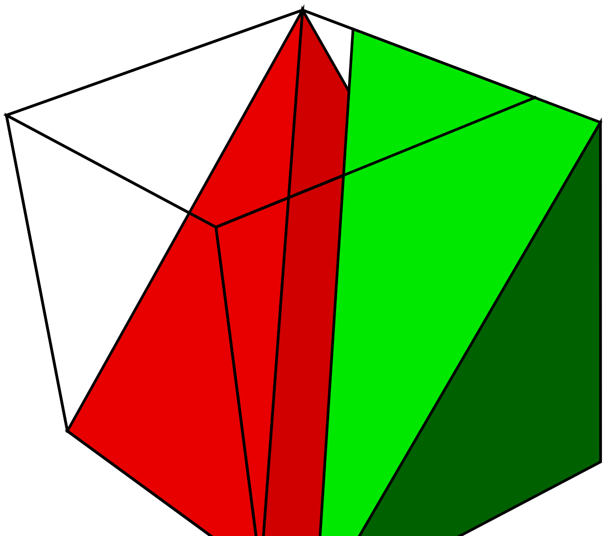
Pieh-nao Volume = 1/6 x Base x Width x Height

Let’s have a closer look at the Pieh-nao.

The second object created by cutting the Ch’ien-tu into two pieces is seen on the left. This solid is called a Yang-ma, which translates into Male Horse. The volume of a Yang-ma was found, by Liu Hui, to equal one third of the volume of the unit cube from which it was created. We will see how Liu Hui proved this relationship in a moment.



A Yang-ma and a Pied-nao can be combined to make a Ch’ien-tu. Let’s see how this combination works.



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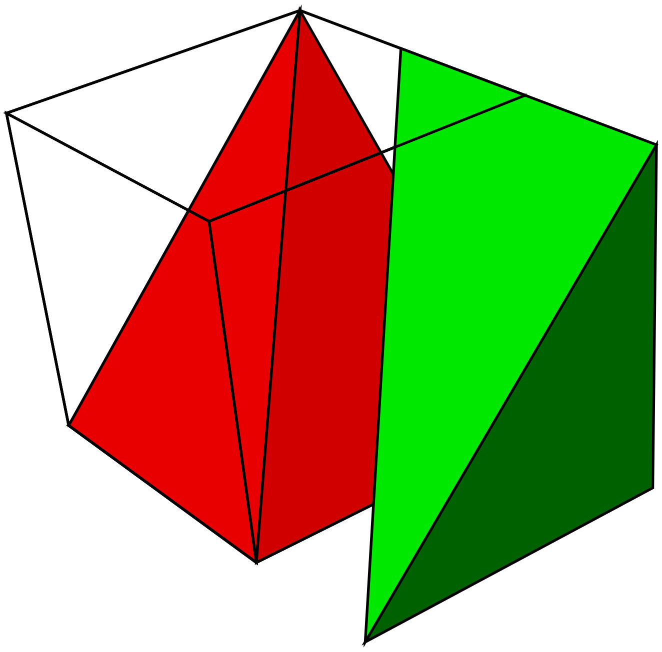

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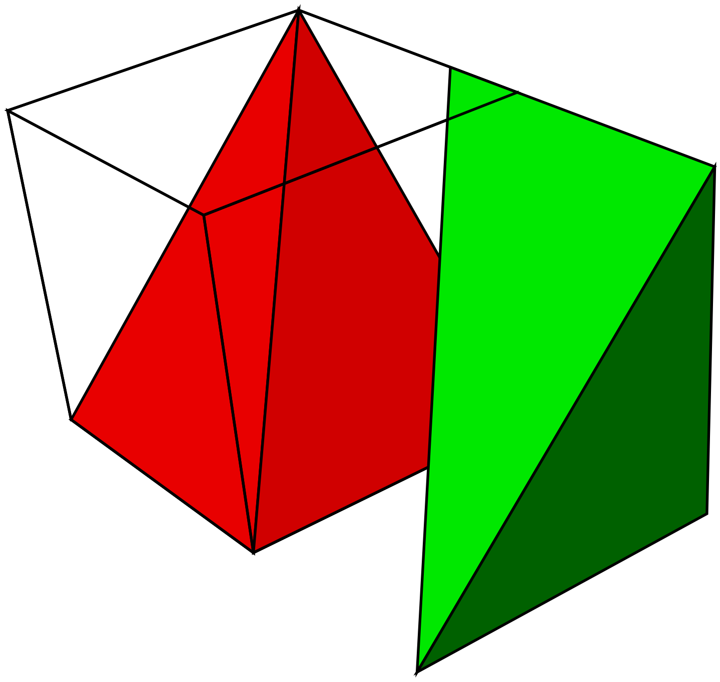

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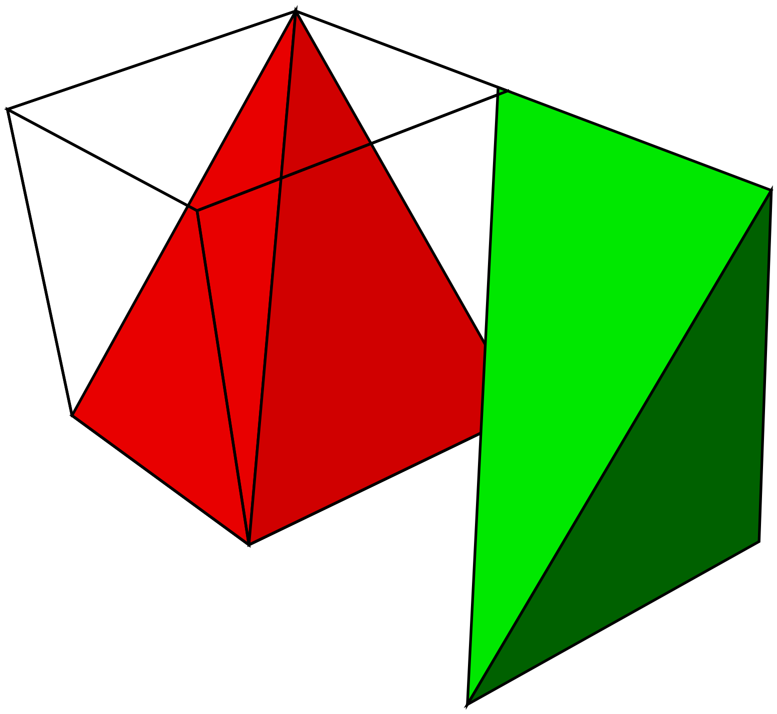

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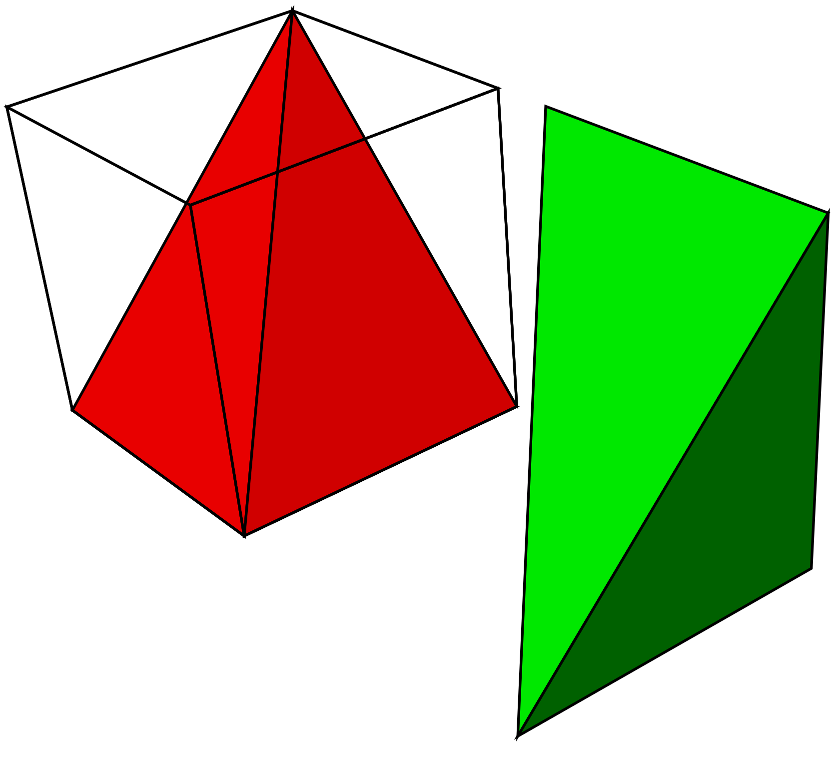

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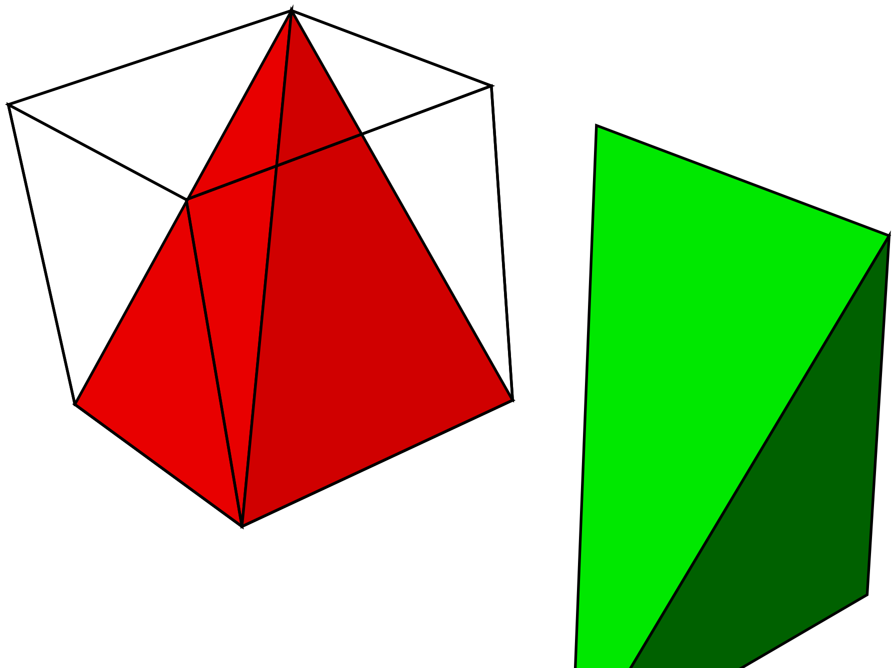
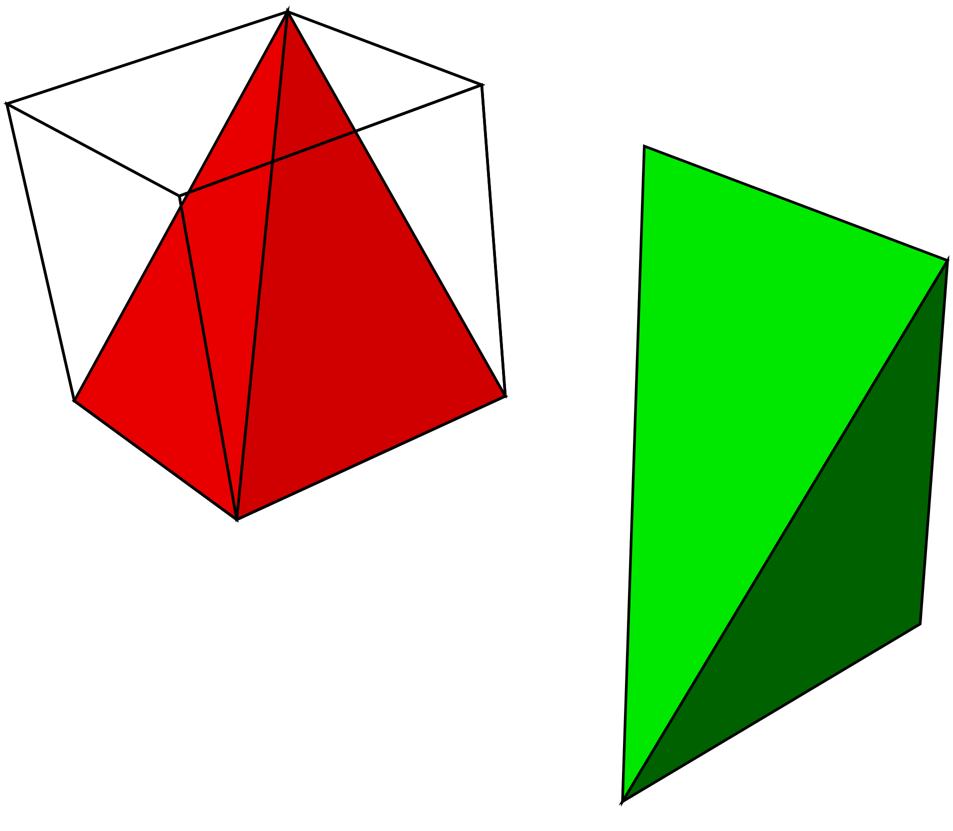
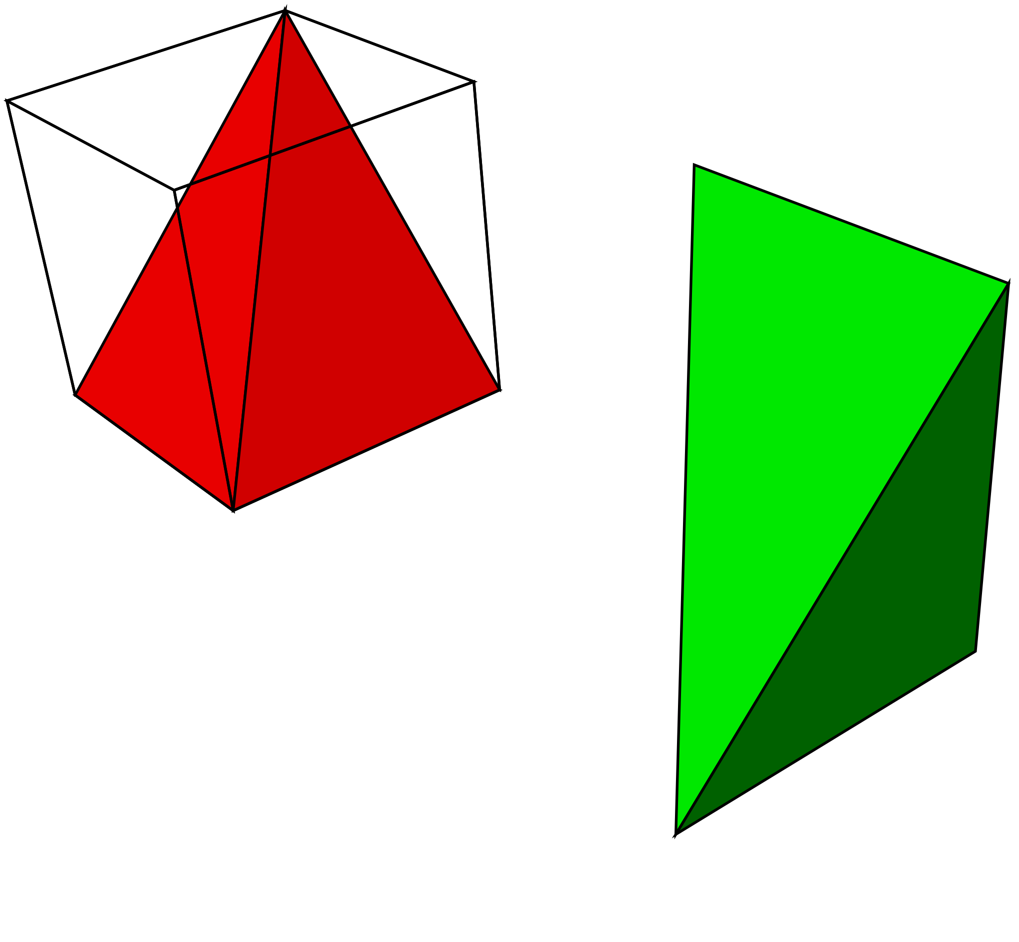


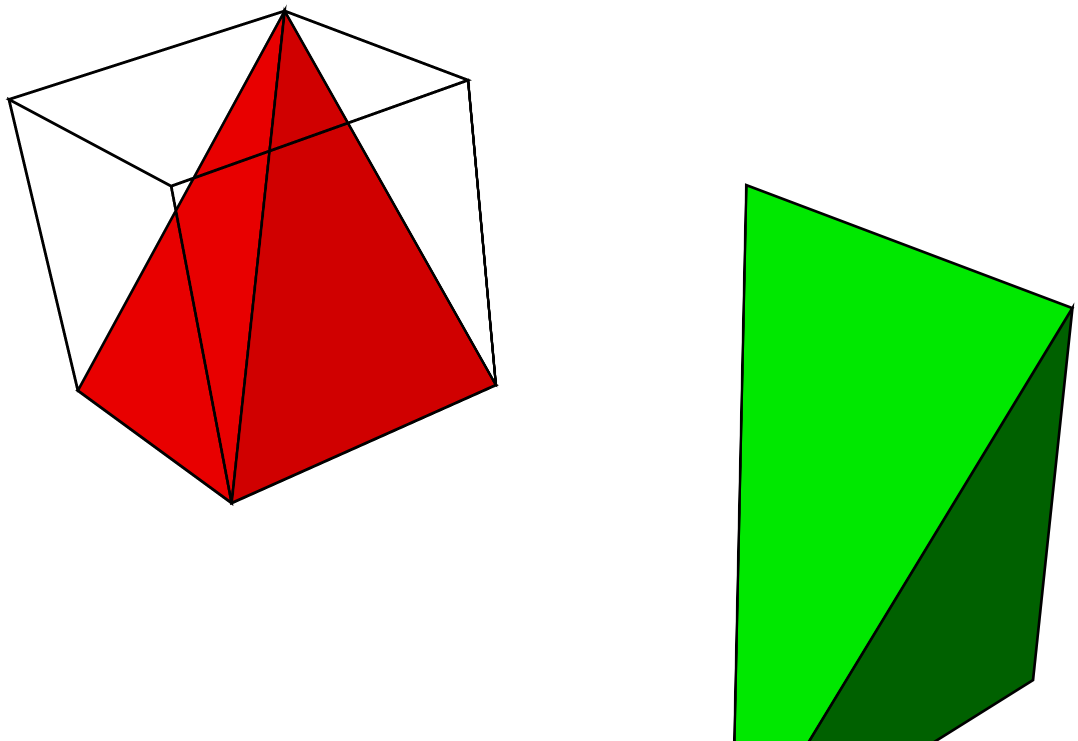
A Yang-ma and a Pied-nao can be combined to make a Ch’ien-tu. Let’s see how this combination works.



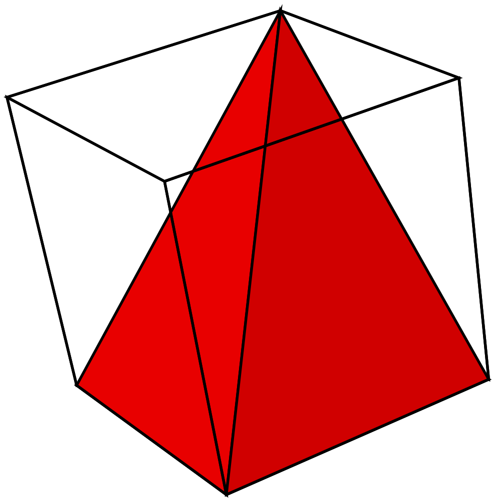
A Yang-ma and a Pied-nao can be combined to make a Ch’ien-tu. Let’s see how this combination works.

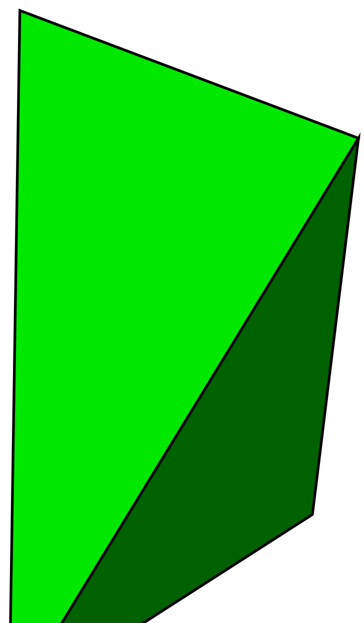


A Yang-ma and a Pied-nao can be combined to make a Ch'ien-tu. Let's see how this combination works.

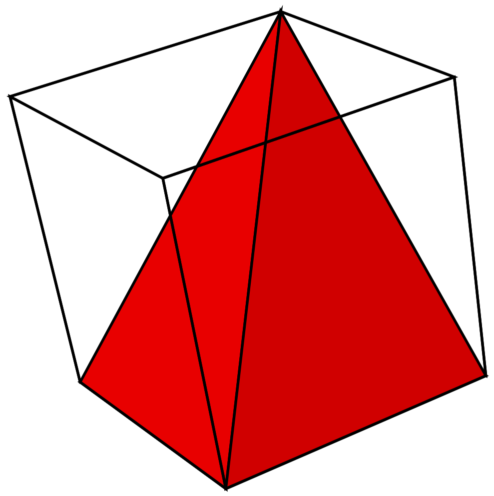


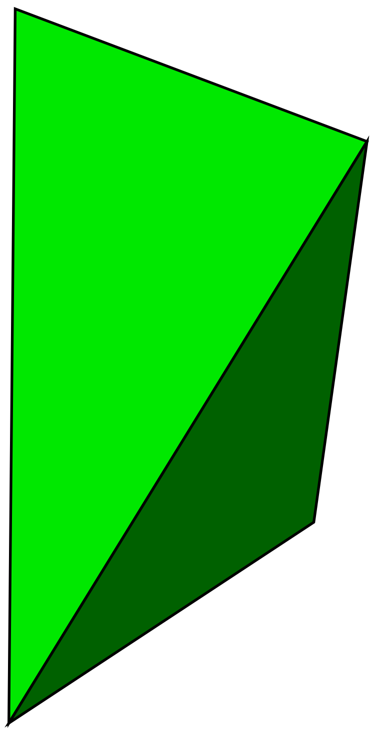
A Yang-ma and a Pied-nao can be combined to make a Ch’ien-tu. Let’s see how this combination works.



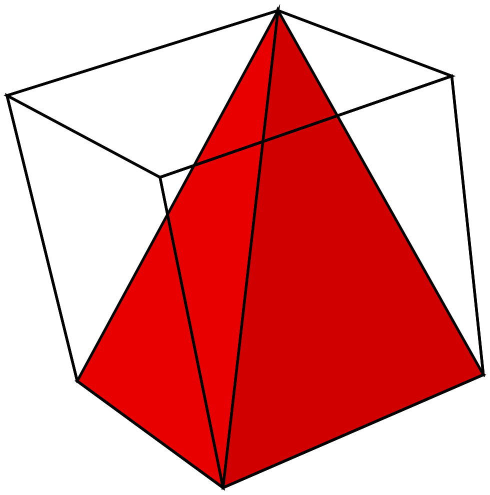


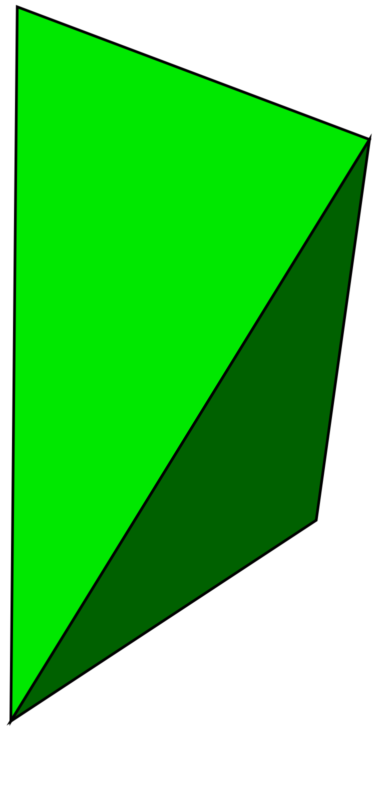
A Yang-ma and a Pied-nao can be combined to make a Ch’ien-tu. Let’s see how this combination works.



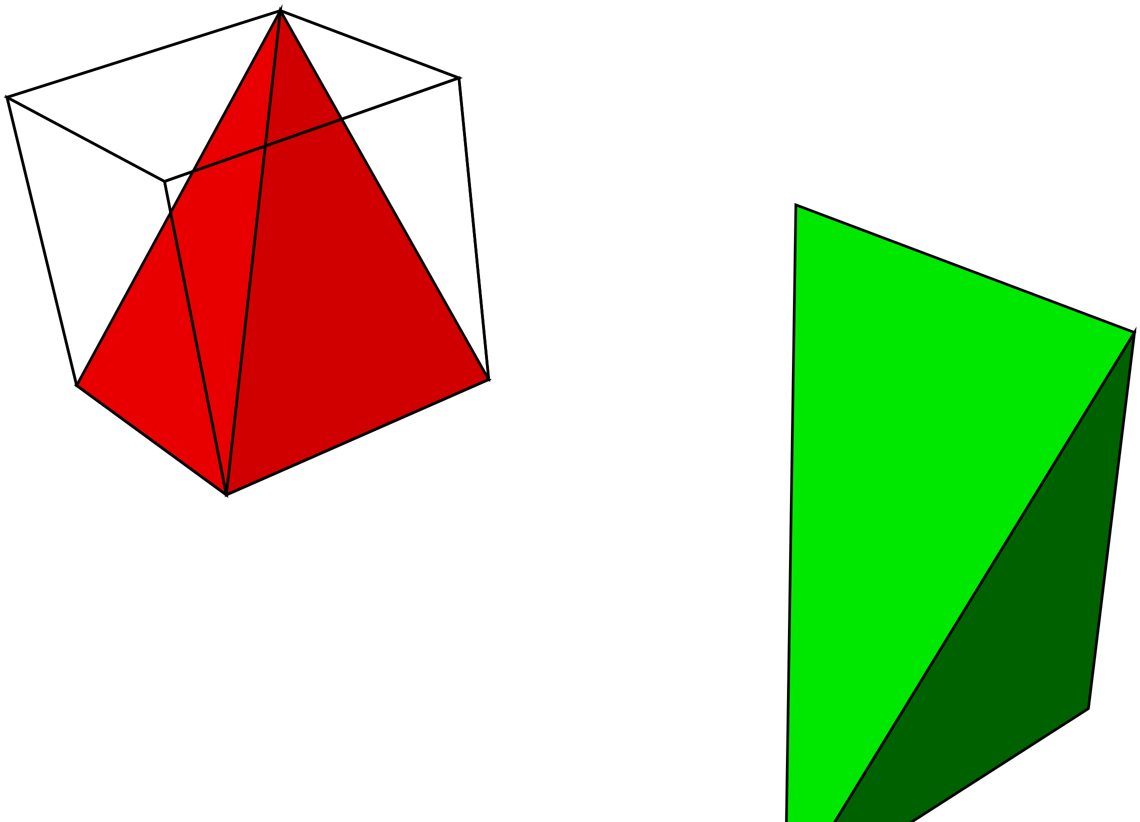


A Yang-ma and a Pied-nao can be combined to make a Ch’ien-tu. Let’s see how this combination works.

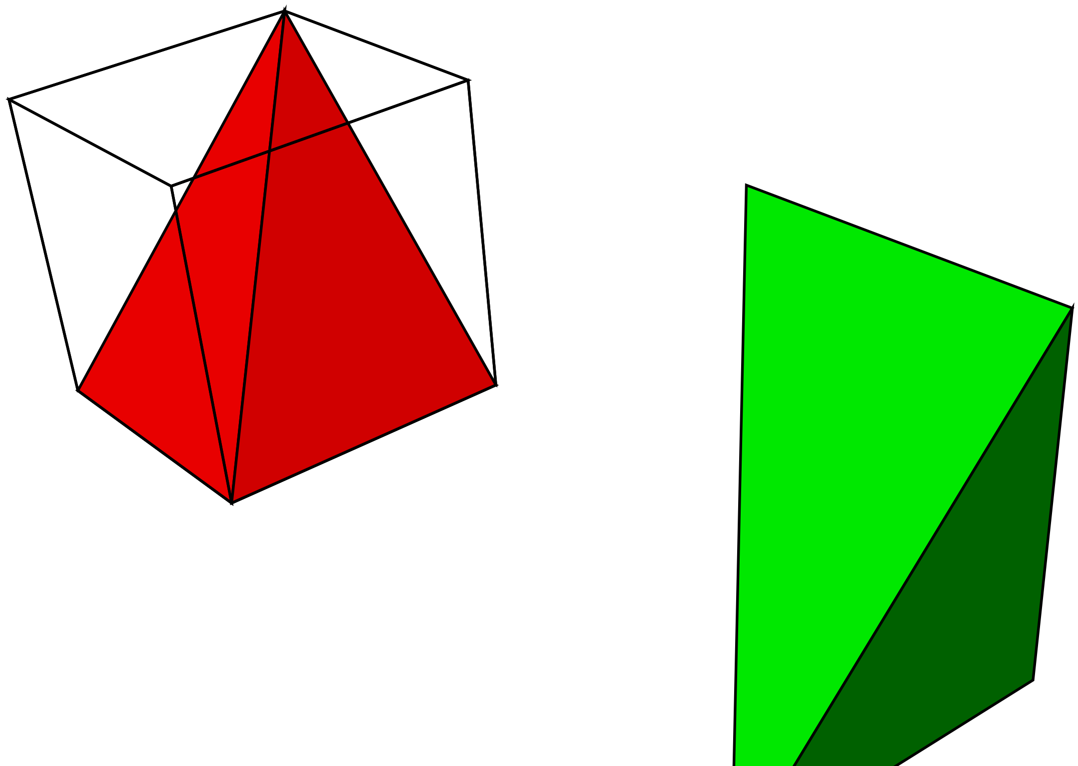




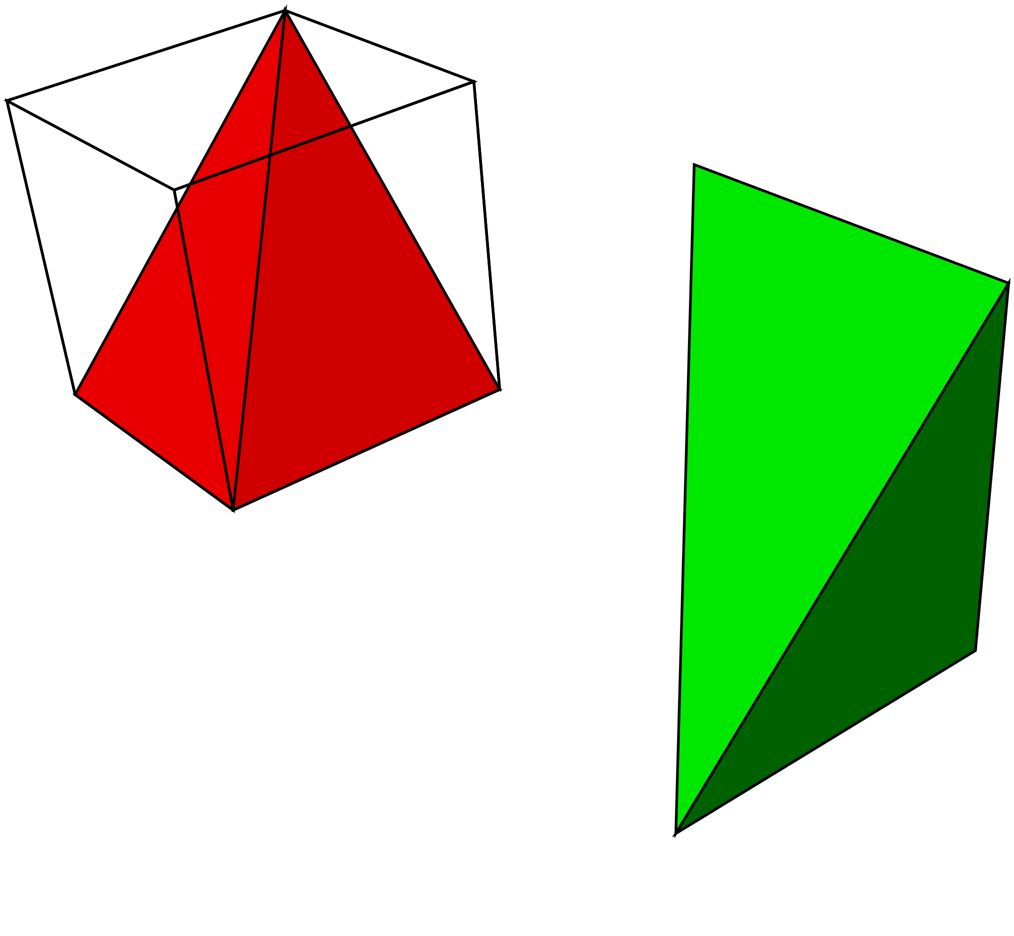
A Yang-ma and a Pied-nao can be combined to make a Ch’ien-tu. Let’s see how this combination works.



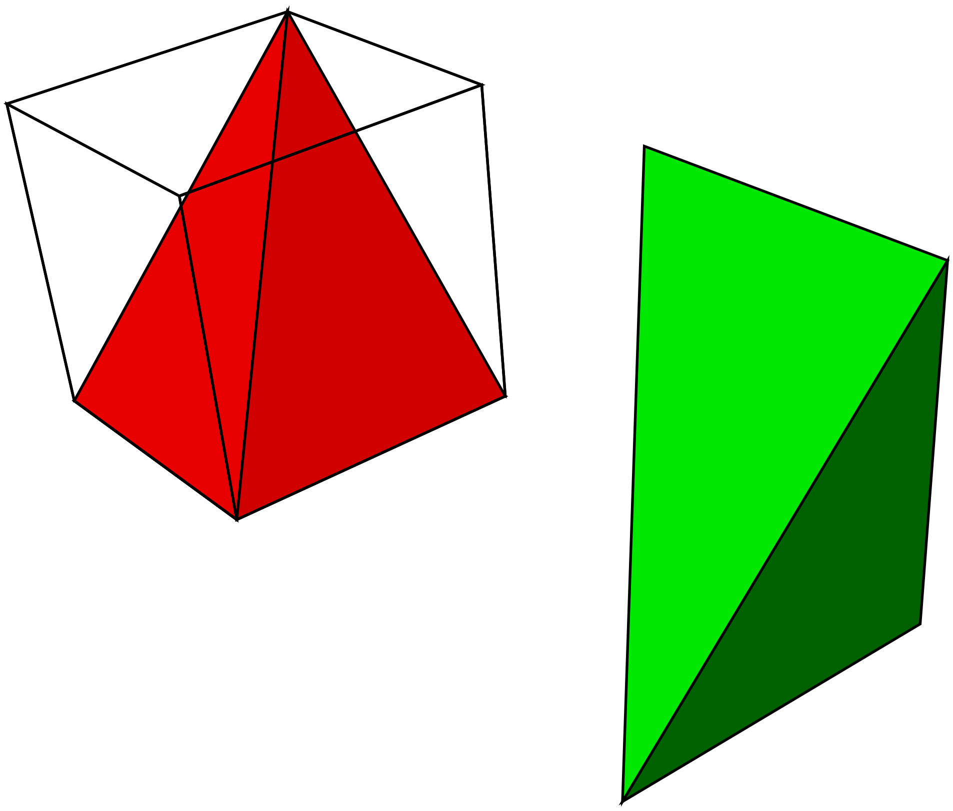
A Yang-ma and a Pied-nao can be combined to make a Ch’ien-tu. Let’s see how this combination works.



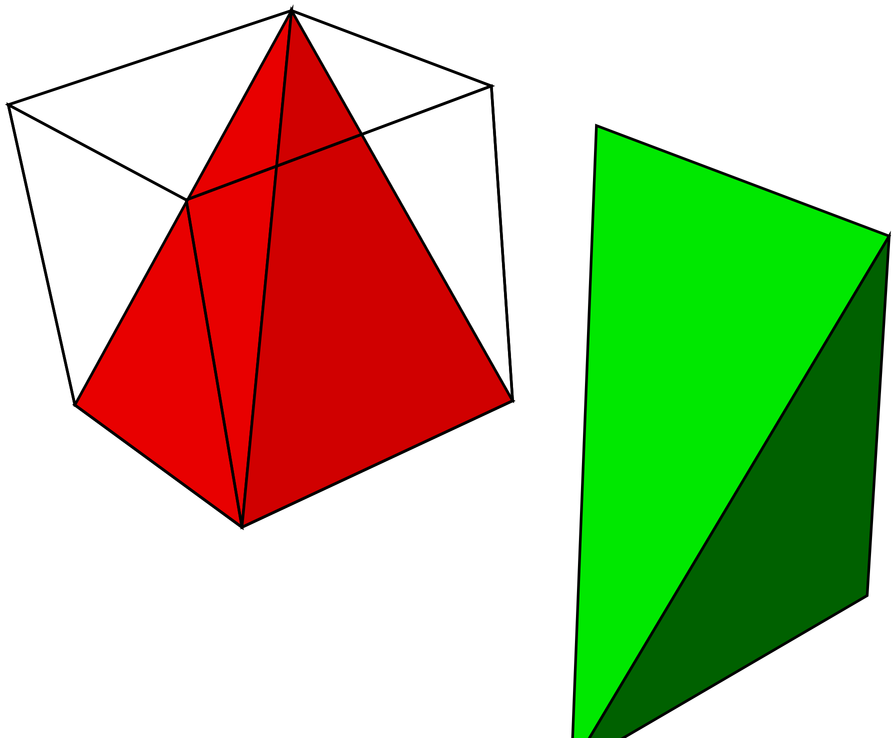
A Yang-ma and a Pied-nao can be combined to make a Ch’ien-tu. Let’s see how this combination works.



A Yang-ma and a Pied-nao can be combined to make a Ch’ien-tu. Let’s see how this combination works.



A Yang-ma and a Pied-nao can be combined to make a Ch’ien-tu. Let’s see how this combination works.



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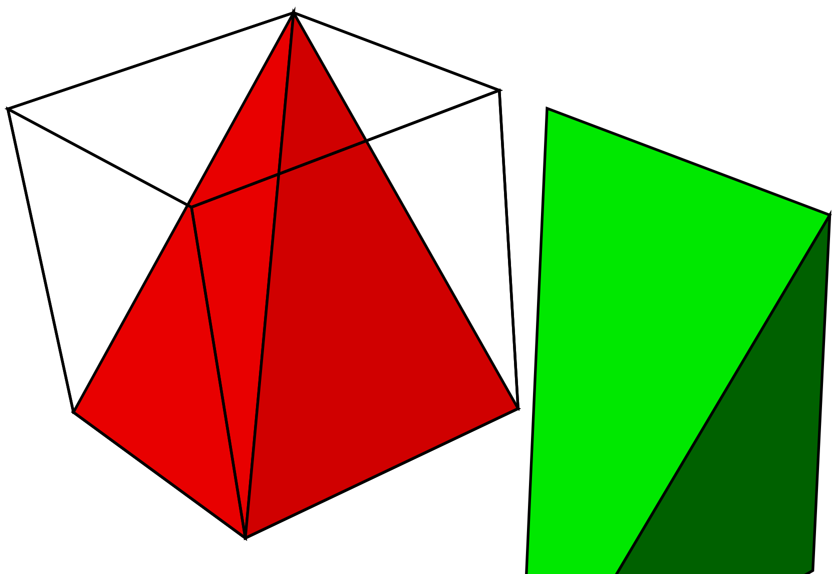
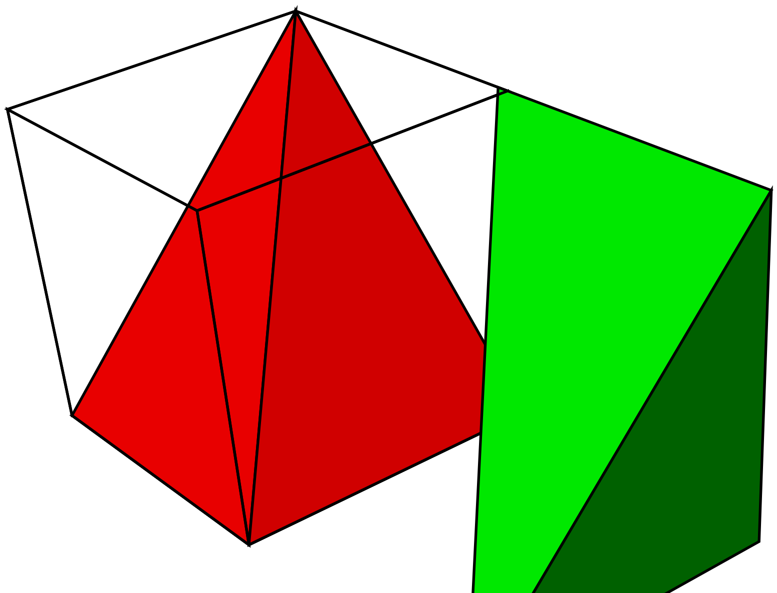
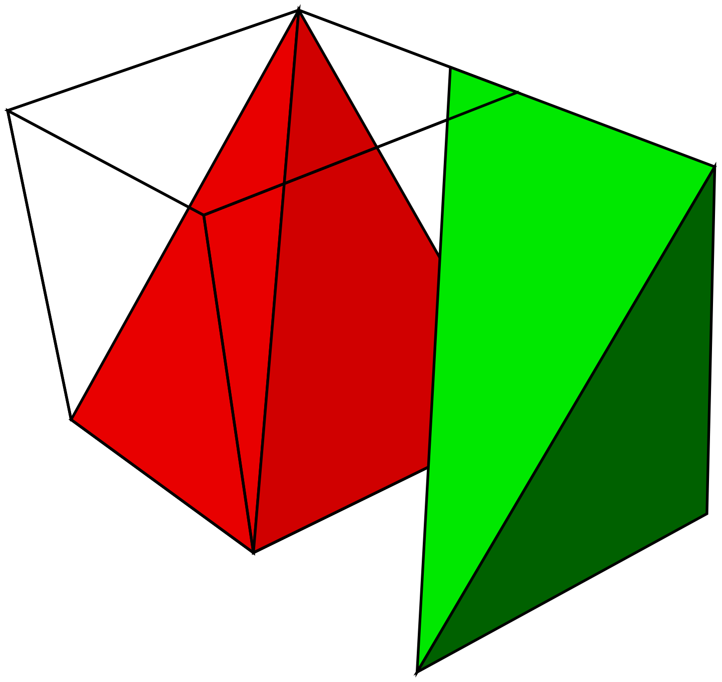


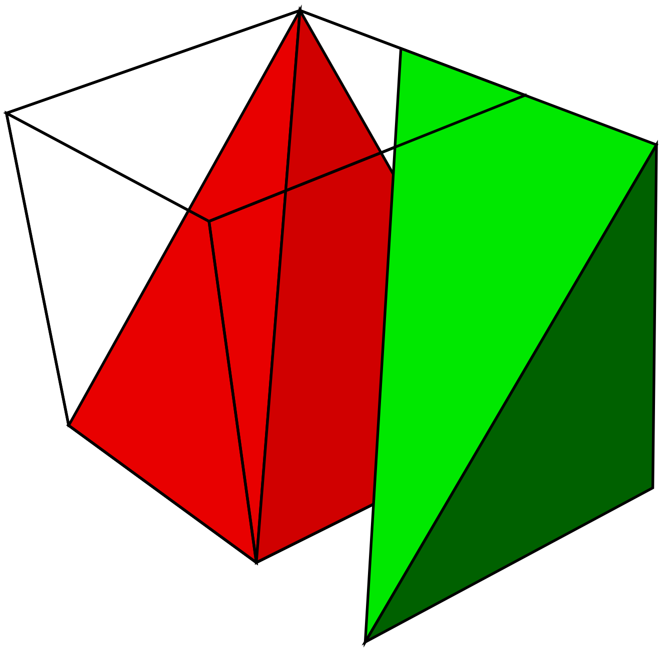
A Yang-ma and a Pied-nao can be combined to make a Ch’ien-tu. Let’s see how this combination works.



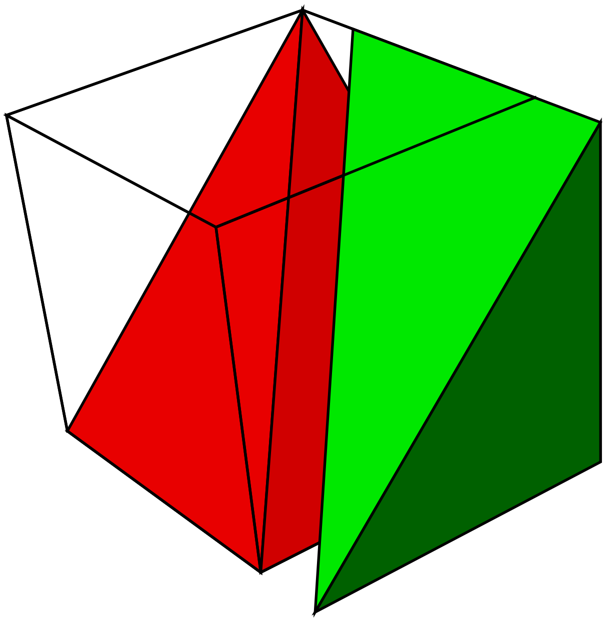
A Yang-ma and a Pied-nao can be combined to make a Ch’ien-tu. Let’s see how this combination works.



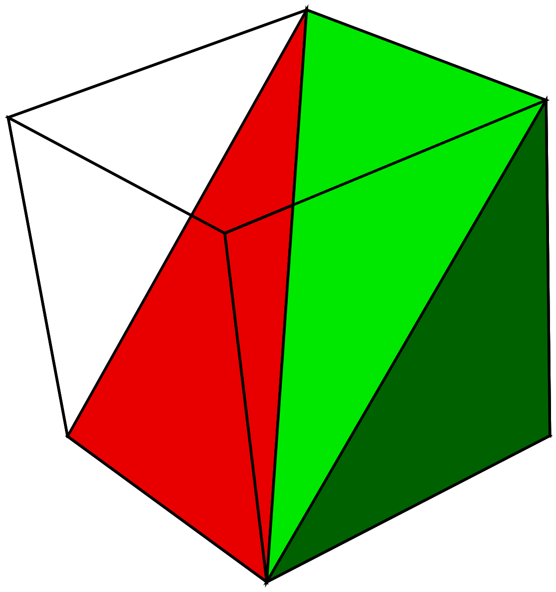
A Yang-ma and a Pied-nao can be combined to make a Ch’ien-tu. Let’s see how this combination works.



A Yang-ma and a Pied-nao can be combined to make a Ch’ien-tu. Let’s see how this combination works.

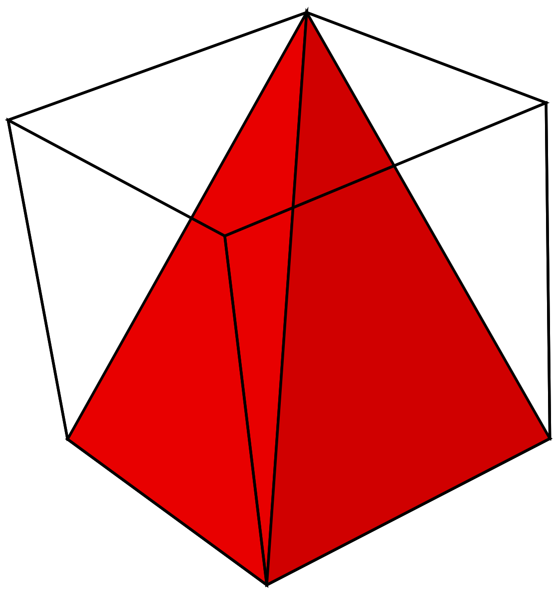


A Yang-ma and a Pied-nao can be combined to make a Ch'ien-tu. Let's see how this combination works.



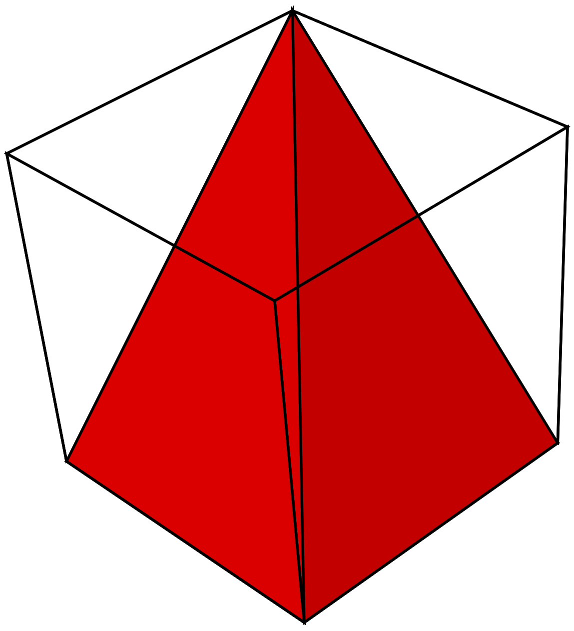
A Yang-ma and a Pied-nao can be combined to make a Ch’ien-tu. Let’s see how this combination works.

The Yang-ma is the final of the four basic objects used by Liu Hui. It is a pyramid with a square base and one lateral edge perpendicular to the base.



Yang-ma Volume = 1/3 x Base x Width x Height

Let's have a closer look at this object.



Yang-ma Volume = 1/3 x Base x Width x Height

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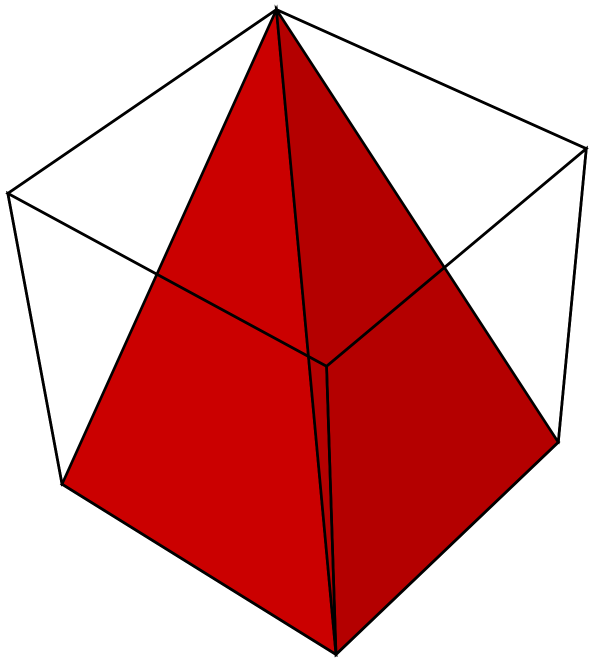

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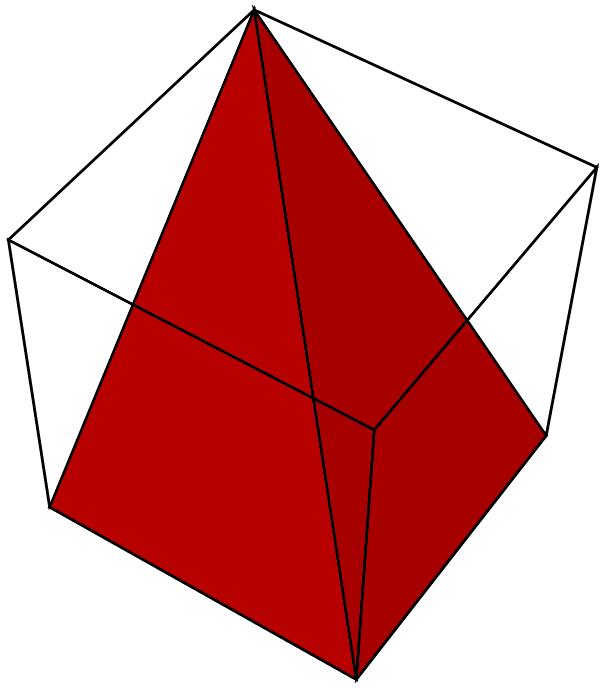

Yang-ma Volume = 1/3 x Base x Width x Height

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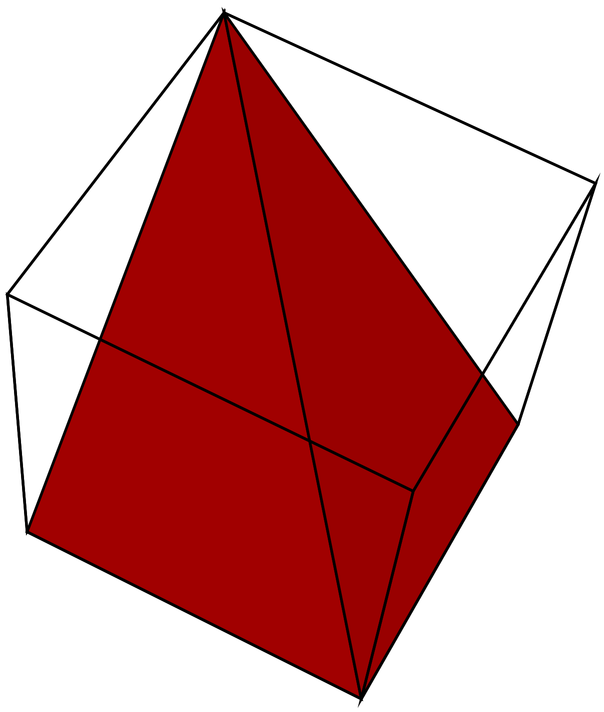

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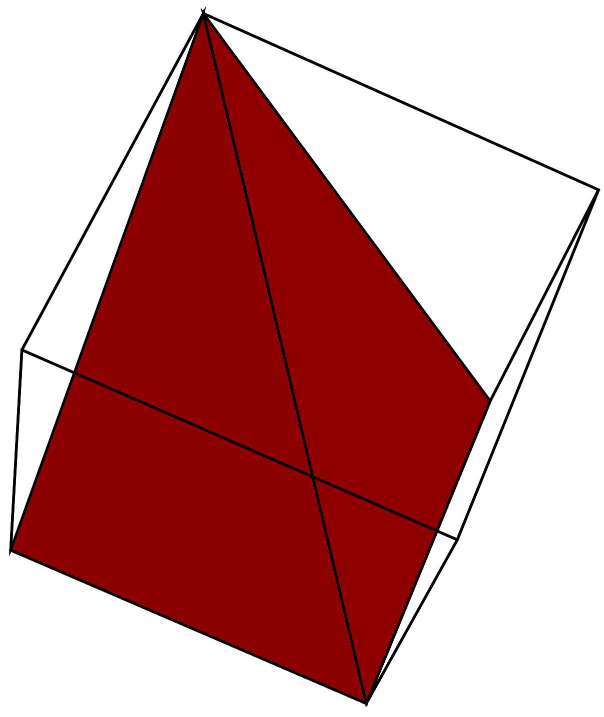

Yang-ma Volume = 1/3 x Base x Width x Height

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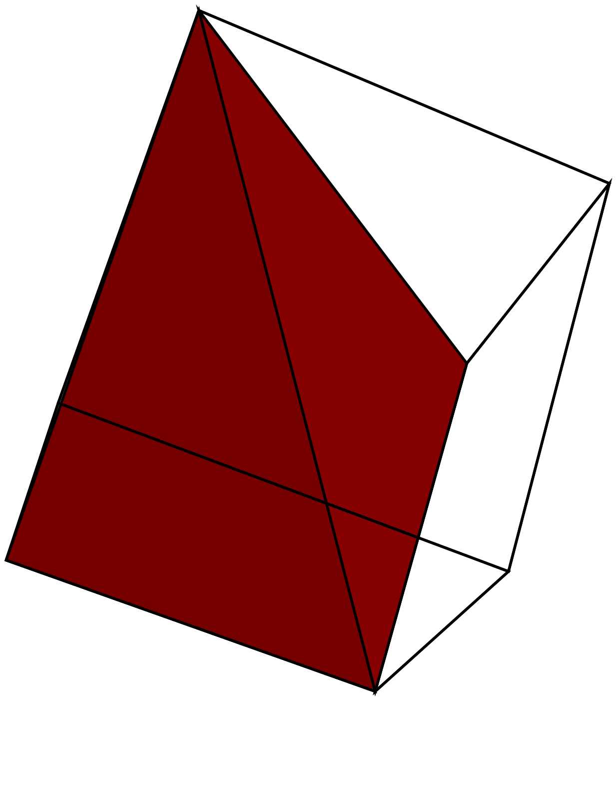

Yang-ma Volume = 1/3 x Base x Width x Height

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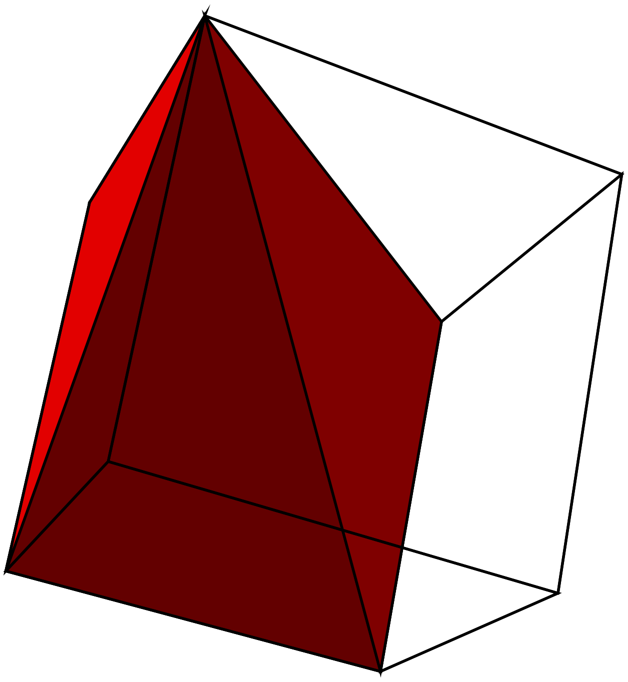

Yang-ma Volume = 1/3 x Base x Width x Height

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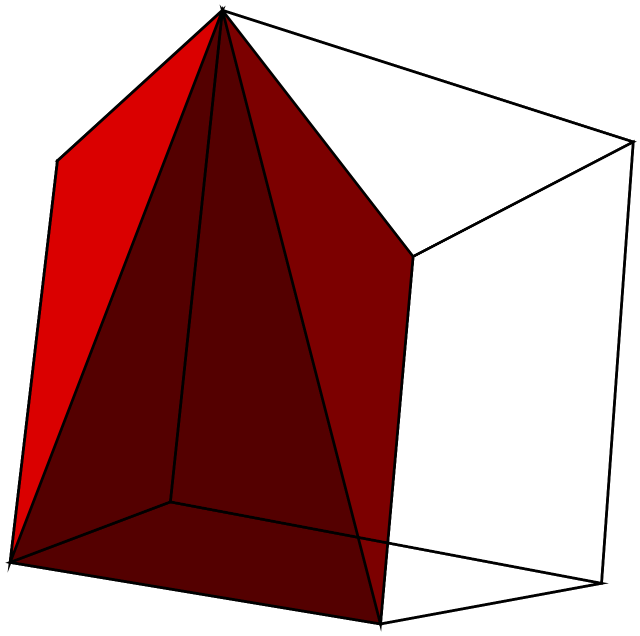
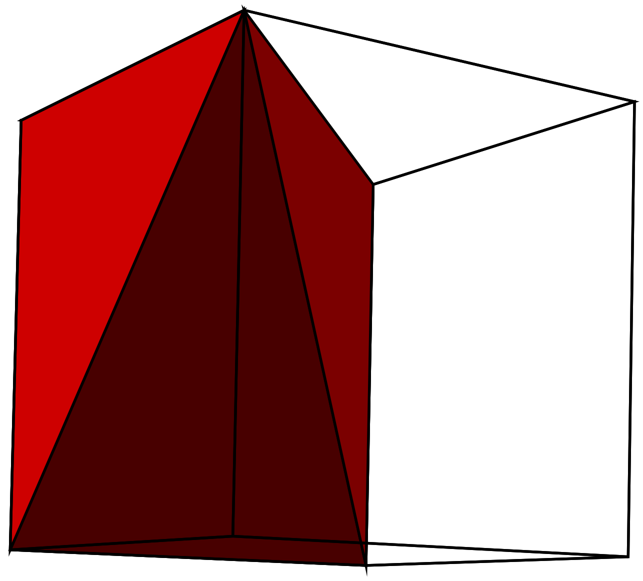


Yang-ma Volume = 1/3 x Base x Width x Height

Let’s have a closer look at this object.



Yang-ma Volume = 1/3 x Base x Width x Height

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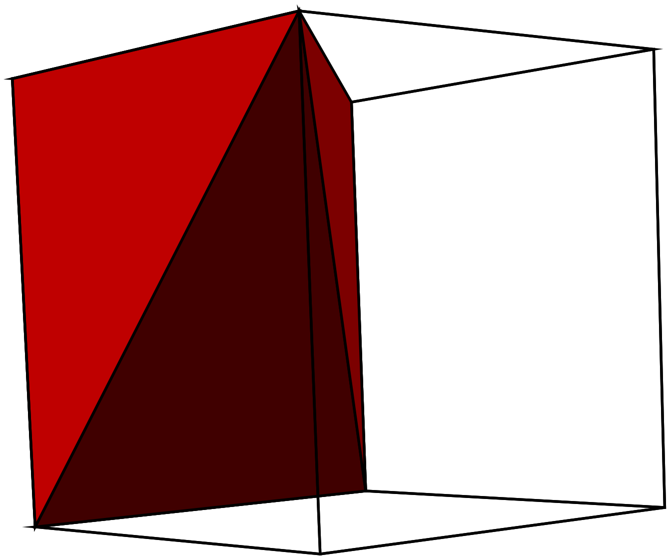

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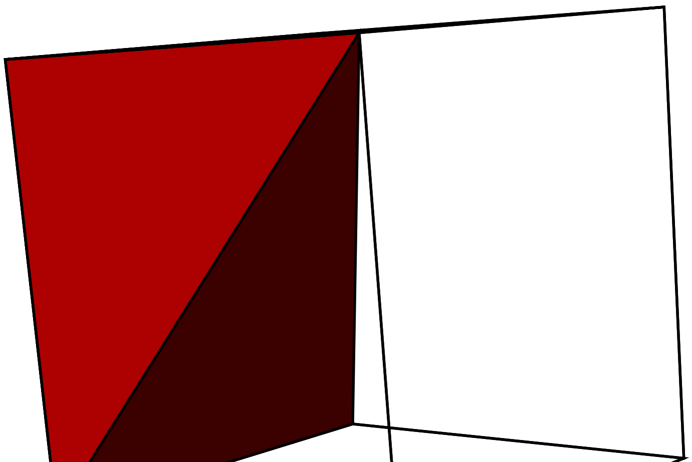

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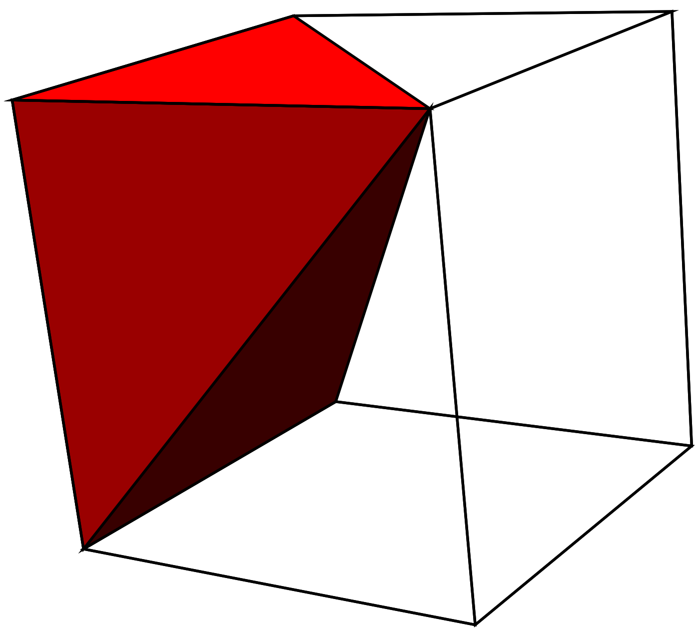

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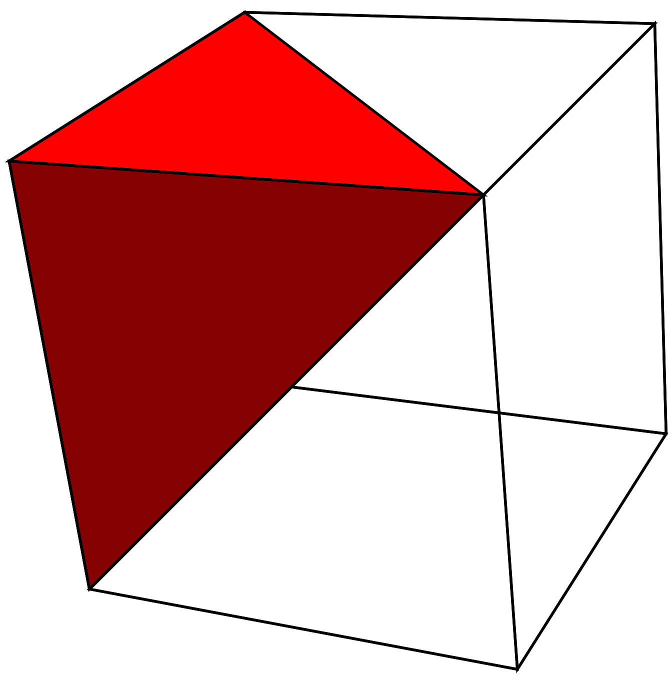

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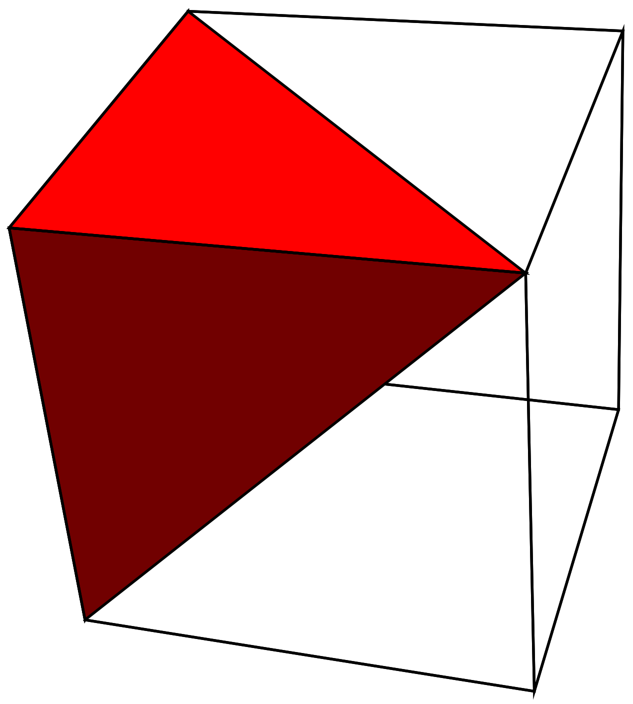

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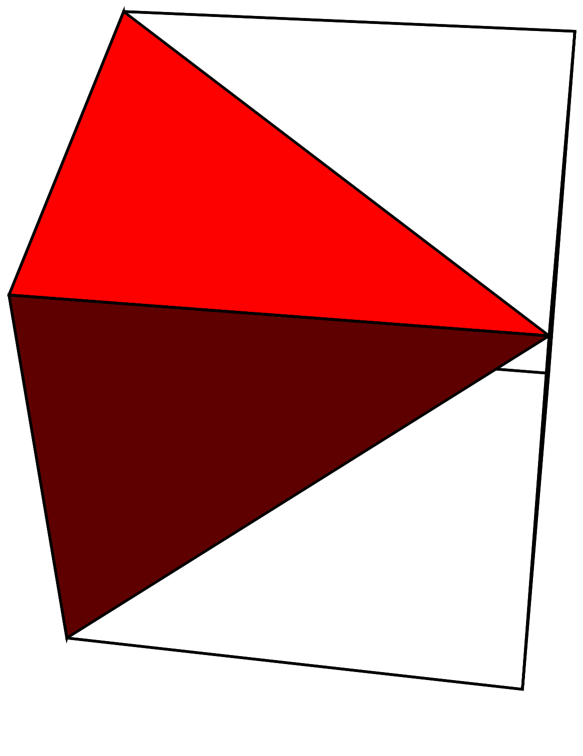

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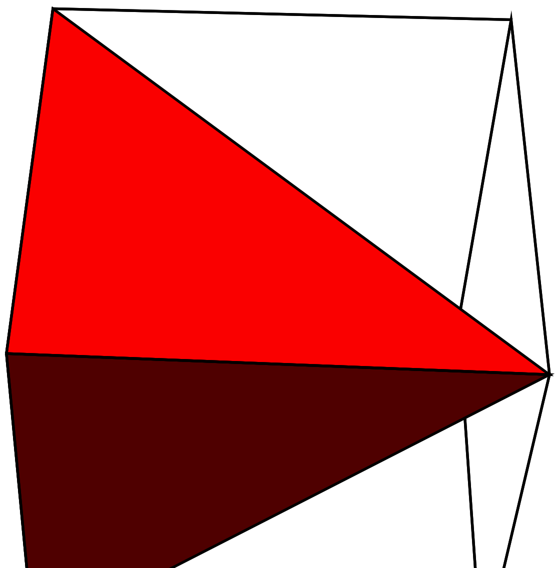

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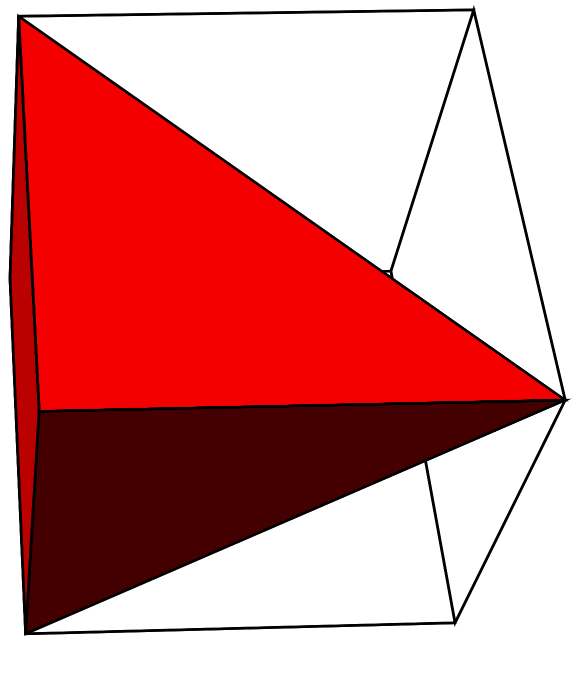

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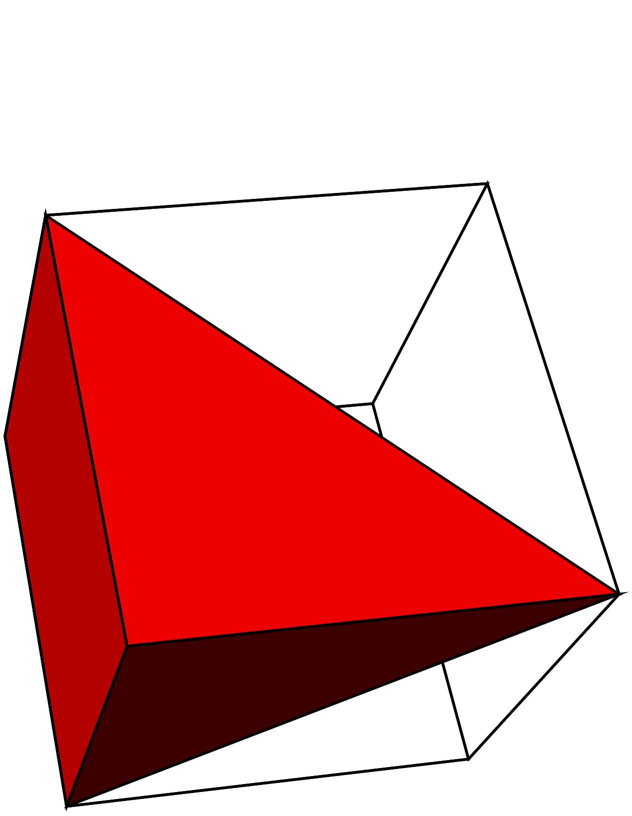

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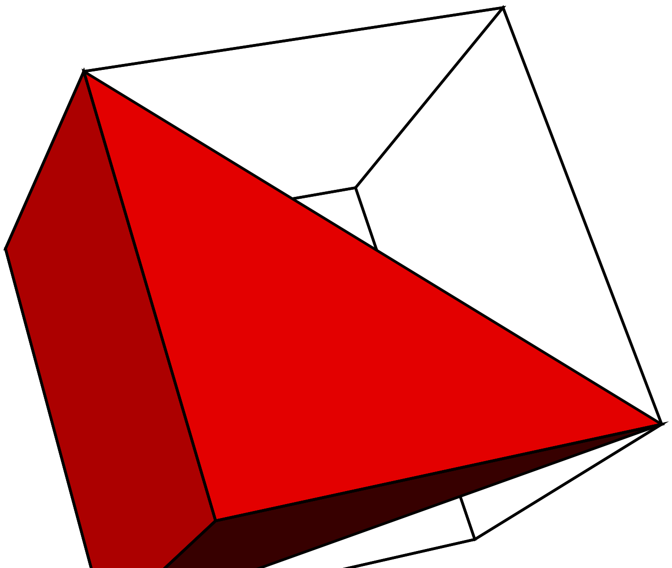

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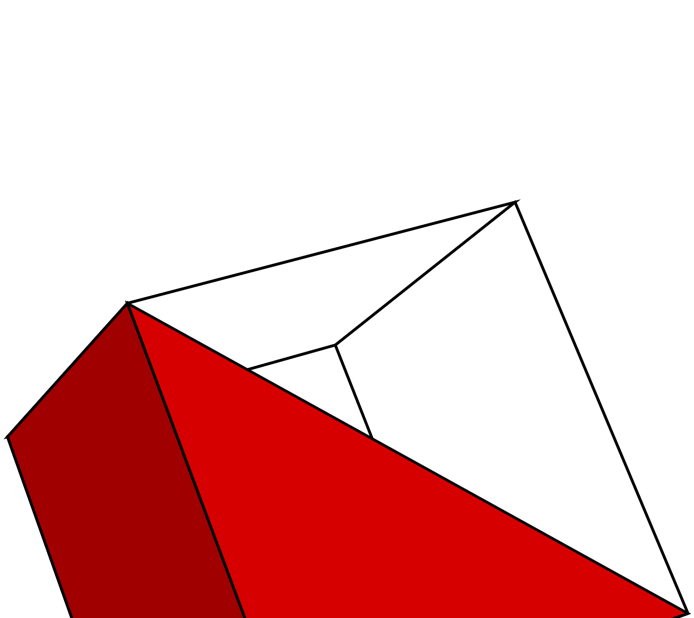

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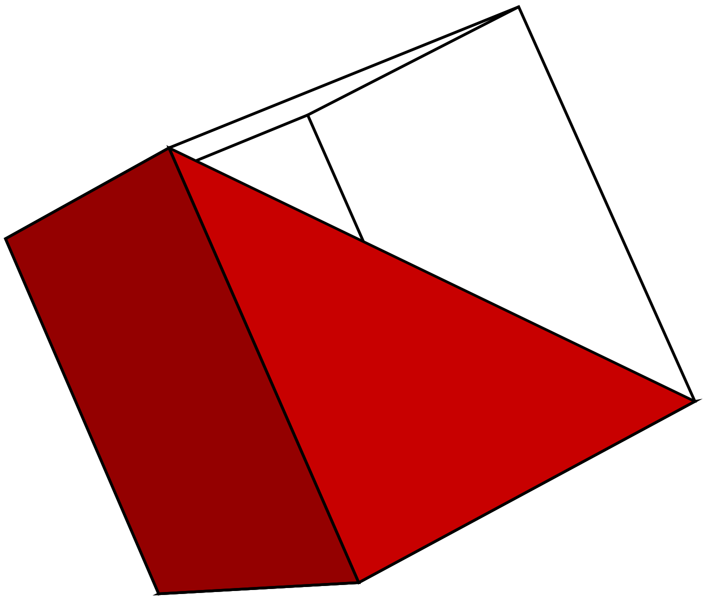

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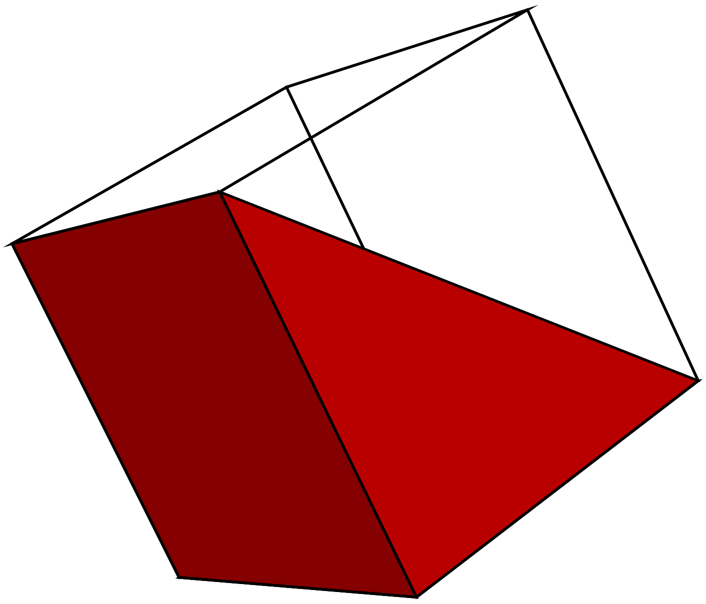

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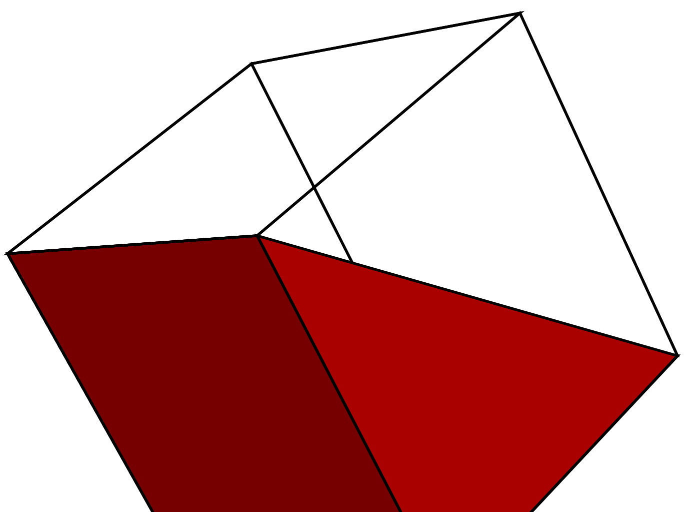

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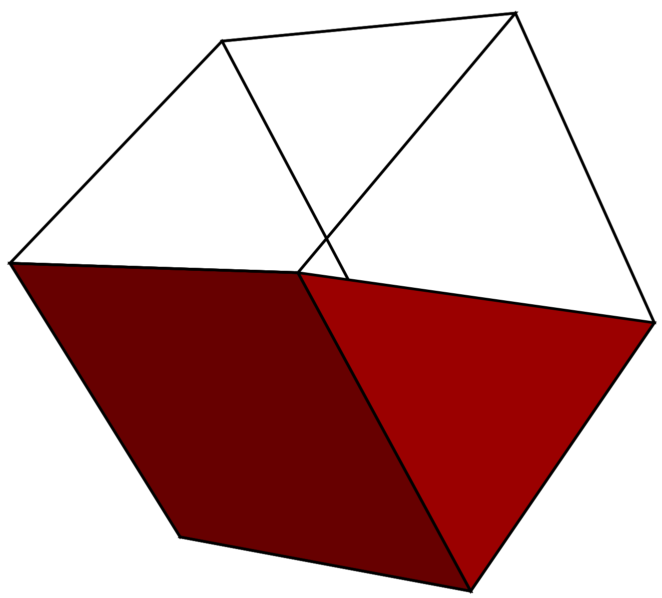

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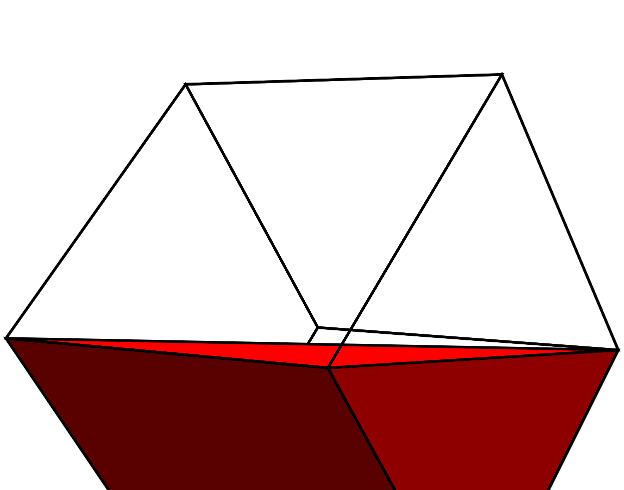

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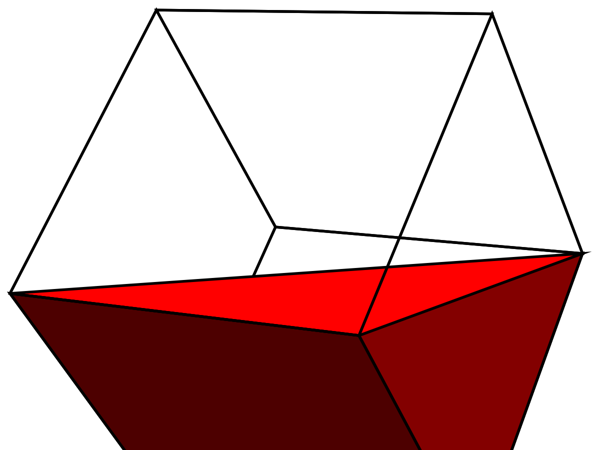

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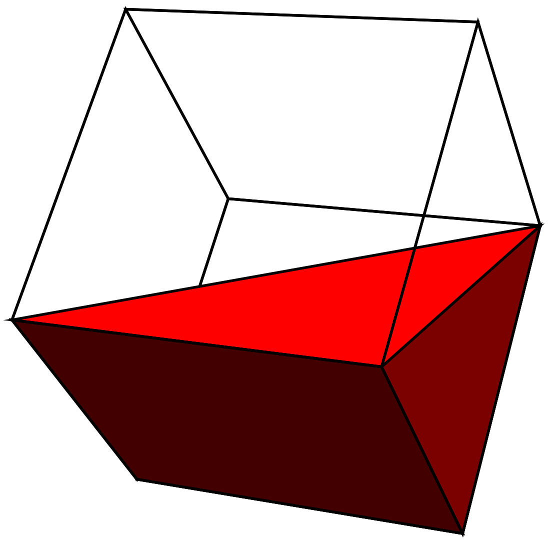

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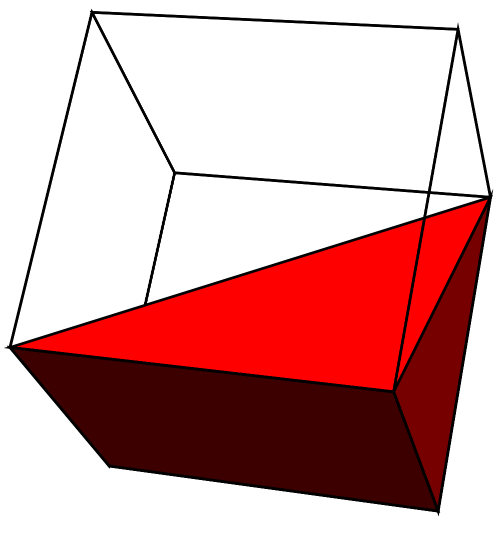

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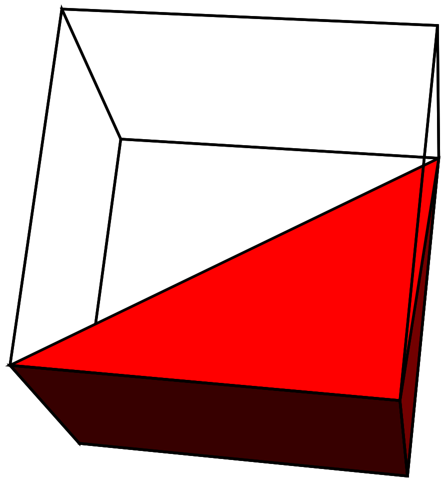

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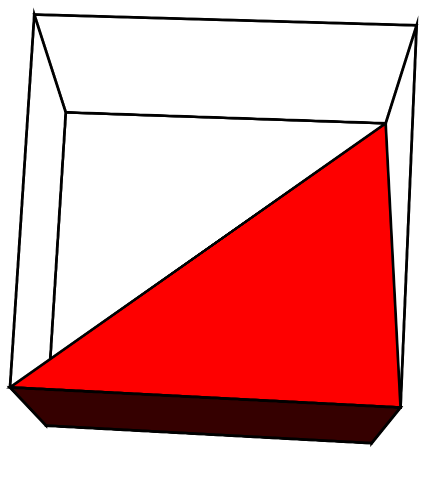

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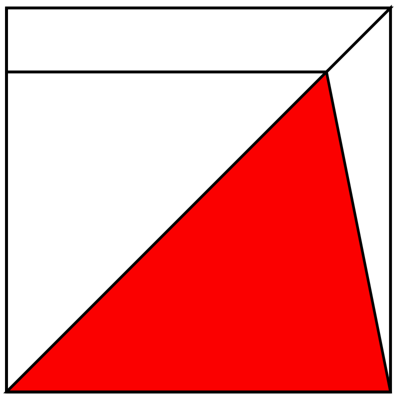

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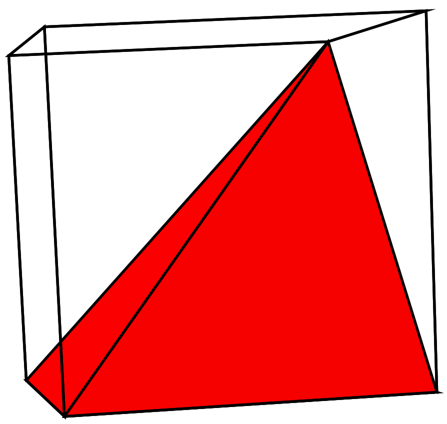

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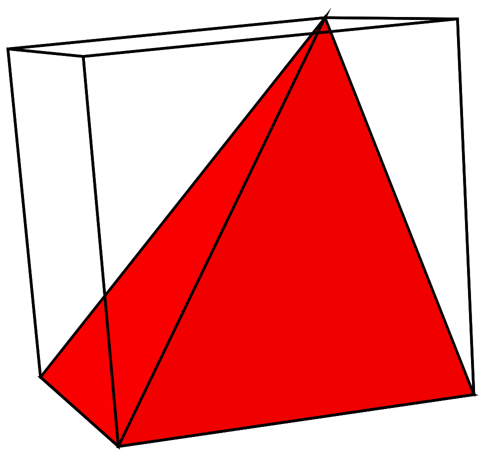
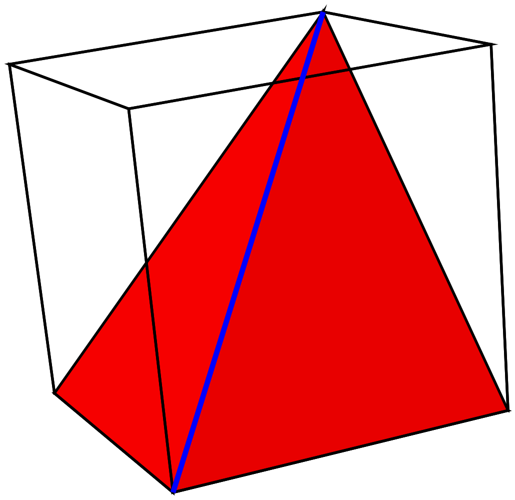


Yang-ma Volume = 1/3 x Base x Width x Height

Let's have a closer look at this object.

Finally Liu Hui cut up a Yang-ma by slicing it into two along the spine. The result is two solids that while not identical are mirror images of each other, and as a result they have equal volume. Liu Hui also noticed that one of the two solids is identical to the Pied-nao. It should also be noted



that Liu Hui did not differentiate between a Pied-nao and the mirror image which he also called a Pied-nao. So the result of splitting the Yang-ma into two was the the creation of two more Pieh-nao.

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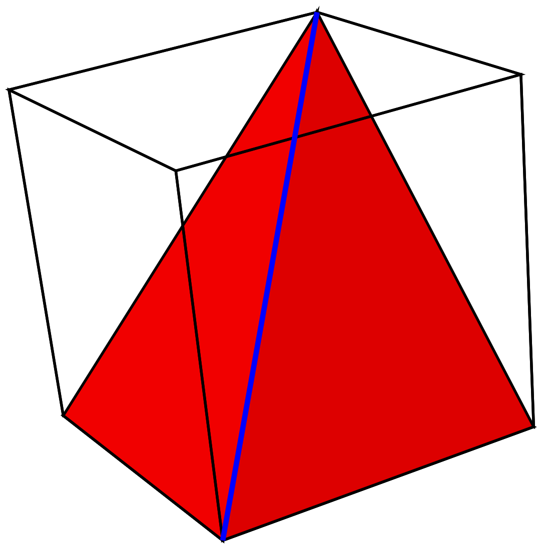

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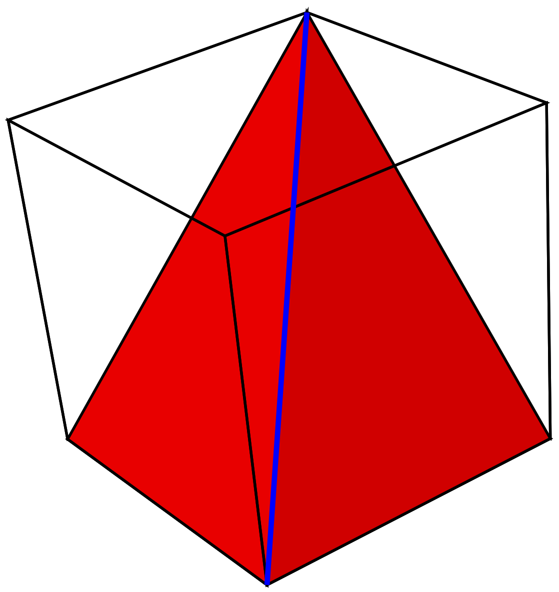
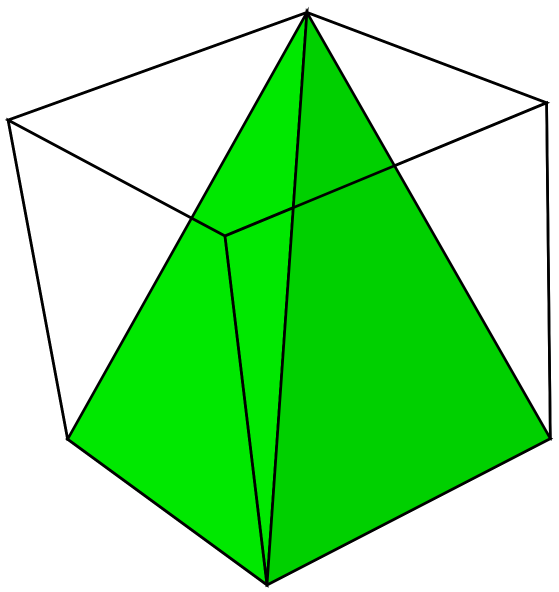
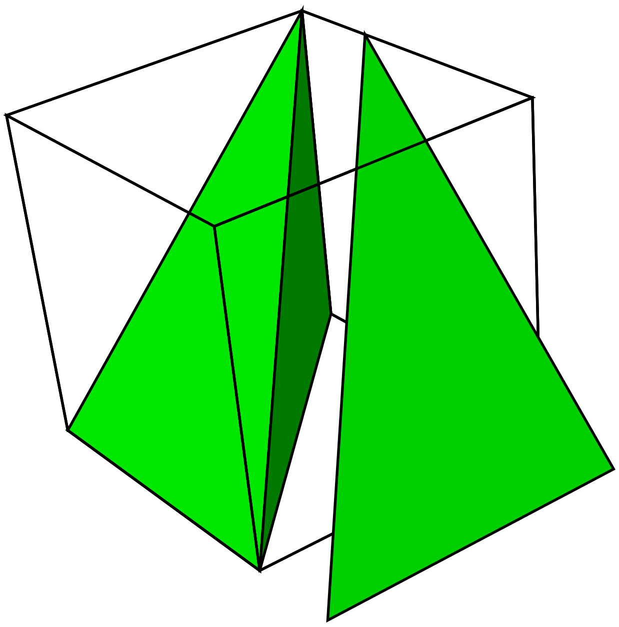


that Liu Hui did not differentiate between a Pied-nao and the mirror image which he also called a Pied-nao. So the result of splitting the Yang-ma into two was the the creation of two more Pieh-nao.

But how did Liu Hui conclude that a Yang-ma is one third the volume and the Pieh-nao is one sixth the volume of the unit cube. Well to start a Yang-ma is made from two Pieh-nao. Therefore he concluded that a Yang-ma is twice the volume of a Pieh-nao.



Two Pieh-nao can be combined to create a single Yang-ma. Let's see how this works.



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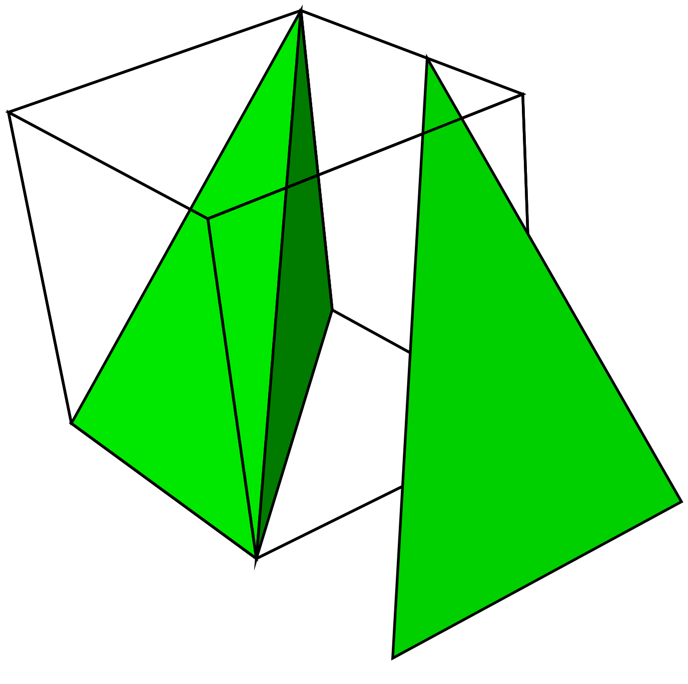

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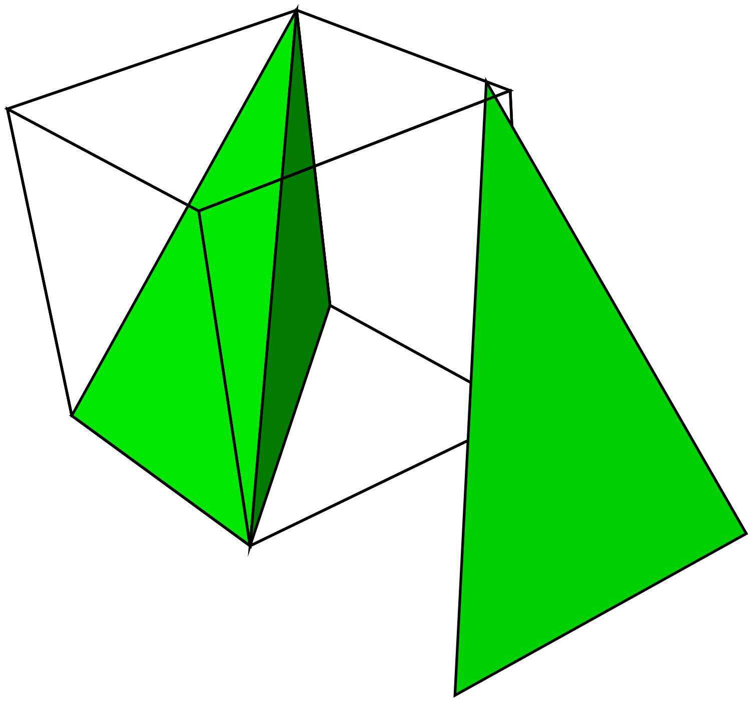

Two Pieh-nao can be combined to create a single

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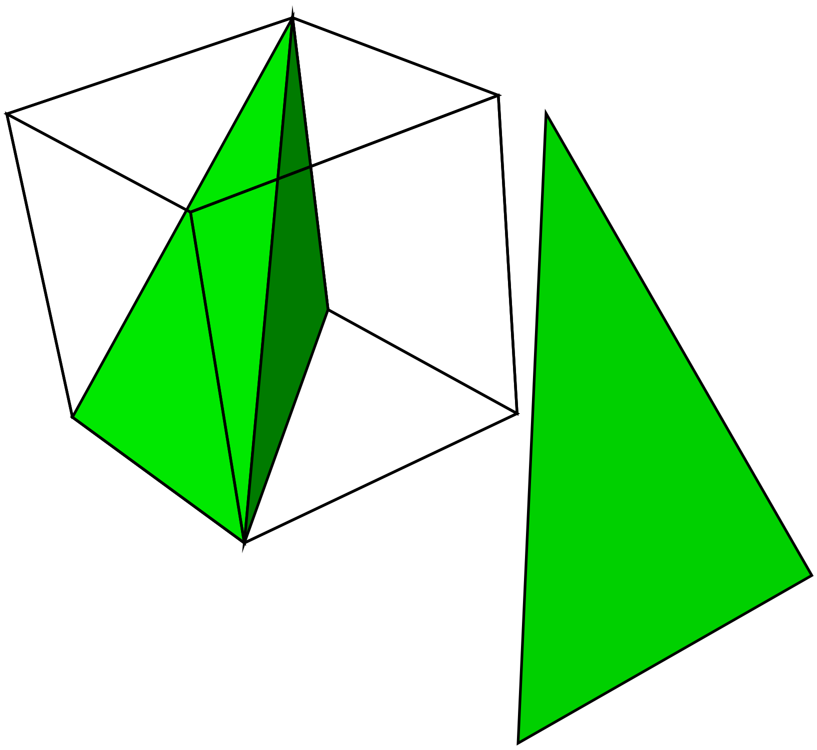
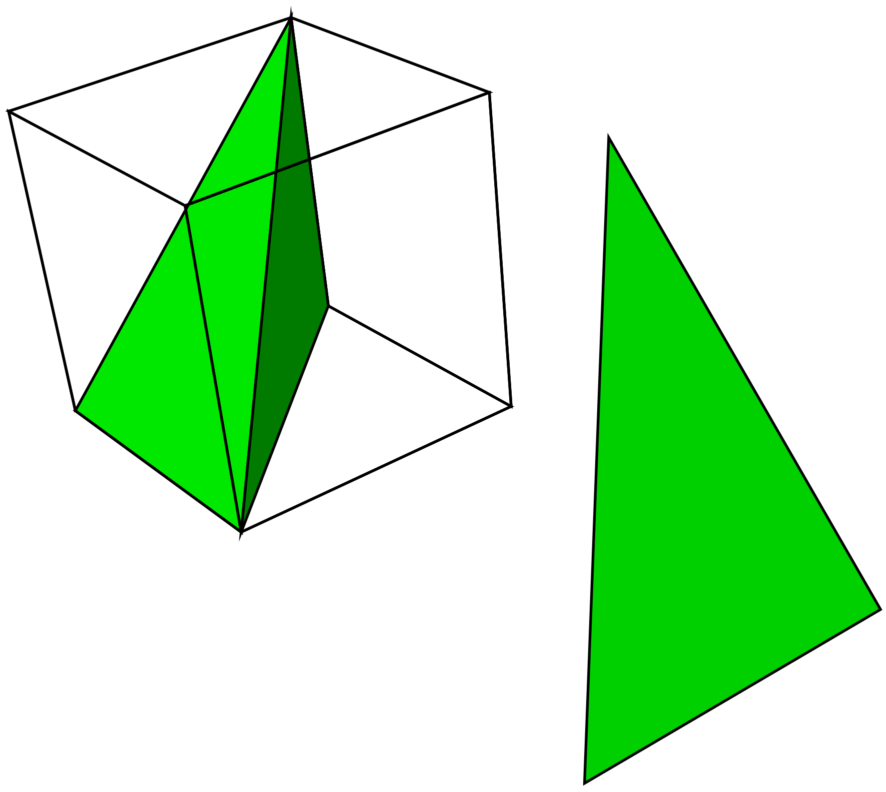


Two Pieh-nao can be combined to create a single Yang-ma. Let’s see how this works.



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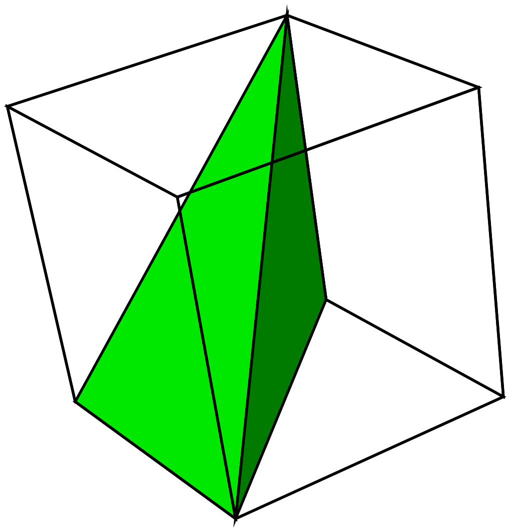

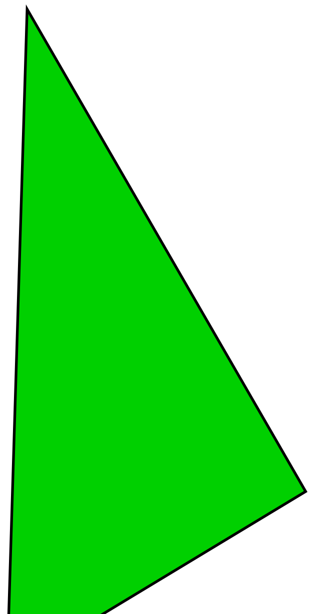

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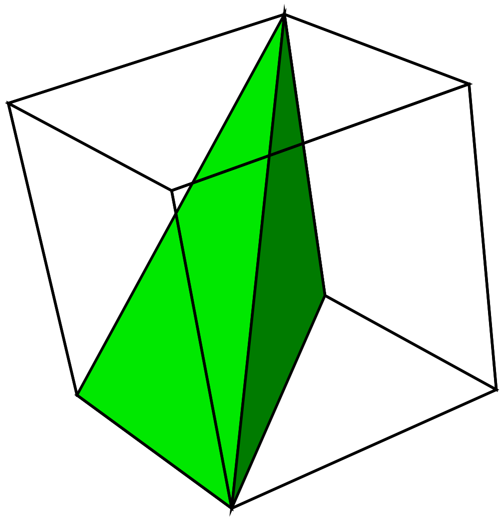
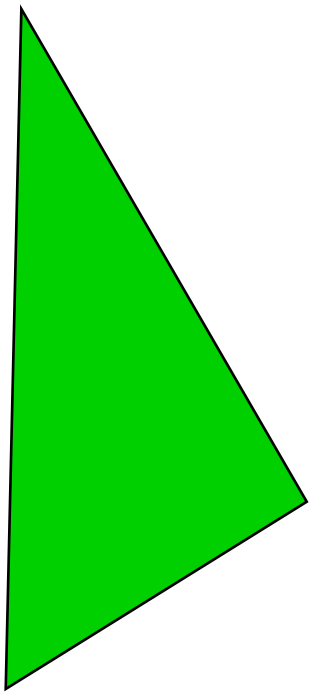
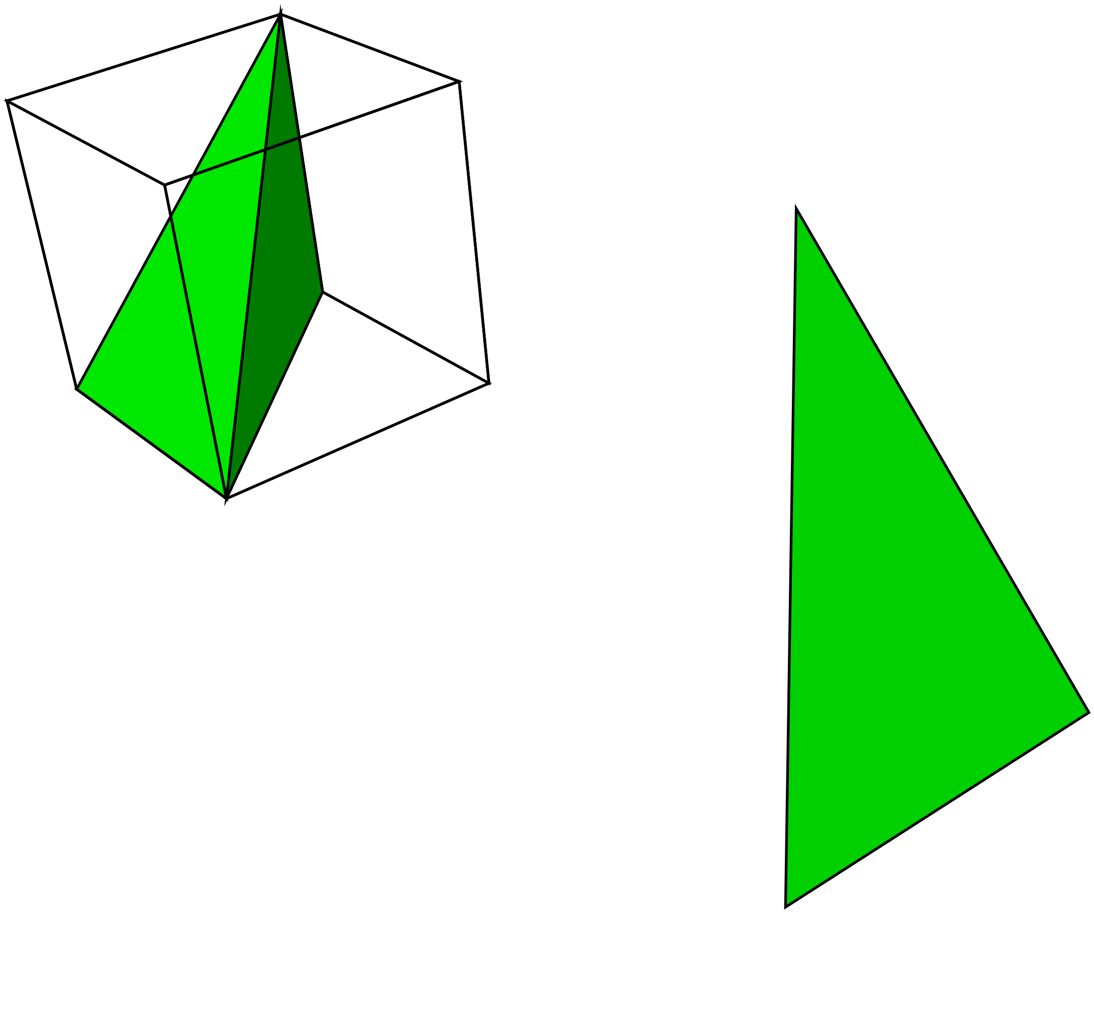
Yang-ma. Let’s see how this works.





Two Pieh-nao can be combined to create a single Yang-ma. Let's see how this works.



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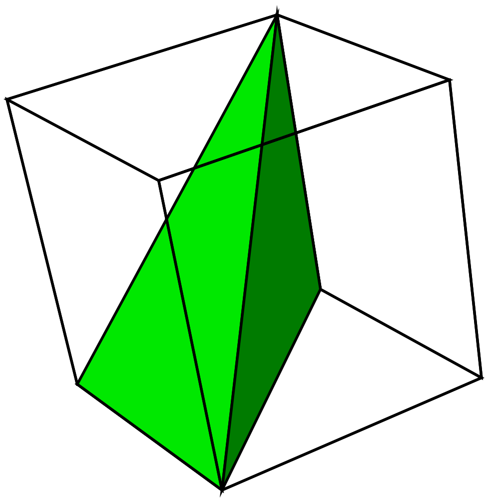

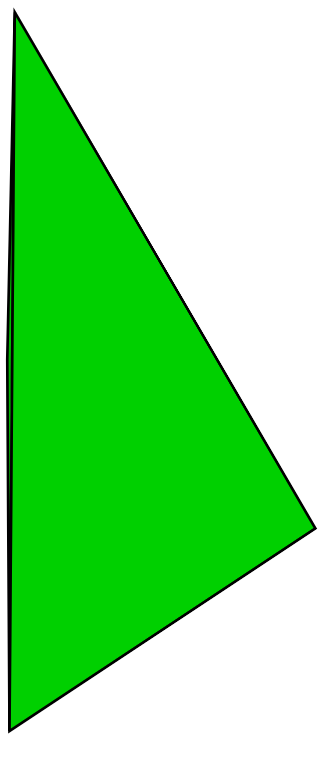

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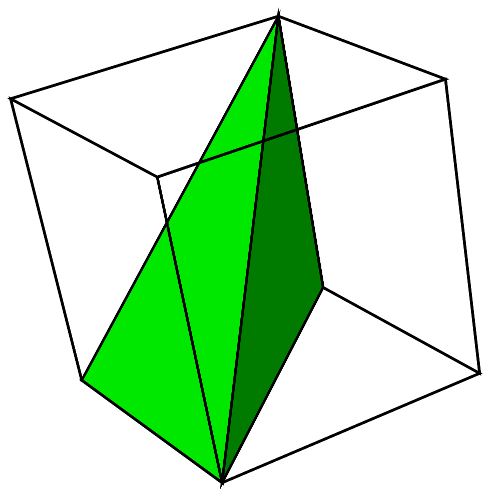

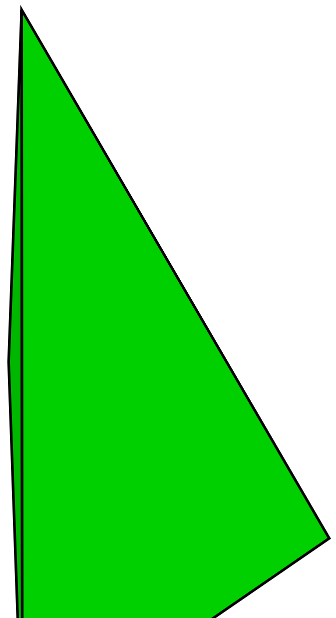

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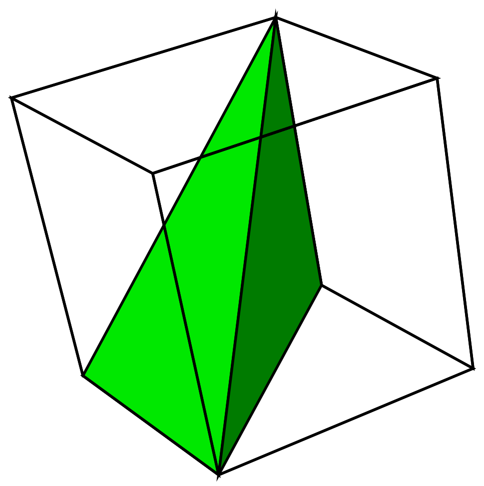

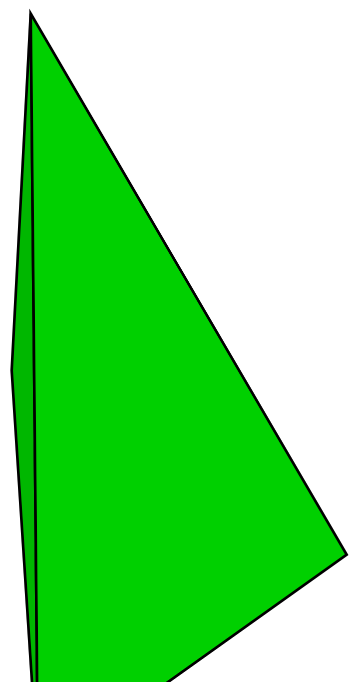

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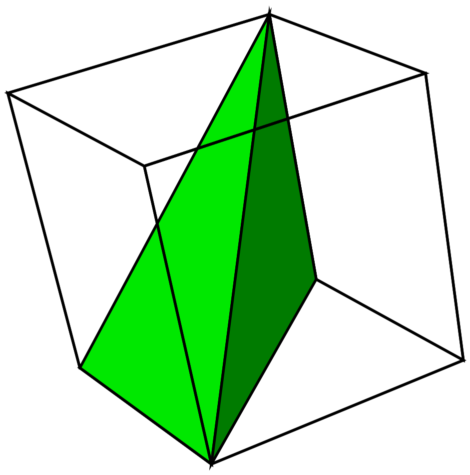

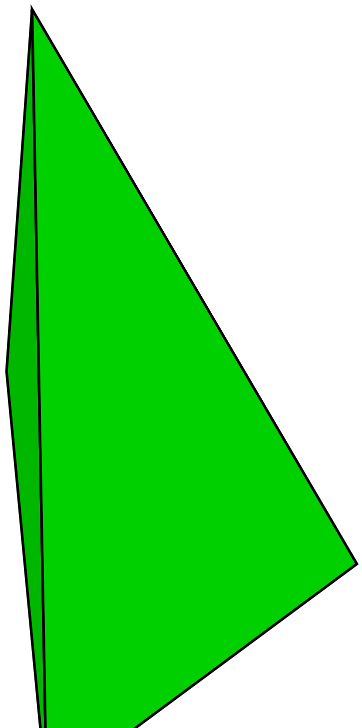

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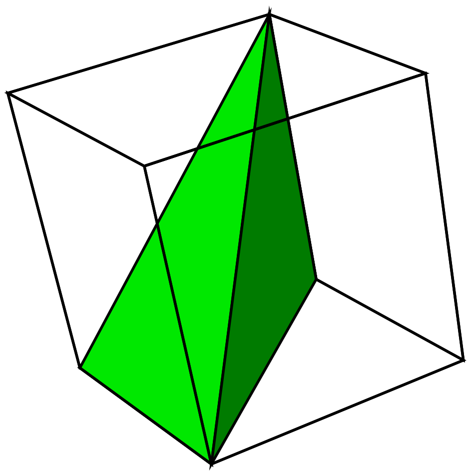

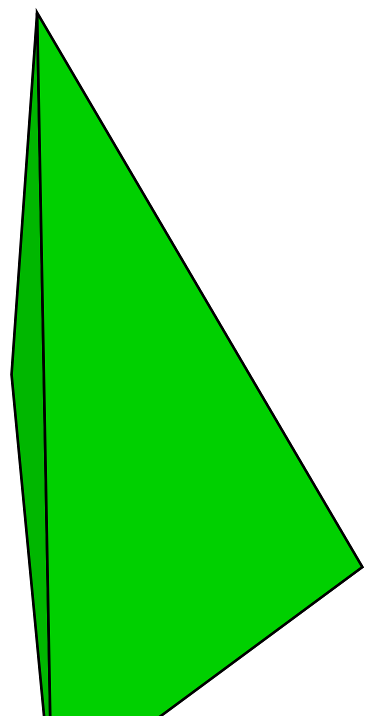

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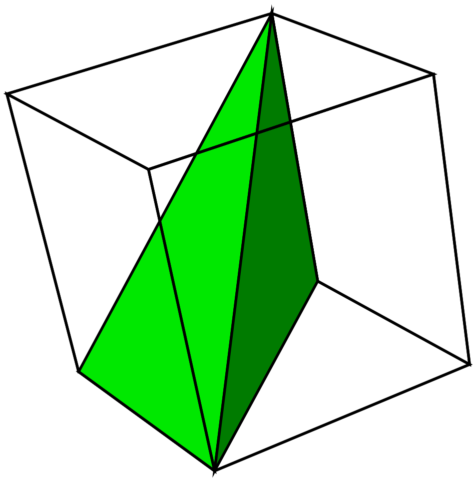

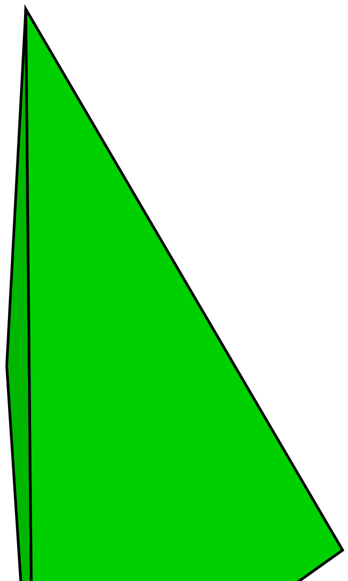

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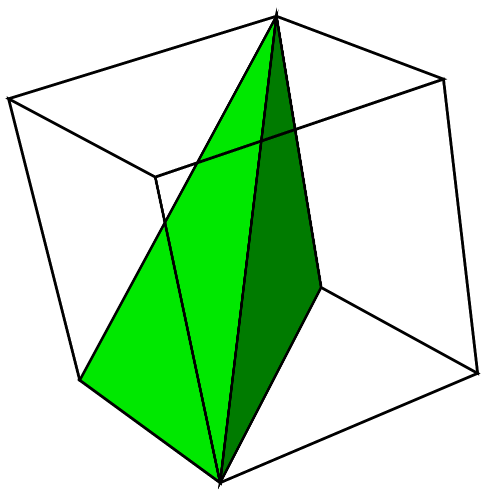

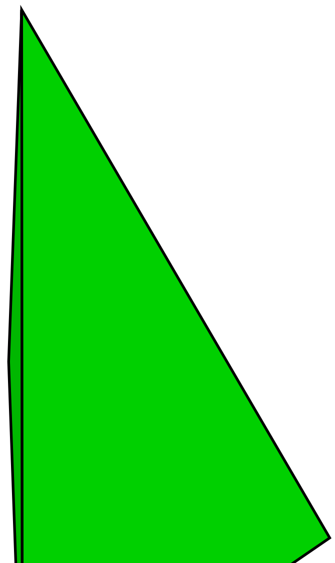

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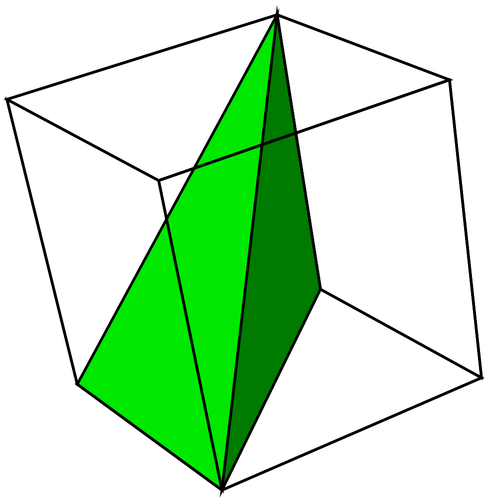

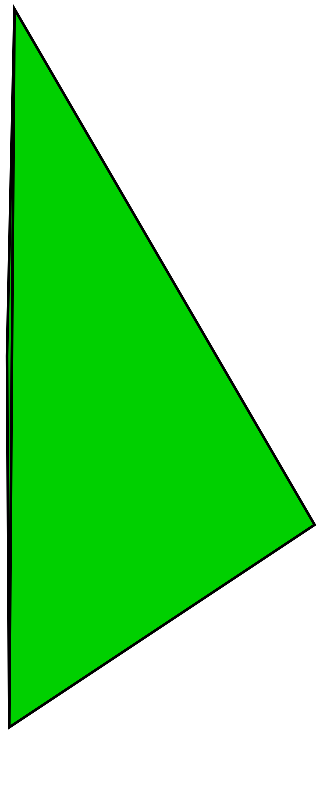

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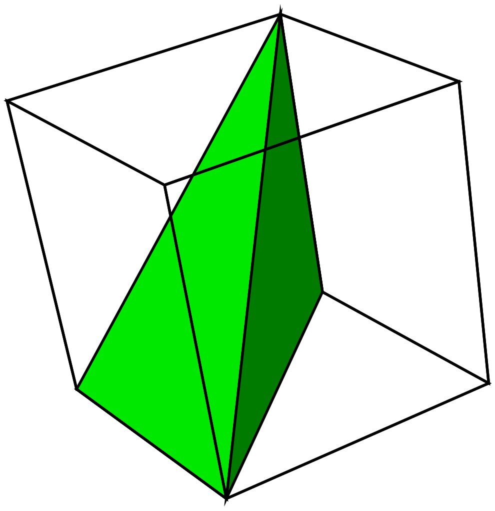

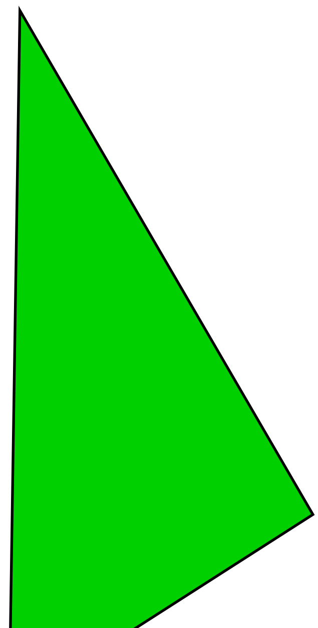

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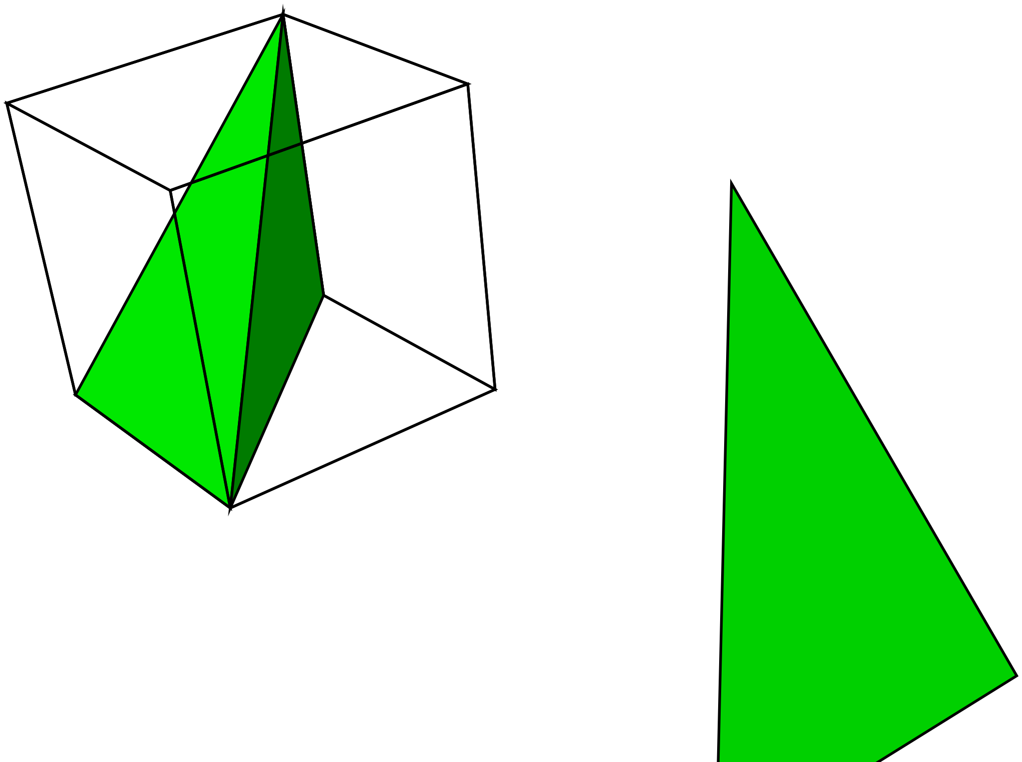
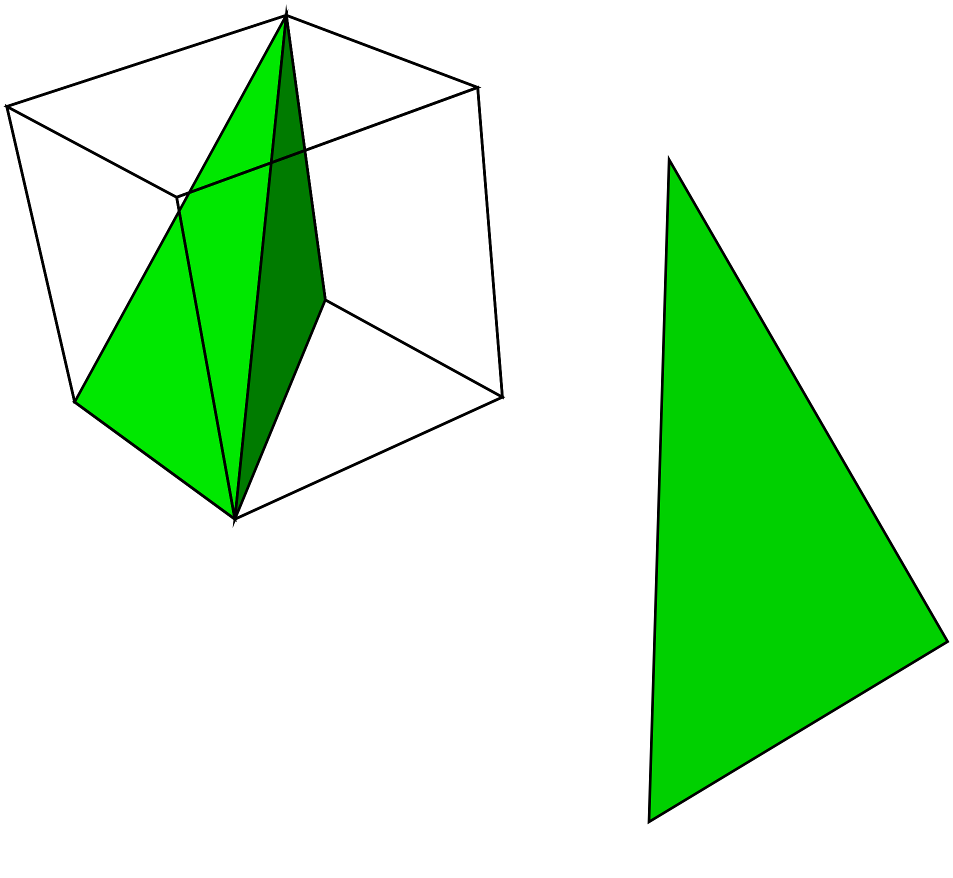
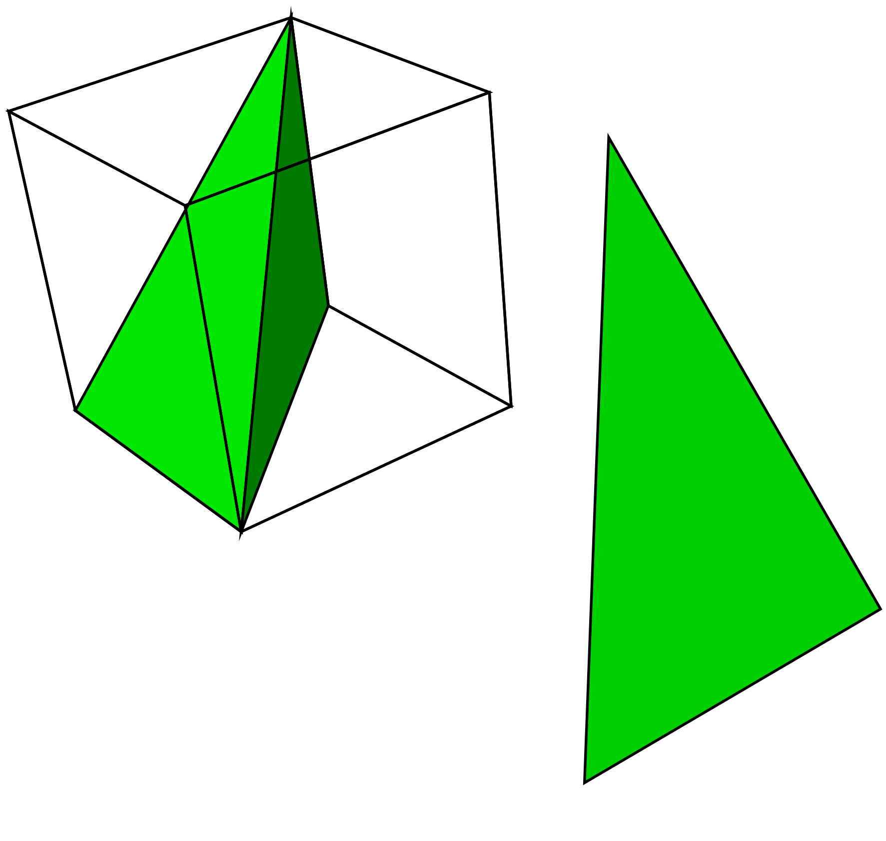


Two Pieh-nao can be combined to create a single Yang-ma. Let's see how this works.



Two Pieh-nao can be combined to create a single Yang-ma. Let's see how this works.



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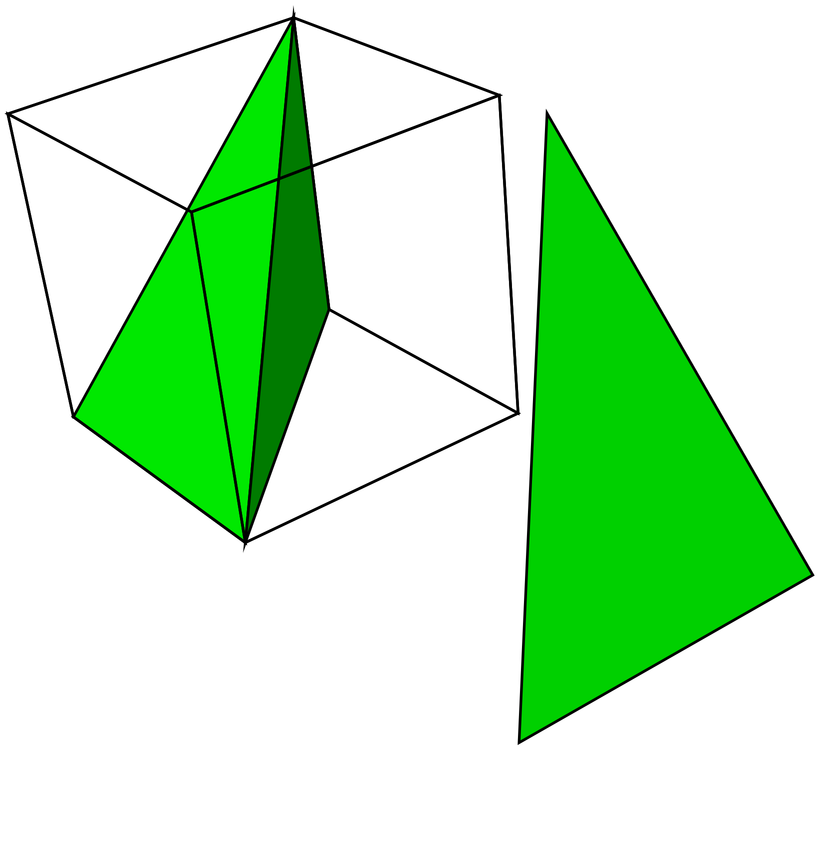
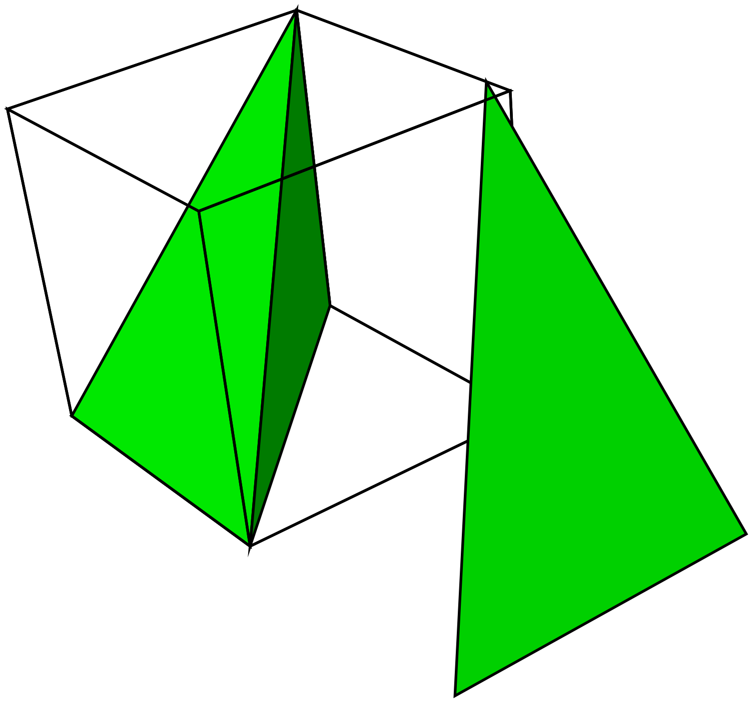


Two Pieh-nao can be combined to create a single Yang-ma. Let’s see how this works.



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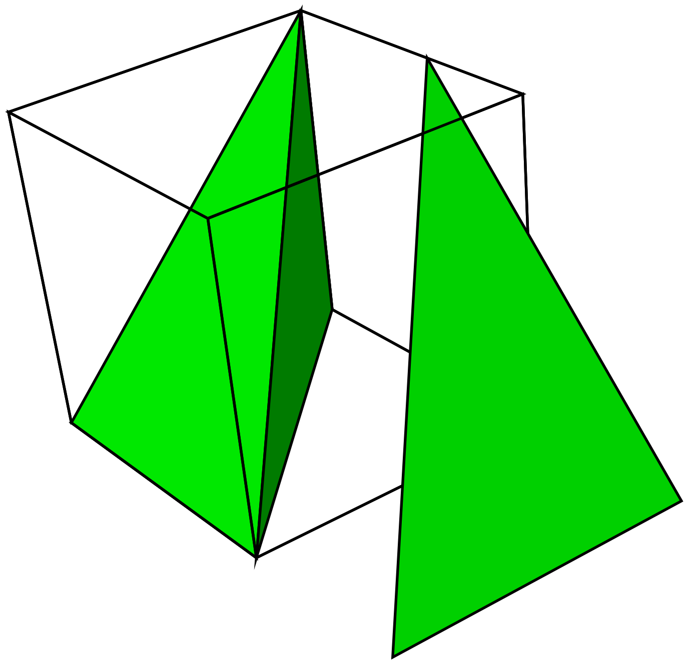

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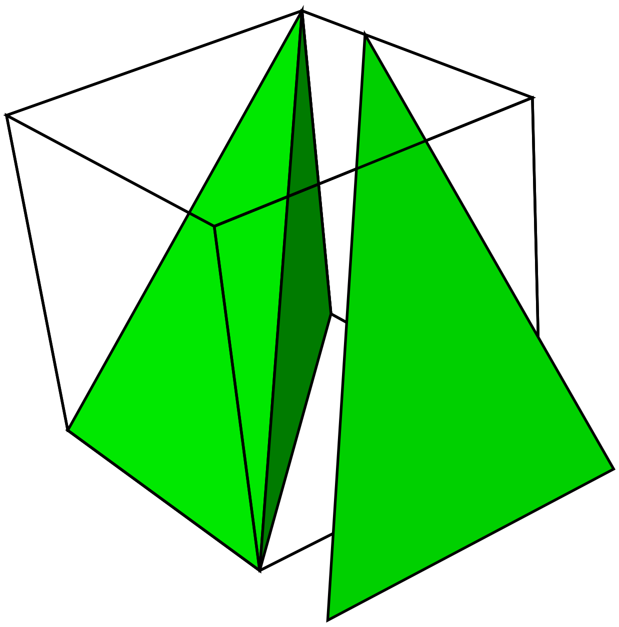
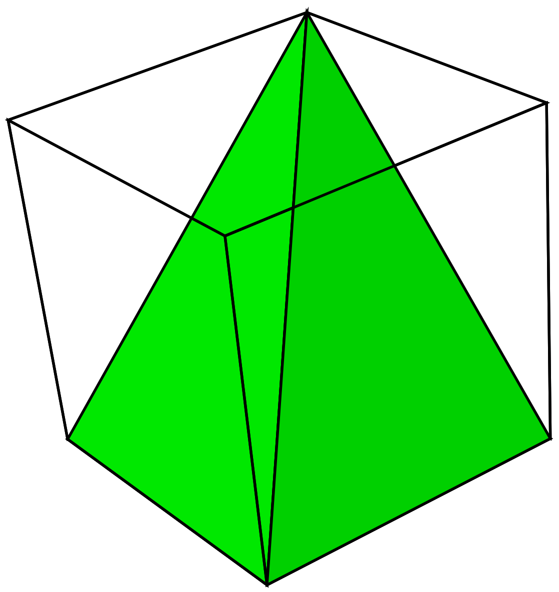
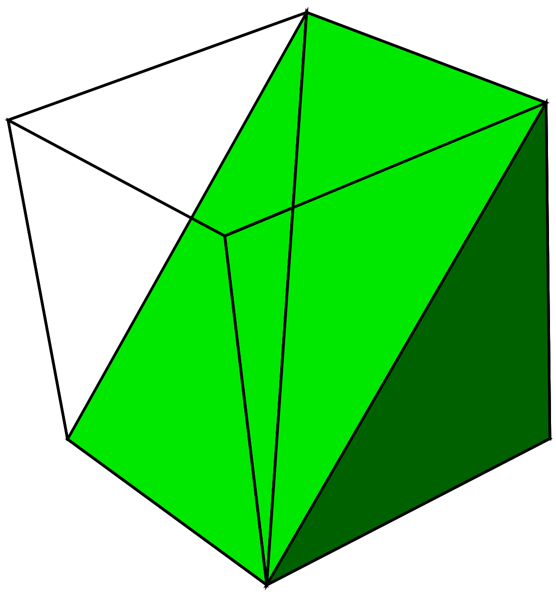


Two Pieh-nao can be combined to create a single Yang-ma. Let's see how this works.



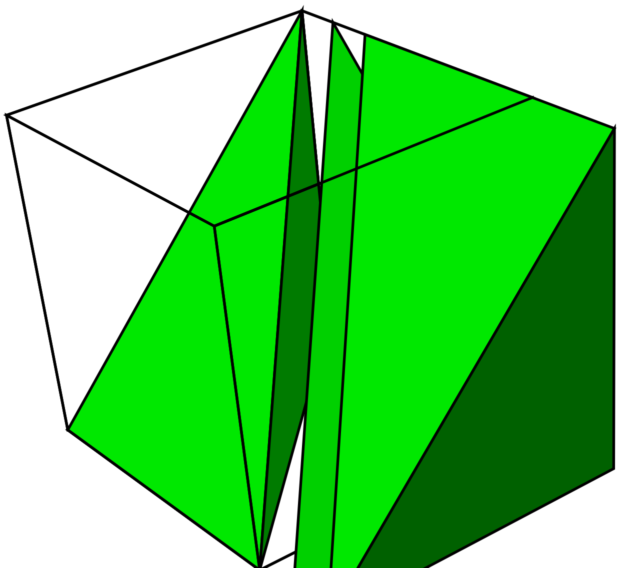
Two Pieh-nao can be combined to create a single Yang-ma. Let’s see how this works.

However a Ch’ien-tu is one Pied-nao combined with one Yang-ma, which is a combination of two more Pieh-nao. Therefore a Ch’ien-tu is three Pied-nao and thus a Pied-nao must equal one third the volume of the Ch’ien-tu. This means the Pied-nao is one sixth the volume of the unit cube as the Ch’ien-tu is half the volume of the unit cube. Similarly as the



Yang-ma is double the volume of the Pied-nao, Liu Hui knew that the Yang-ma is a third the volume of the cube.

However a Ch’ien-tu is a Pieh-nao and a Yang-ma. Thus a Ch’ien-tu is also three Pieh-nao combined. Let’s watch.



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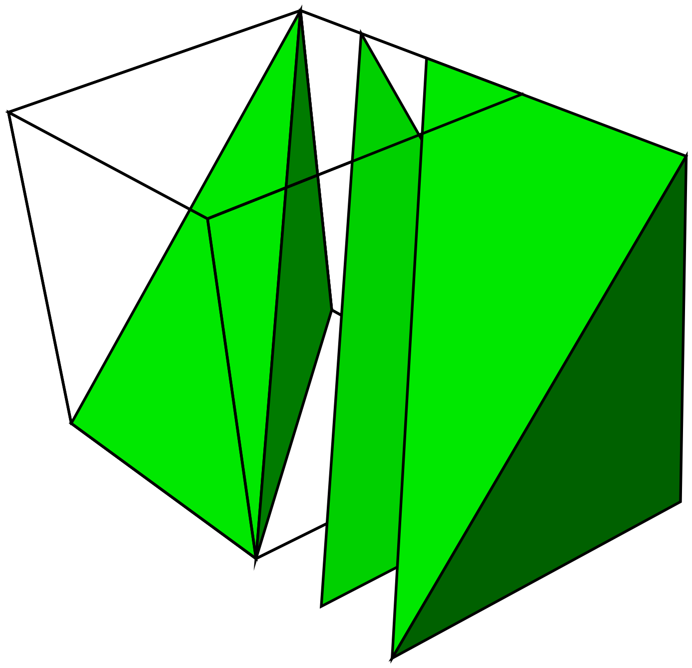

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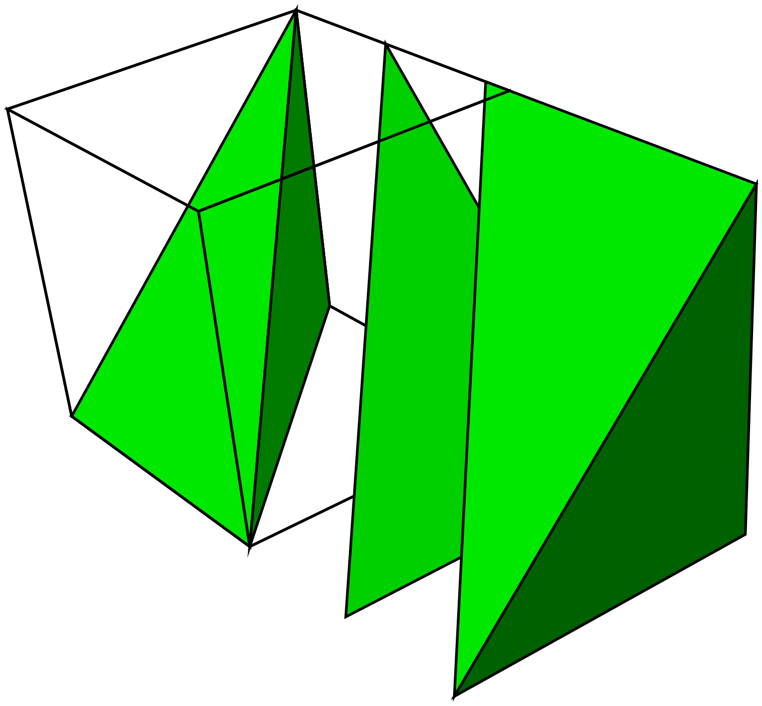

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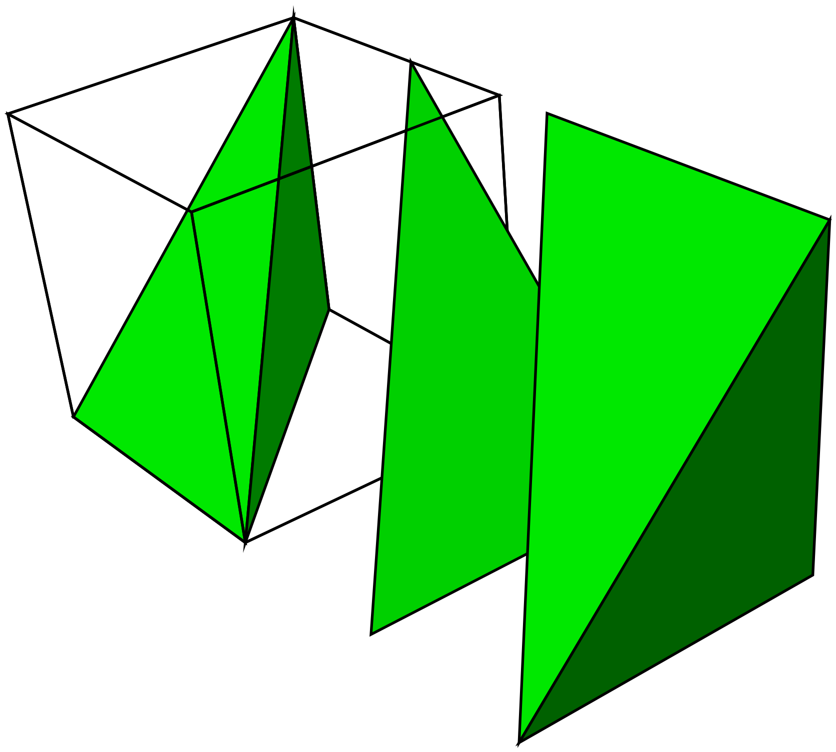

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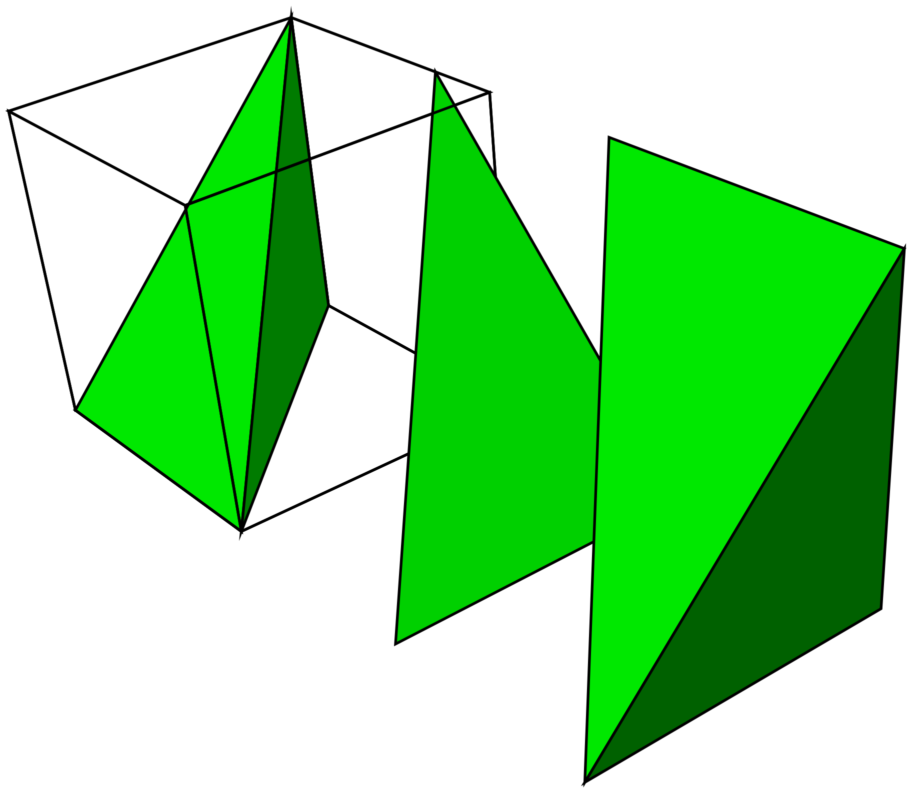
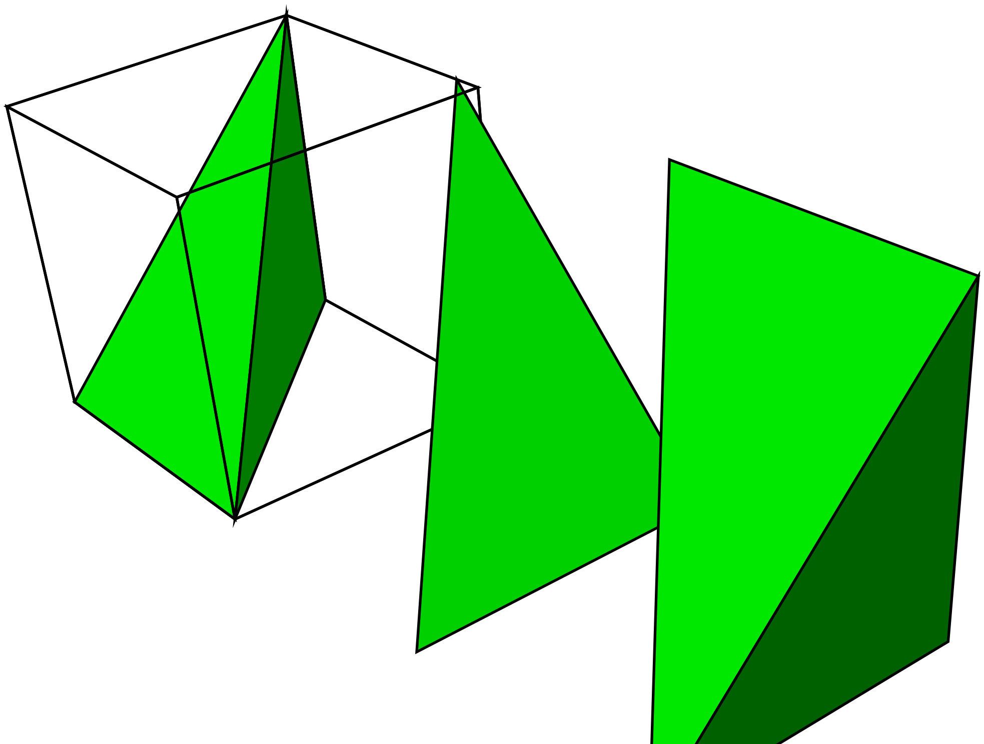


However a Ch’ien-tu is a Pieh-nao and a Yang-ma. Thus a Ch’ien-tu is also three Pieh-nao combined. Let’s watch.



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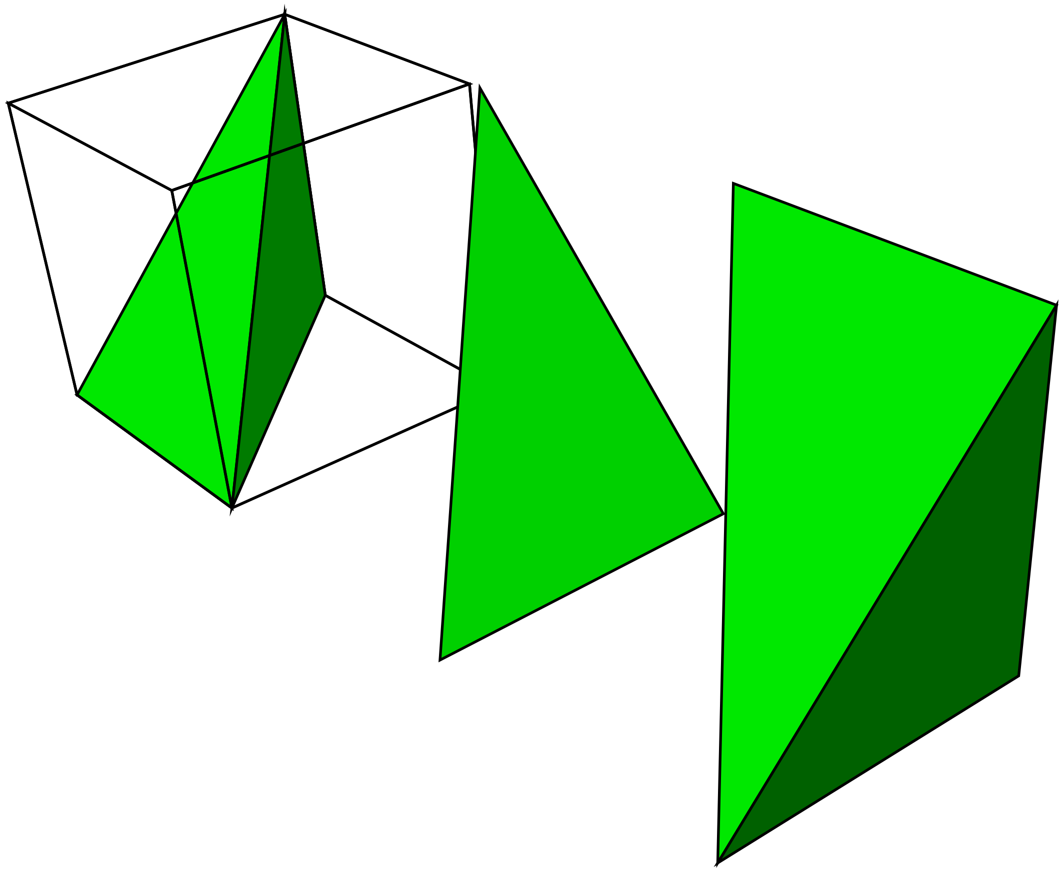

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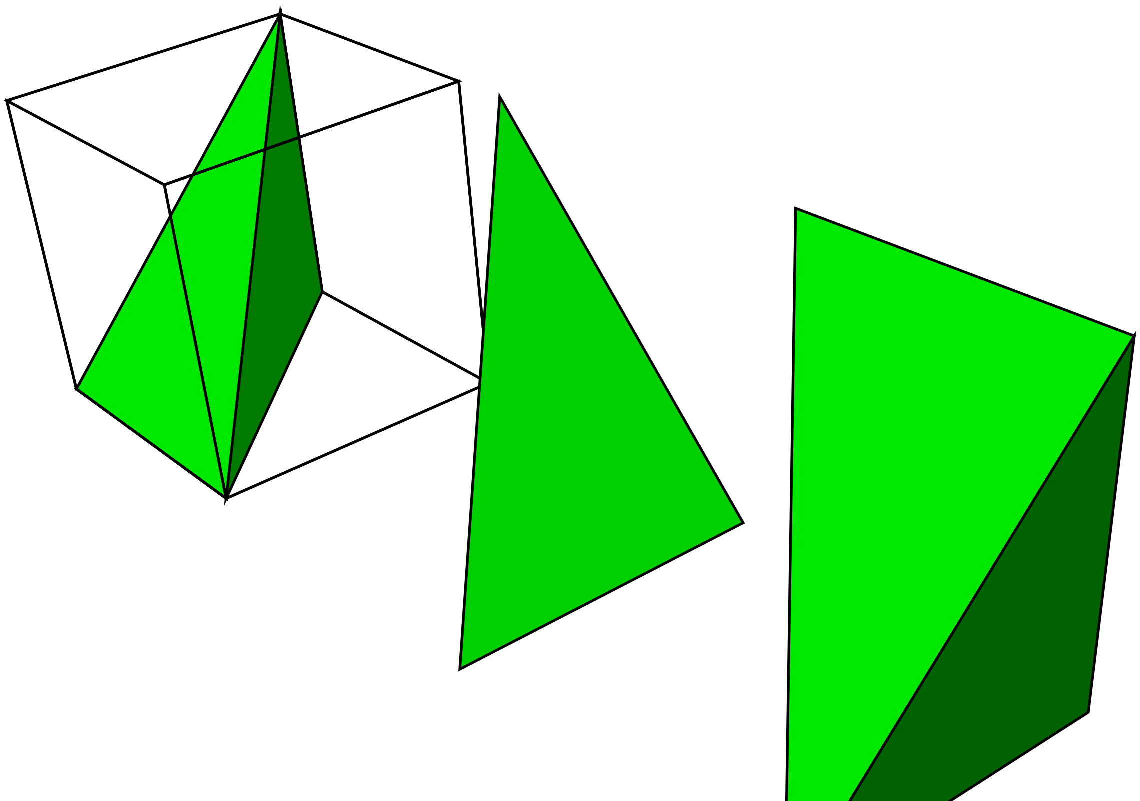

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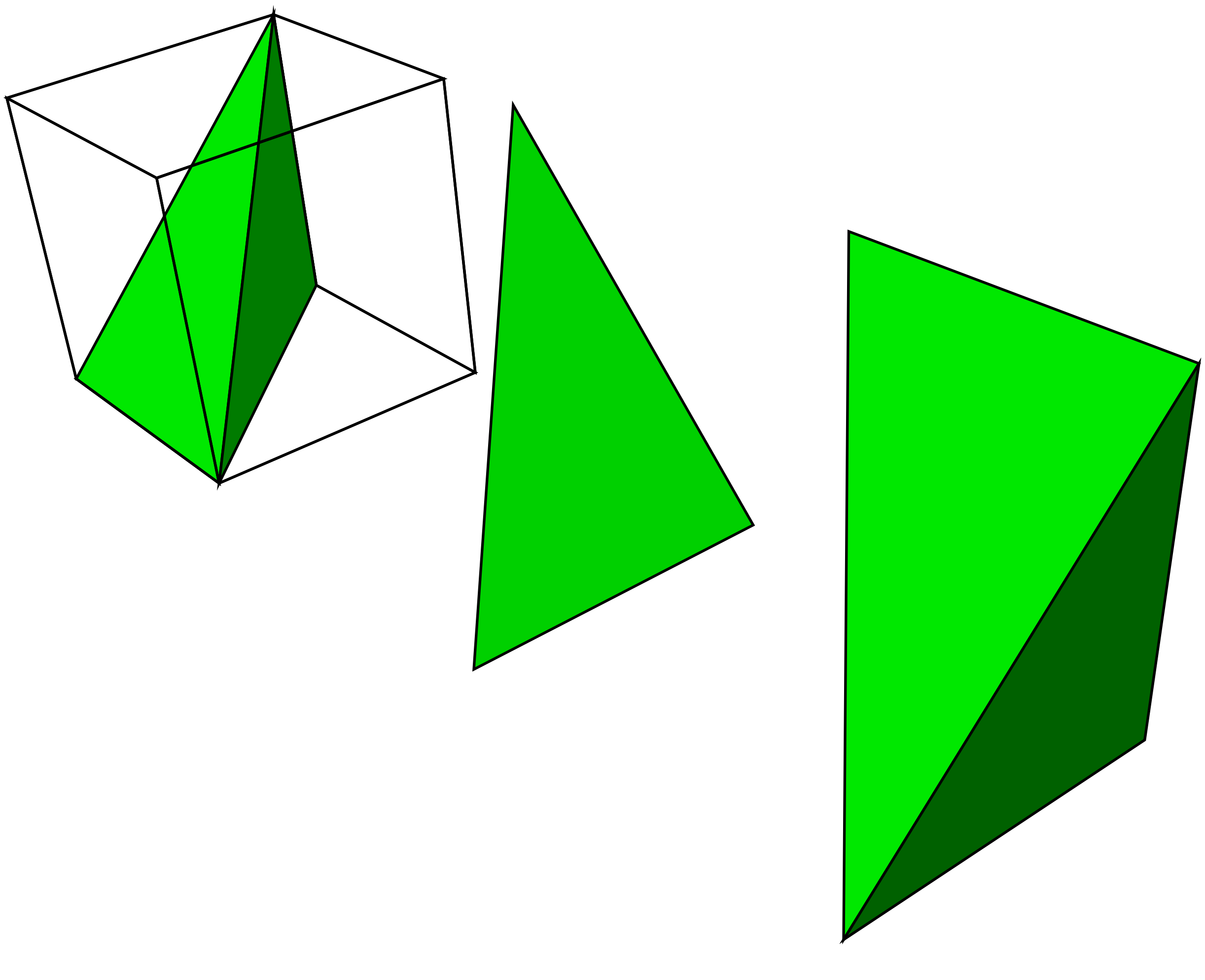

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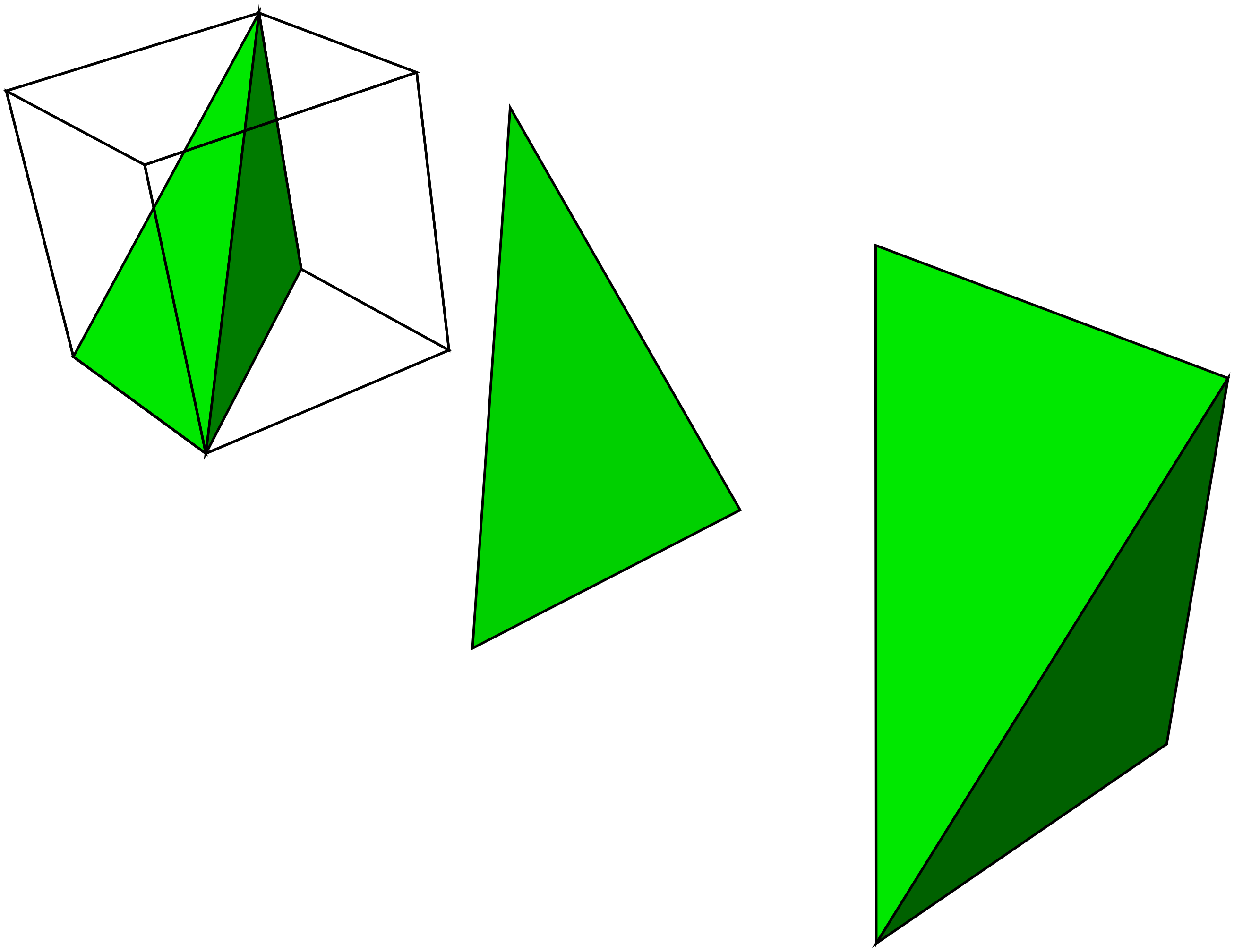

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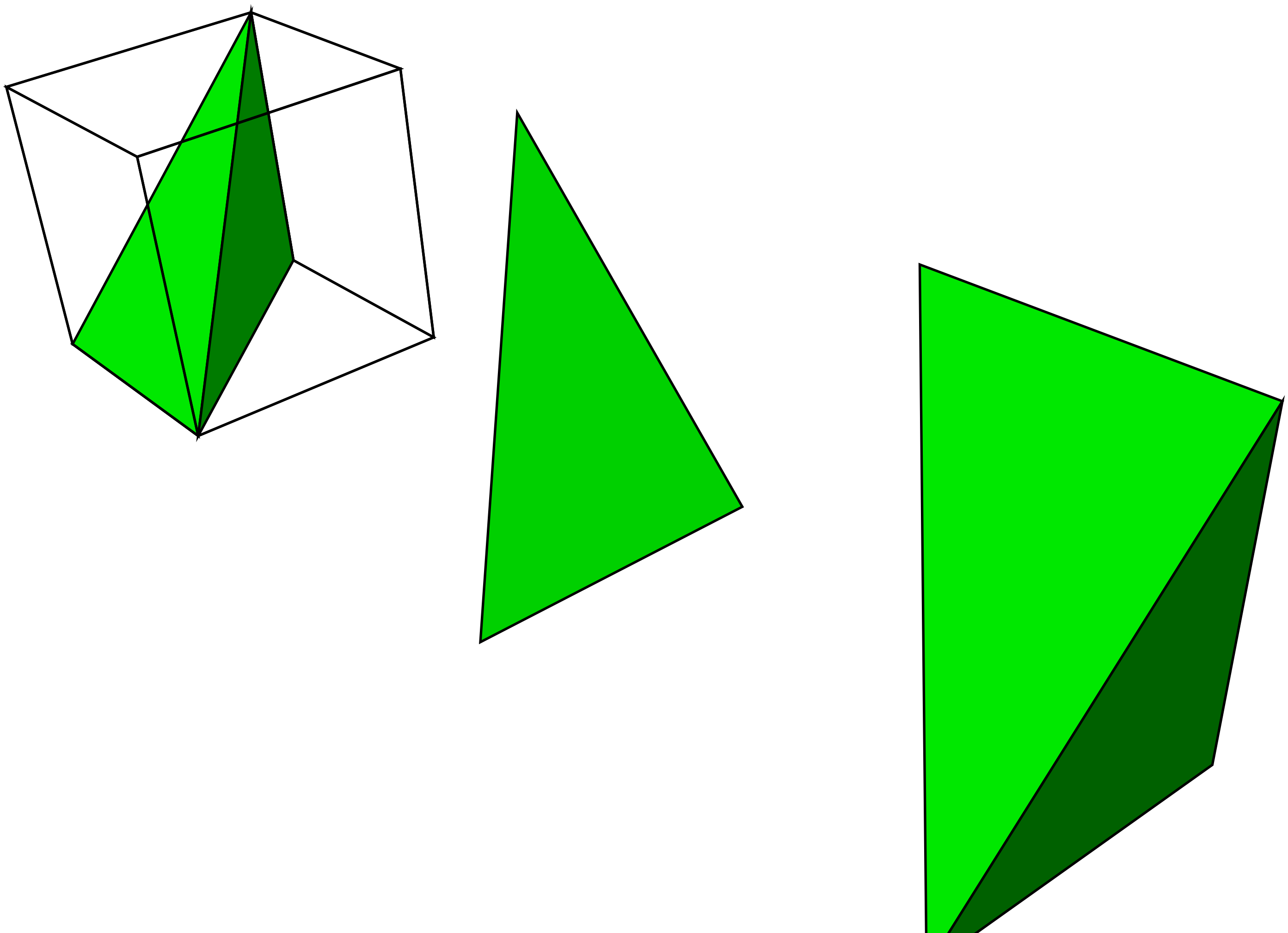

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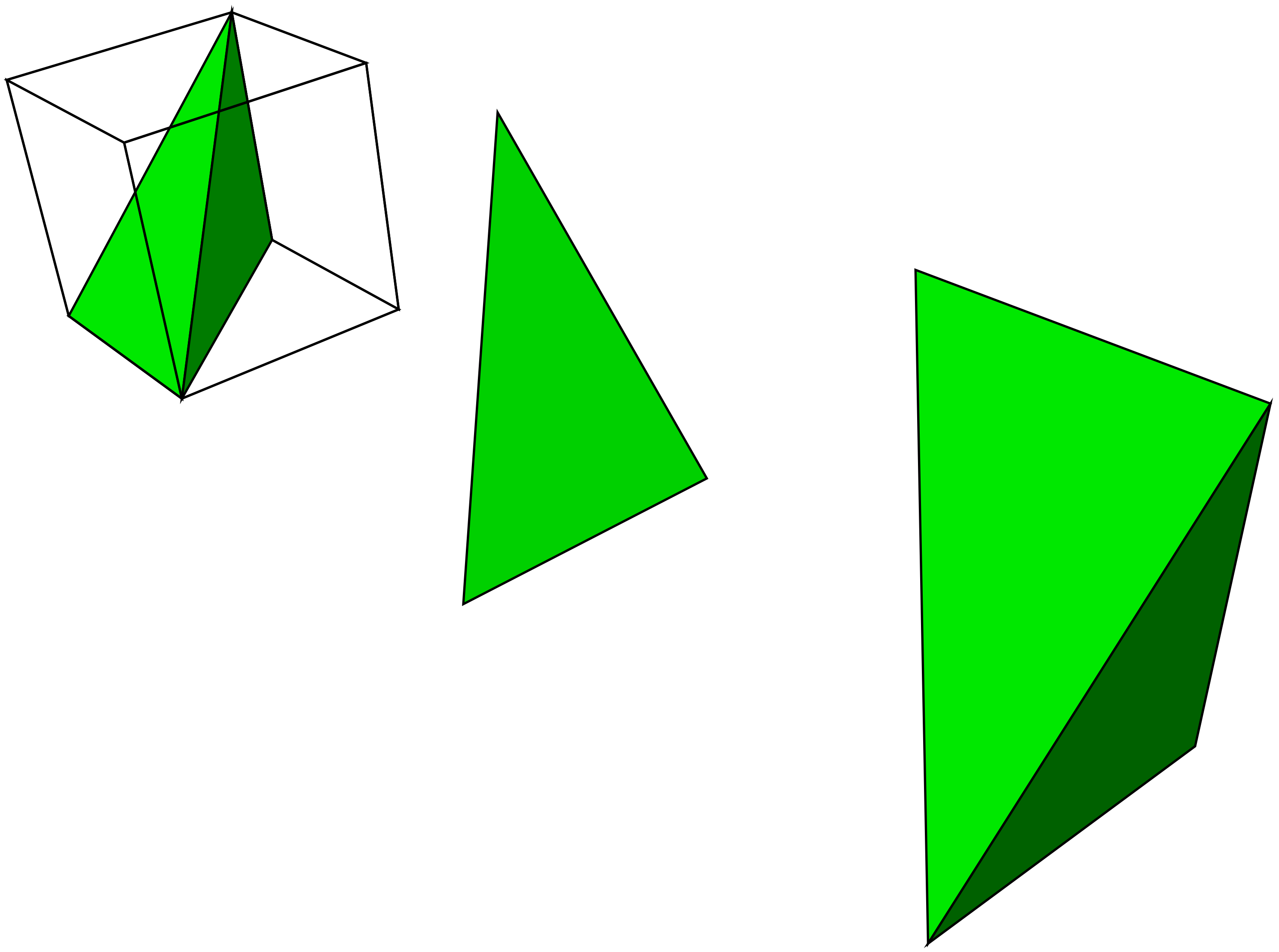

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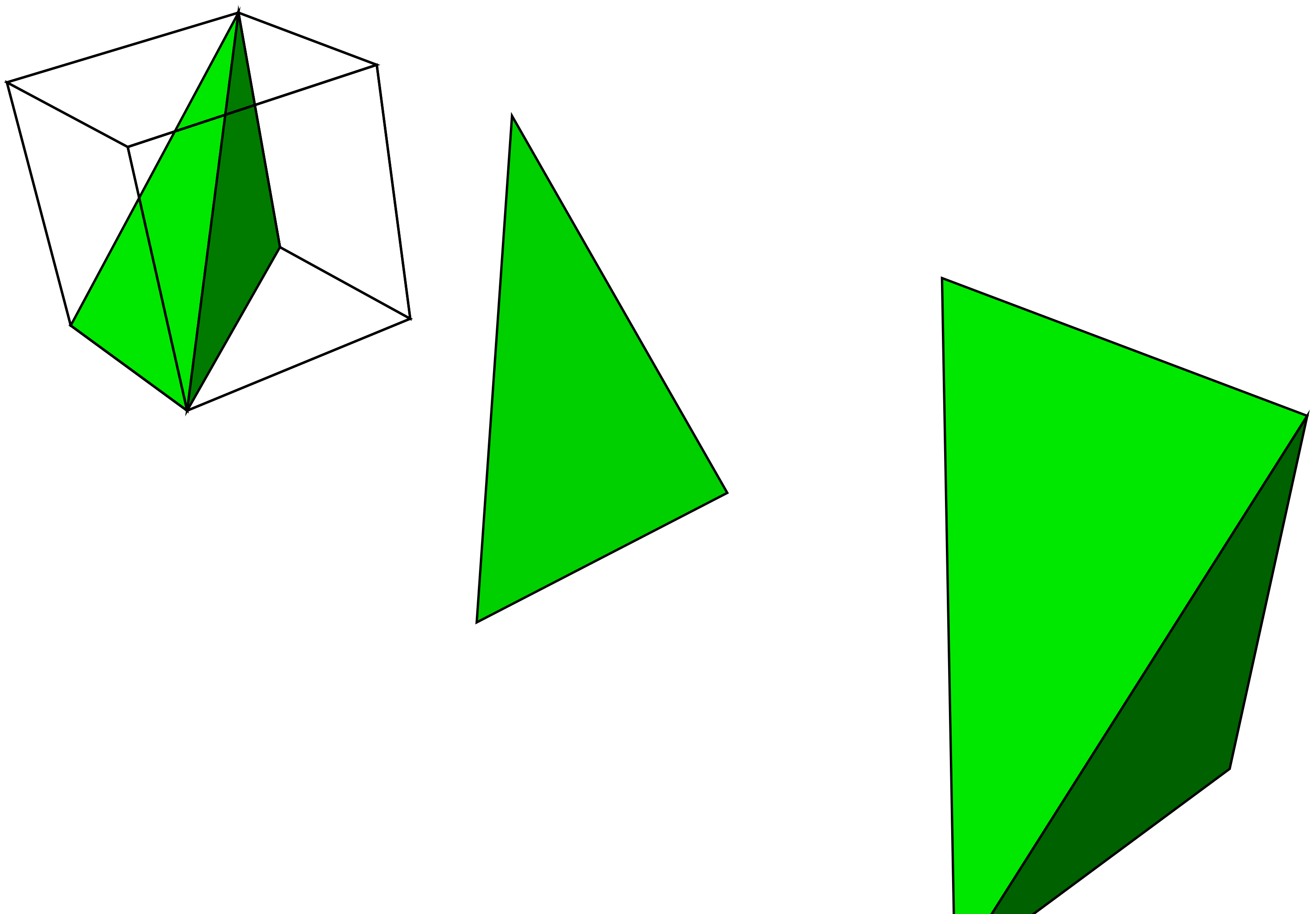

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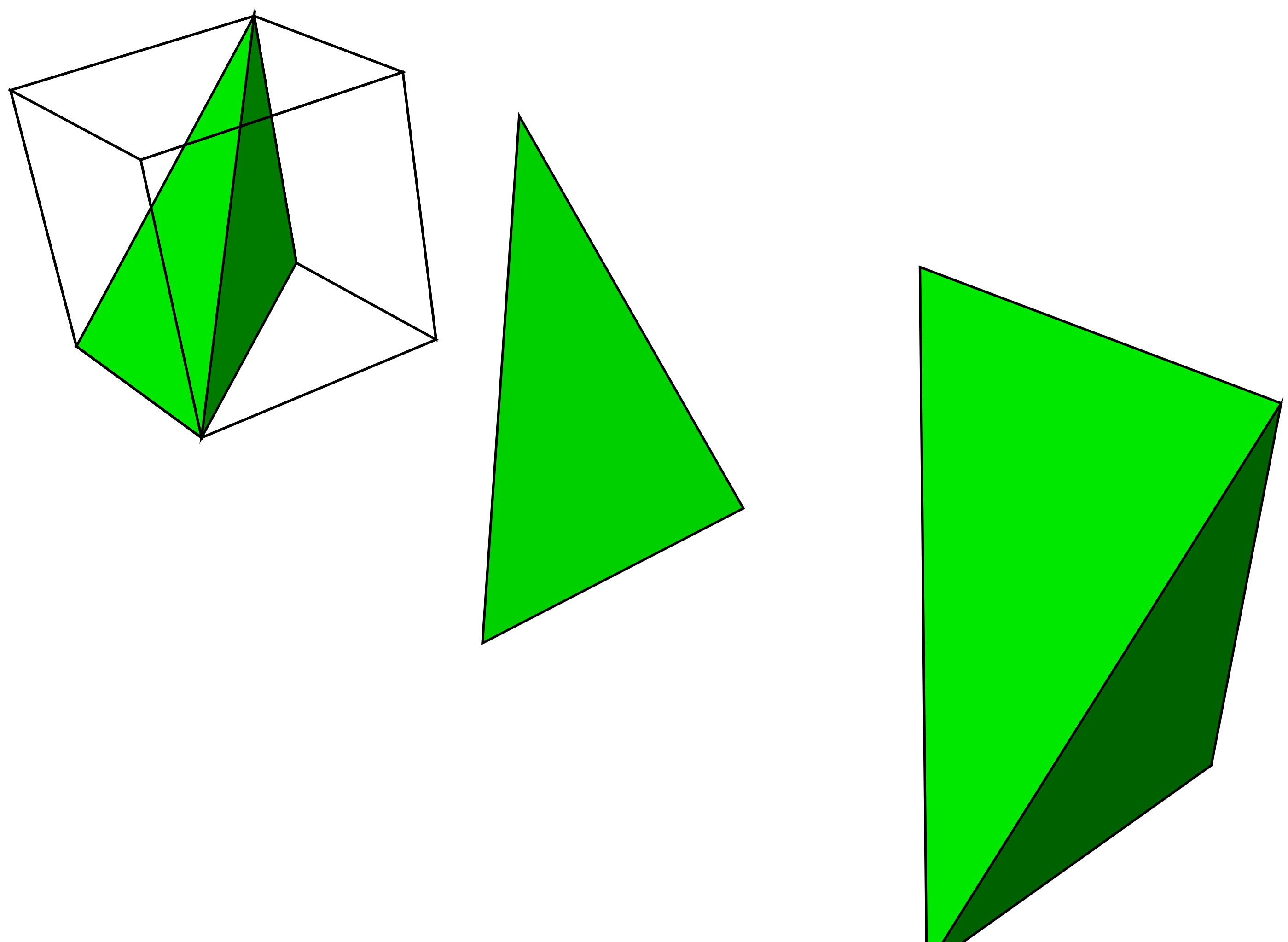

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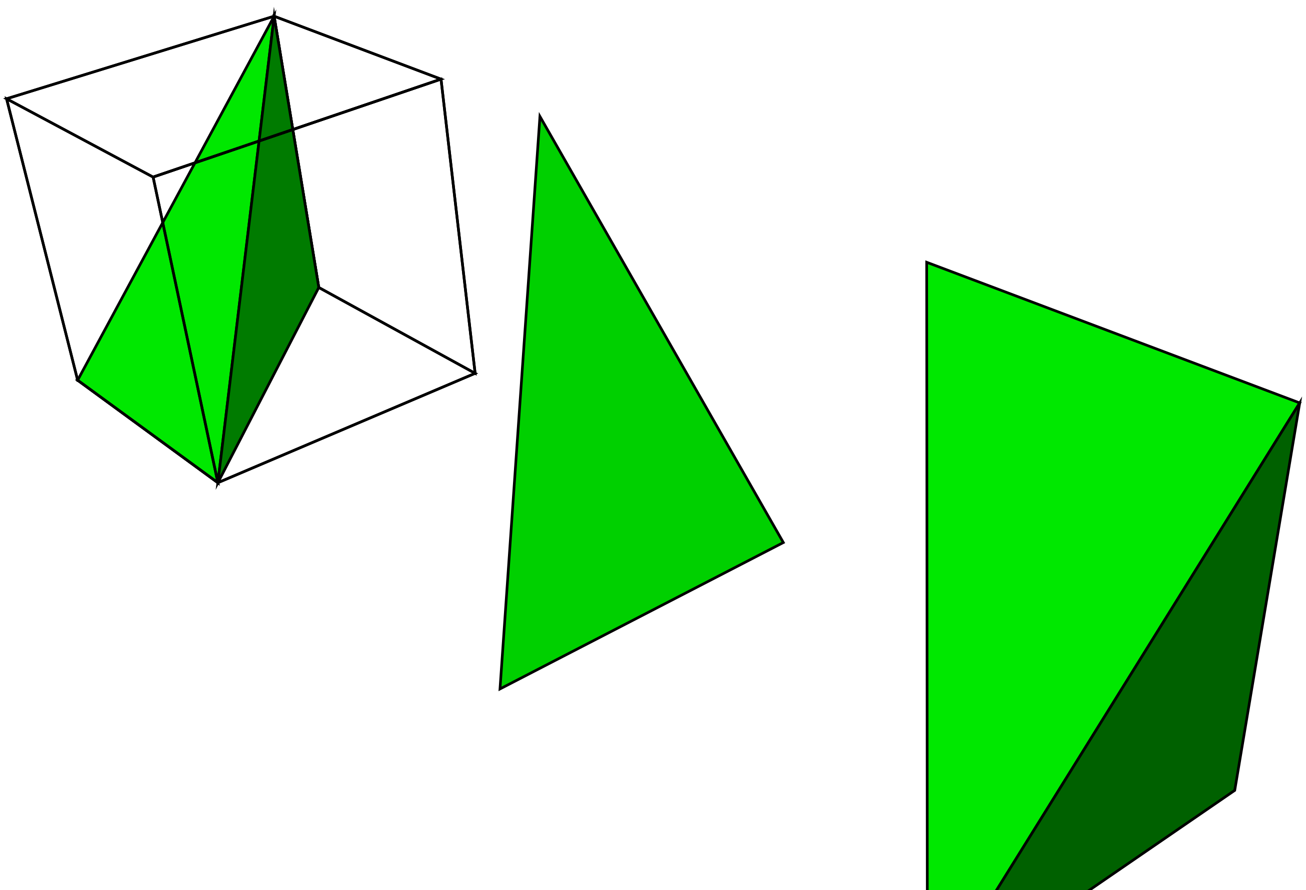

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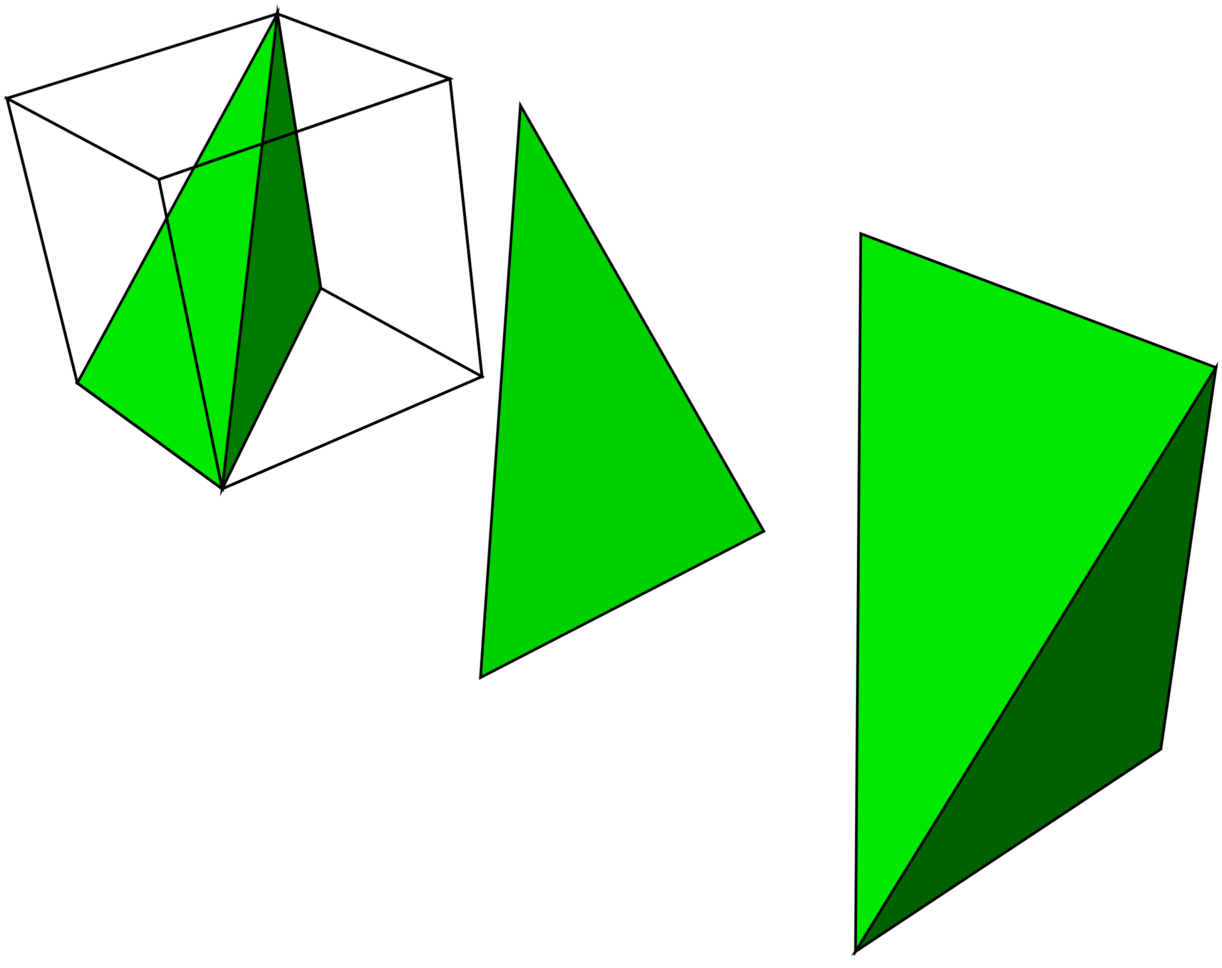

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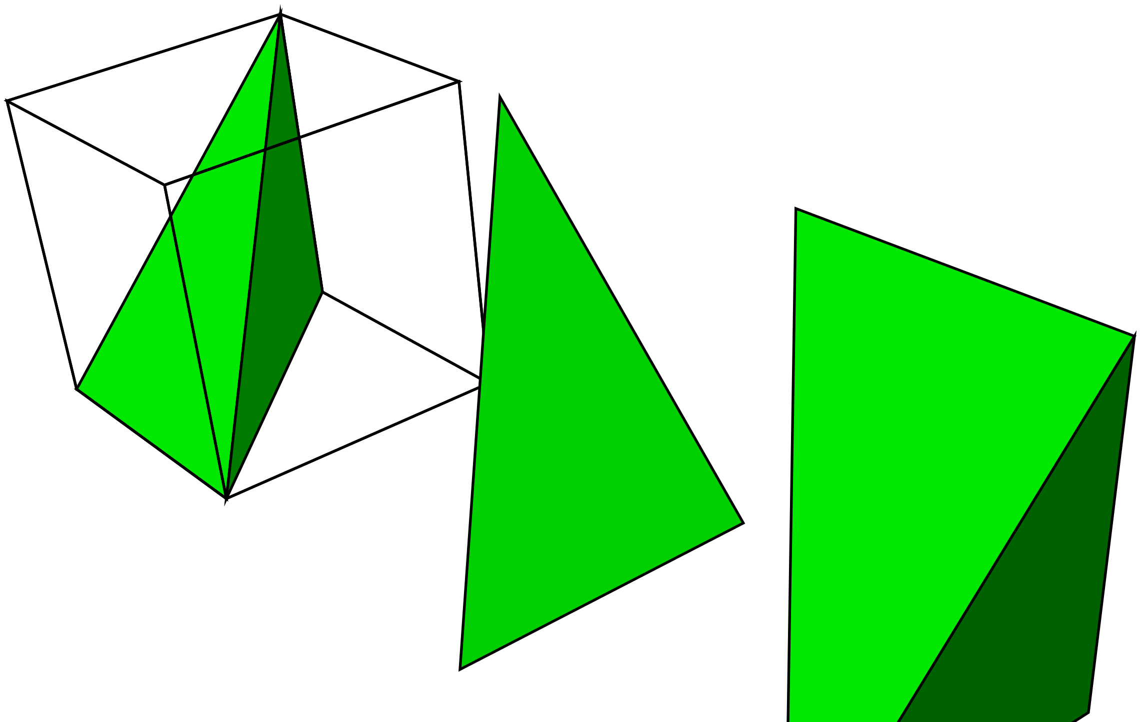

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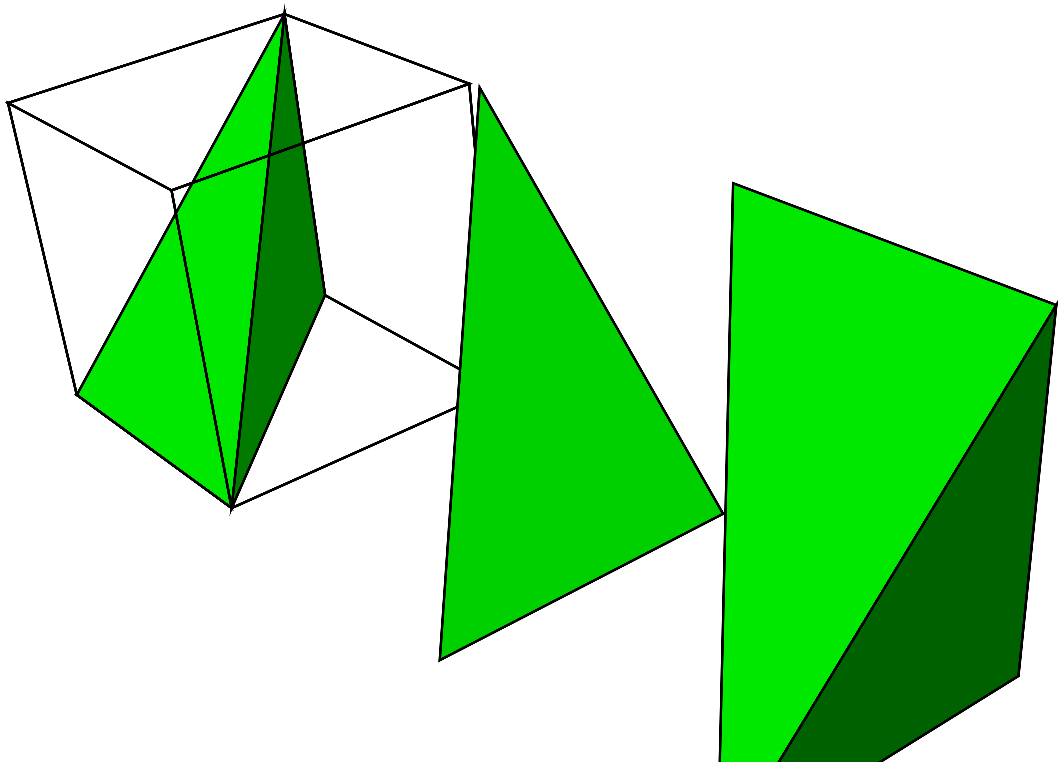

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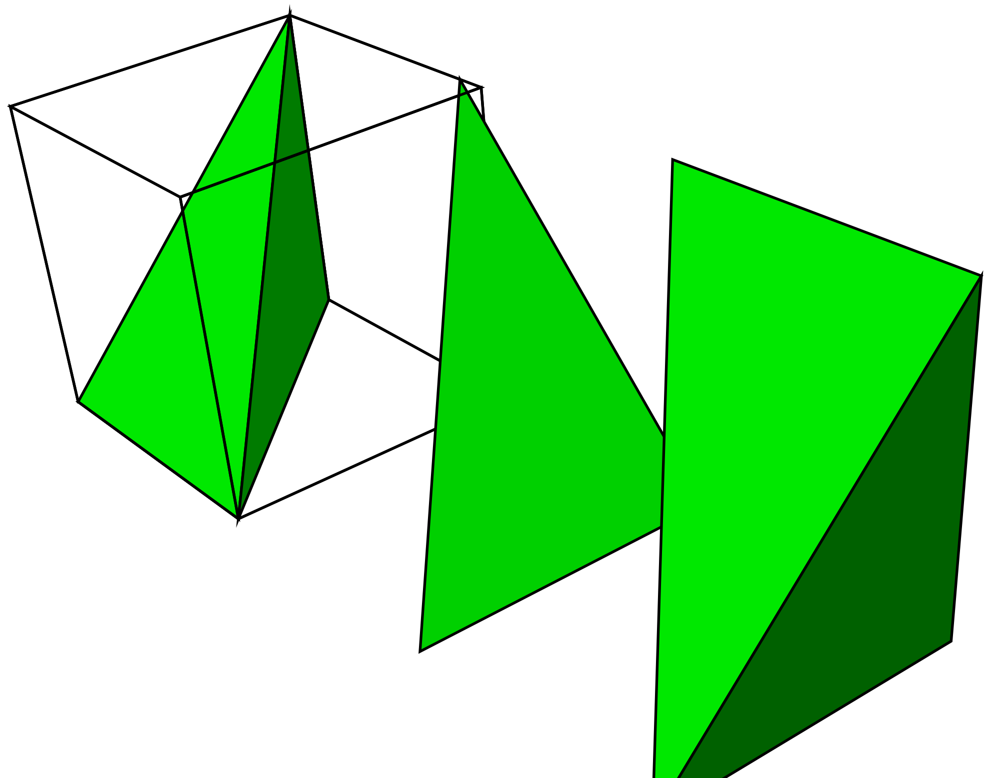

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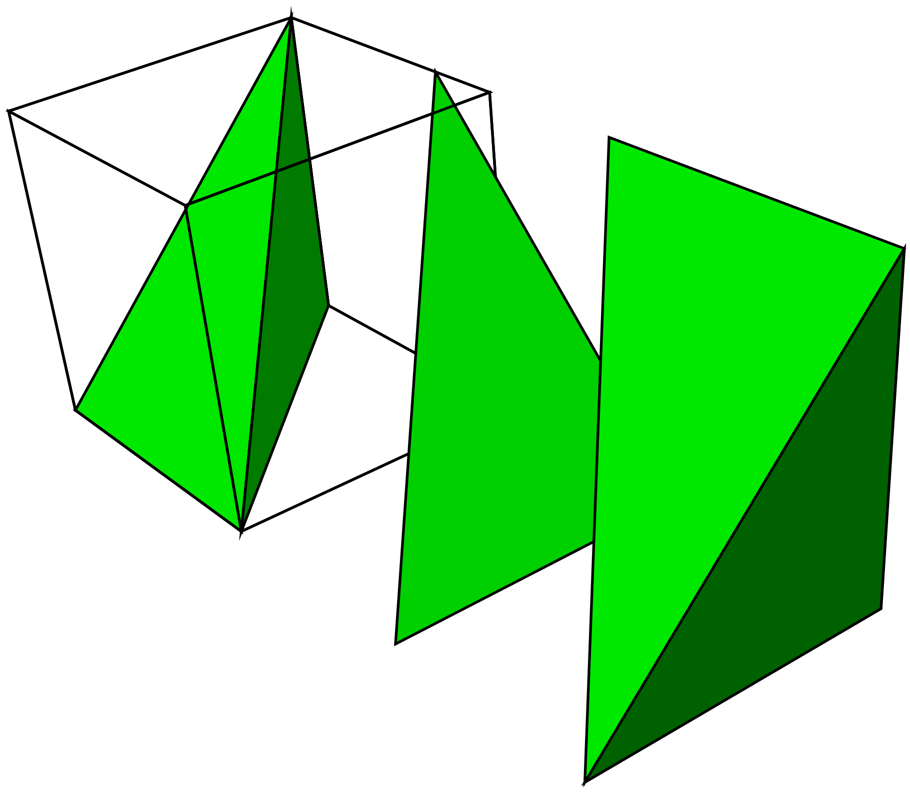

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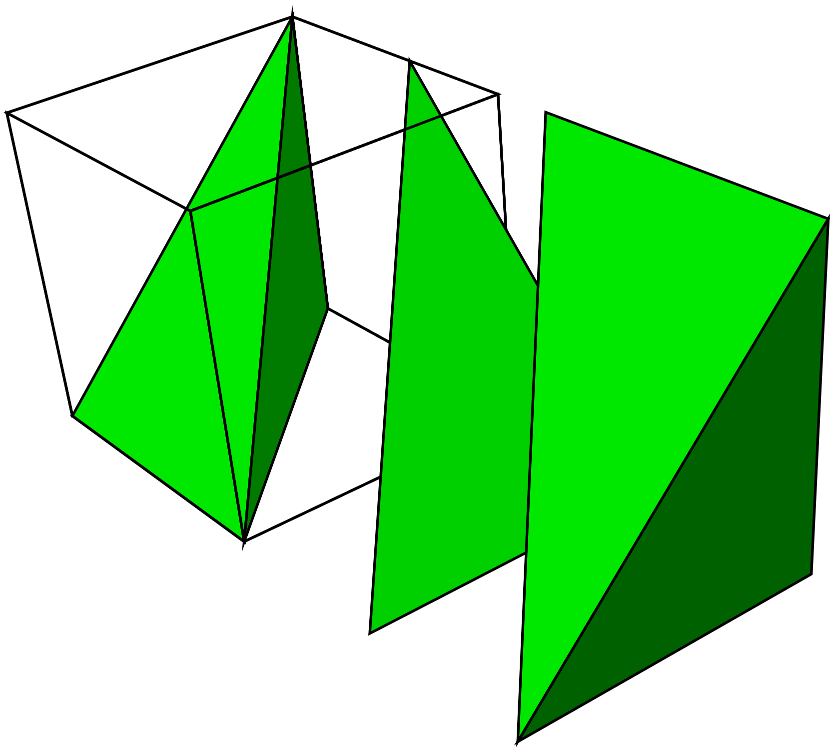

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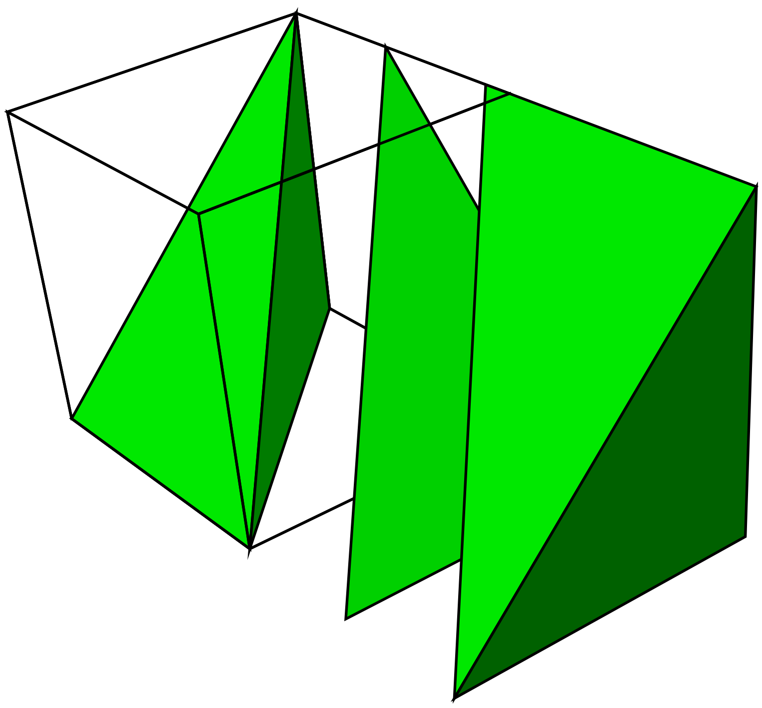

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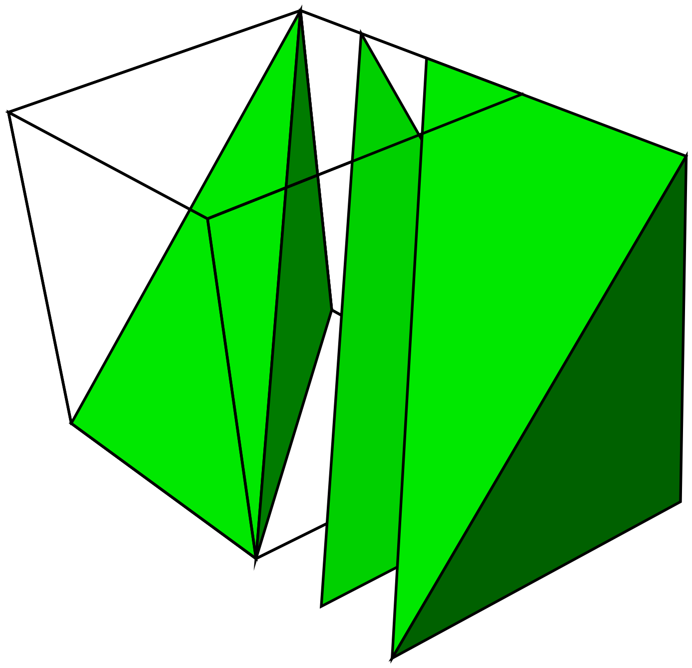

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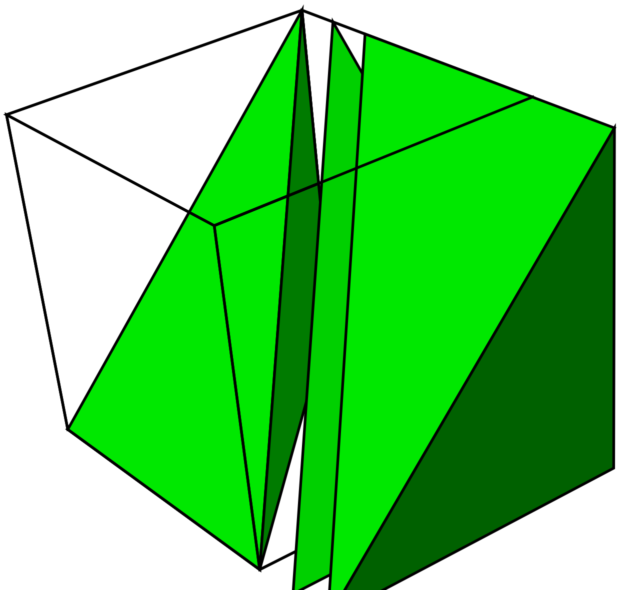

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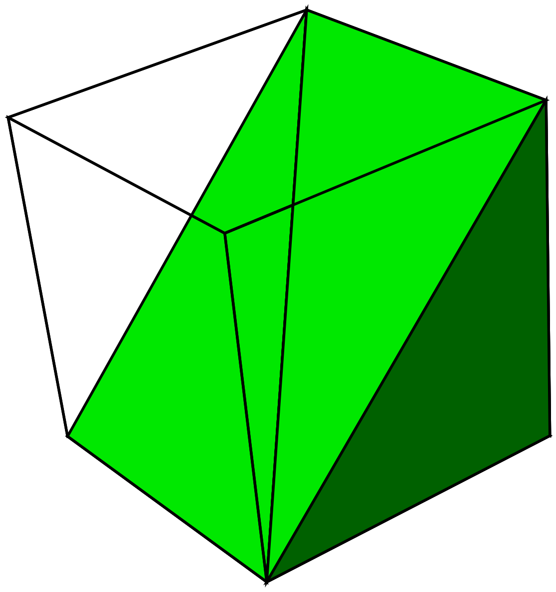
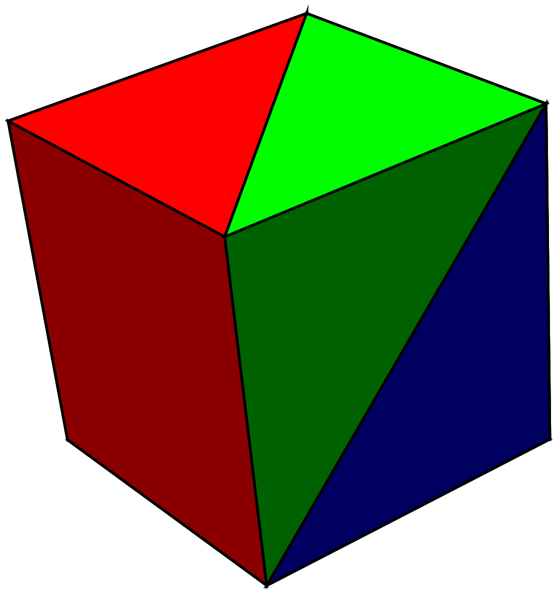
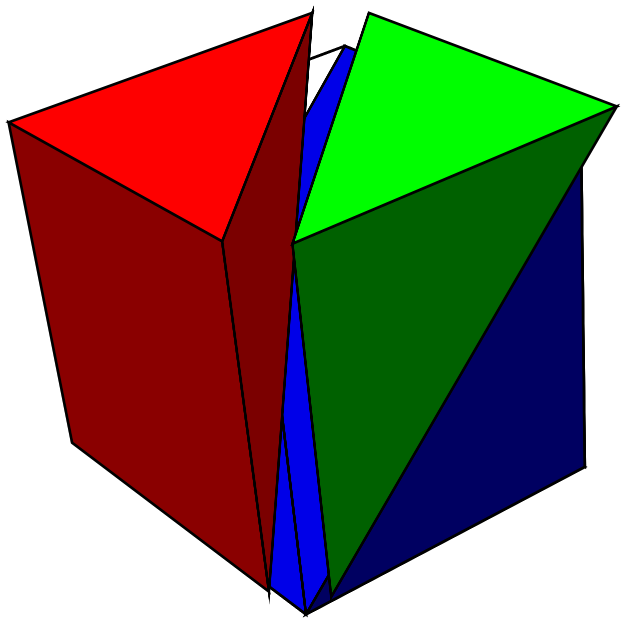


However a Ch’ien-tu is a Pieh-nao and a Yang-ma. Thus a Ch’ien-tu is also three Pieh-nao combined. Let’s watch.

Now since a Ch’ien-tu is half the volume of the unit cube and a Yang-ma combined with a Pieh-nao is also half the volume of the unit cube; Liu Hui concluded that one of each of the three objects could be combined to create a unit cube.



A Ch’ien-tu, Pieh-nao, and Yang-ma can be combined to create a single Unit Cube. Let’s see how this works.



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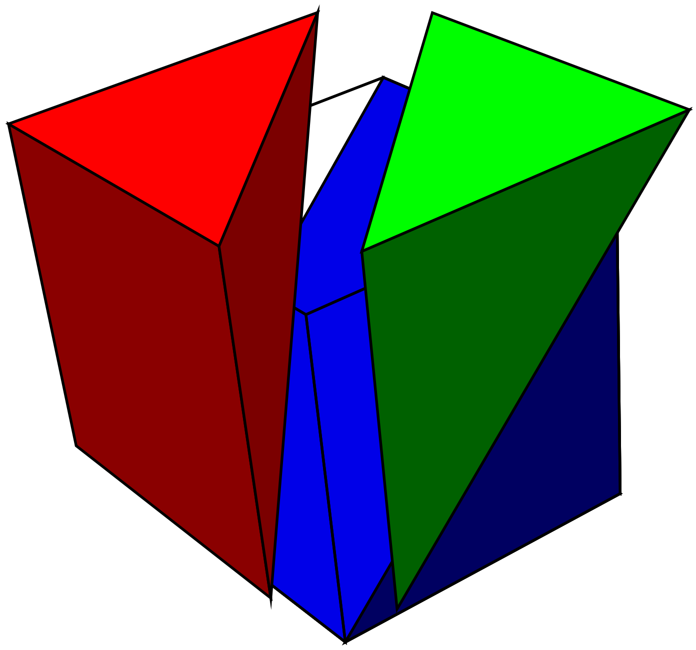

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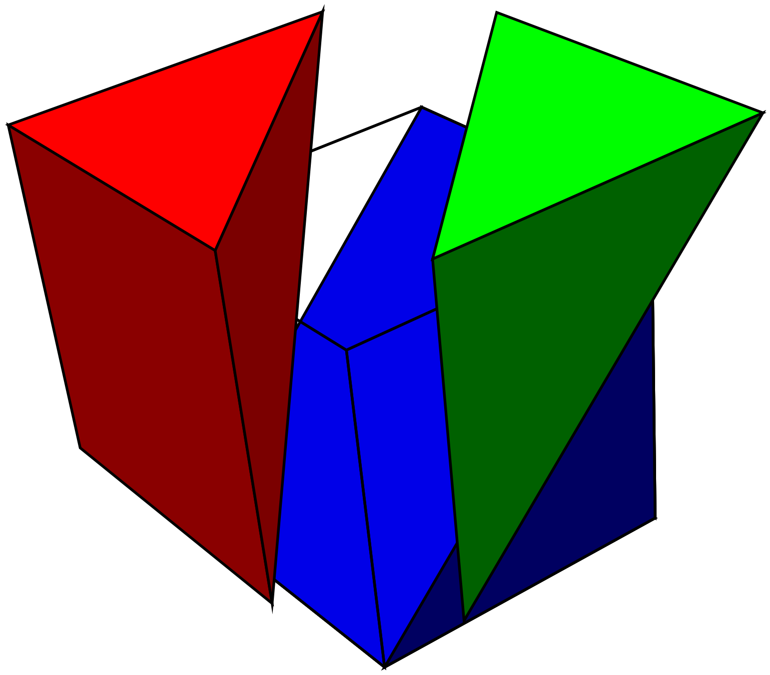

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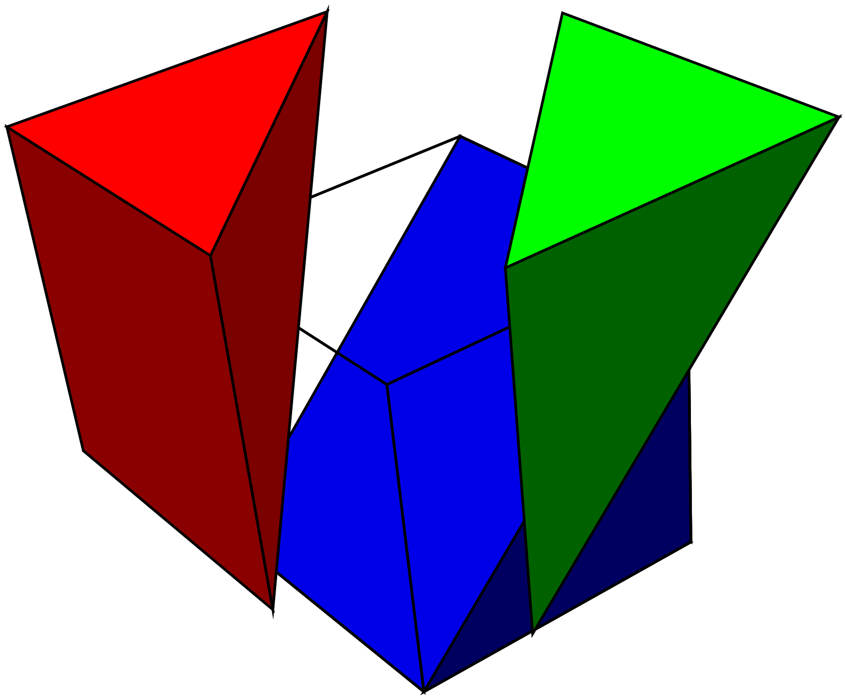

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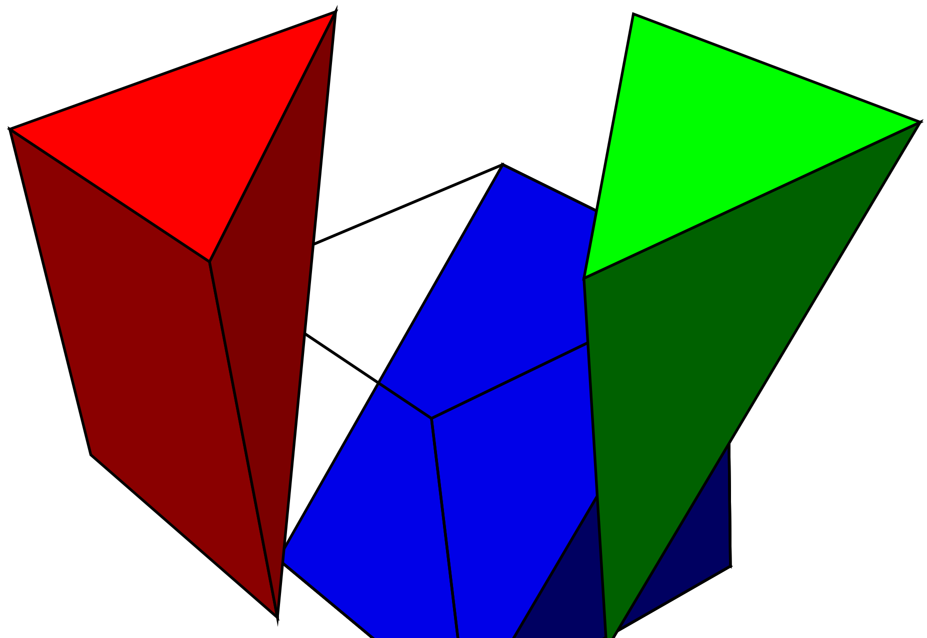

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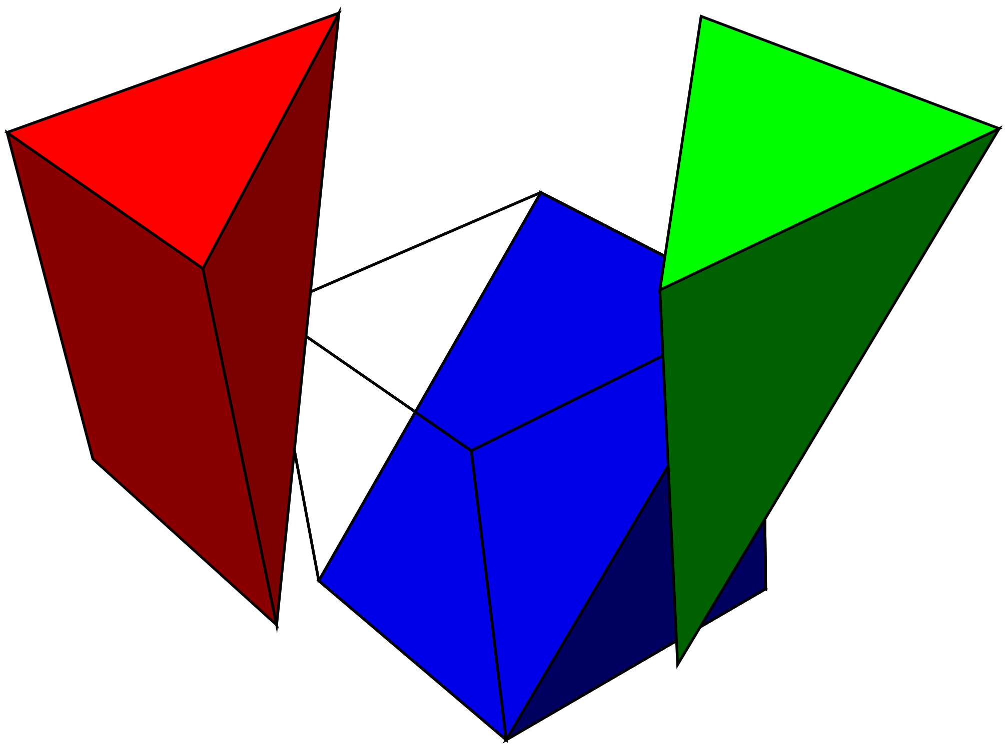

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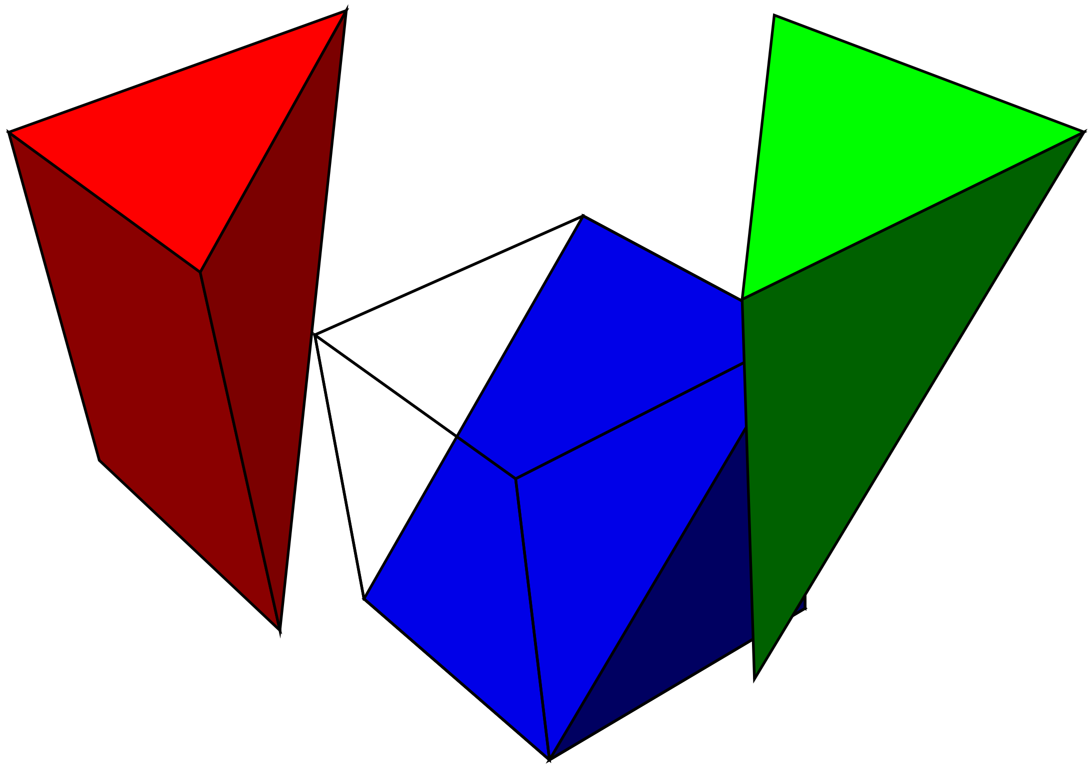

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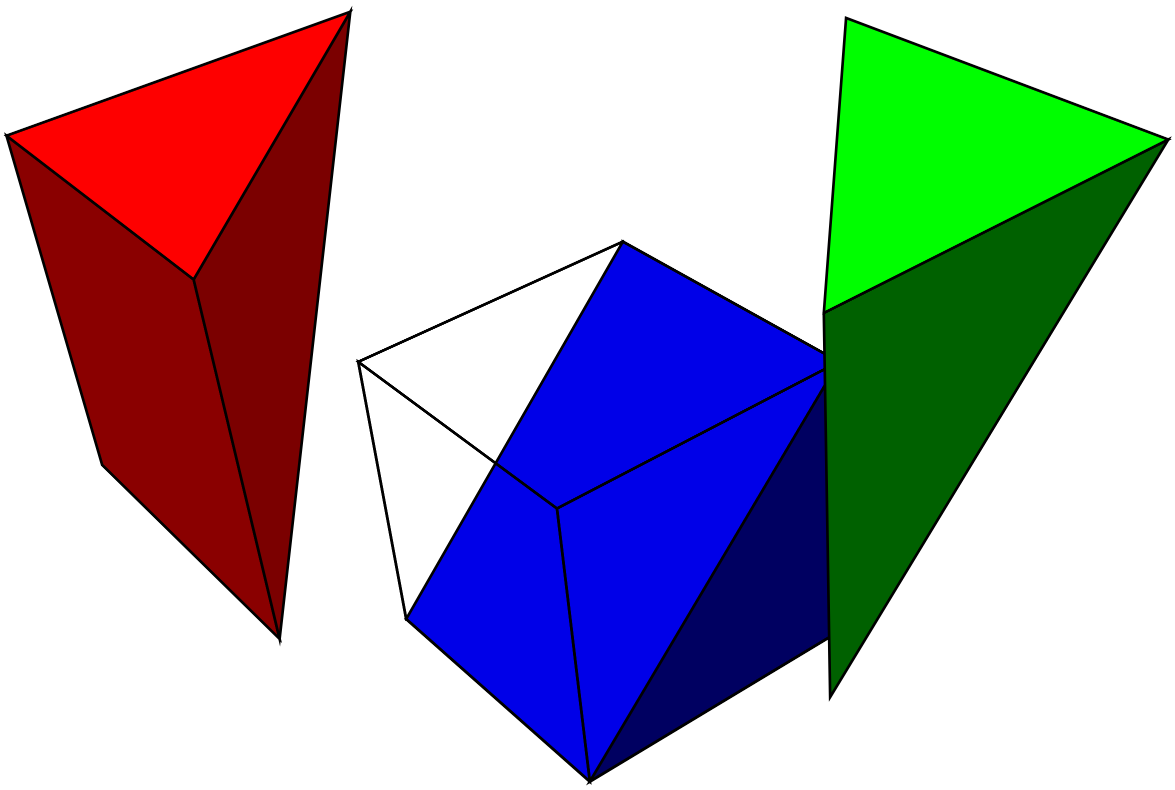

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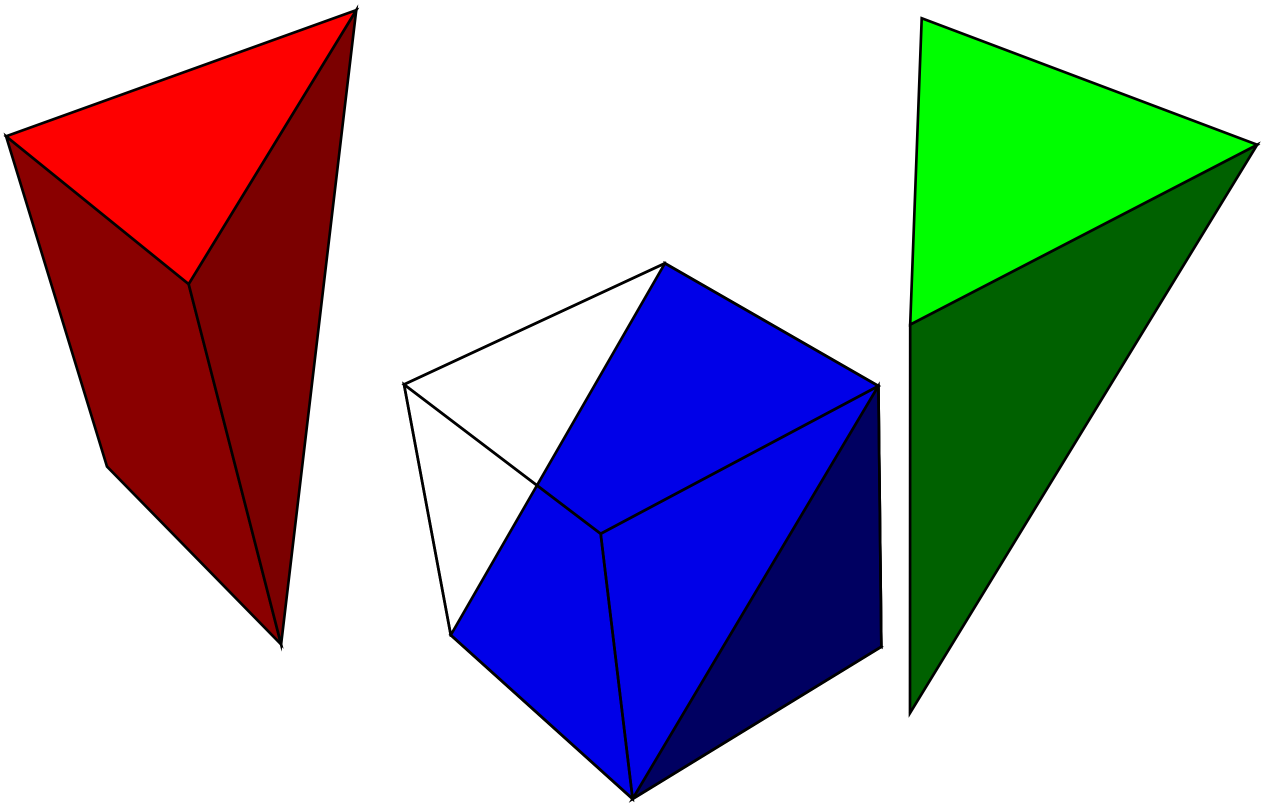

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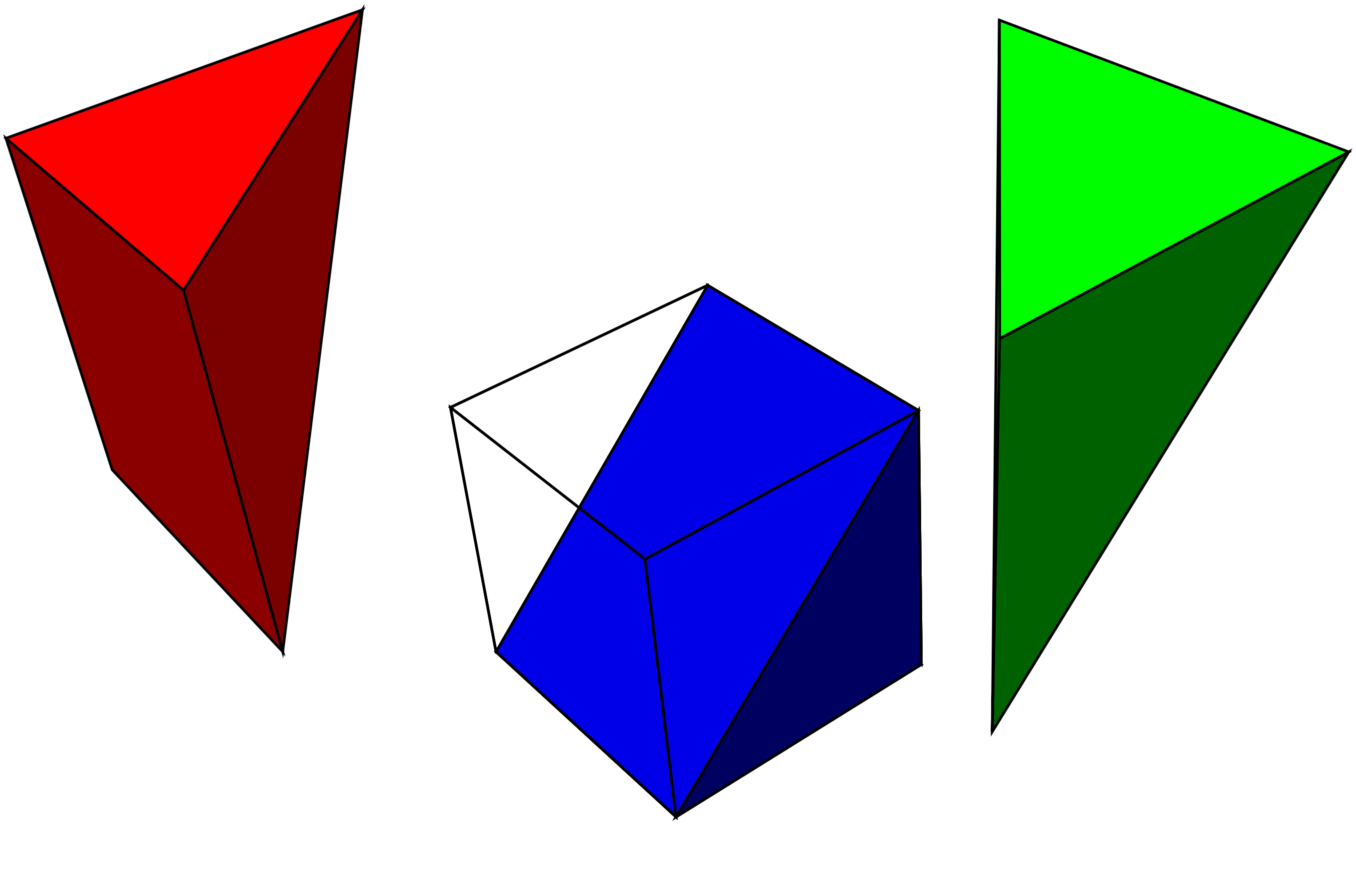

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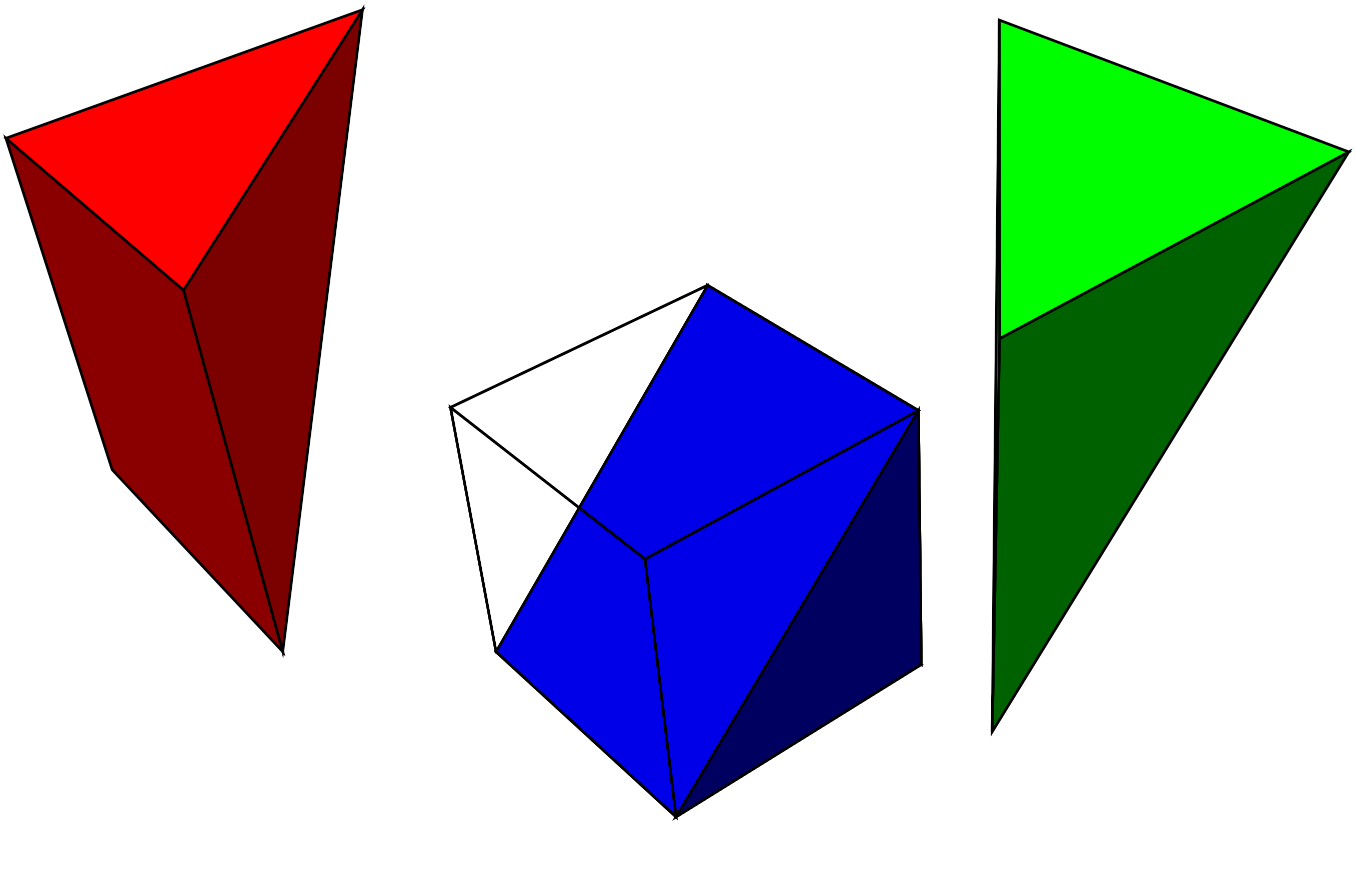

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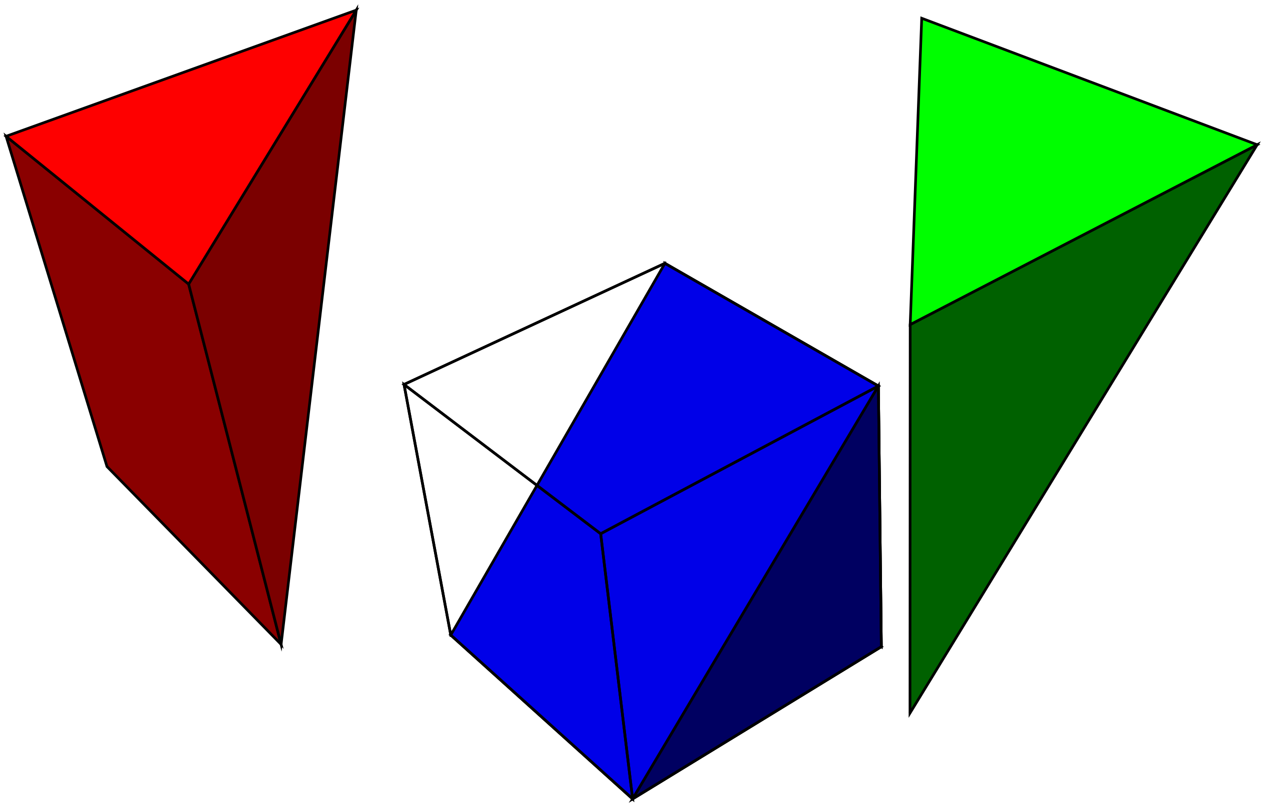

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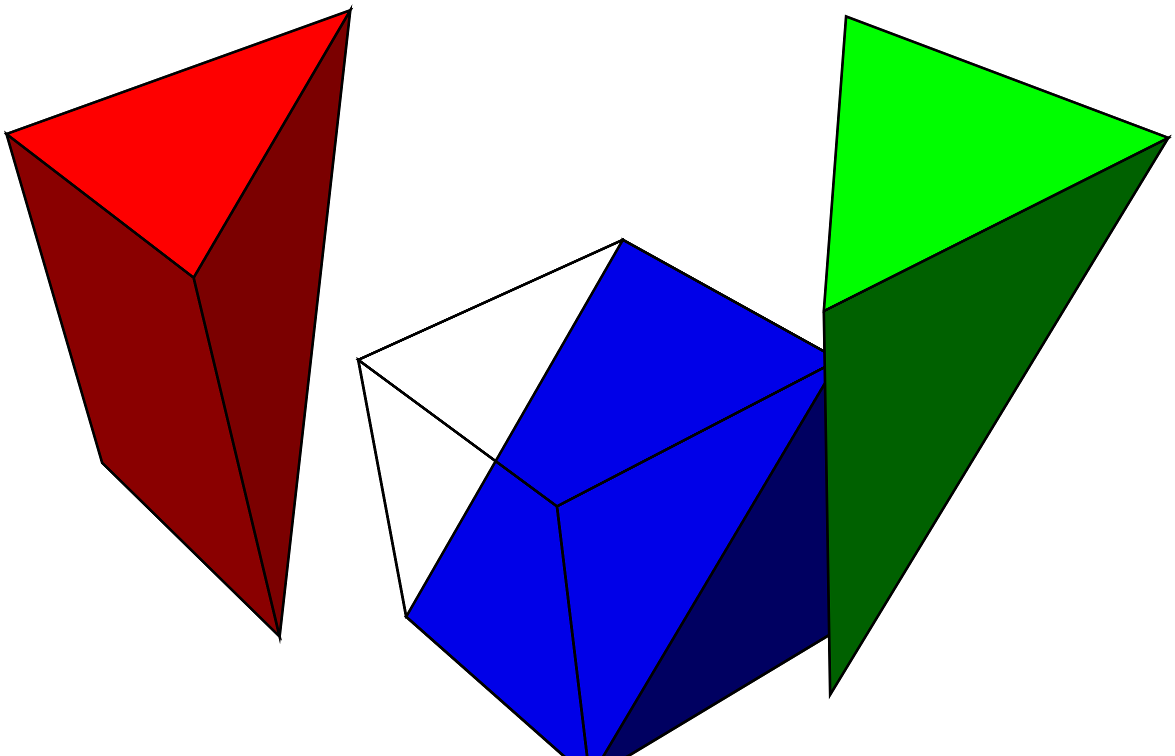

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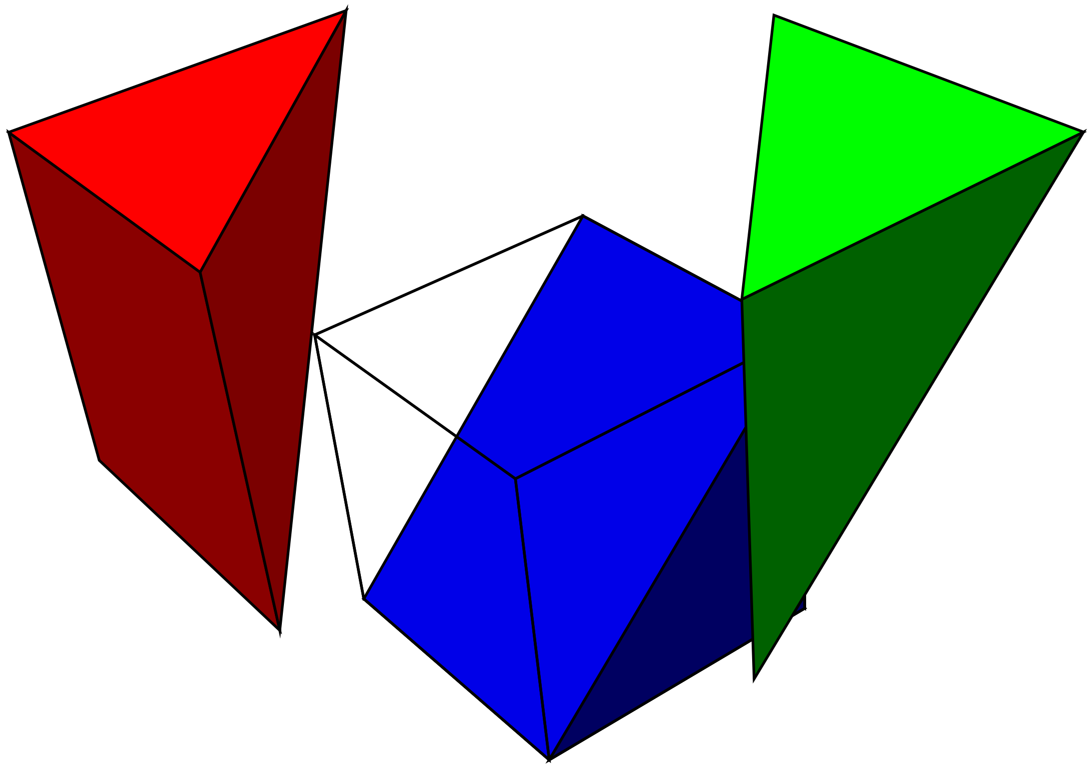

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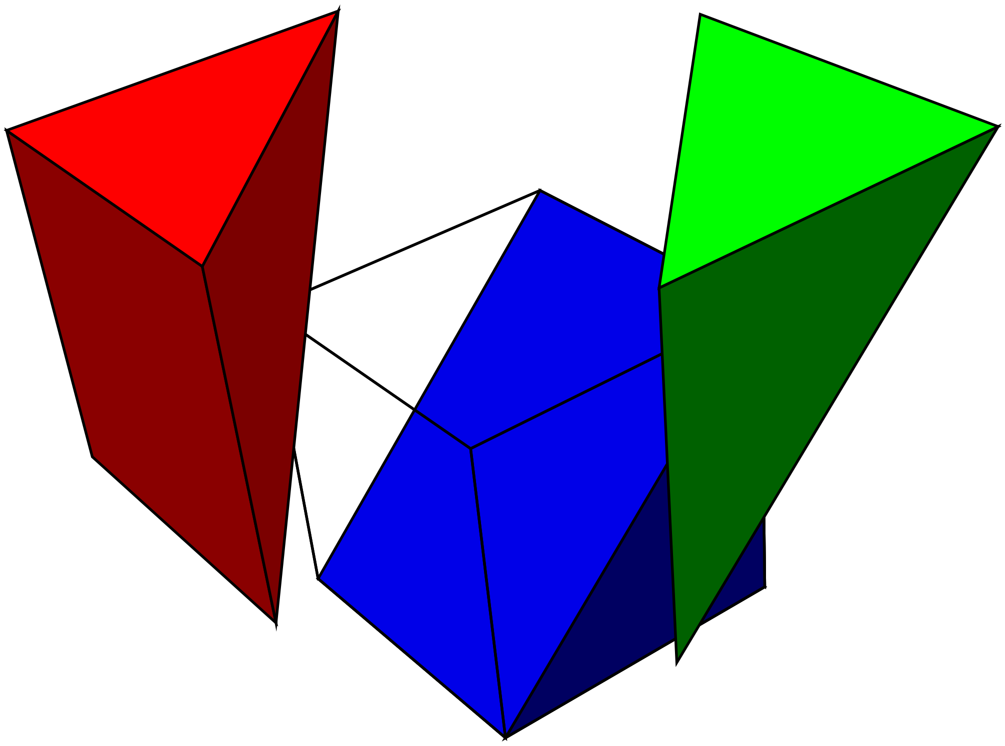

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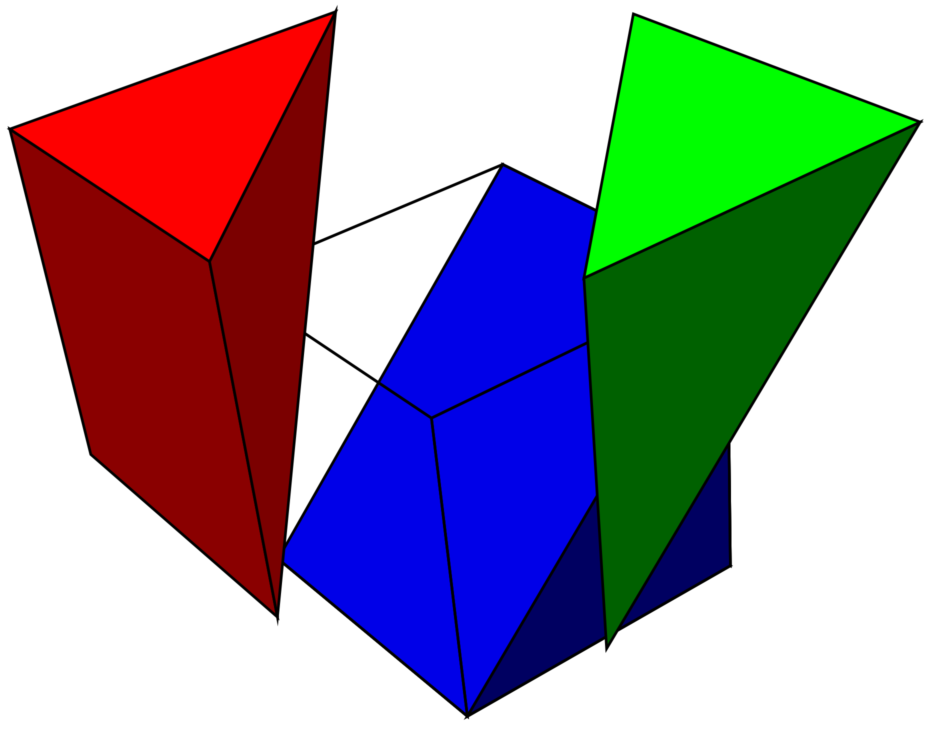

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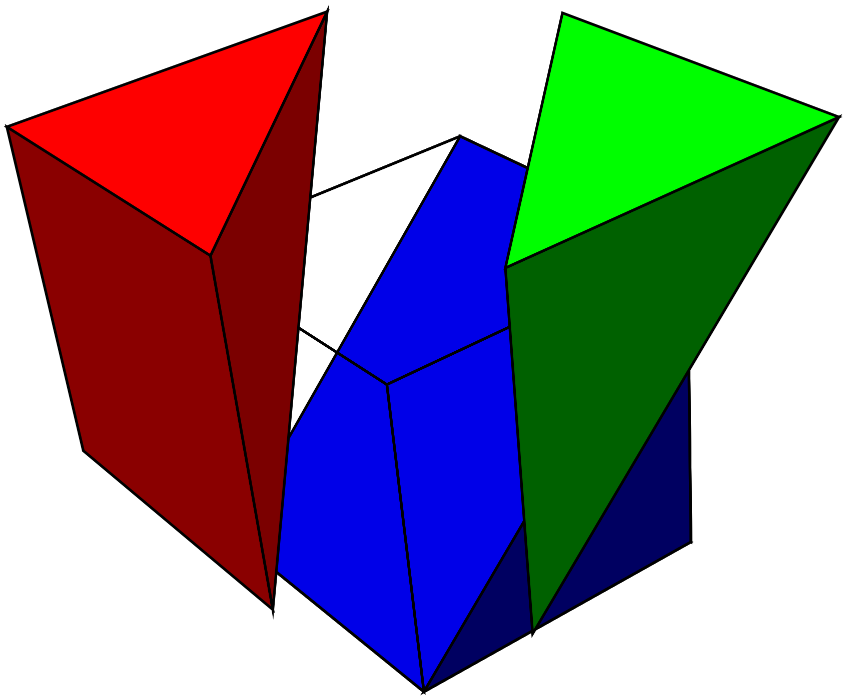

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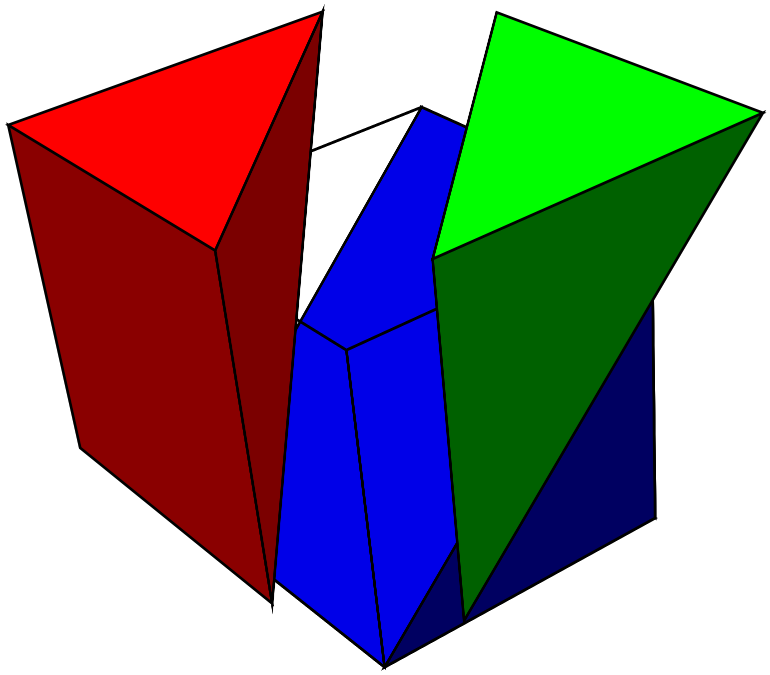

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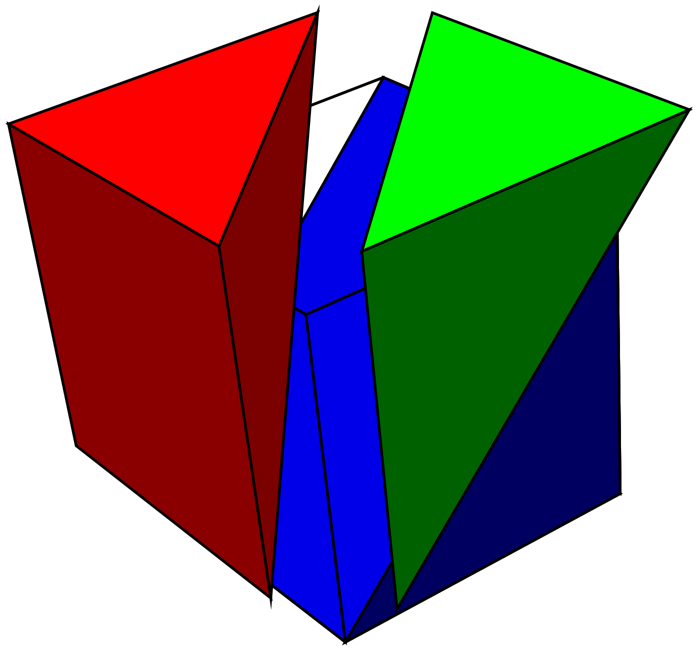

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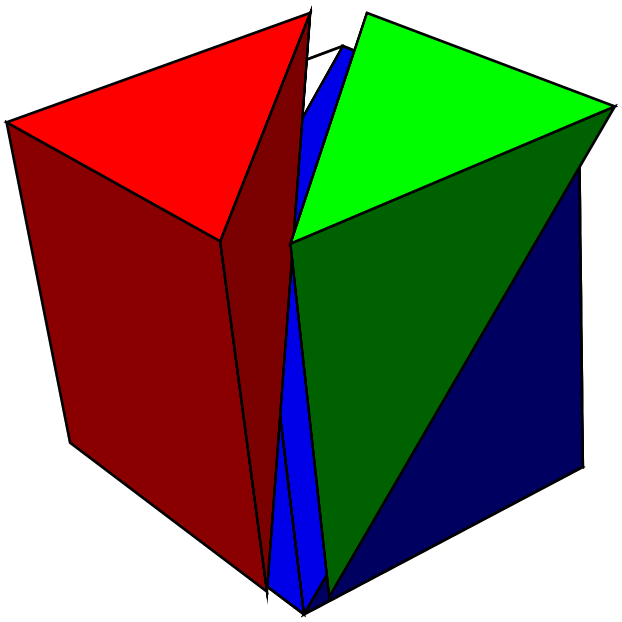

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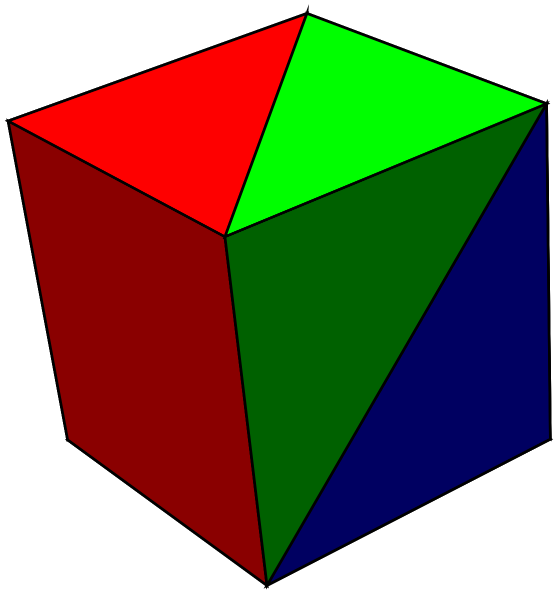
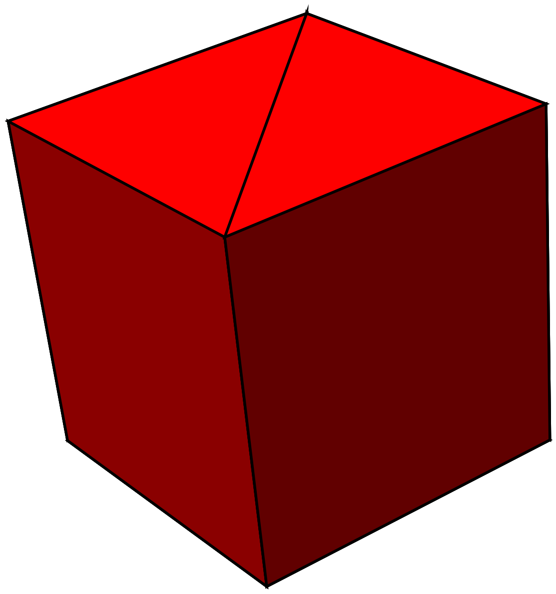
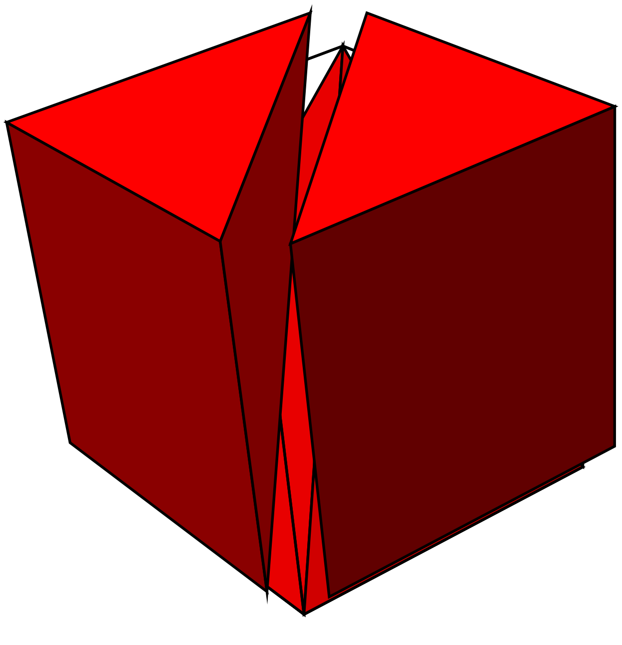


A Ch’ien-tu, Pieh-nao, and Yang-ma can be combined to create a single Unit Cube. Let’s see how this works.

Finally as a Yang-ma is equal to one third the volume of the unit cube, Liu Hui determined that three Yang-ma should be able to combine to form a unit cube. This would be further proof that a Yang-ma is one third the volume of the unit cube and that the Pied-nao is one sixth the volume as it is half the volume of a Yang-ma.



Three Yang-ma can be combined to create a single Unit Cube. Let’s see how this works.



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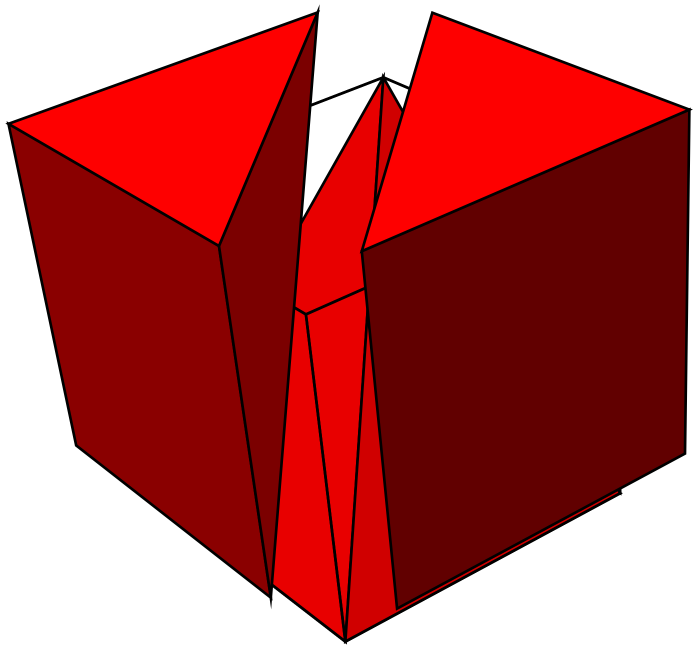
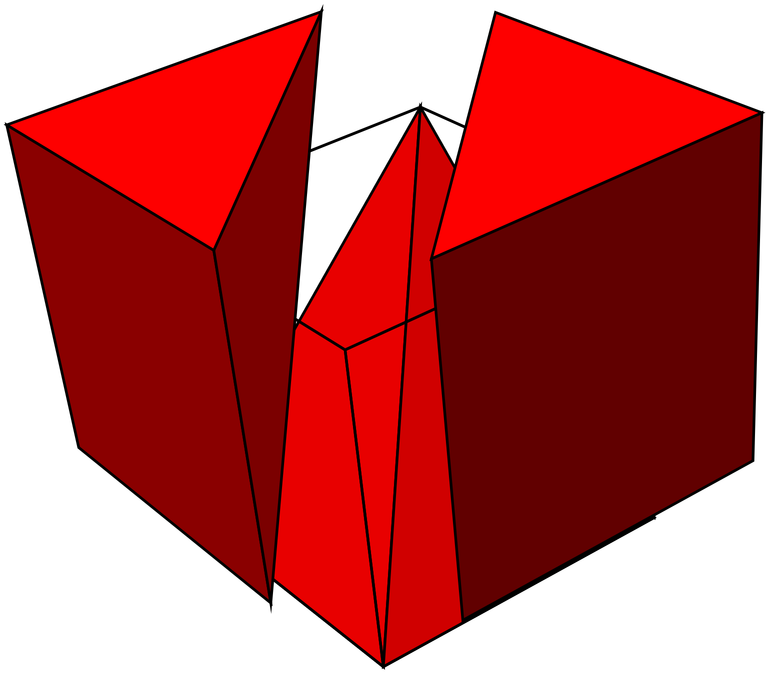


Three Yang-ma can be combined to create a single Unit Cube. Let’s see how this works.



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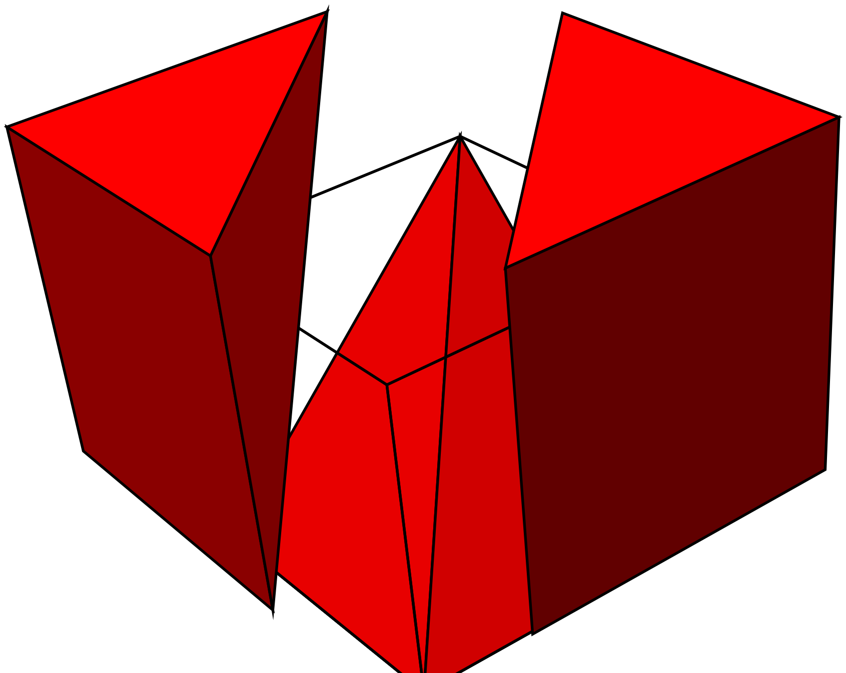

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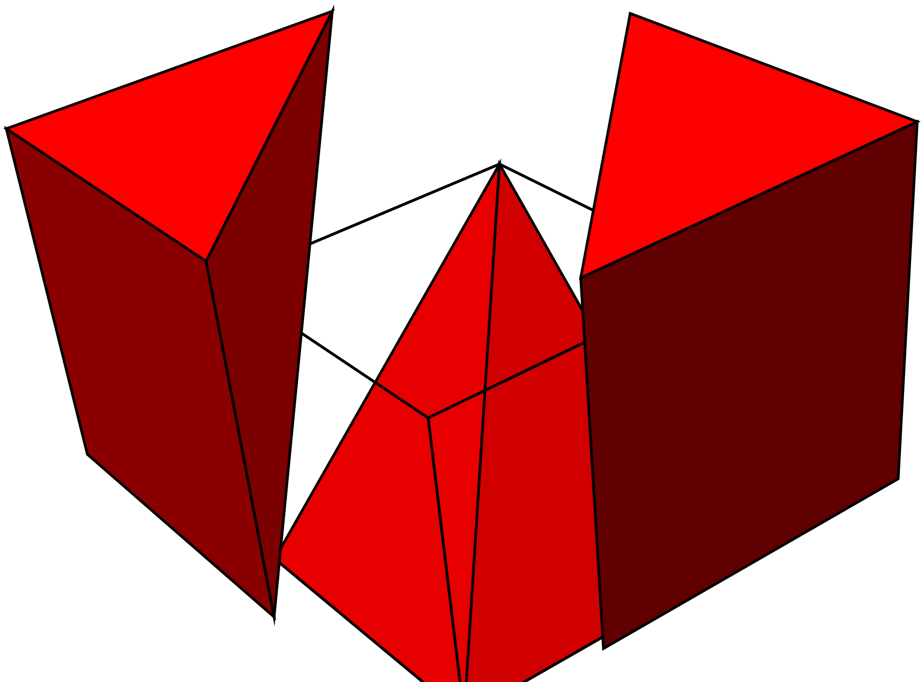

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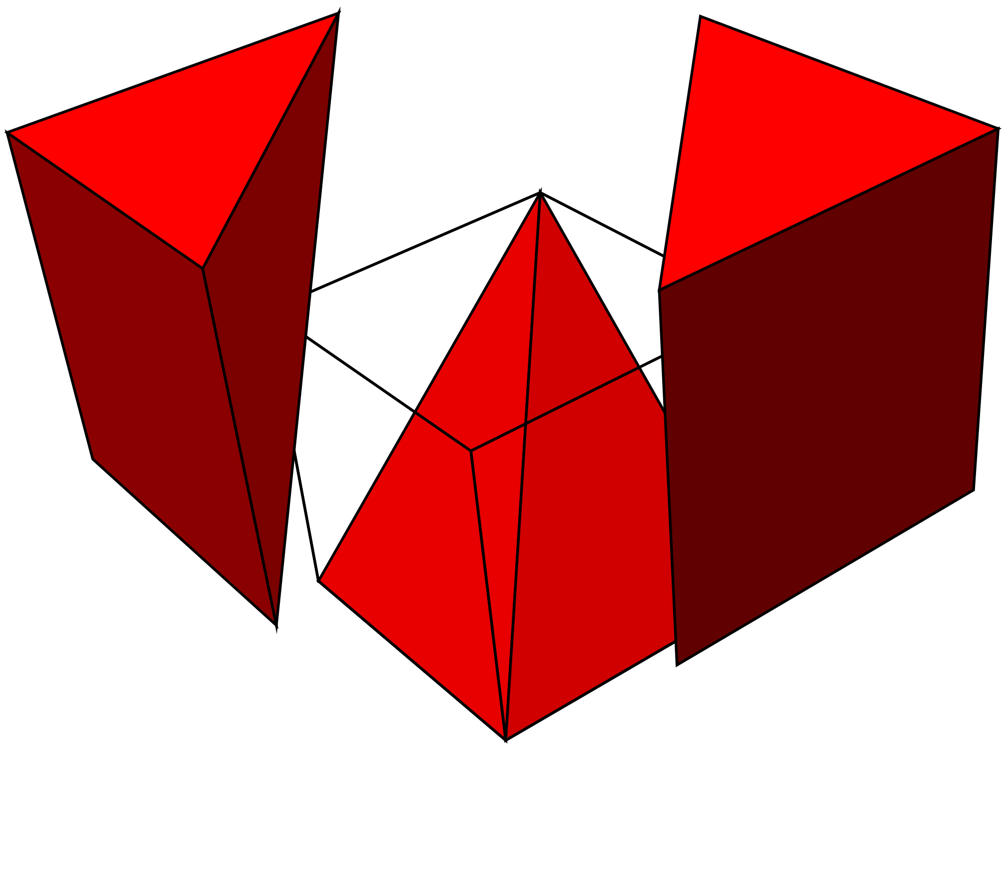

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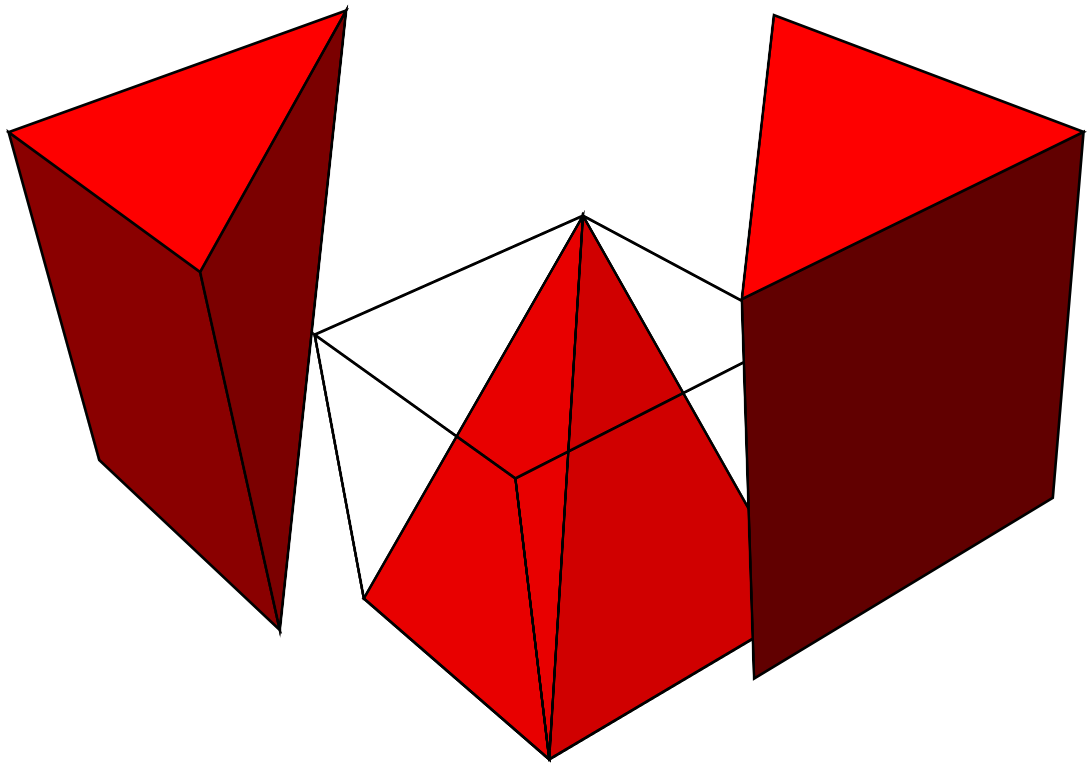

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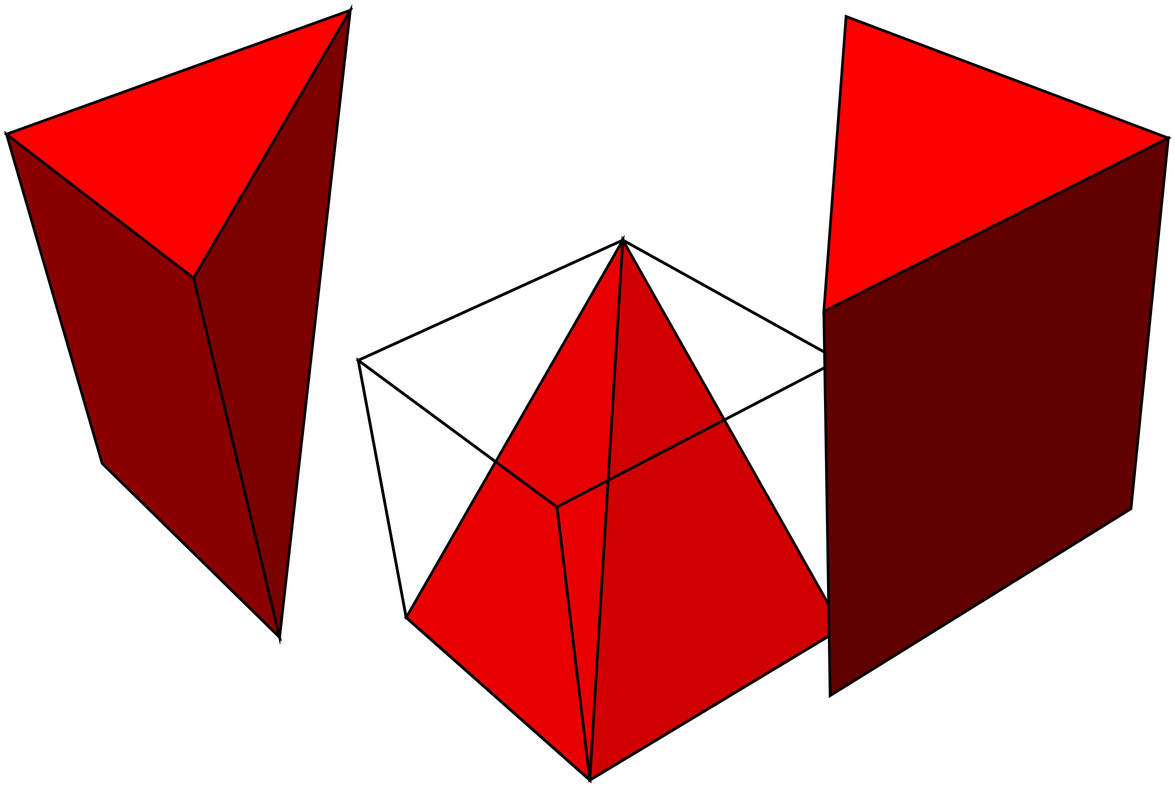

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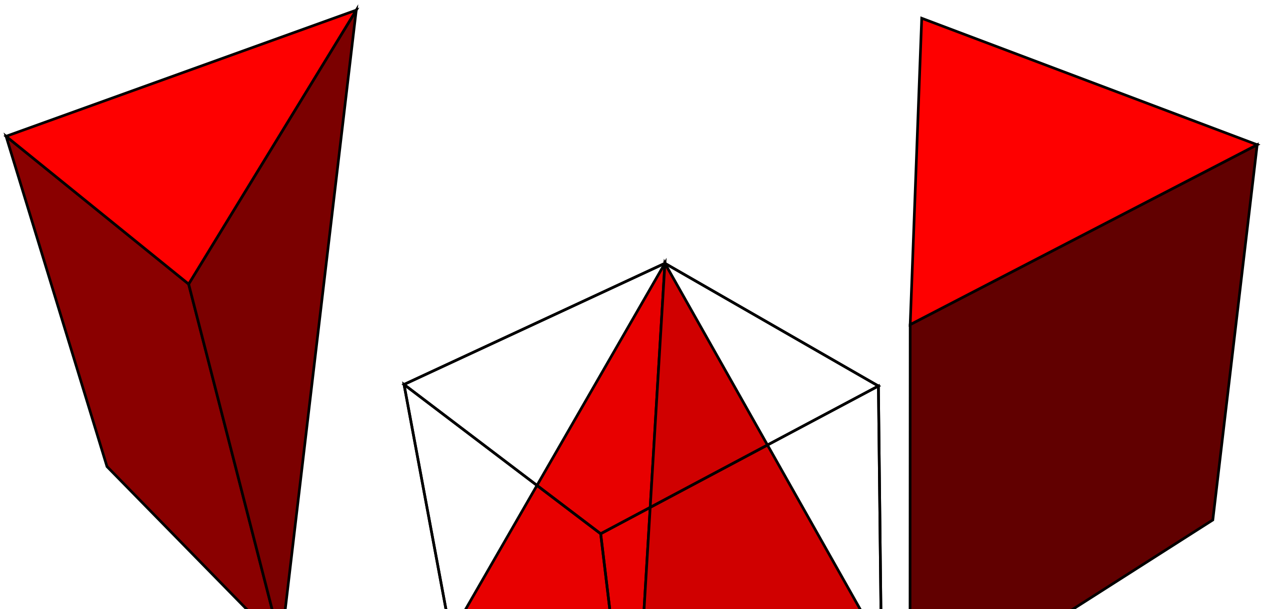

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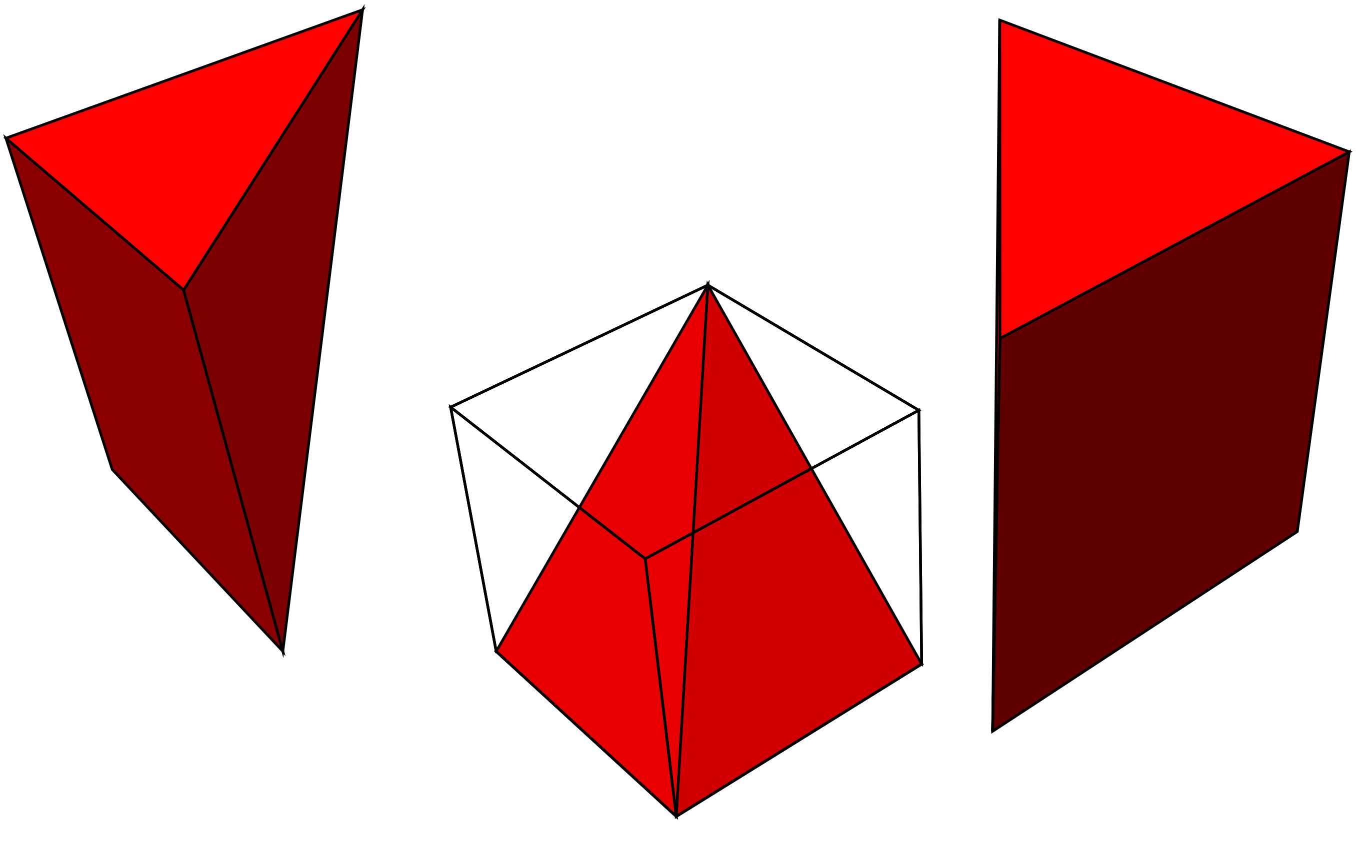

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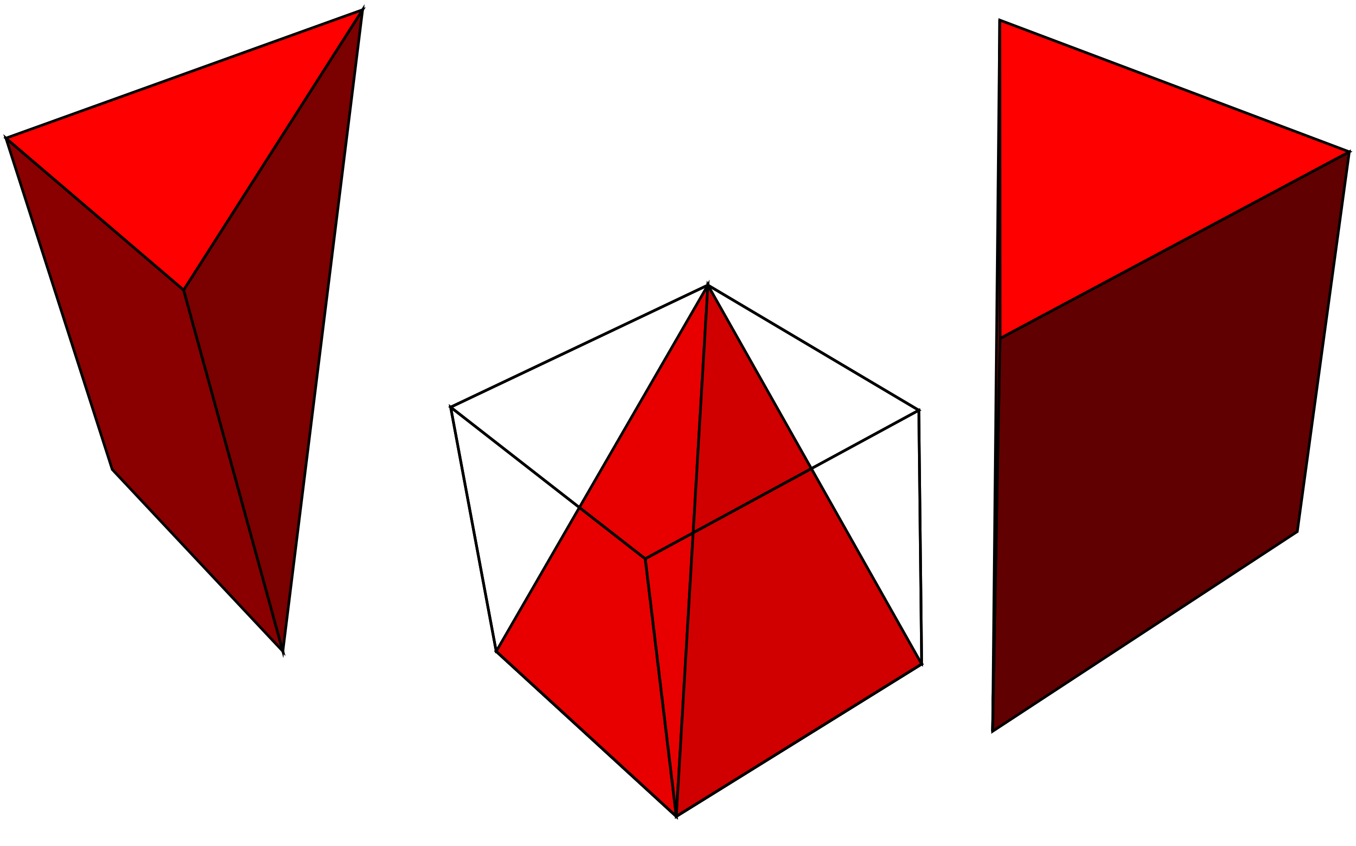

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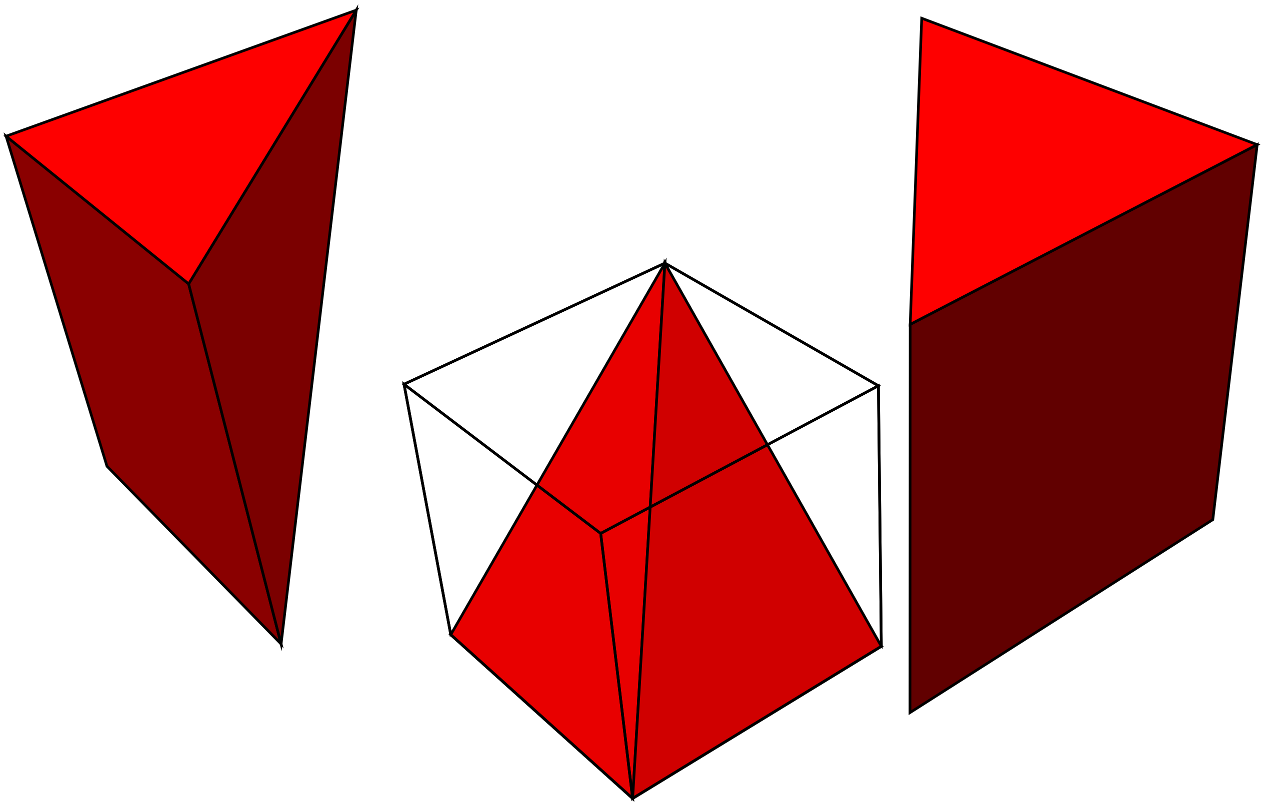

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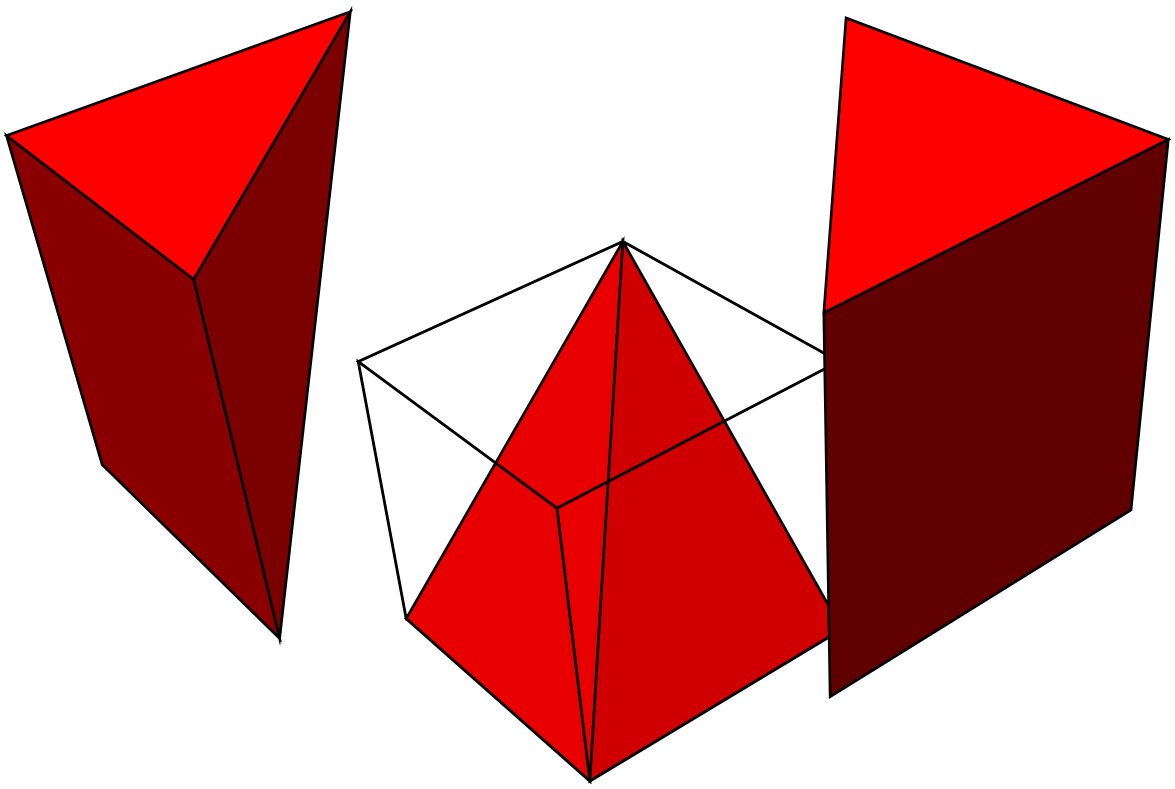

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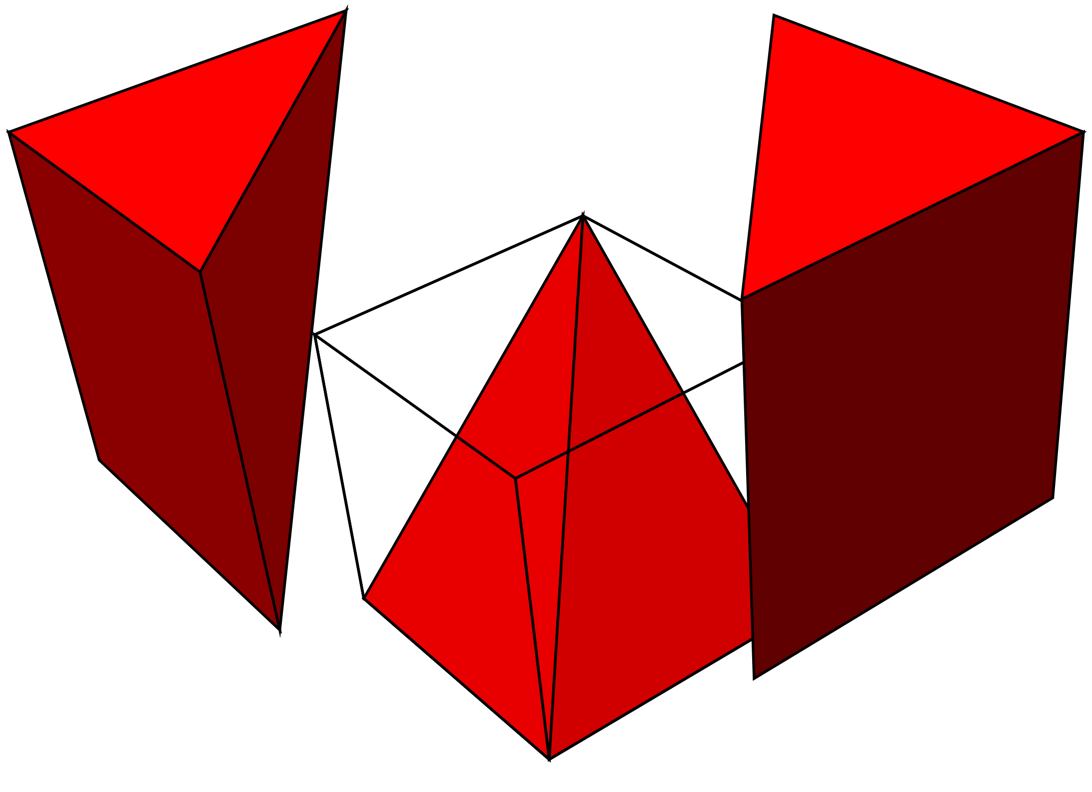

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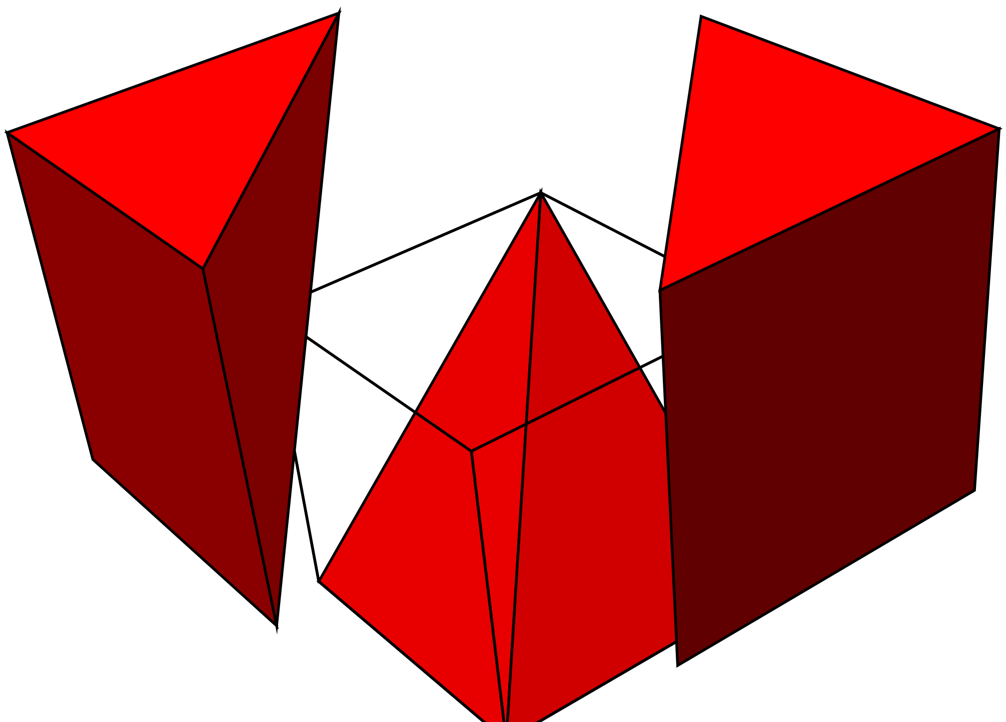

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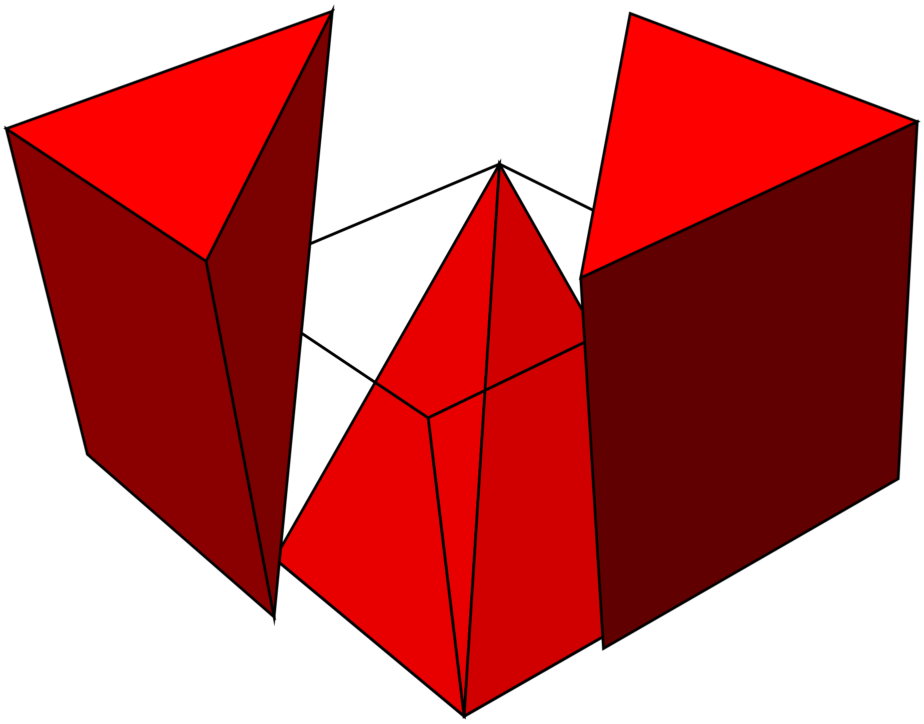

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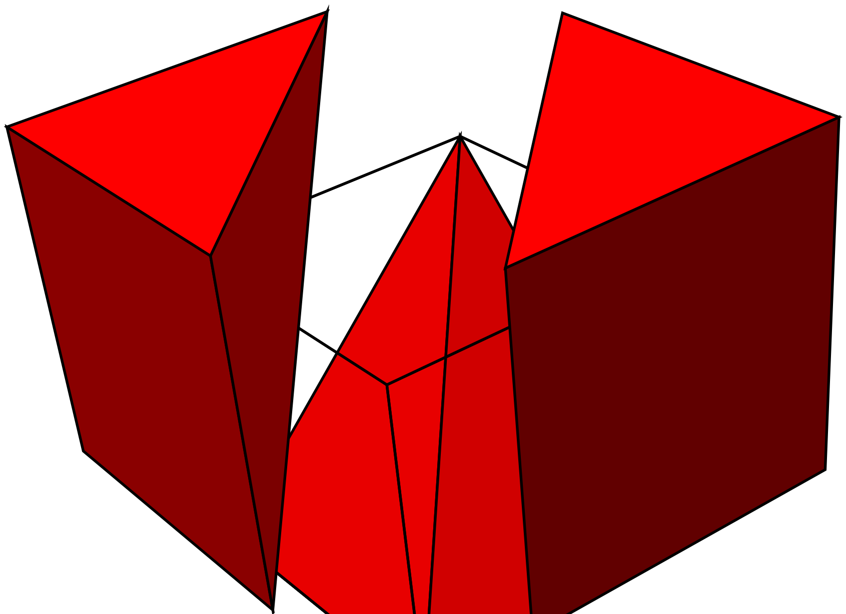

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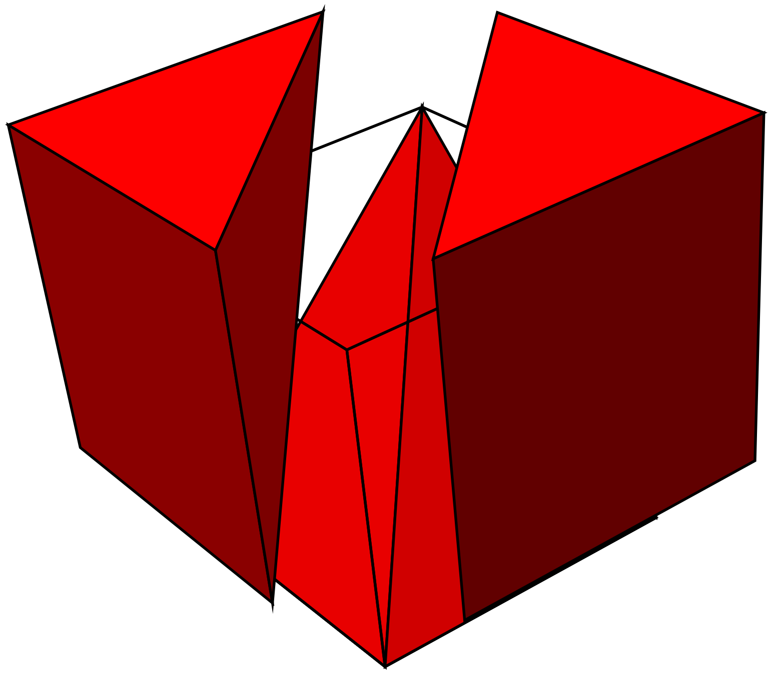

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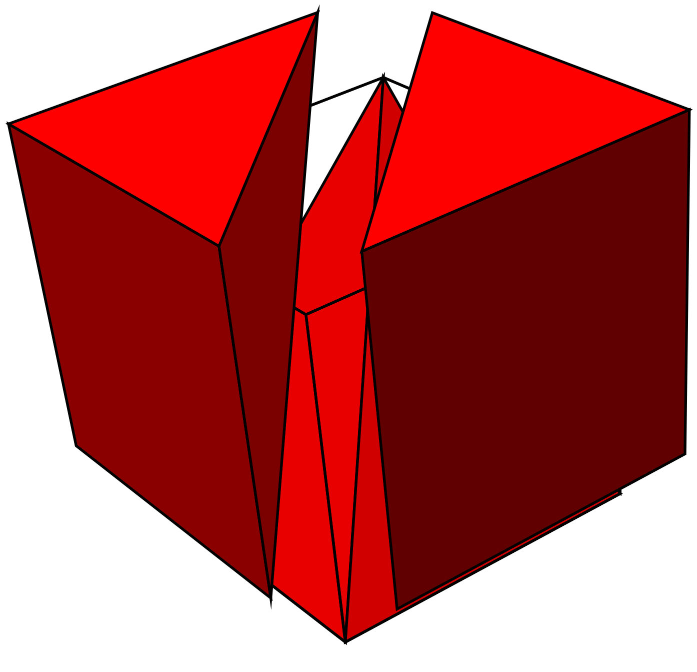

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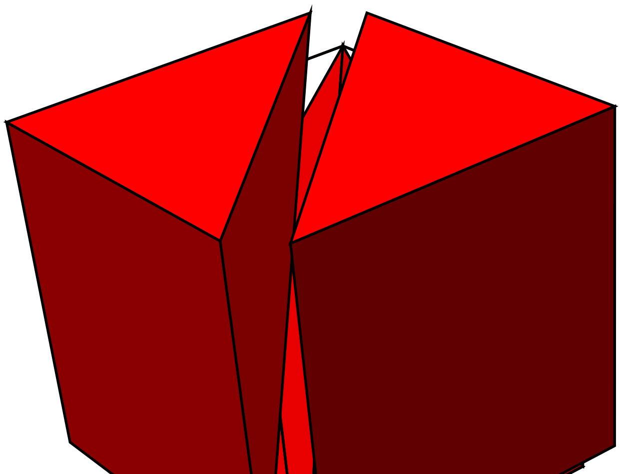

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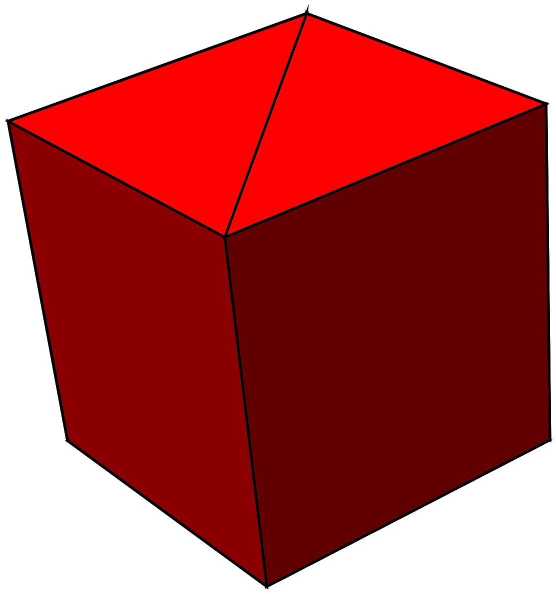


Three Yang-ma can be combined to create a single Unit Cube. Let’s see how this works.

Now let's review the four basic solids that Liu Hui had to use with his method. The first is the unit cube which has a volume equal to the base multiplied by the width times the height. The three remaining solids are created from the unit cube. The Ch'ien-tu is a wedge that has half the volume of the cube. The Yang-ma is a pyramid with one third the cube volume and finally the Pied-nao is a tetra-hedron that has one sixth the volume of the cube. The unit cube can also be constructed by combining the other three solids in various ways.

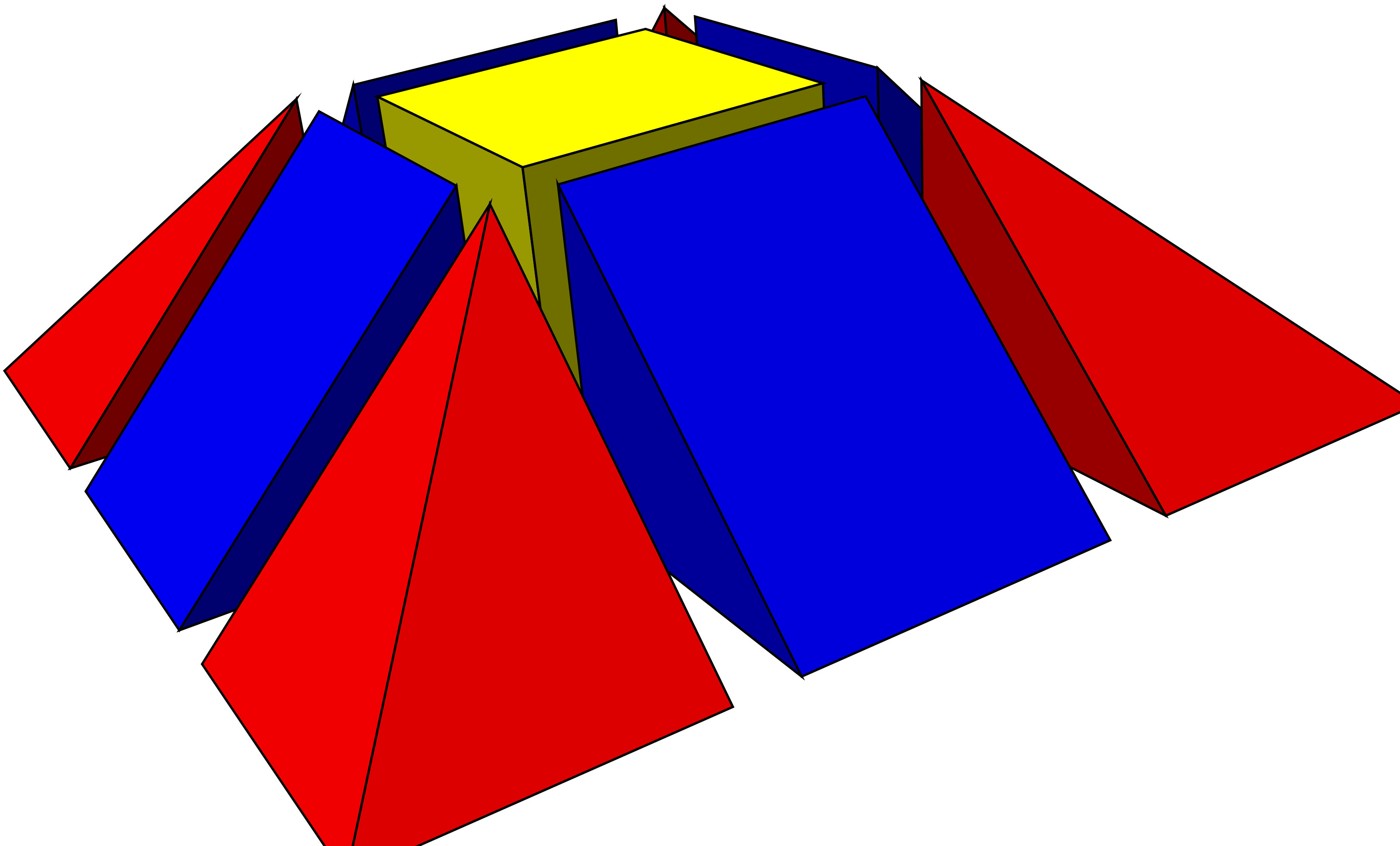
So how did Liu Hui use these objects to prove the volume formulas that are stated in the book Nine Chapters of. Mathematical Arts? Let's consider an example.

For our example lets show how Liu Hui proved the formula for the volume of a truncated right pyramid with a square base. This solid is named Fang-T'ing which literally means a Square Pavilion. The Nine Chapters of Mathematical Arts states the formula for the volume of a Fang-T'ing as:

$$\text{Vol.} = 1/3 \times ((B \times B \times H) + (B \times A \times H) + (A \times A \times H)).$$

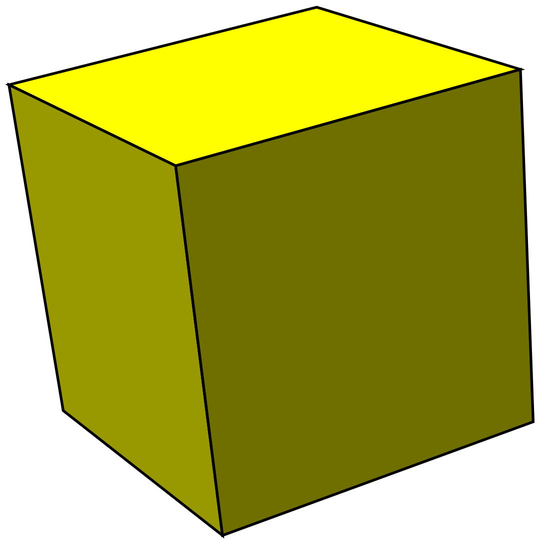
In this formula "B" is the length of the base of the Fang-T'ing, "A" is the length of the truncated top, and "H" is the height. Liu Hui constructed a Fang-T'ing of Base equal to 3, Height equal to 1, and Top equal to 1 from his basic solids.

Liu Hui used one unit cube, four Yang-ma and four Pieh-nao to construct the Fang-T'ing of our example. The next step in his method was look at the 3 seperate terms of the volume formula. He constructed solids that are equal to (B x B x H), (B x A x H), and (A x A x H) from the basic solids.



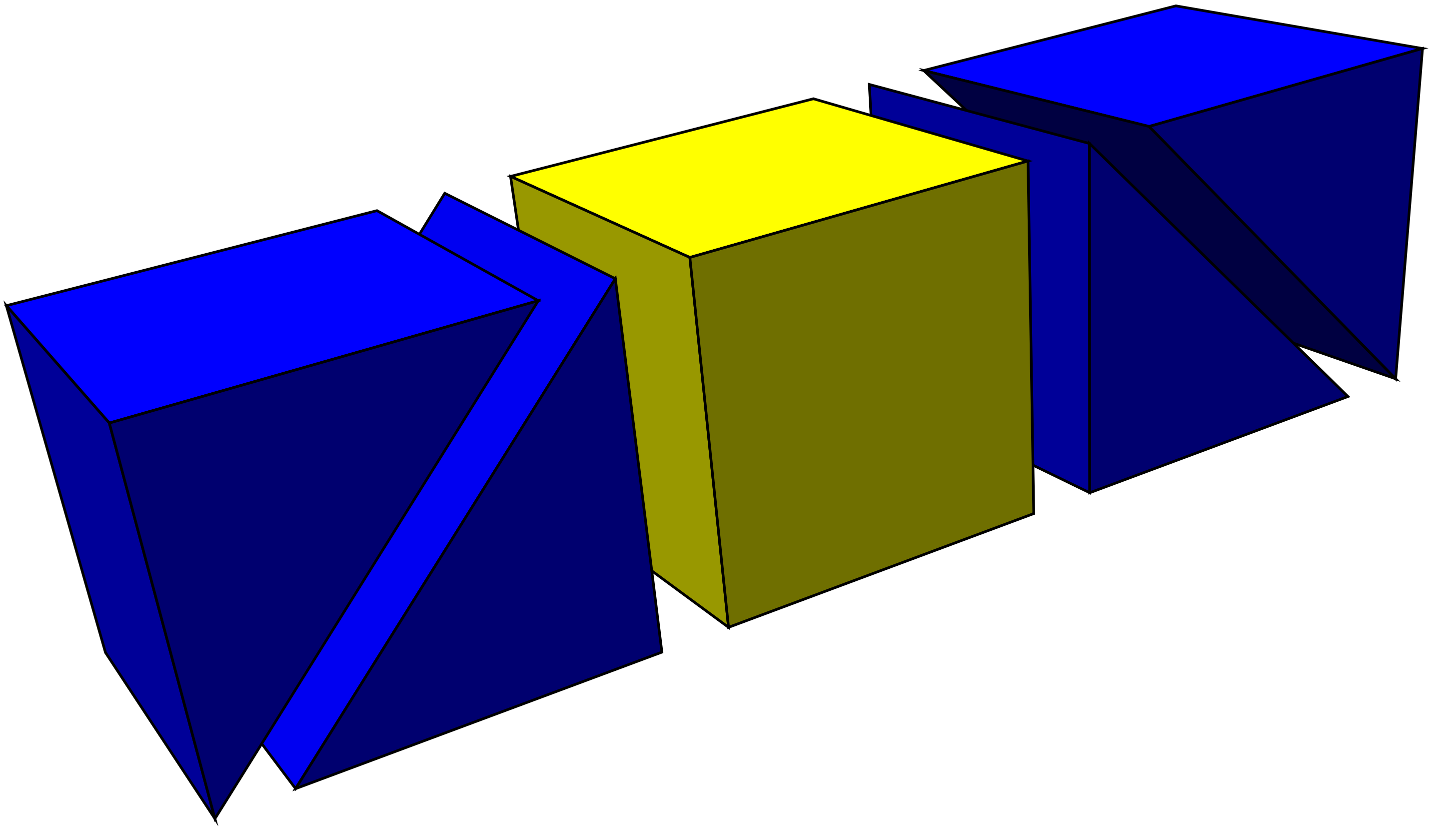
Vol. = 1/3 x ((B x B x H) + (B x A x H) + (A x A x H))

Liu Hui used one unit cube to create the solid that has a volume equal to the top squared times the height of the Fang-T'ing.



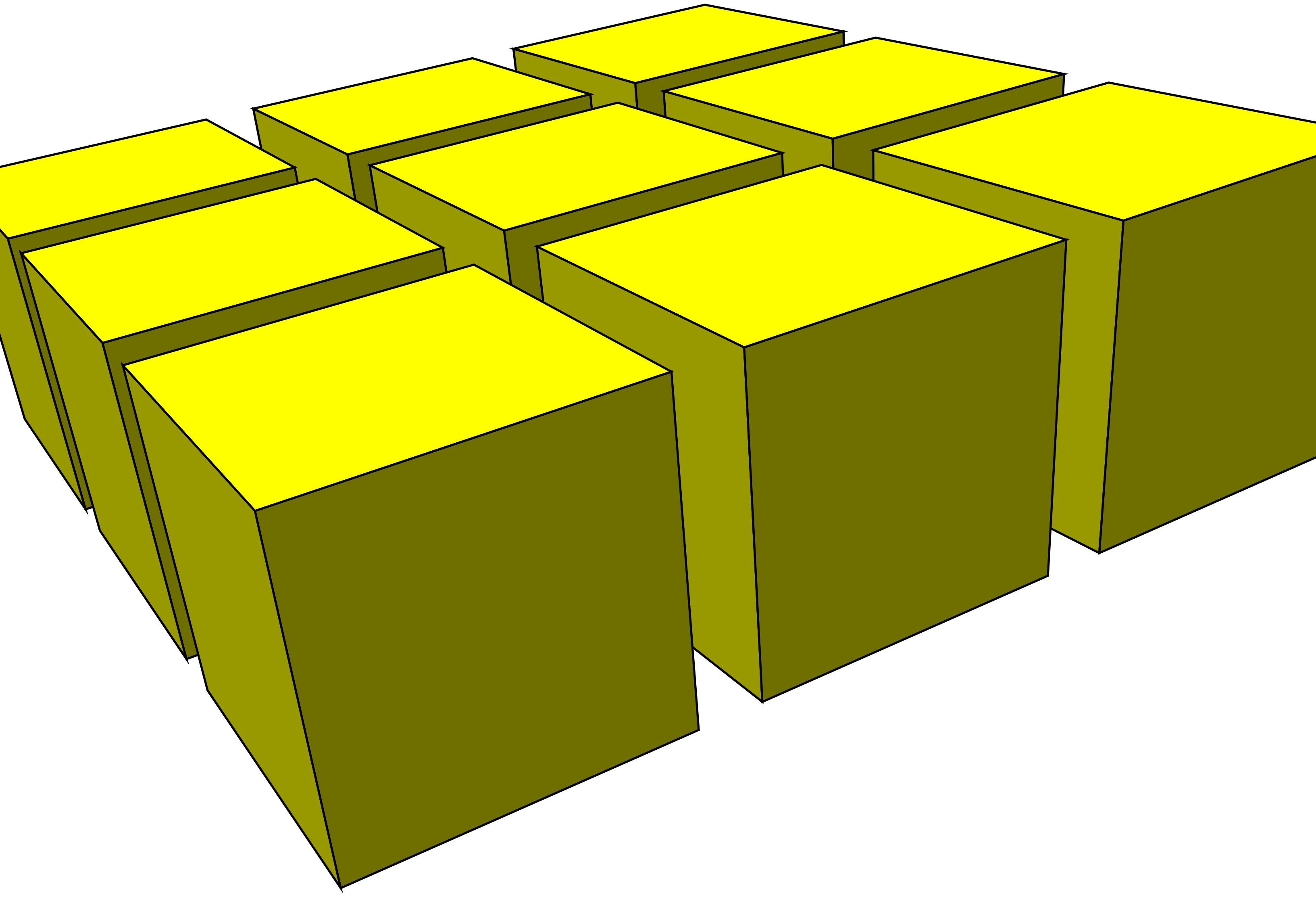
Vol. = (A x A x H)

Liu Hui used one unit cube, and four Ch’ien-tu to make a solid which has volume equal to the base times the height times the top of the Fang-T’ing. He could also have used three unit cubes as two Ch’ien-tu can be combined to create a unit cube.



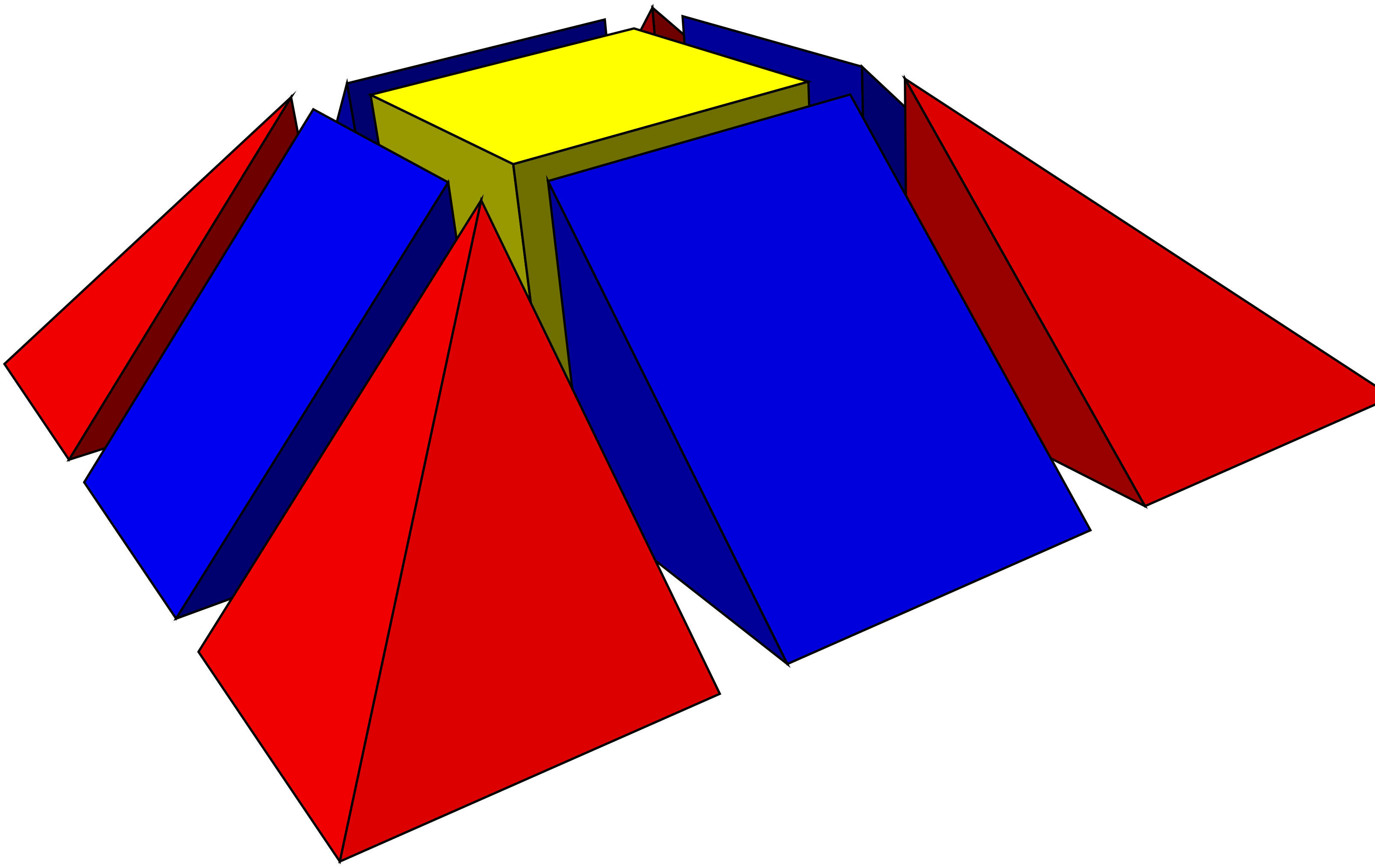
Vol. = (B x A x H)

Liu Hui could have used nine unit cubes to create the solid that is the base squared times the height of the Fang-T'ing. However instead he adds eight Yang-ma, two at each corner, and four Ch'ien-tu, one on each side to the Fang-T'ing. Thus (A x A x H) was created from one unit cube, eight Ch'ien-tu and twelve Yang-ma.



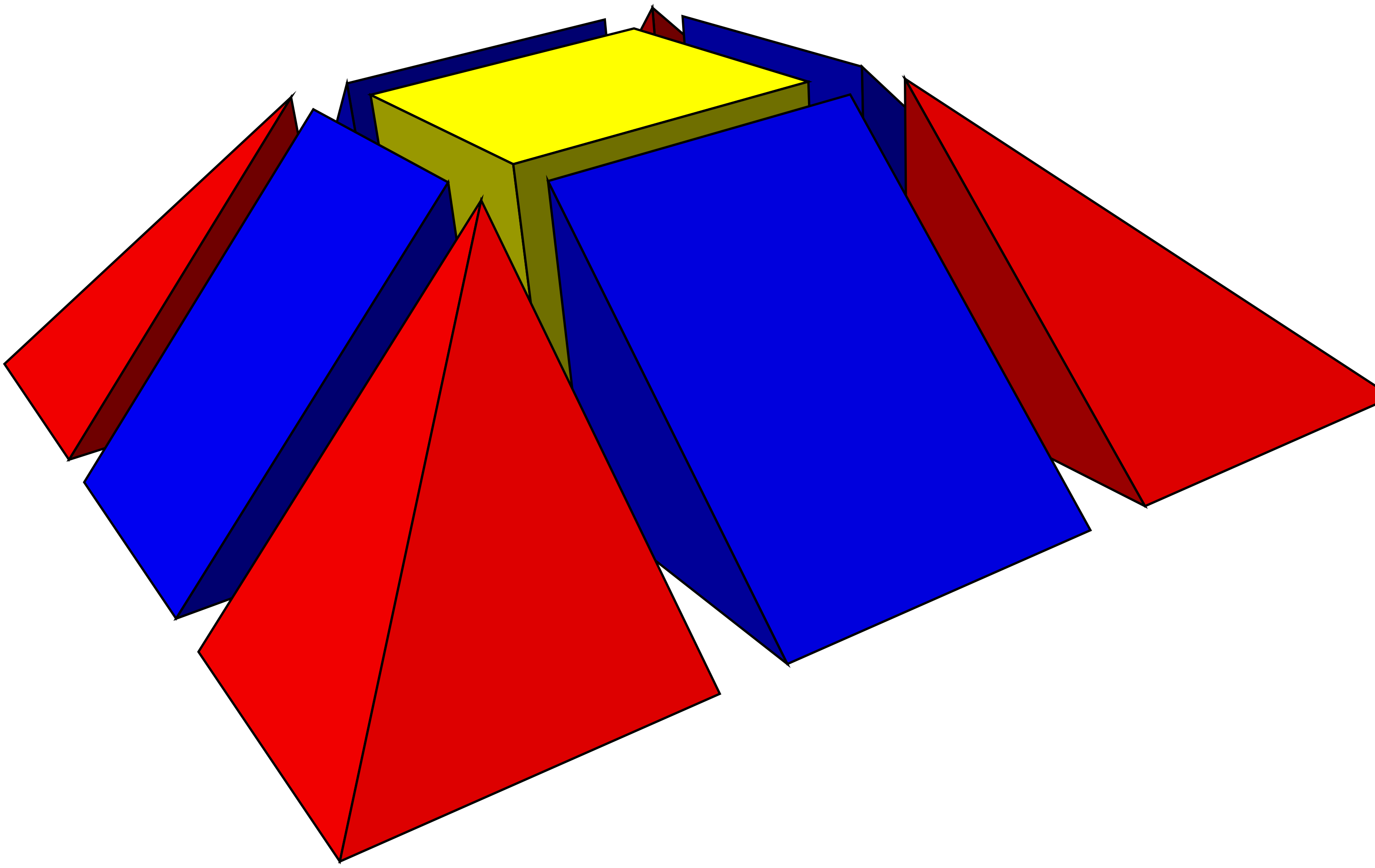
Vol. = (B x B x H)

So (A x A x H) is one unit cube; (B x A x H) is a unit cube and four Ch'ien-tu; and (B x B x H) is a unit cube, twelve Yang-ma and eight Ch'ien-tu. Thus adding up all three terms, (B x B x H) + (B x A x H) + (A x A x H), gives three unit cubes, twelve Ch'ien-tu and twelve Yang-ma.



Vol. = 1/3 x ((B x B x H) + (B x A x H) + (A x A x H))

So (B x B x H) + (B x A x H) + (A x A x H) equals three unit cubes, twelve Ch’ien-tu and twelve Yang-ma. However, we can see that the example Fang-T’ing is one unit cube four Ch’ien-tu and four Yang-ma. Notice this is one third the number of each object from (B x B x H) + (B x A x H) + (A x A x H). Thus the volume formula for a Fang-T’ing has been confirmed. This concludes the example.



Vol. = 1/3 x ((B x B x H) + (B x A x H) + (A x A x H))

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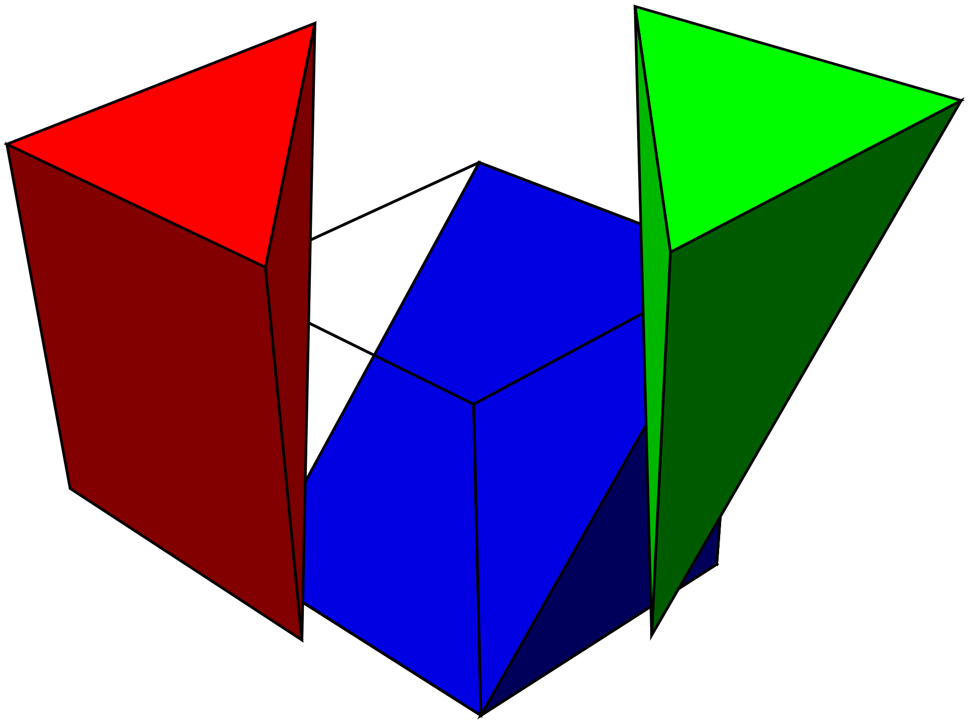
The End



Math 308, section 101
TermProject
Jonathan Pilley
Std # 23110877