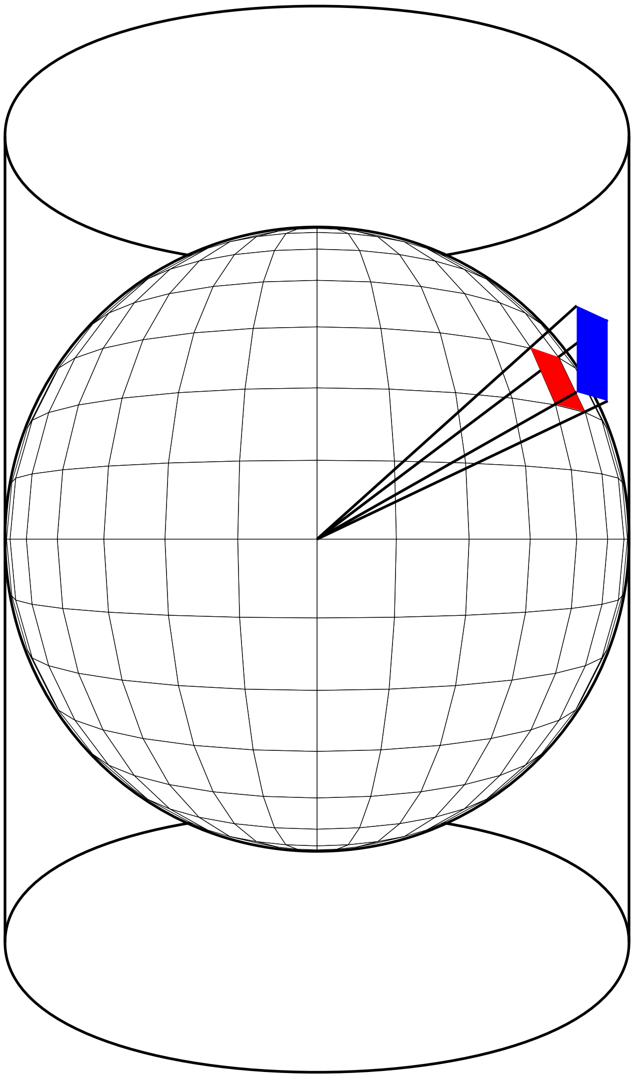
Fung Tsui Alvin So
82398017
MATH 308
Term Project
fungtsui@interchange.ubc.ca

MATH 308 Term Project

Map Making

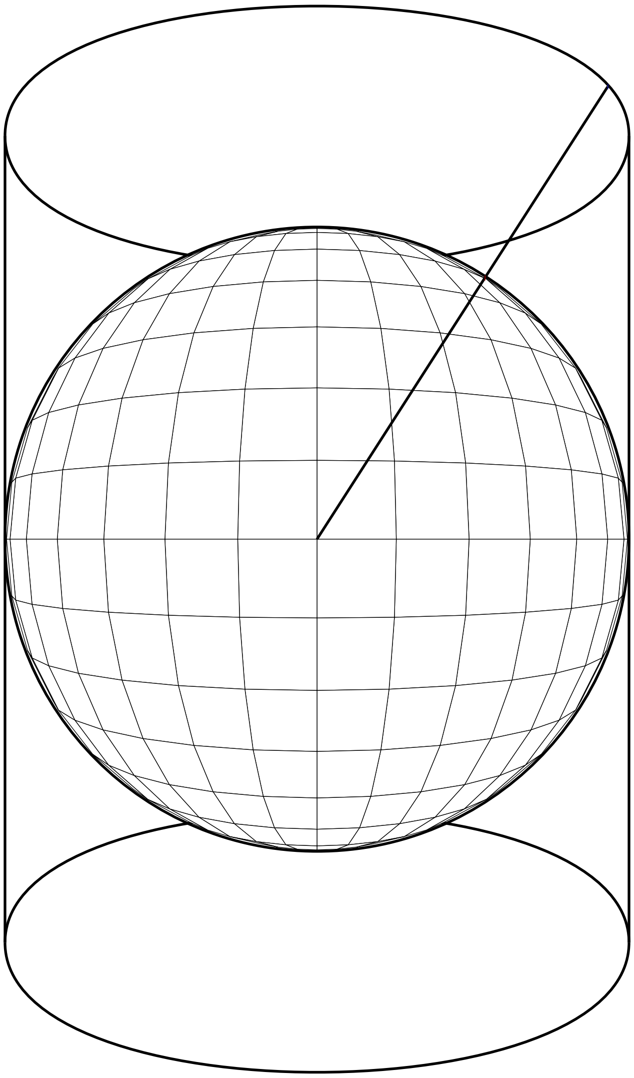
I. Cylindrical Projection

Assuming the globe is a perfect sphere, the cylindrical projection projects points on the surface of the sphere to the surface of the cylinder.



One type of cylindrical projection is to project from the center of the sphere. Imagine there is a light source sitting at the center of the sphere. Therefore, the blue area on the surface of the cylinder in the figure is the shadow of the red area on the surface of the sphere.

We now project points on the sphere with the same longtitude.



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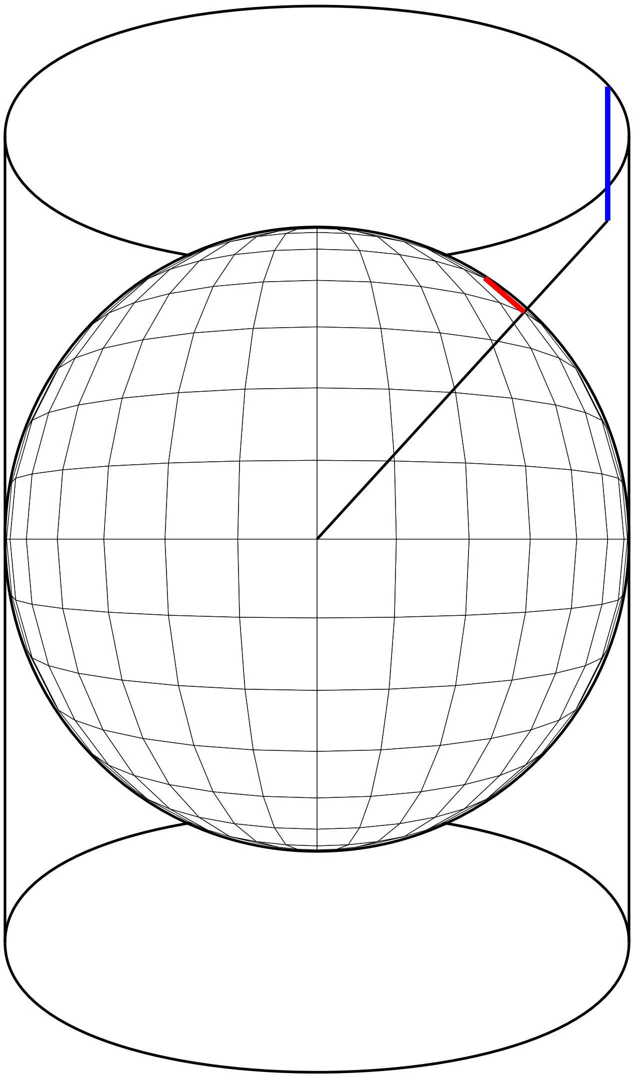

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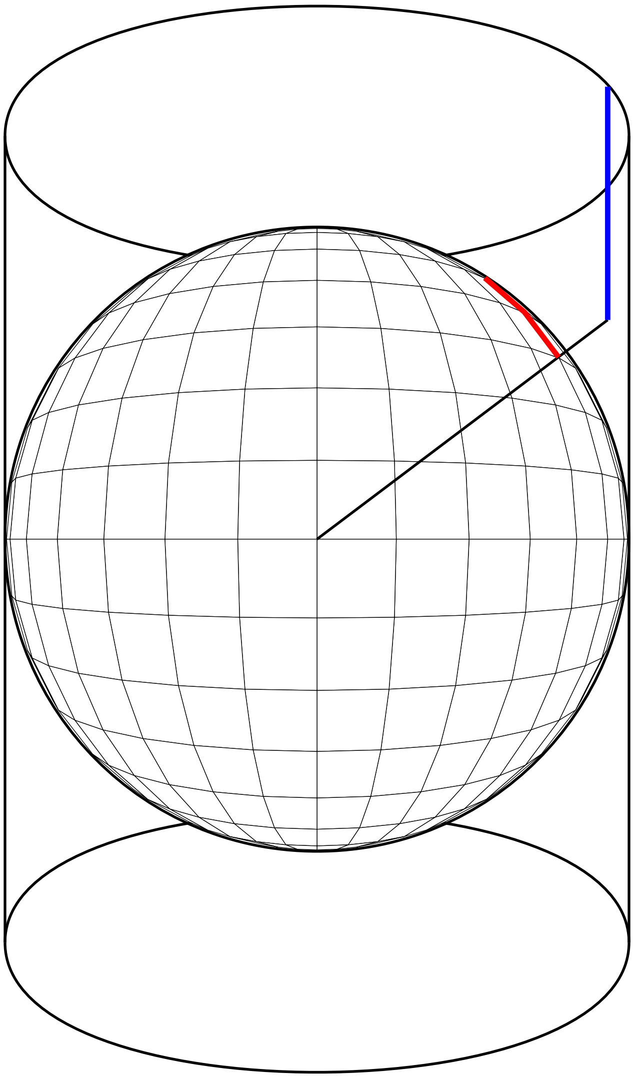

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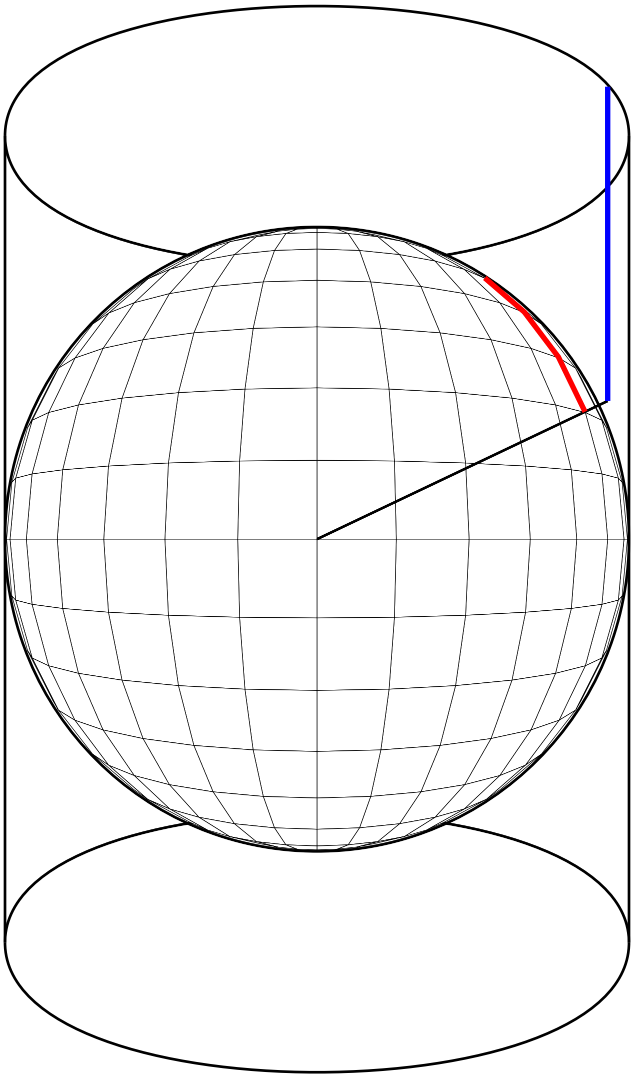

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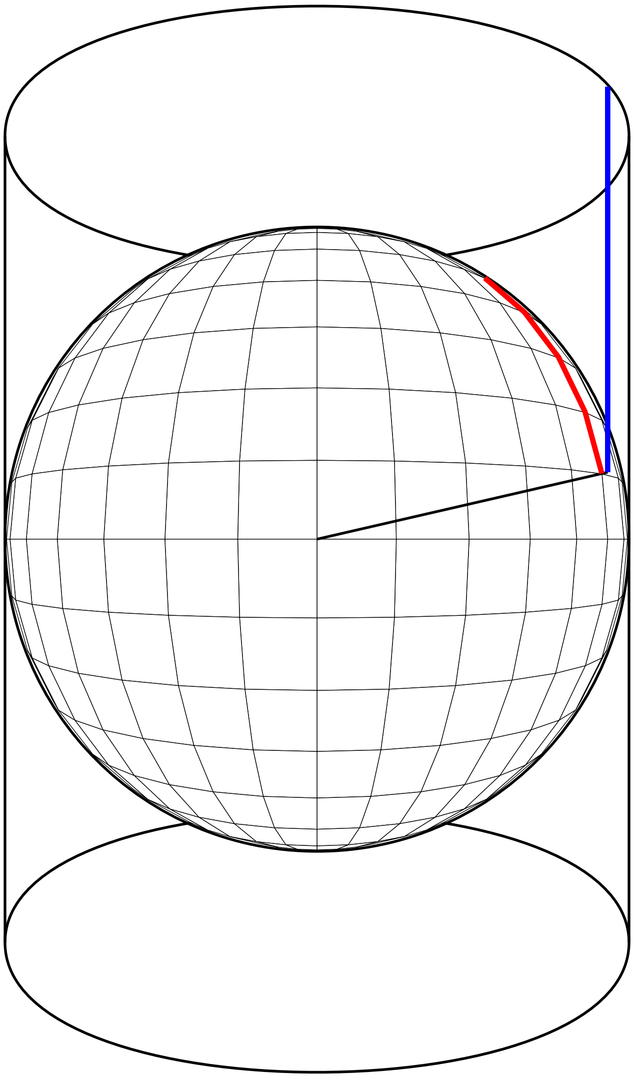

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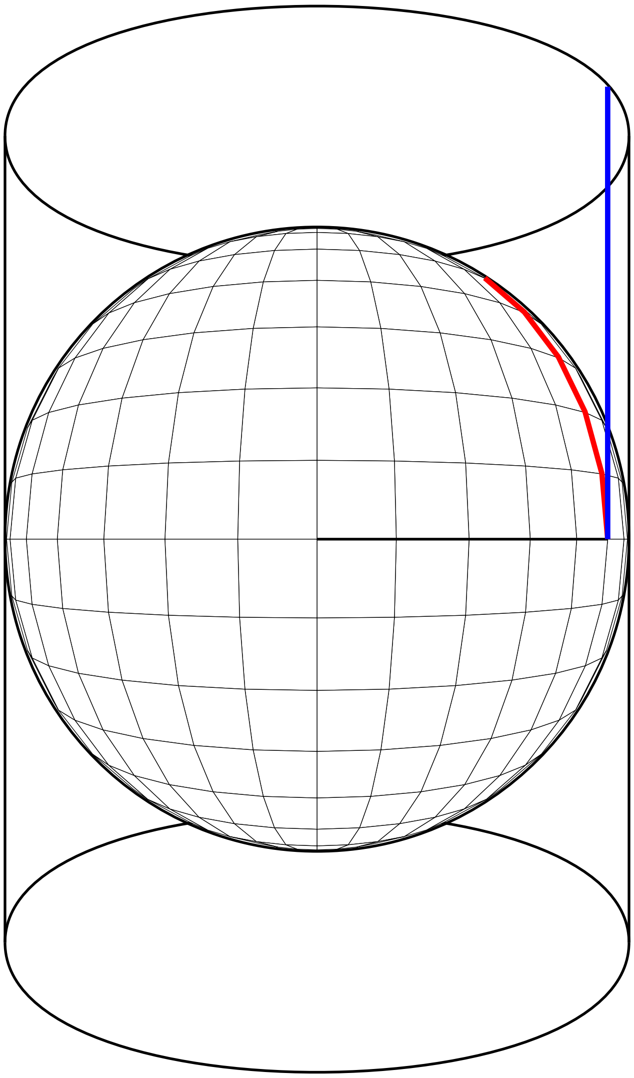

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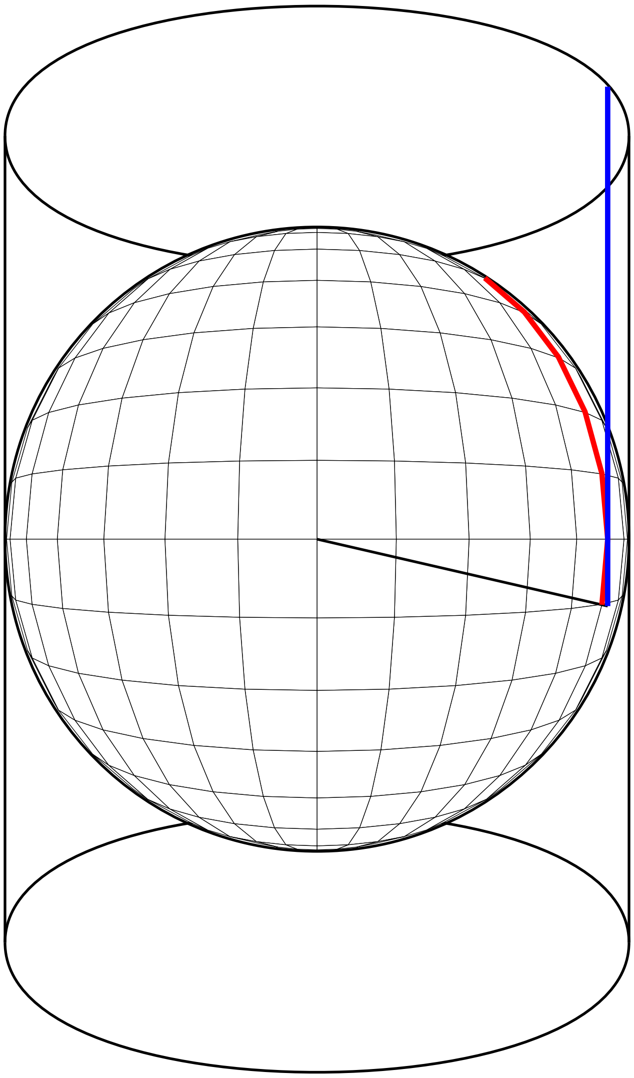

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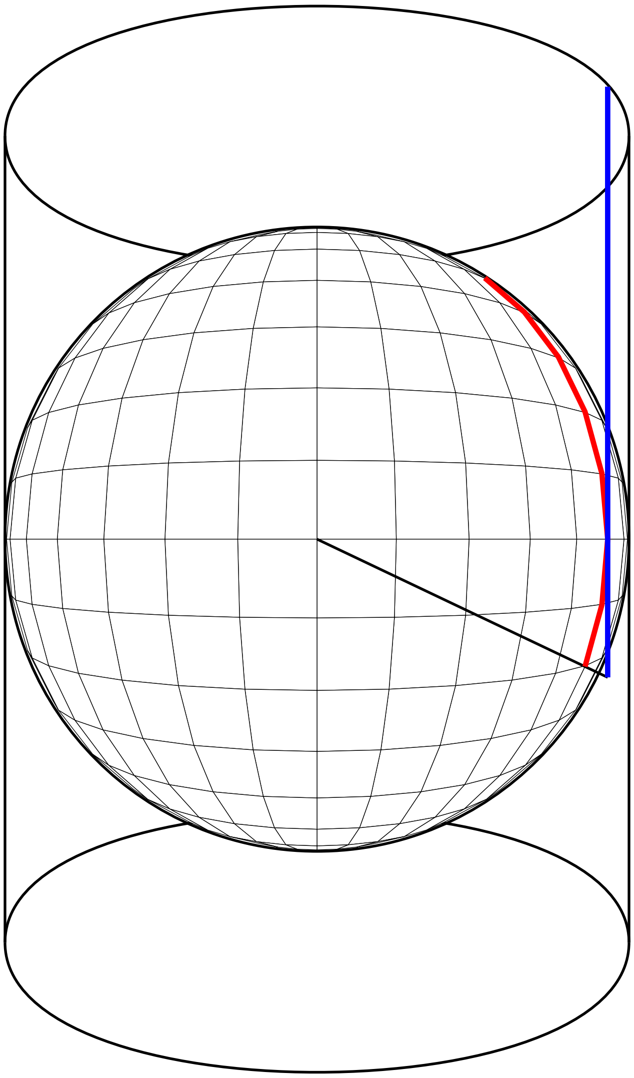

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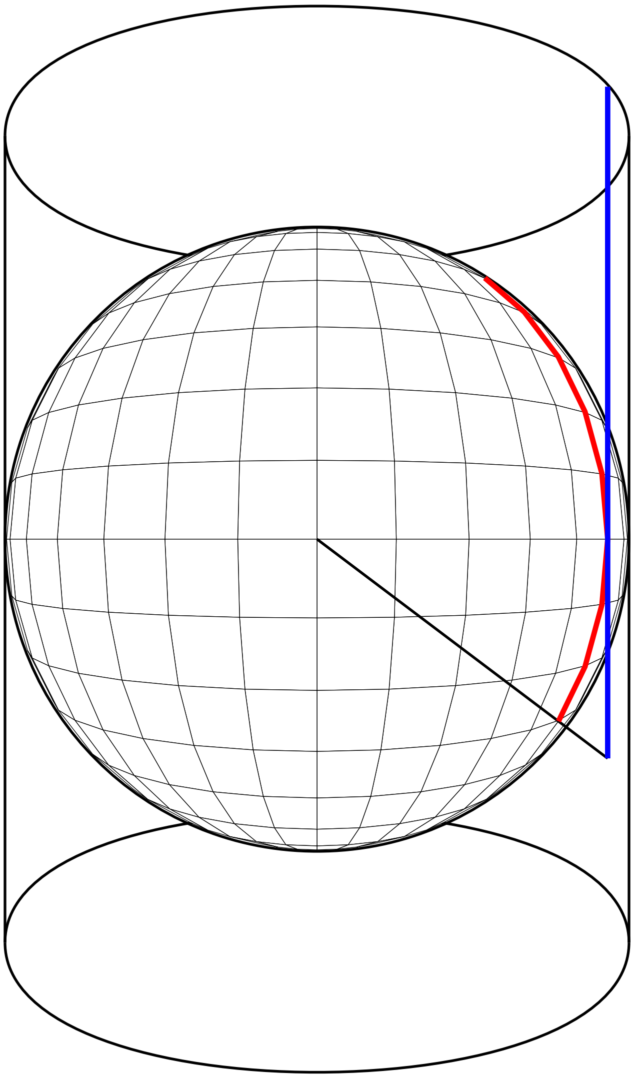

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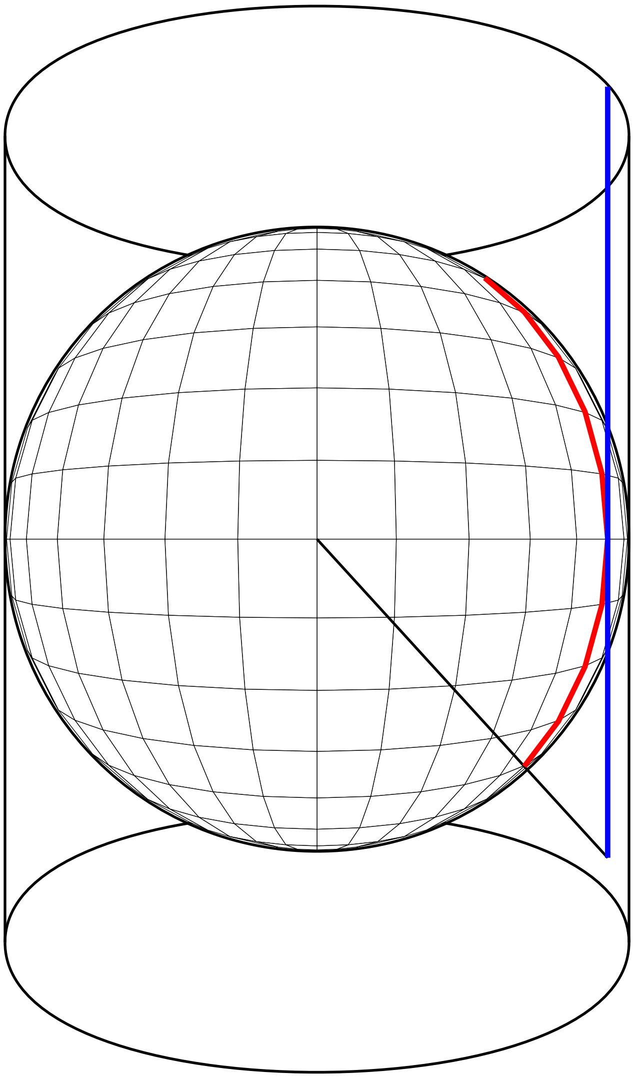

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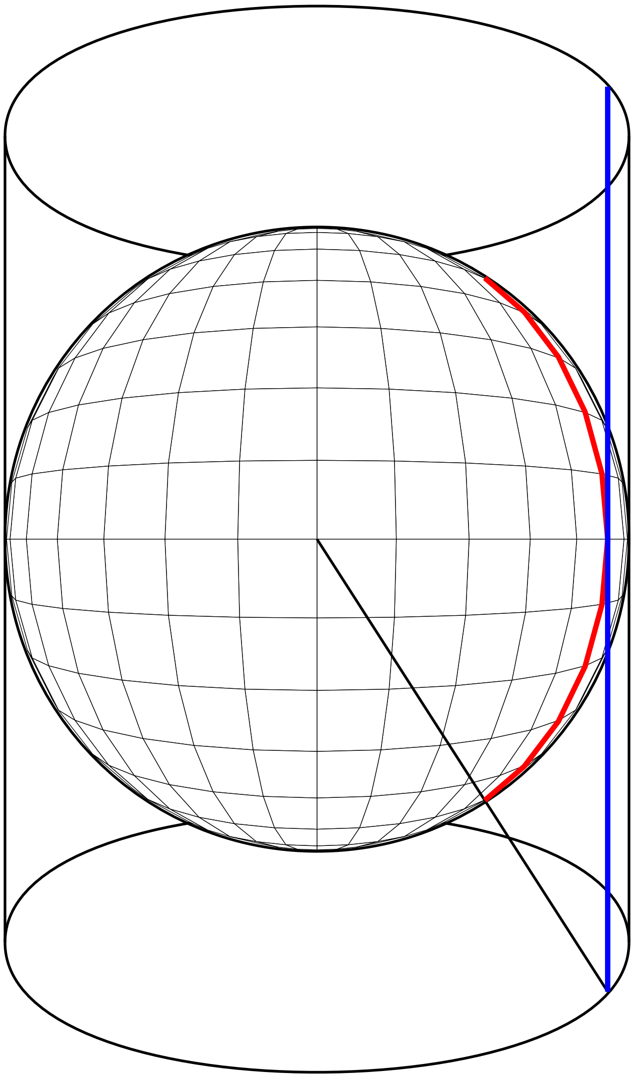
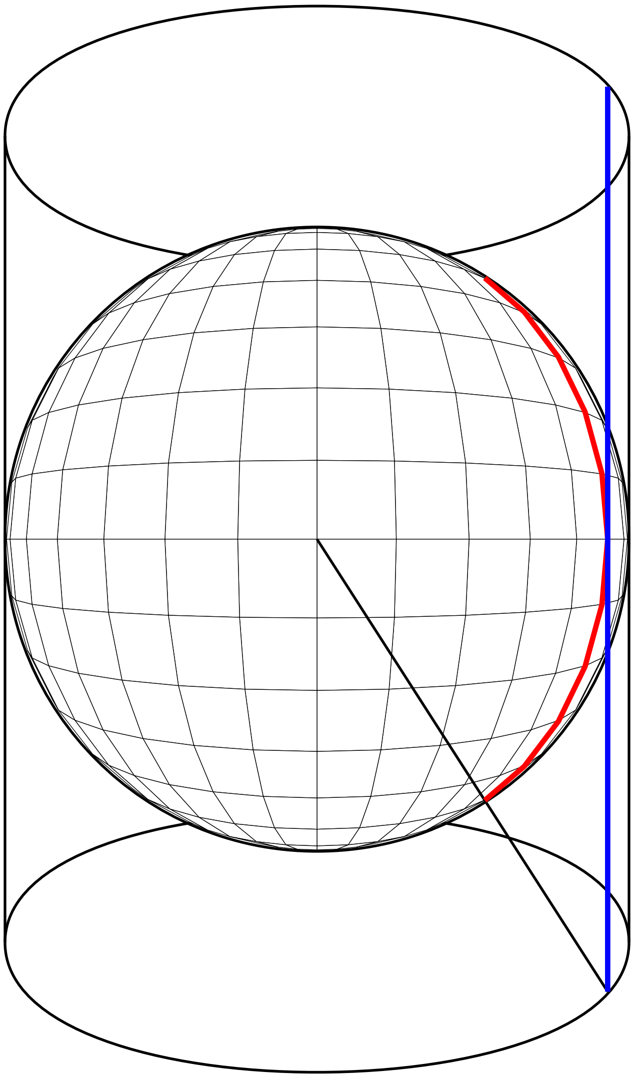
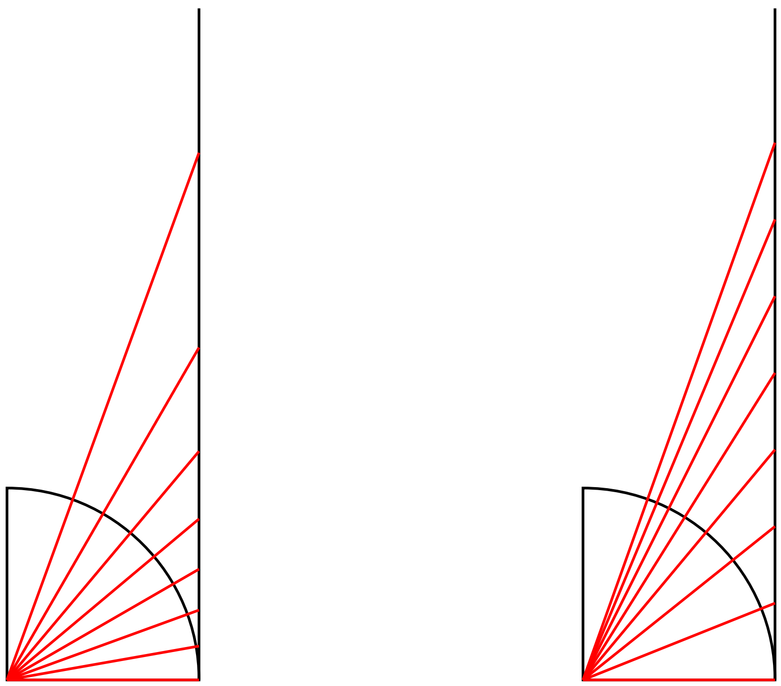


We can see that the projection of points with the same longtitude will be a straight vertical line. Since the angle between longtiudes are the same, the projection of longtitudes will be equally spaced.



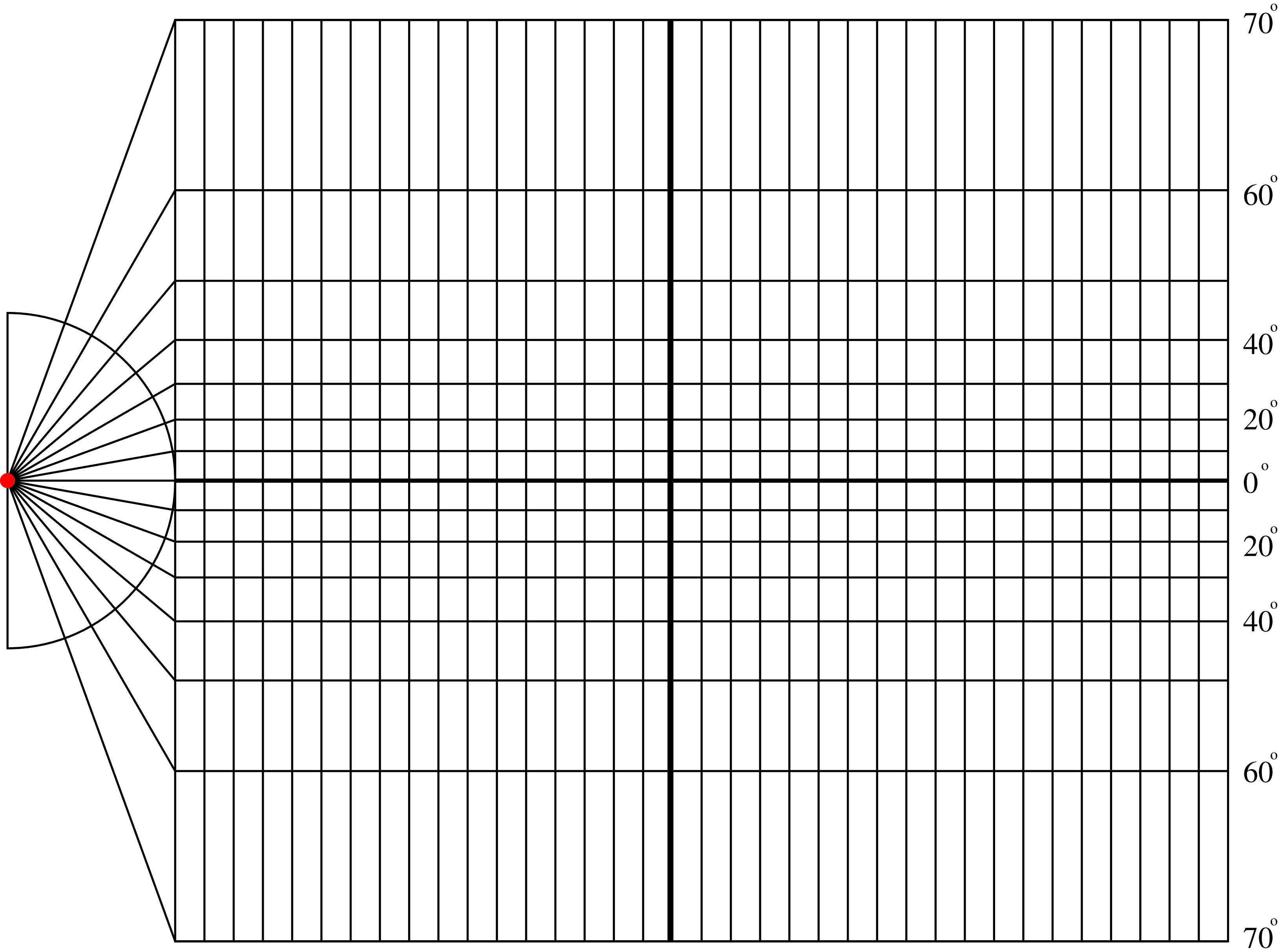
Because they are equally space, the x-component of the projected points will be a multiple of a constant. Let R be the radius of the sphere, A be the longitude, and B be the latitude, then x=2*pi*R*A/360. Note that this x-component does not depend on the latitude.

Now we look at the projection of latitudes. Assuming the figures are a quadrant of a circle. In the left figure, the angles between each projection are the same. We can see that the distance



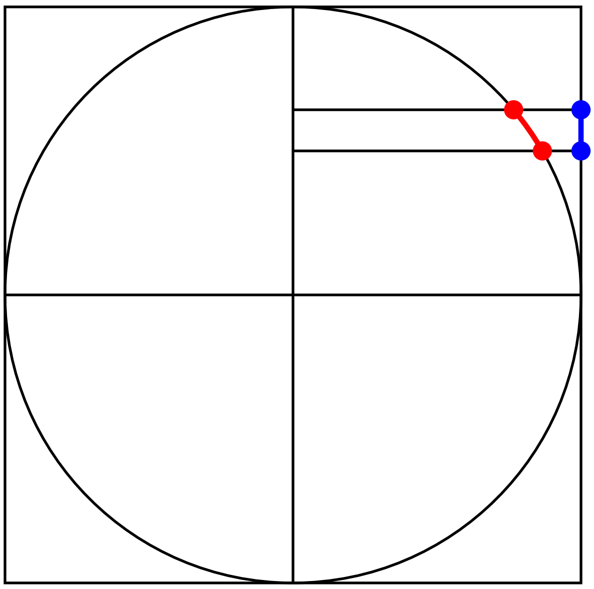
between the projected points are different. The larger the latitude, the longer the distance between the projected points. In the right figure, the distance between the projected points are the same. But the angles between the lines are different. Therefore, the projected map will not have latitudes equally spaced since the angles between latitudes are the same. However, points with the same latitudes will still form a straight horizontal lines on the map as projected points with the same latitudes have the same y-component. Again, let R be the radius of the sphere, A be the longitude, B be the latitude. From the left figure, it can be seen that y=R*sin(B)/cos(B)=R*tan(B). This y-compoenet does not depend on the longtitude either.

This is the map created using cylindrical projection from the center of the sphere. As described, longitudes are equally spaced but latitudes are not. The larger the absolute latitudes, the larger the space between them. As a result, if two lands with

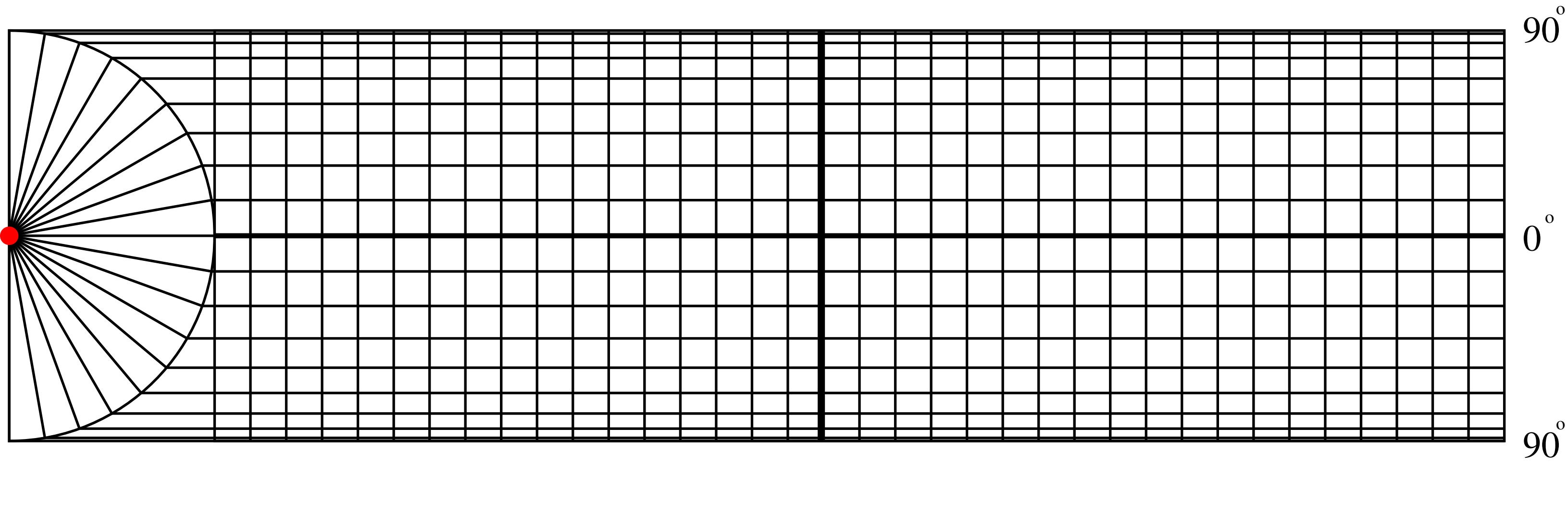


exactly the same shape are one located at the equator and another near the pole, the land near the pole would look a lot larger than the one at the equator on this map.

Another type of cylindrical projection is to project the latitude horizontally from the y-axis. Like projecting from the center of the sphere, projecting from the axis still has equally spaced vertical projected longtitudes.



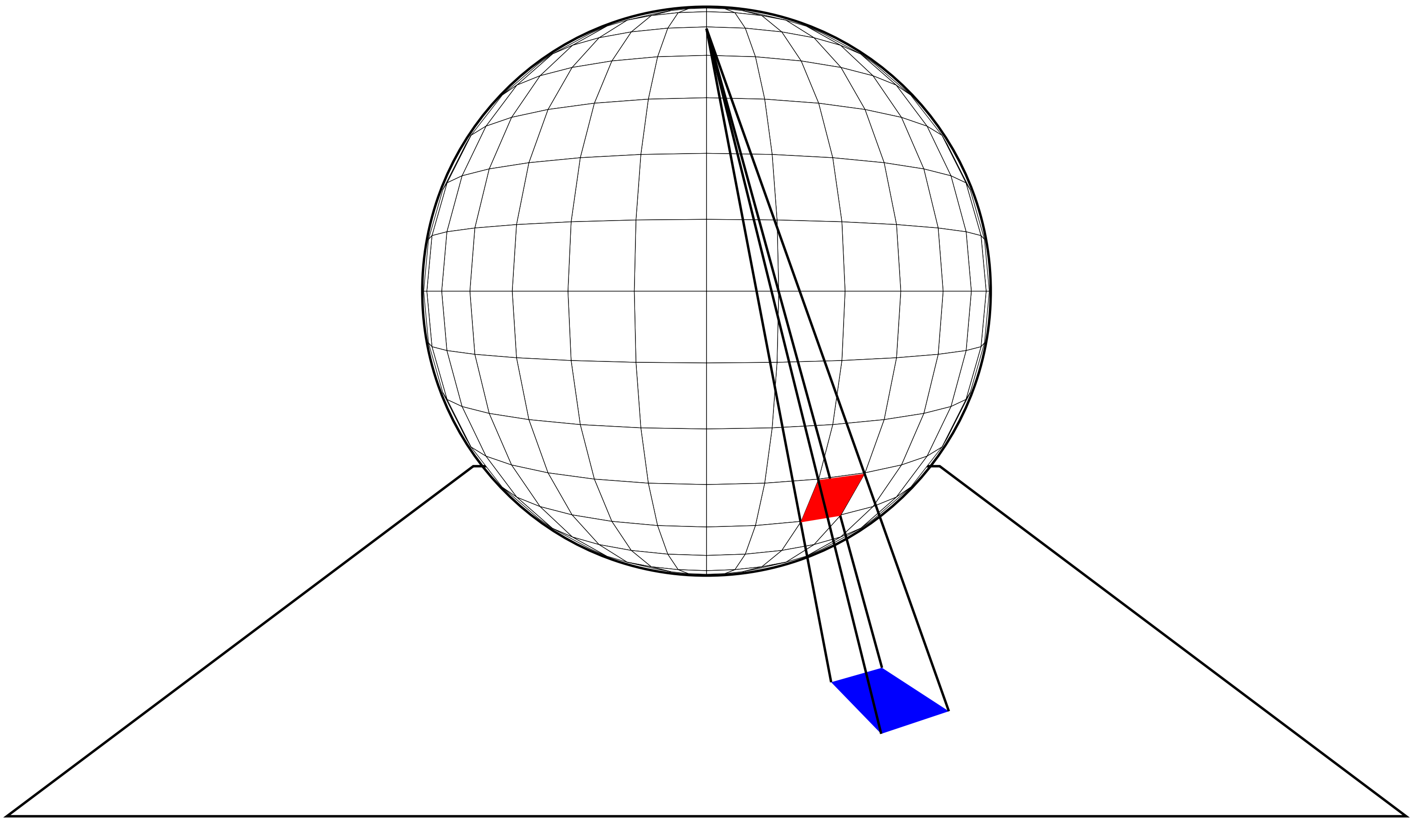
Let R be the radius of the sphere. The area of this map is the surface area of a cylinder minus the top and bottom circles. Therefore, area of the map=2*pi*R*(height of cylinder). But the height of the cylinder is equal to the diameter of the sphere as the top and bottom of the cylinder touch the north and south poles. Hence, area of the map=4*pi*R^2, which is also equal to the surface area of a sphere. Because the area of this map is the same as the surface area of the sphere, this map is area-preserving.



Again, let A be the longtitude and B be the latitude. The x-component is the same as the previous cylindrical projection, which is x=2*pi*R*A/360. The y-componenet is just the y-component of the point, which is y=R*sin(B).

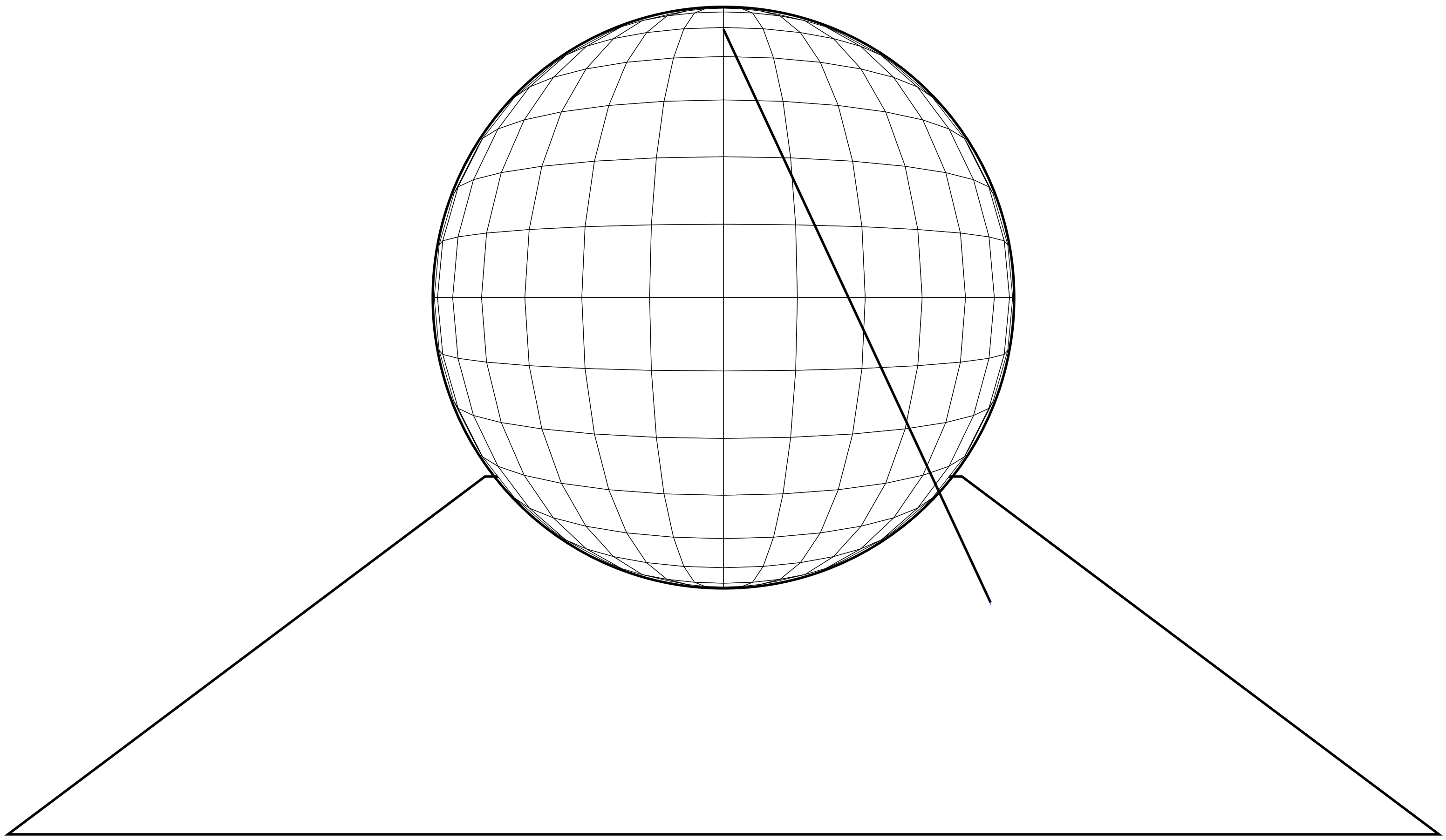
II. Stereographic Projection

Again, assume the globe is a perfect sphere. Stereographic projection can be described as placing the globe on a plane, touching it at one pole, and then projecting from the other pole.



To describe in the same way as the cylindrical projection, it can be said that a plane is tangent to one pole and a light source is sitting at the opposite pole. The blue area on the plane in the figure is the shadow of the red area on the sphere.

We now project points on the sphere with the same latitude.



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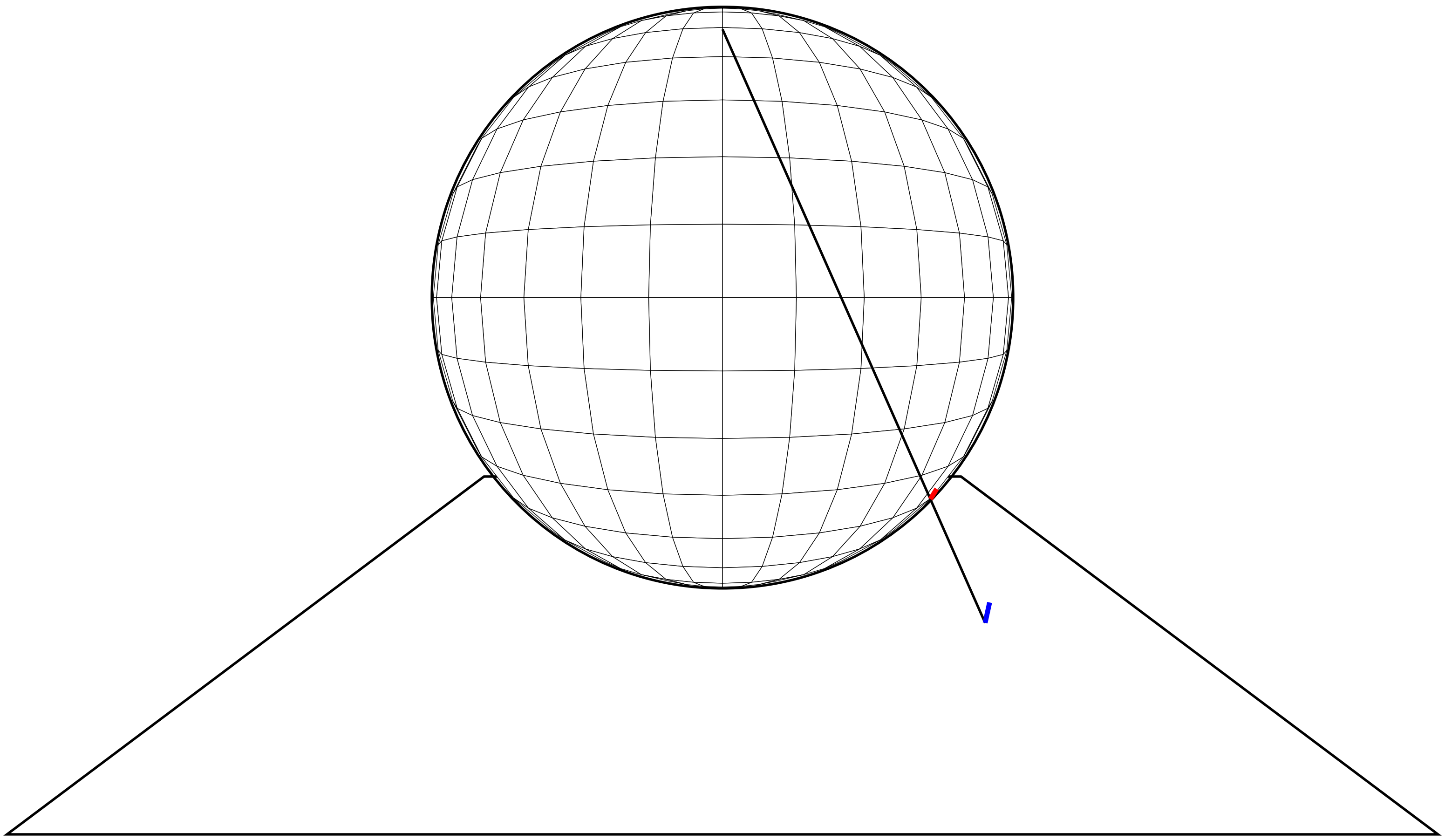

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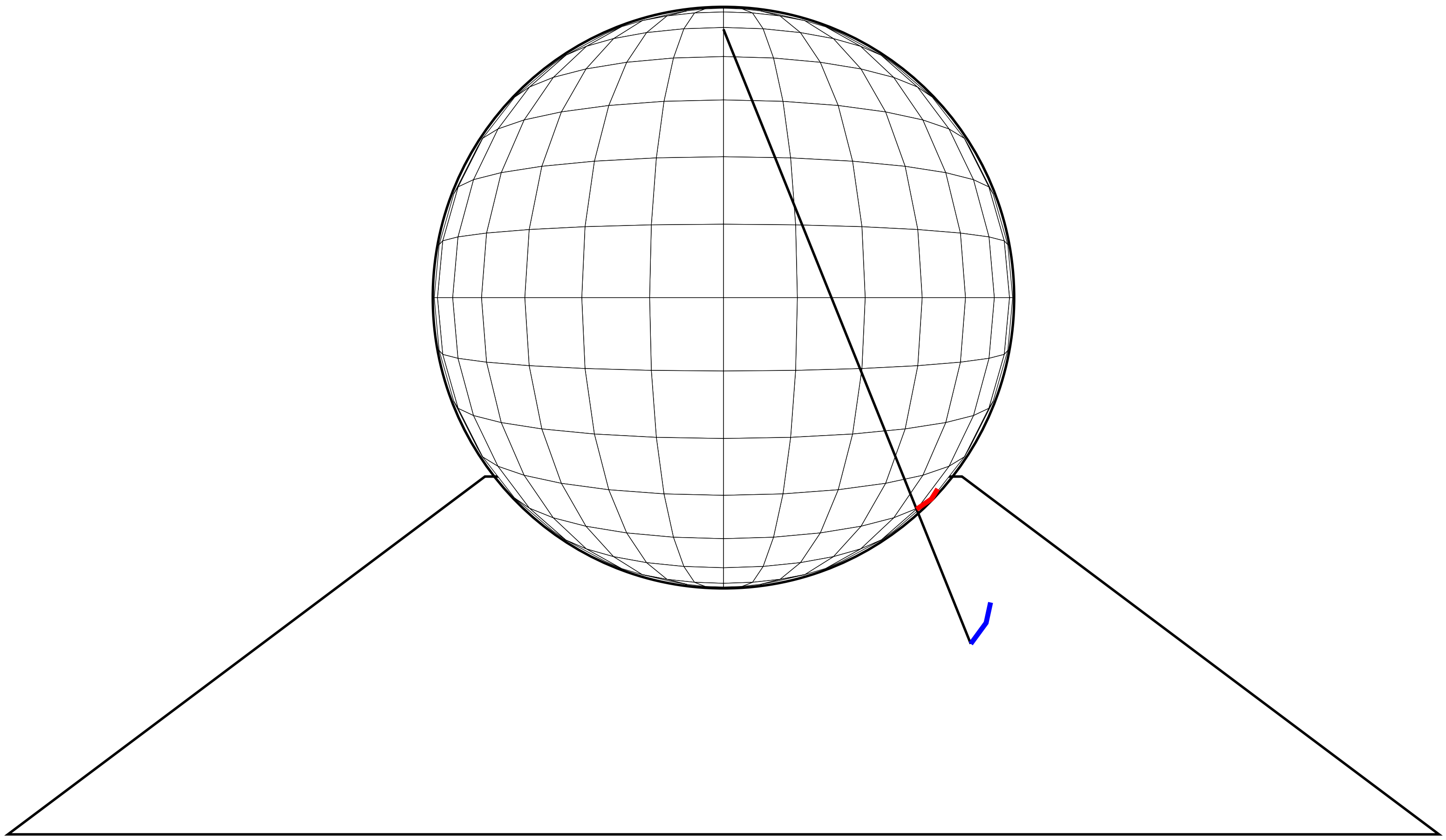

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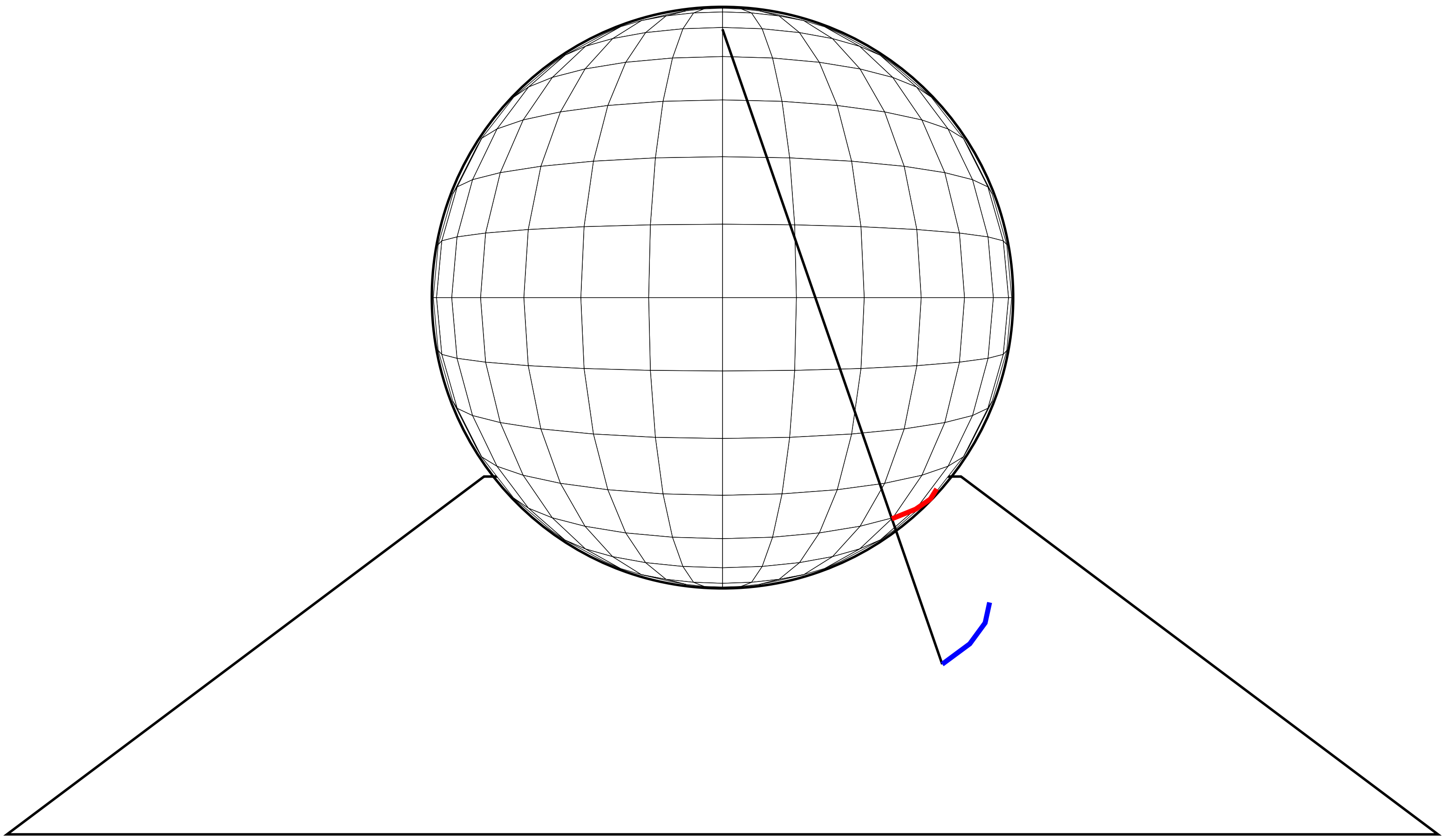

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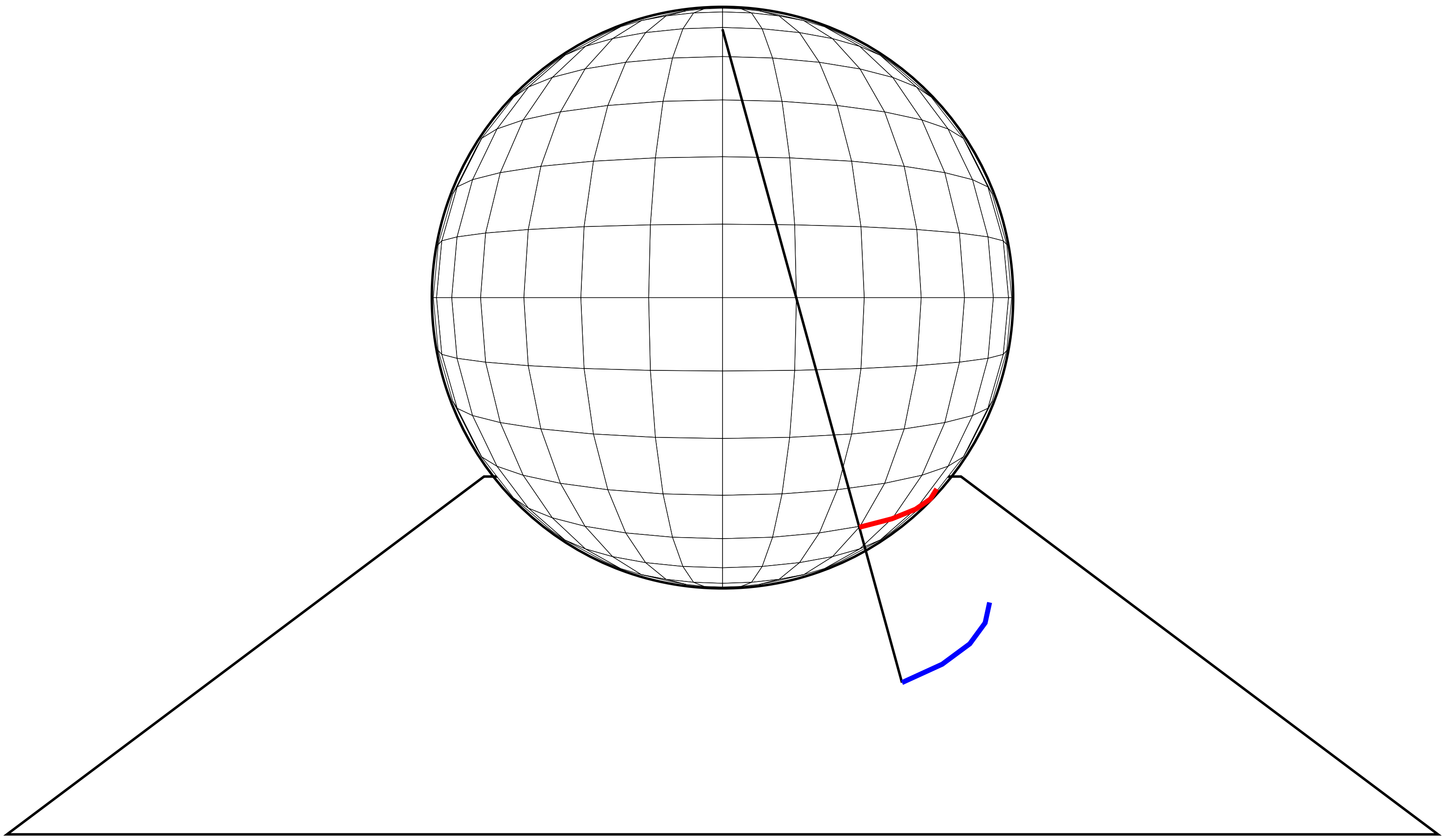

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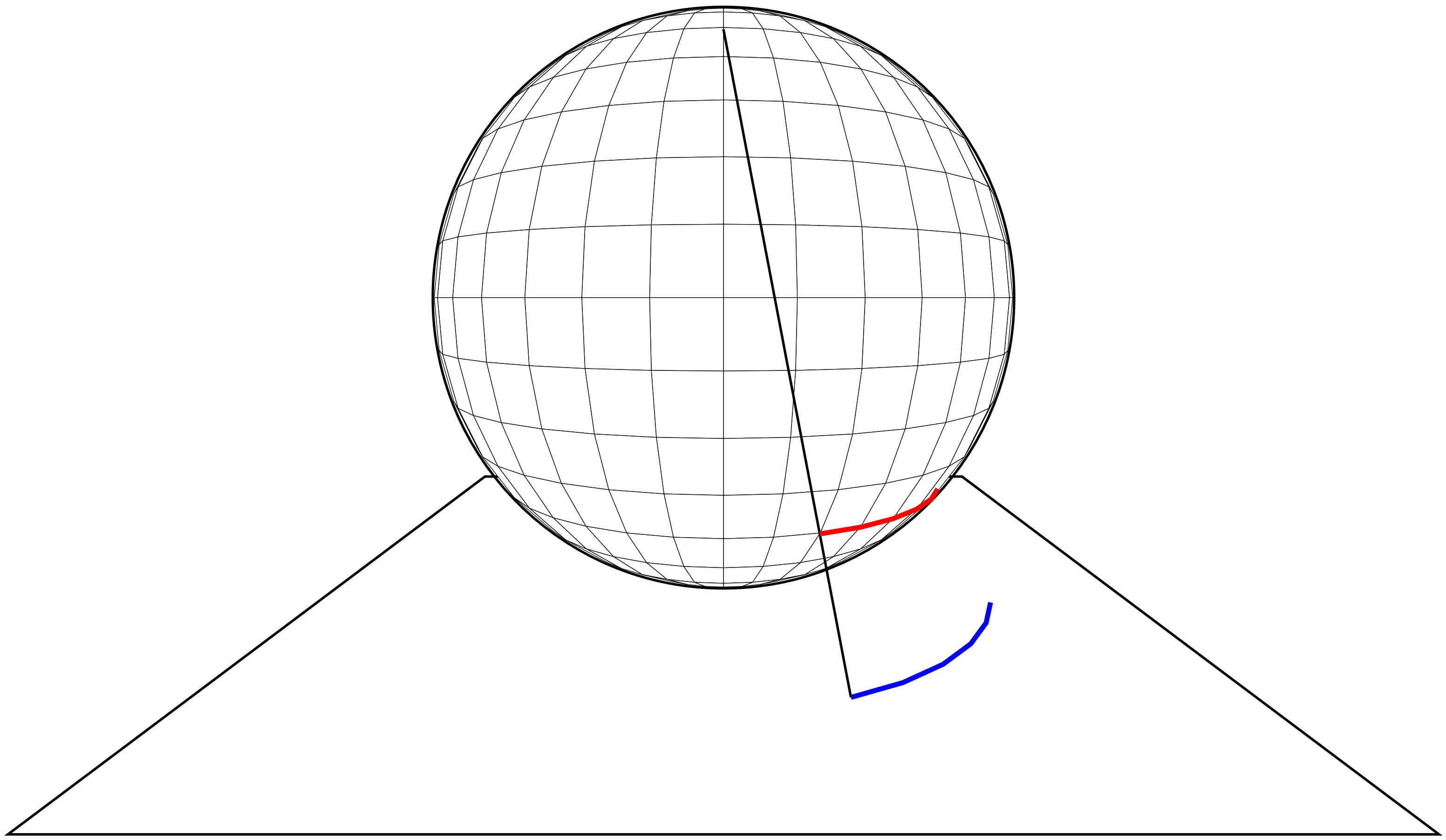

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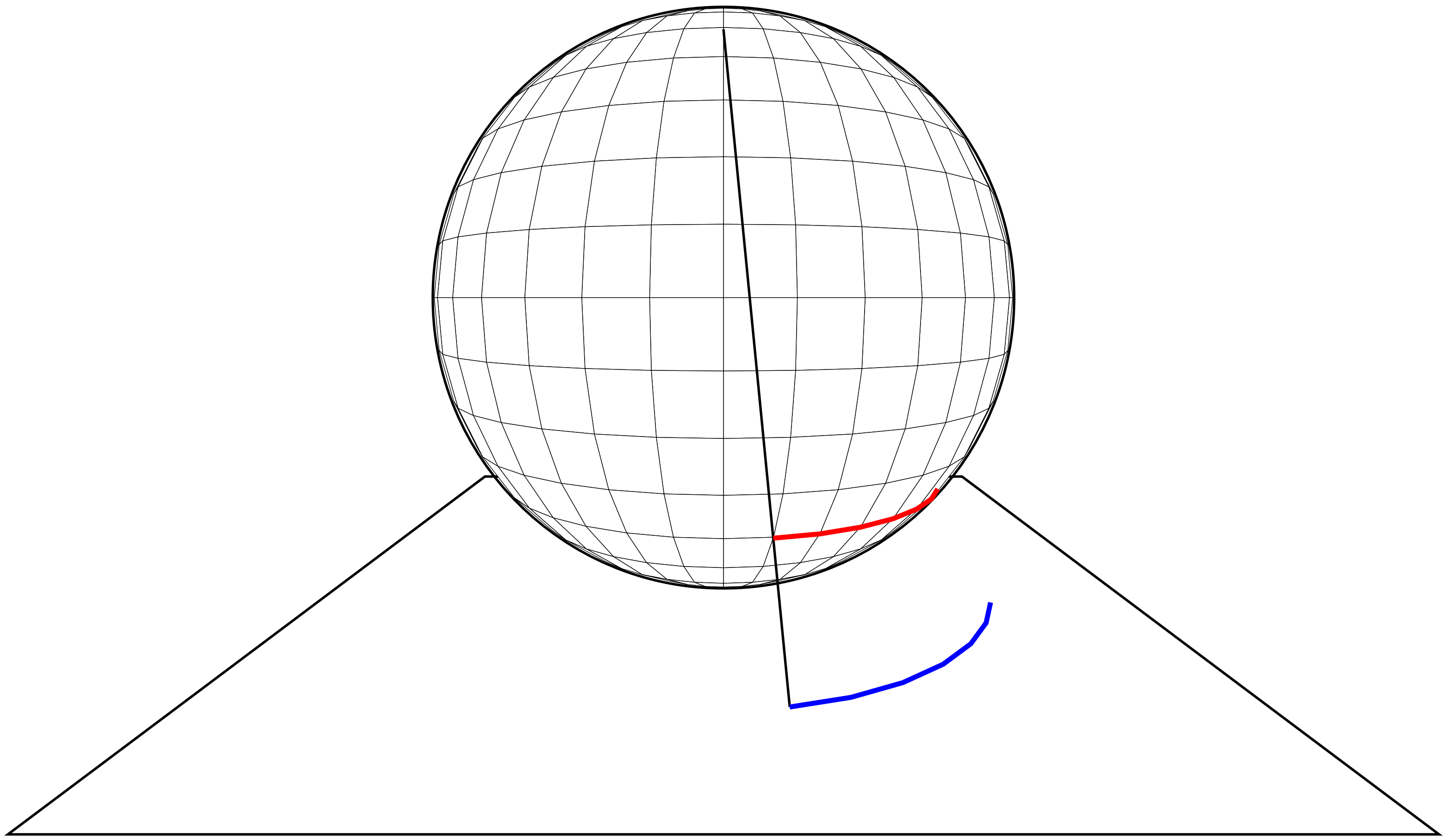

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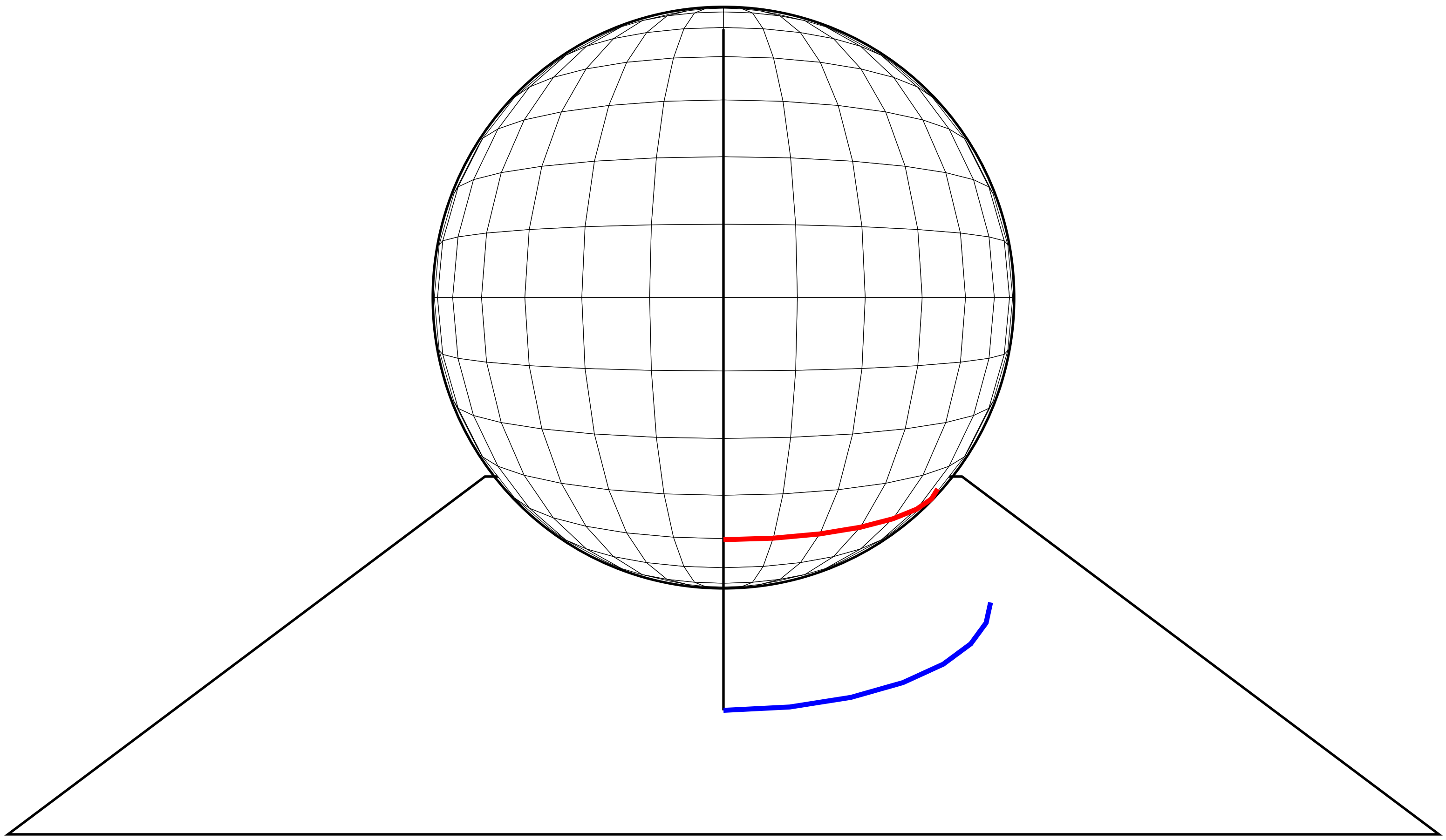

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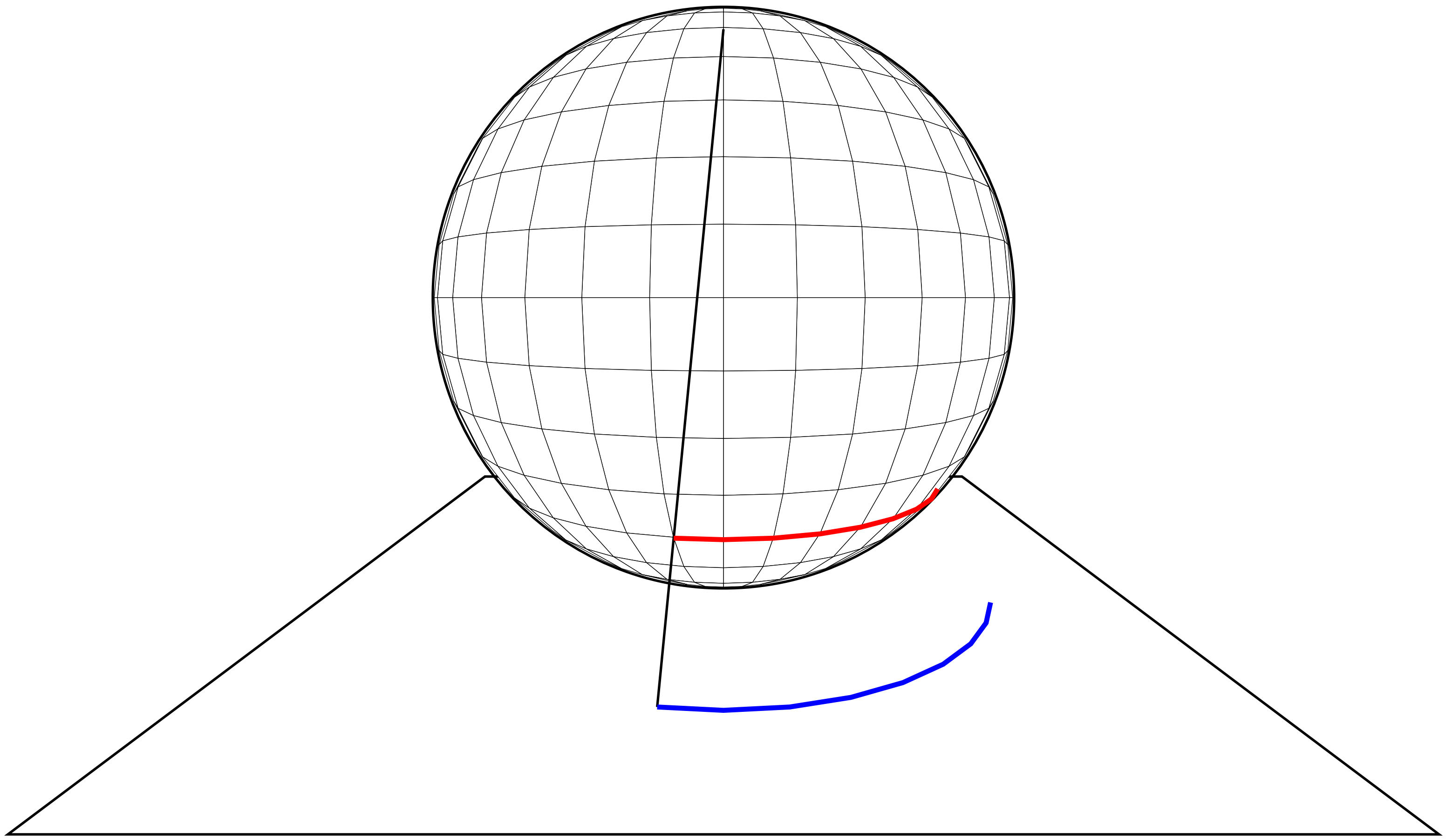

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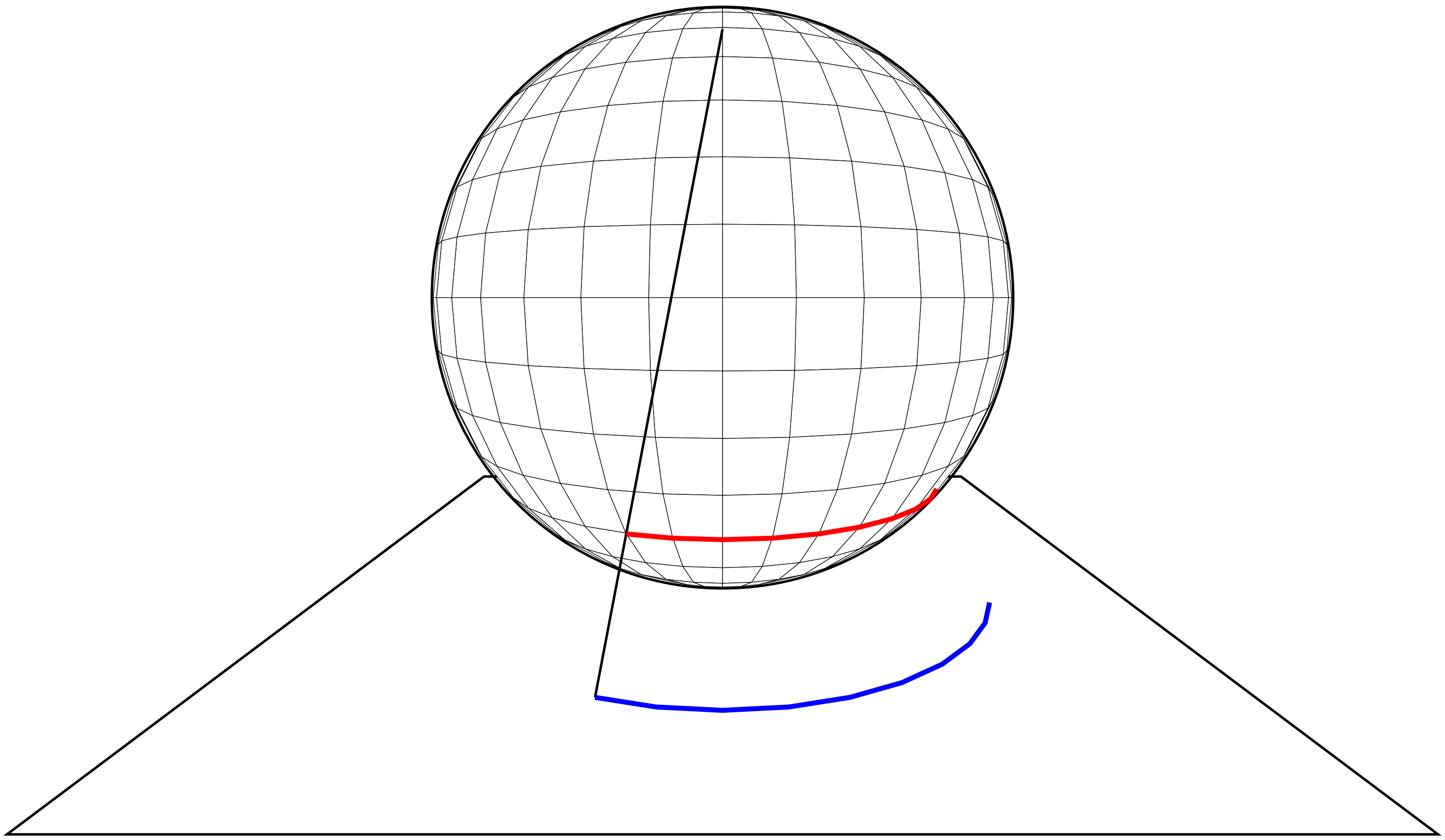

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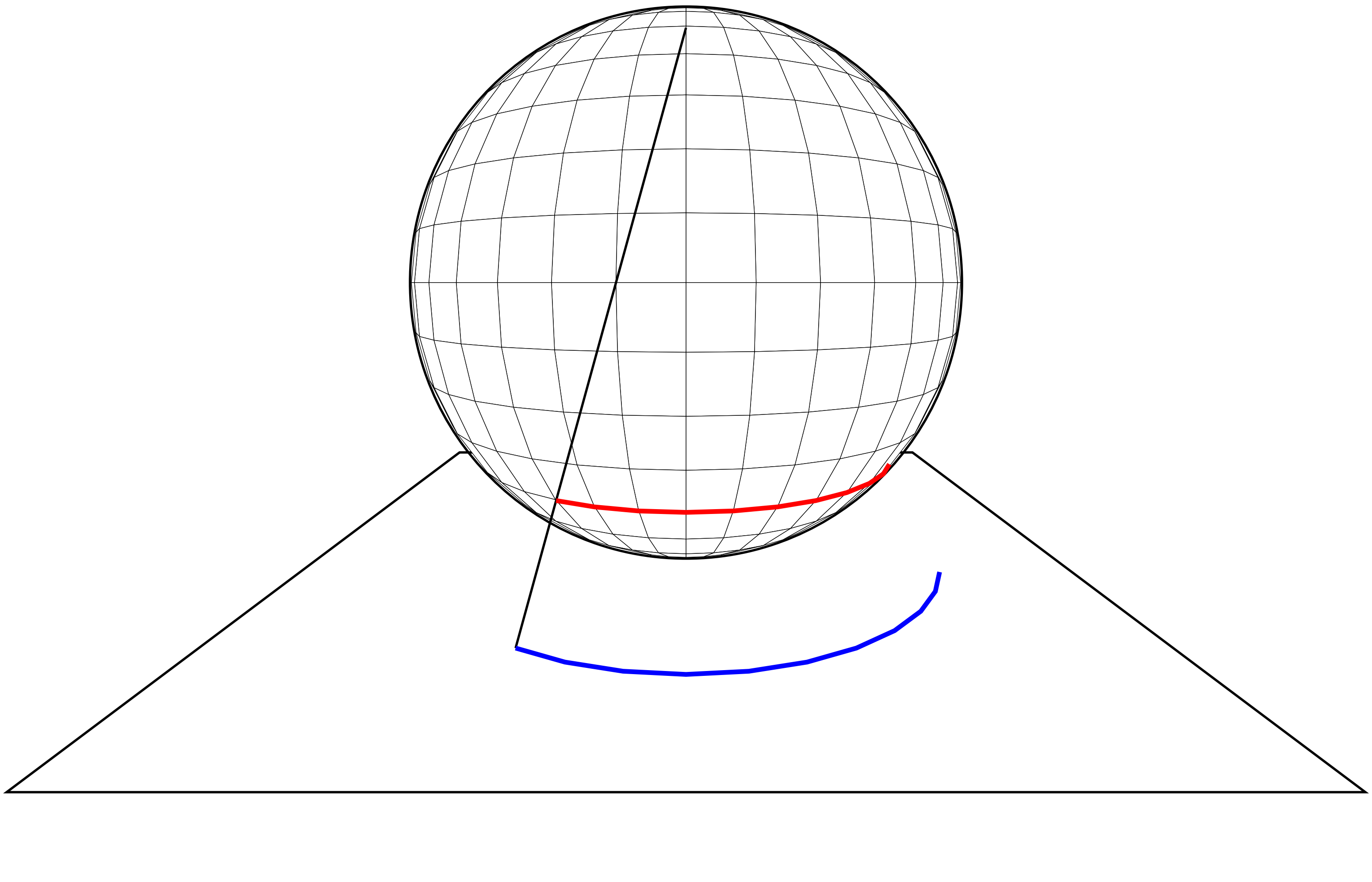

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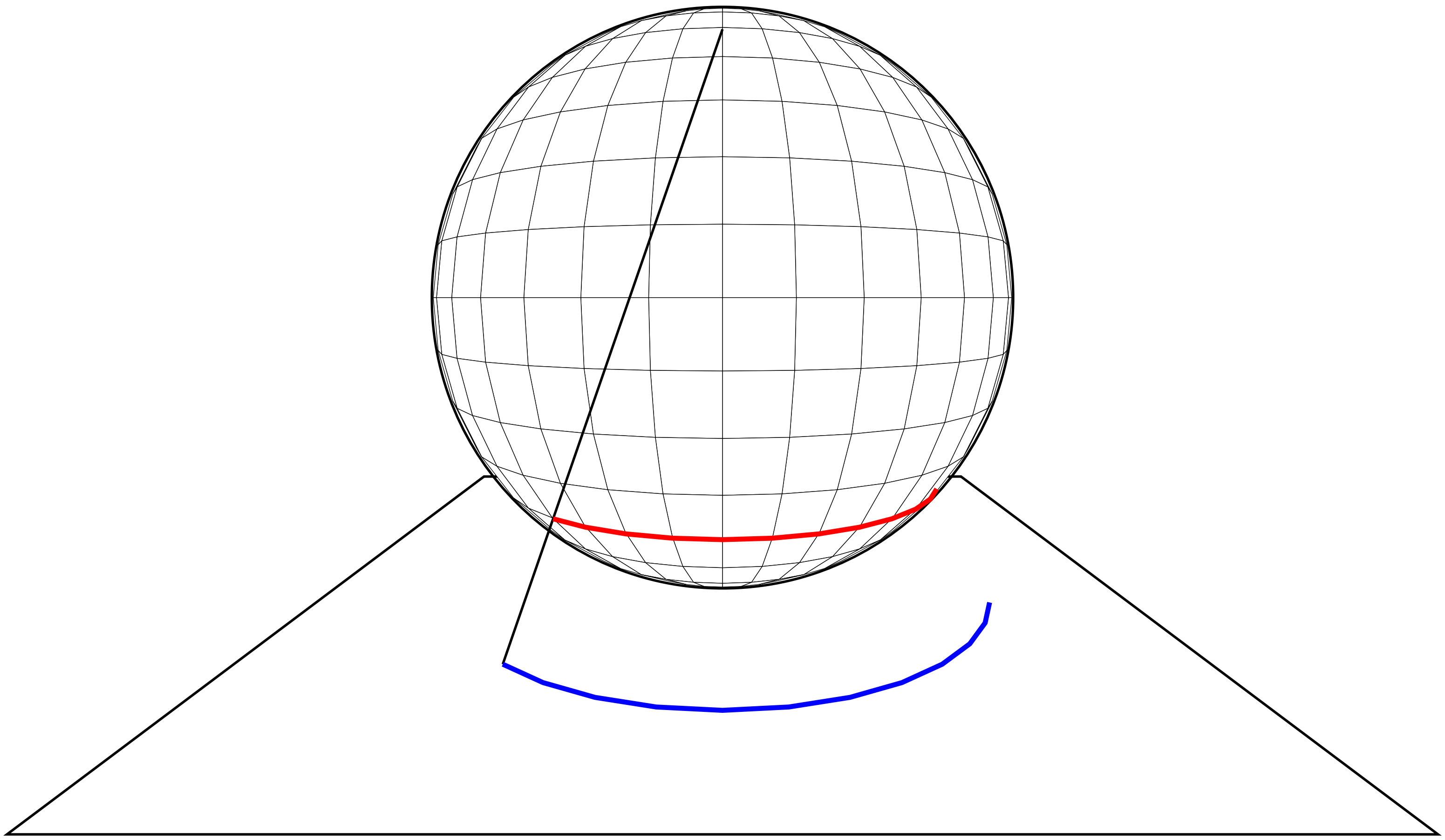

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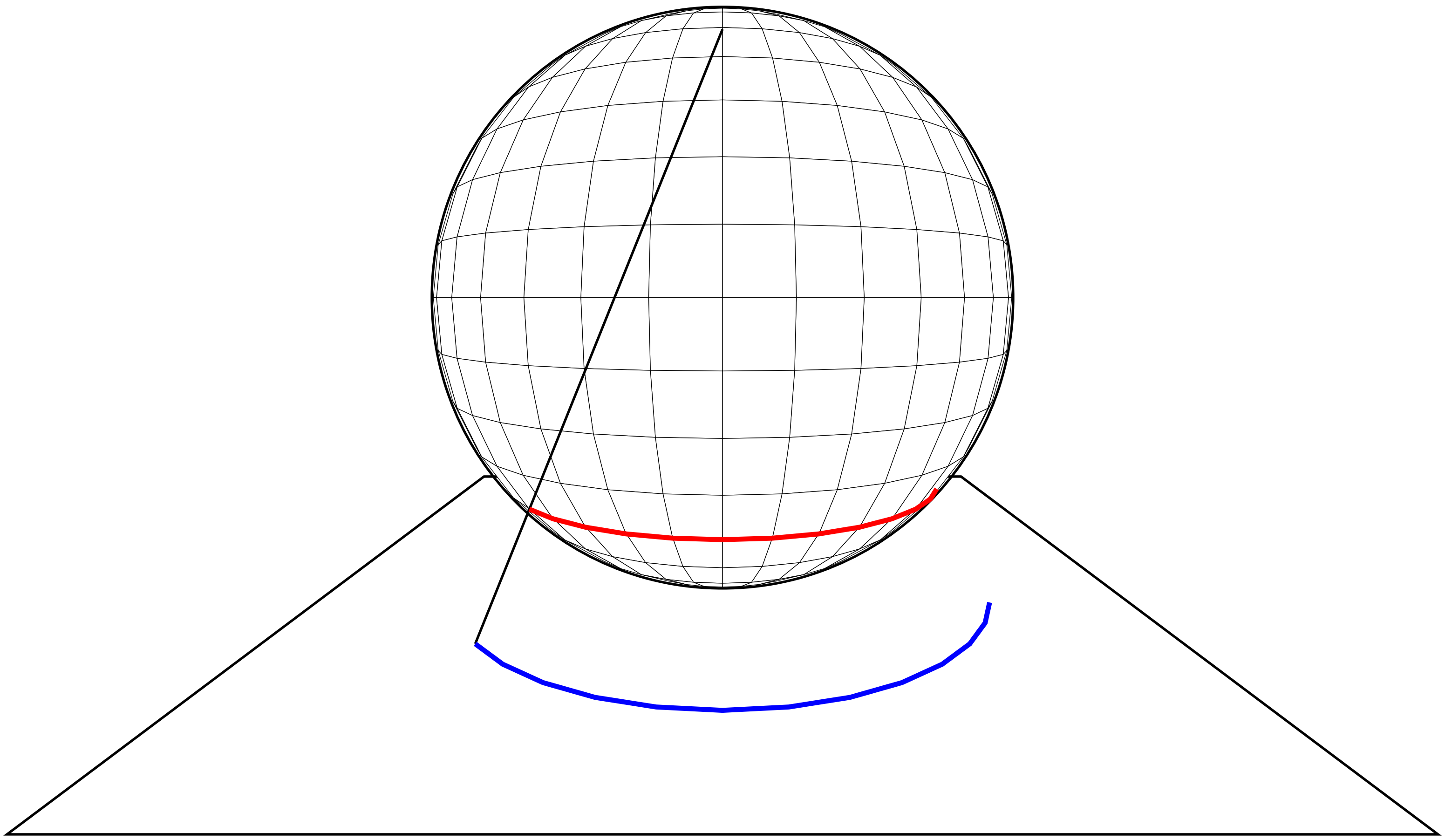

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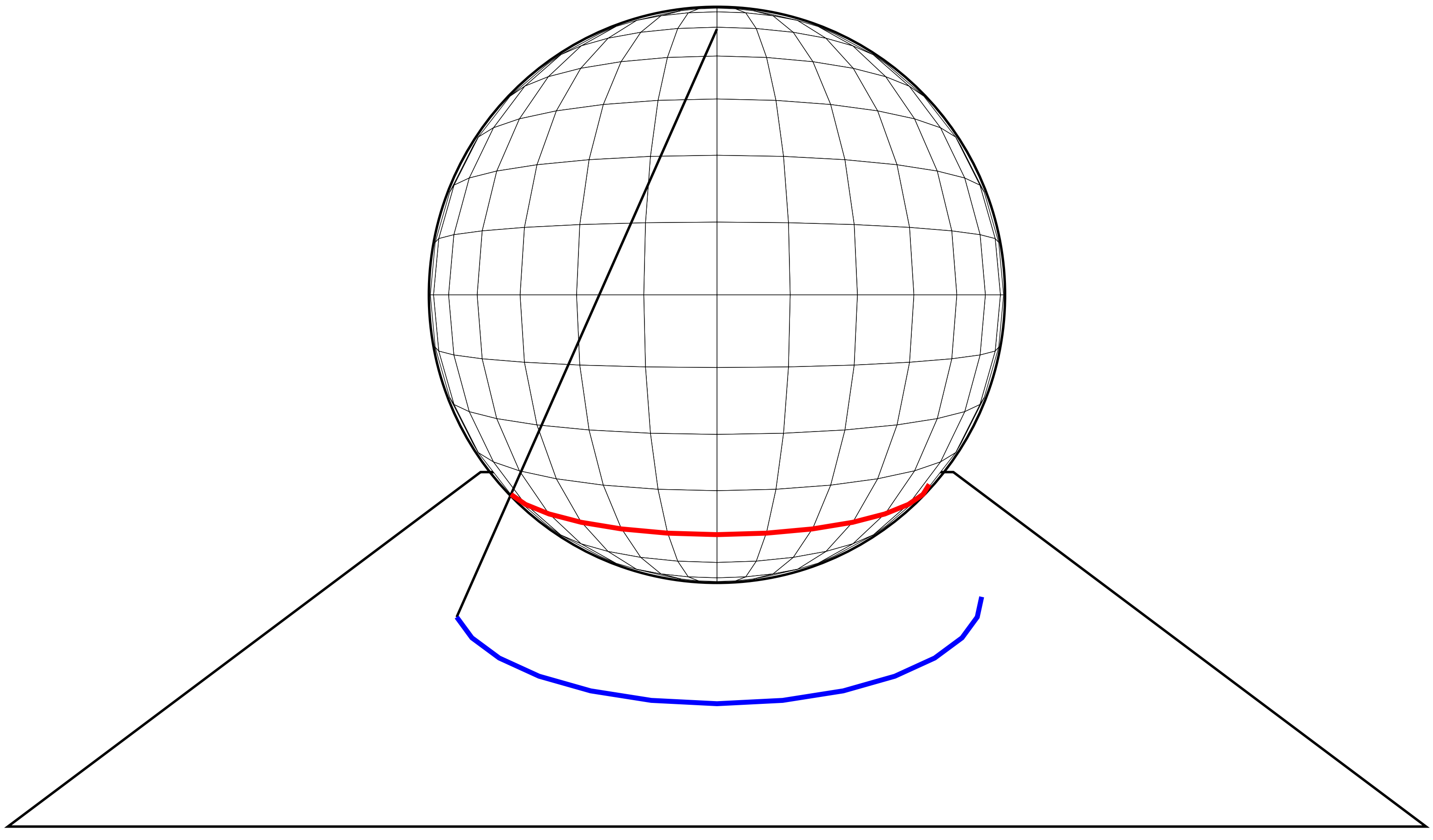

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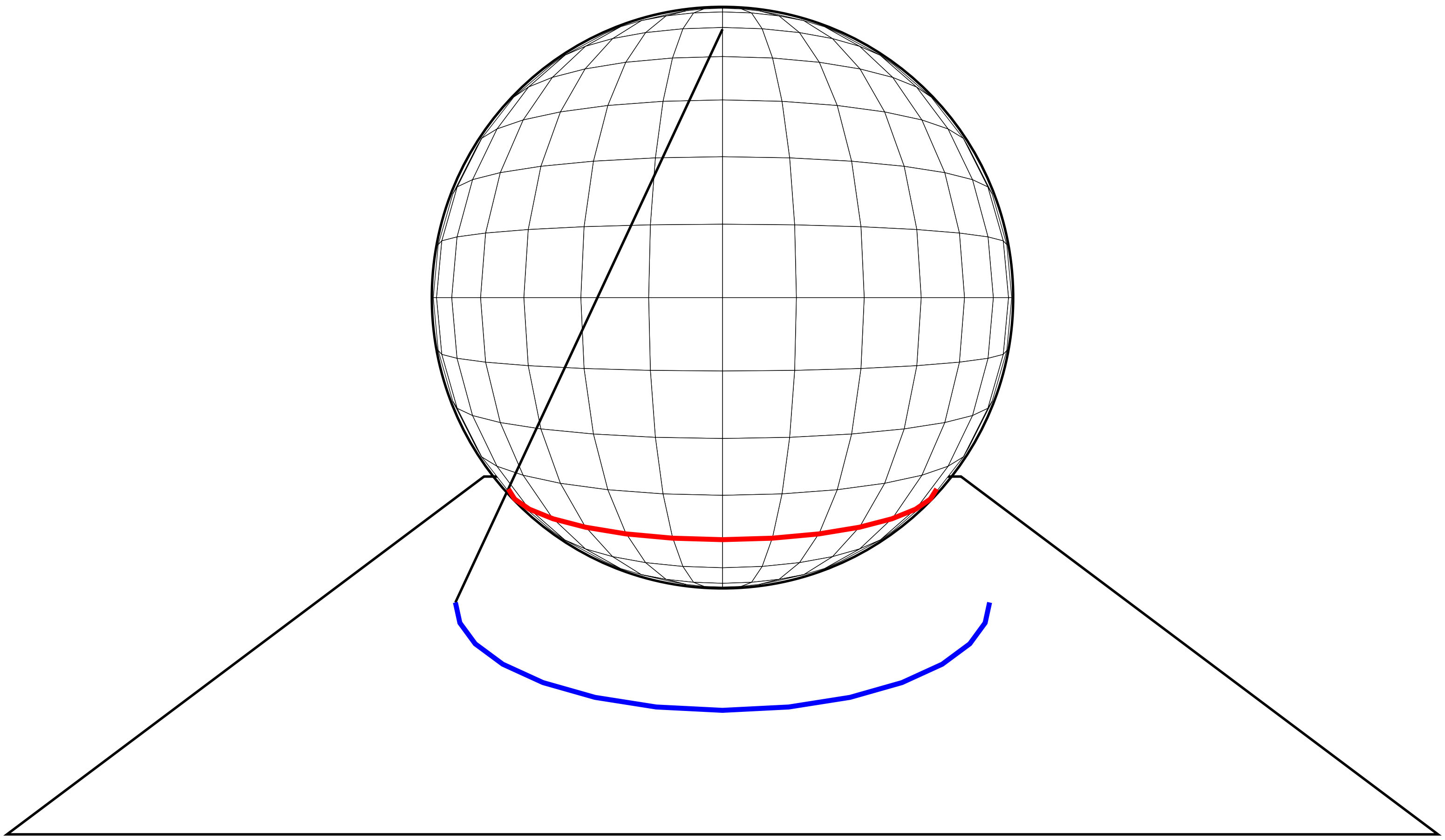
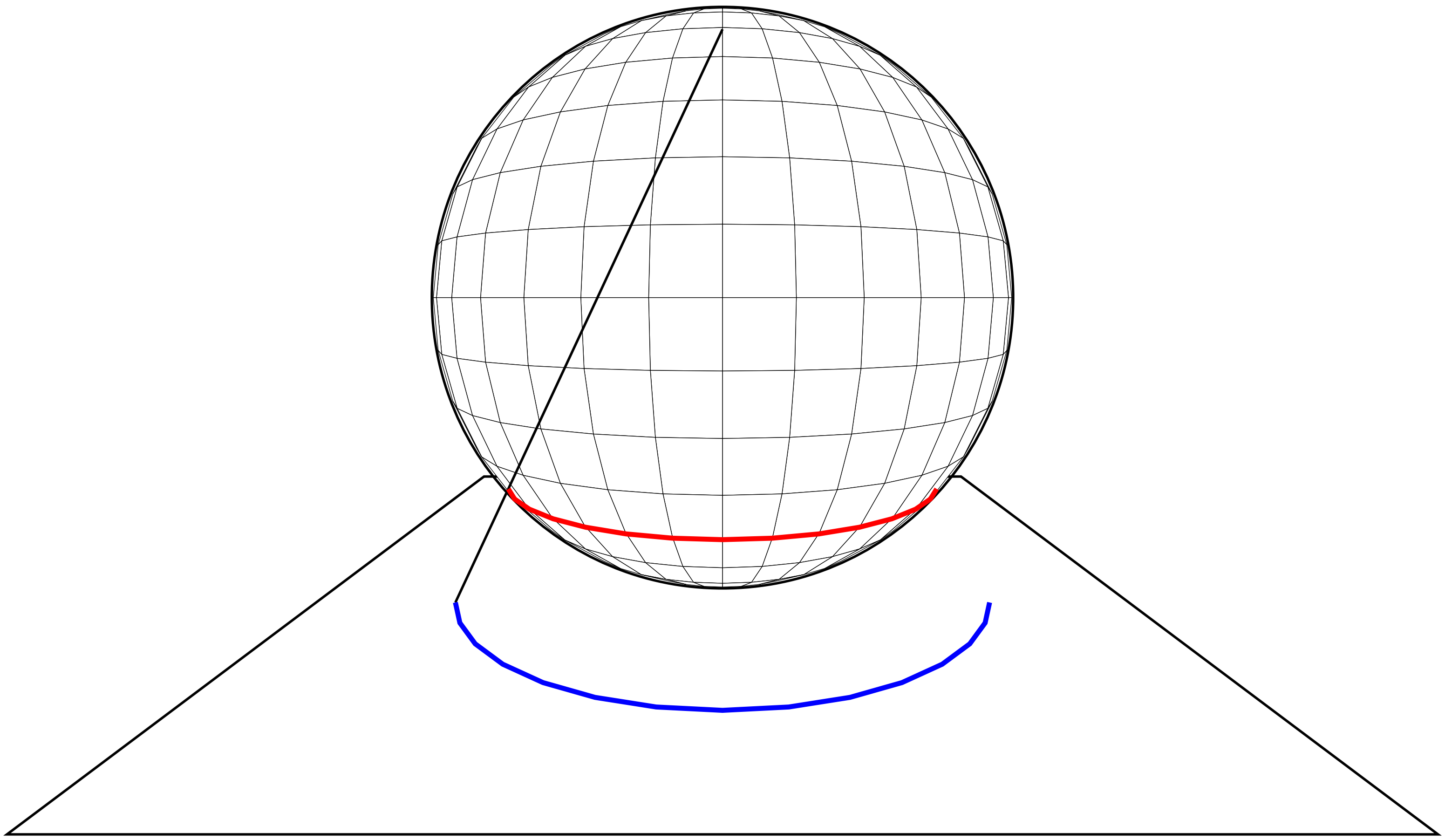
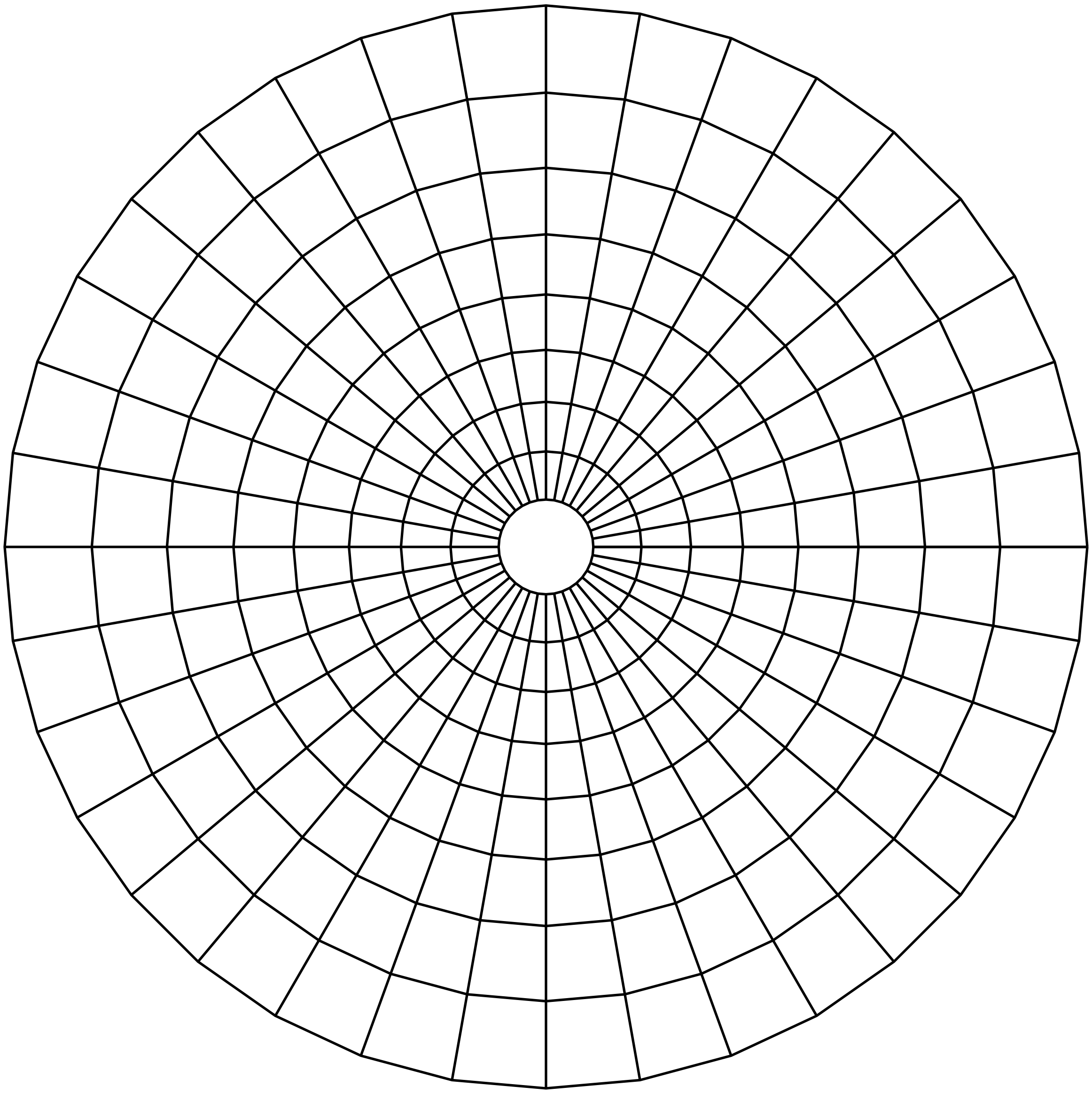


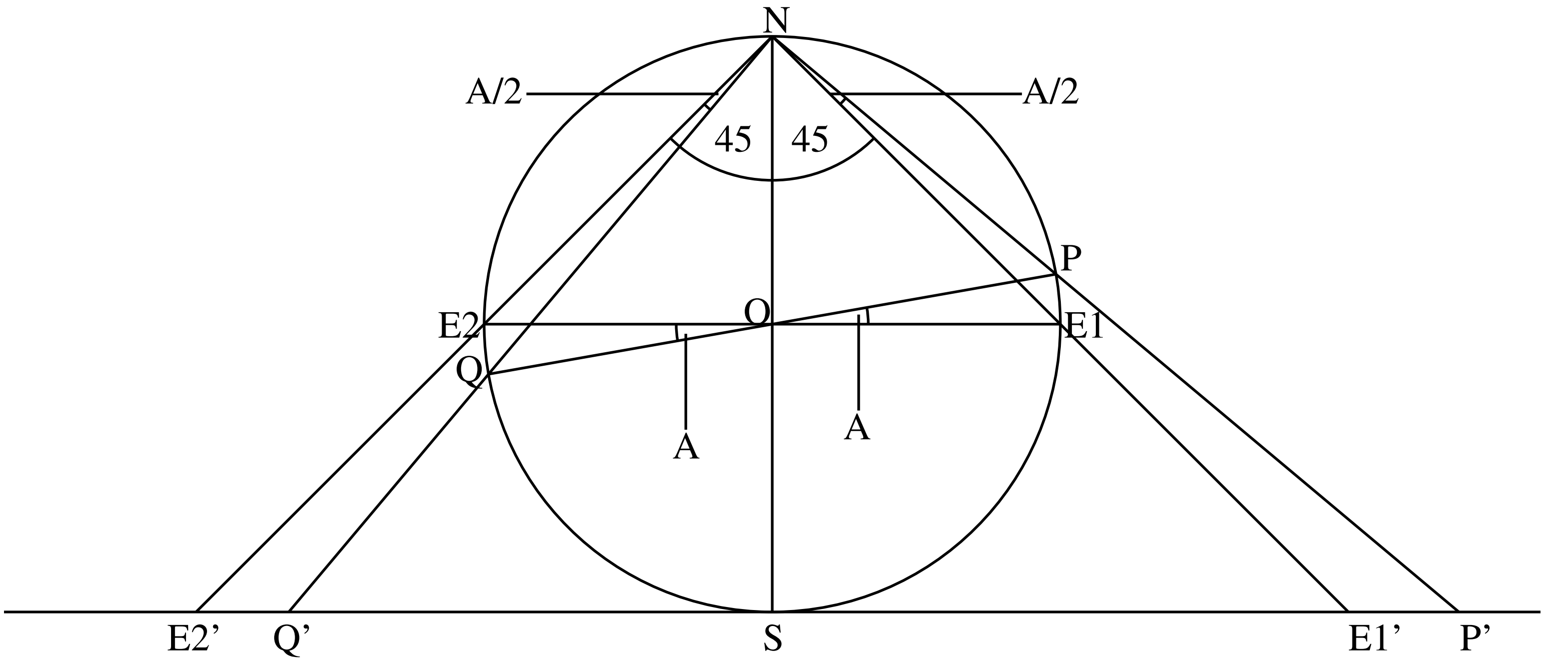
Projection of points with the same latitude will form a circle. The pole will be the center of all circles formed. Since all longitudes meet at the pole and projected latitudes are now circles instead of horizontal lines, the projected longtitude will be straight lines from the center of the circles to the edge of the largest circle projected. The angles between all projected longtitudes are the same.



This is the projection of a hemisphere using stereographic projection. Note that the distance between the equator and latitude=10 is longer than the distance between the pole and latitude=80. Let's examine why.



Let O be the center of the sphere, N be the north pole, S be the south pole, P and Q be points with opposite latitude A, E1 and E2 be points on the equator with same longitude as P and Q respectively, and the sphere has radius R.
Angle ONE1 and Angle ONE2 are 45 degrees. Angle POE1 and Angle QOE2 are A. Angle PNE1 and Angle QNE2 are A/2 because inscribed angle is 1/2 of the central angle if the are cut by the same chord.

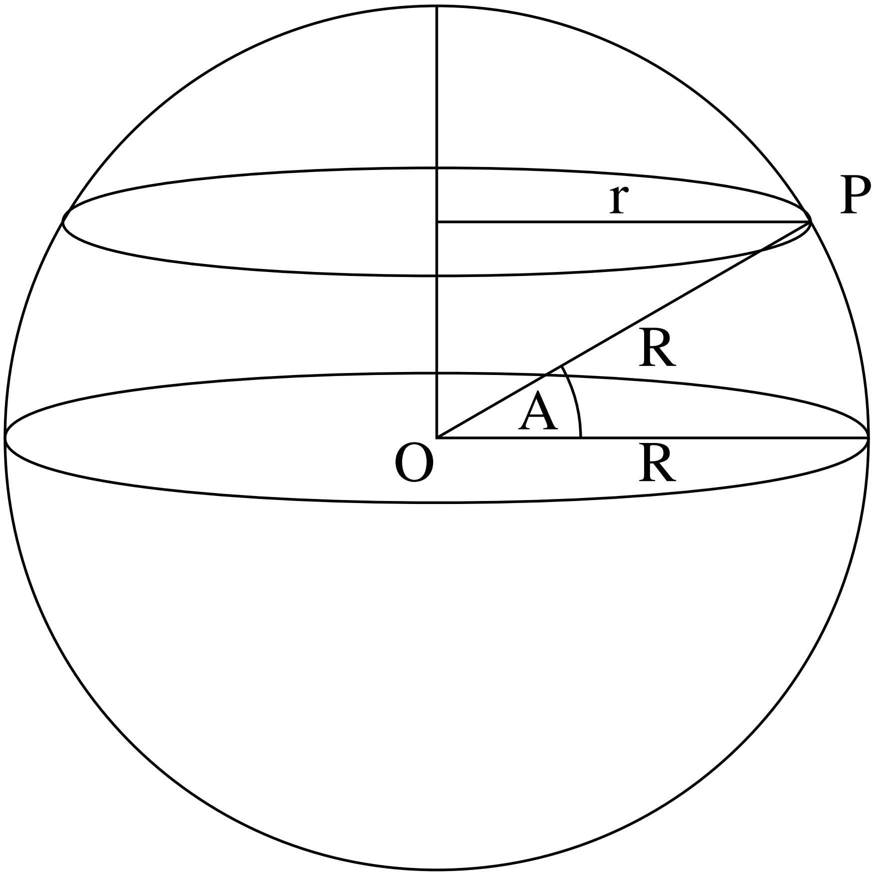


Therefore, Angle ONP'=45+A/2 and Angle ONQ'=45-A/2. Then SP'=2*R*tan(45+A/2) and SQ'=2*R*tan(45-A/2). In the map we created, we used SQ' since we were mapping the hemisphere that touched the plane. Note that tan(45-A/2) is a decreasing function from 0 to 90 degree and is concave up. Hence, the abosulue value of d(SQ')/dA becomes greater as A approaches 0, which means the rate of change in SQ' is greater when the points is closer to the equator, giving longer distance between it and latitude=10.

III. Mercator’s Projection

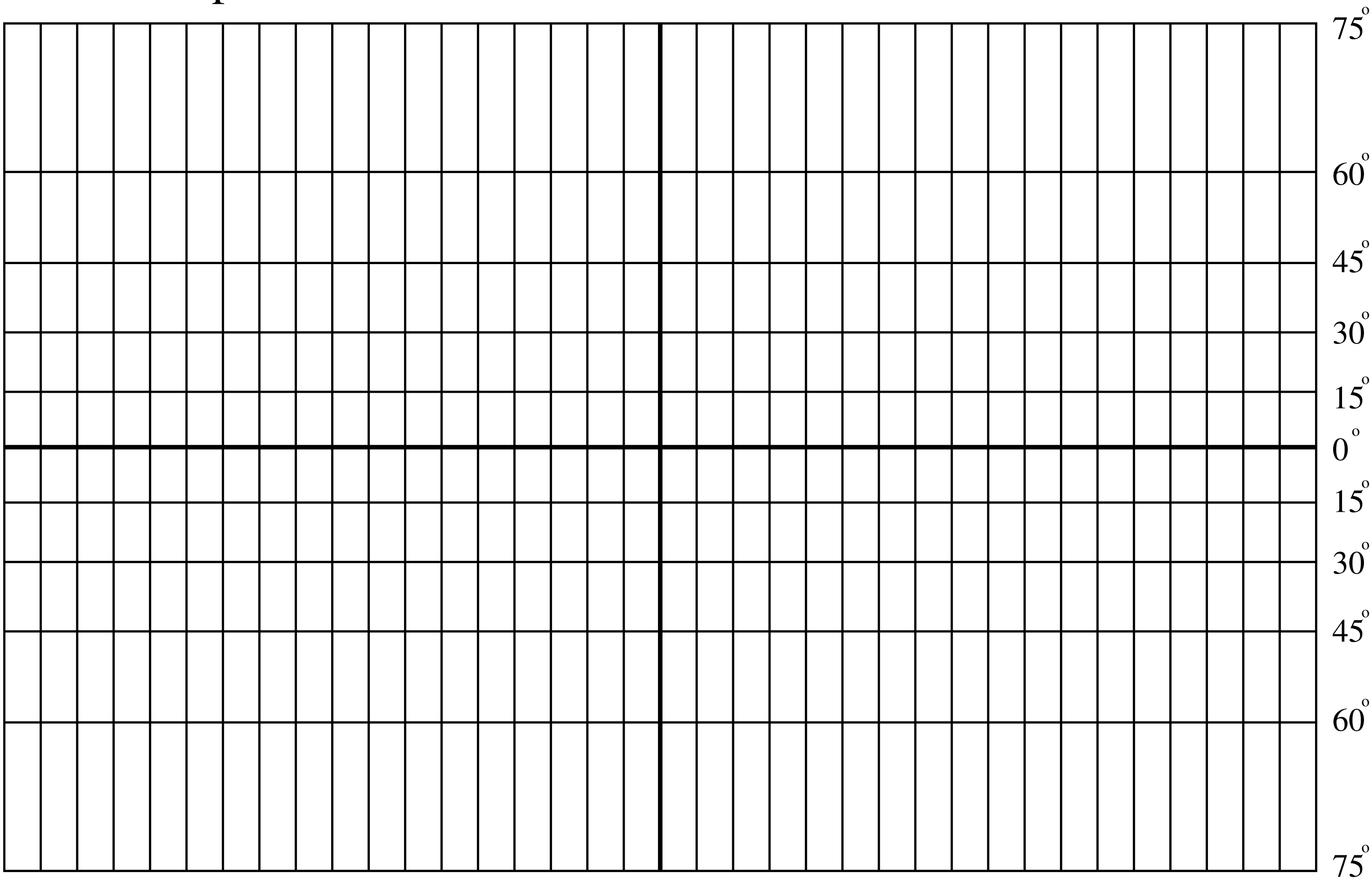
Once again, assume the globe is a perfect sphere. Let R be the radius of the sphere, O be the center of the sphere, P be a point on the sphere with latitude A, r be a line from the y-axis to P that is parallel to the equator.

The circle of latitude decreases in circumference as A increases. But Mercator would like to map the latitudes so that, on his map, they have the same length as the equator. Note that the



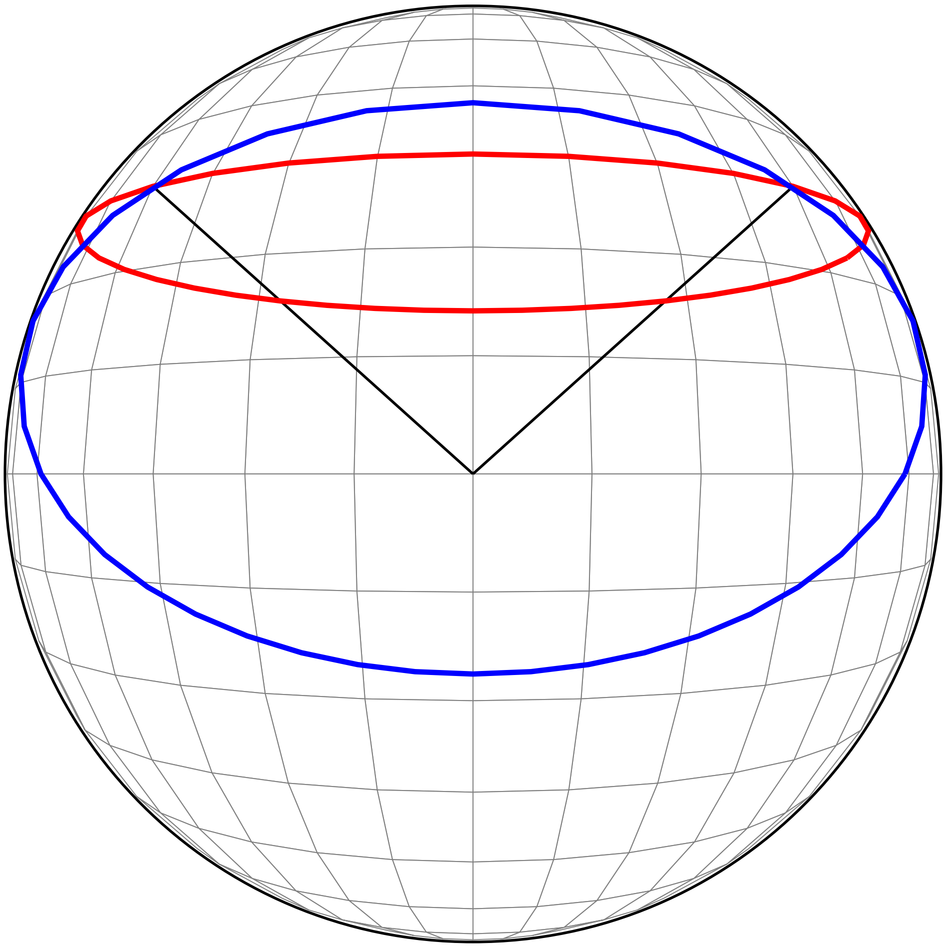
circumference of the small circle is 2*pi*r=2*pi*R*cos(A). And the circumference of the equator is 2*pi*R. So he must scale the latitude by a facter of (2*pi*R)/(2*pi*R*cos(A))=sec(A) on his map. But a east-west stretch must be accompanied by the same amount of north-south stretch. Hence, for the y-component, dy/dA=R*sec(A). Therefore, y=integral of R*sec(t)dt from 0 to A =R*ln(tan(45+A/2)).

This is a Mercator's map. Mercator's projection has often been confused with cylindrical projection. Imagine the globe is a spherical balloon that is blown up inside the cylinder, and sticks to the cylinder when it comes into contact with it. This is a Mercator's map and is different from projecting from the center of the sphere.

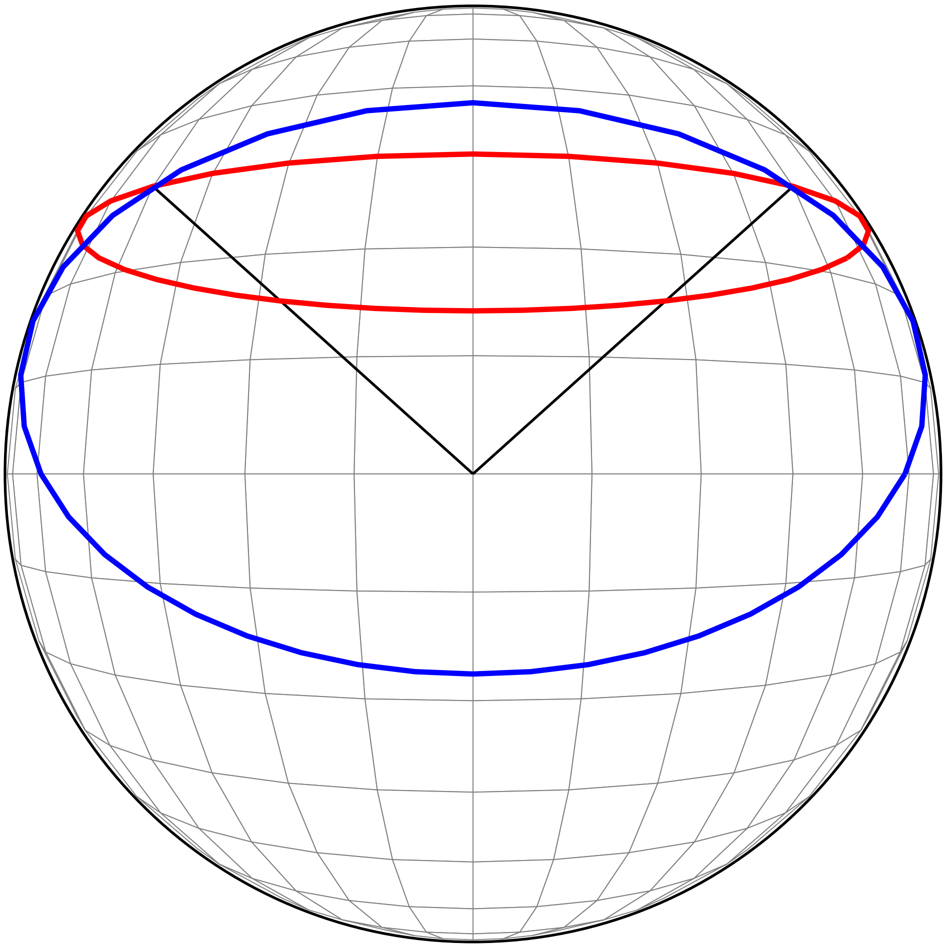


With the scaling and stretching, Mercator made his map direction-preserving. That is, lines of constant compass heading are straight lines on his map. However, the straight line between two points on his map is not the shortest route to travel unless this straight line is on the equator or on a longitude.

Let’s see why the the straight line in Mercator’s map is not the shortest route to travel and that the shortest route is the great circle course.



The black lines connect points on the surface of the sphere to the center of the circle. Let say we want to travel from one of these points to another. Let the red circle consists the compass course (the route in a Mercator's map) and the blue circle consists the great circle course. For the blue circle to consists the great circle course, the center of the blue circle must be the center of the sphere.



We will show that the great circle course is the shortest route through series of animation that involves rotations of the sphere and the red circle.

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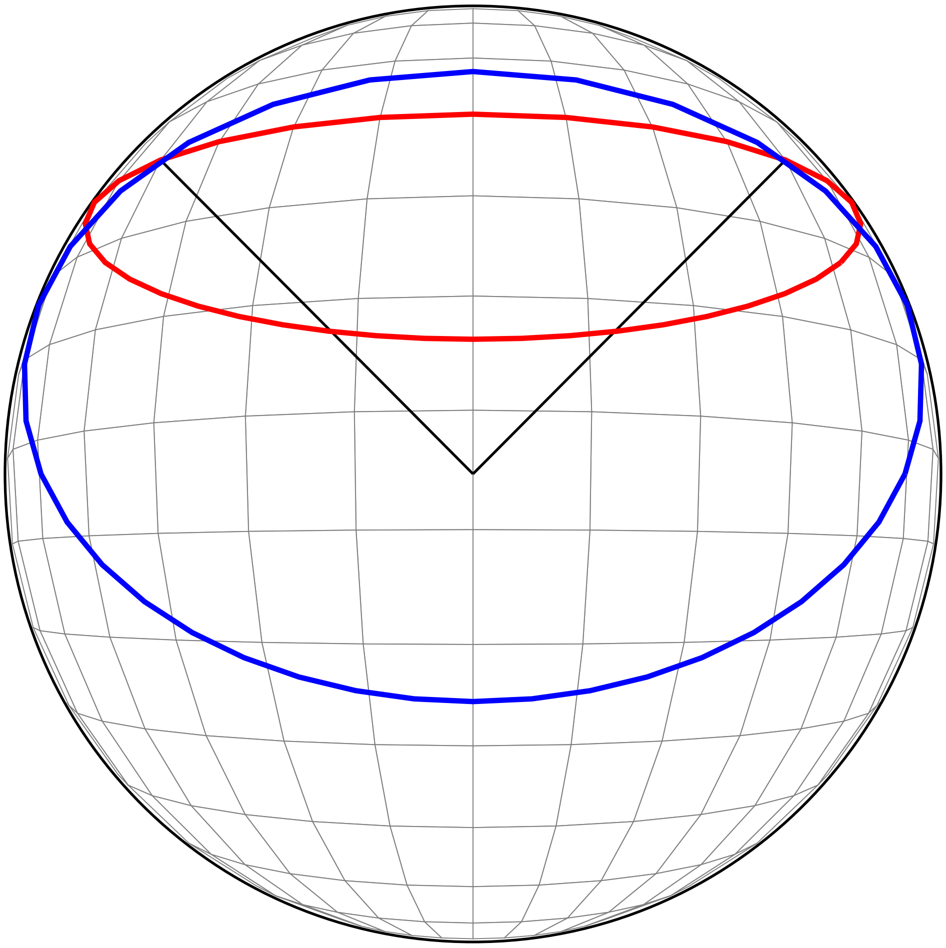

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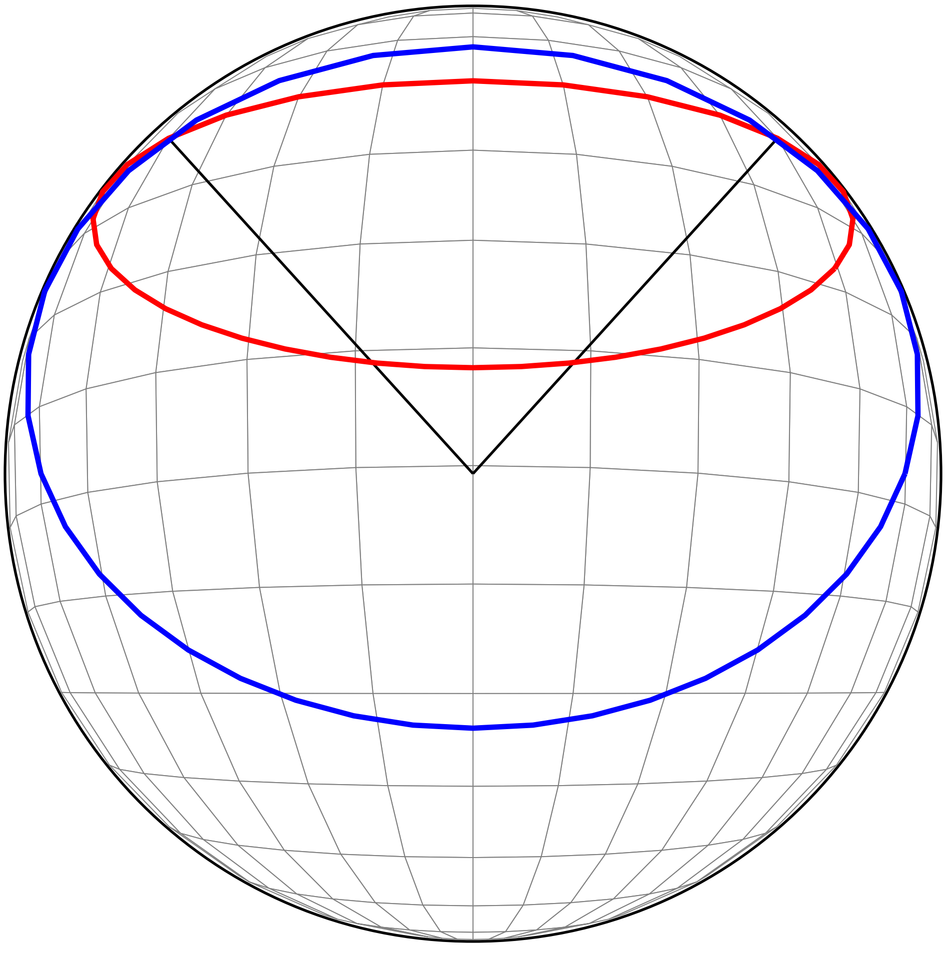

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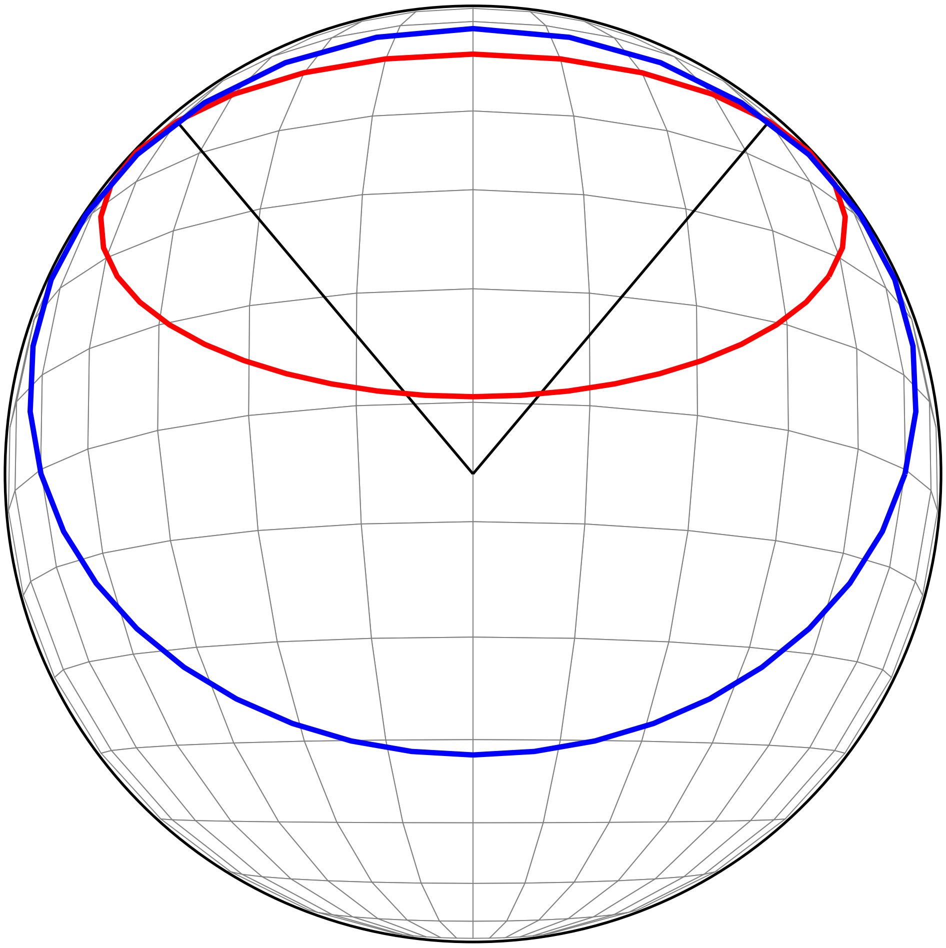

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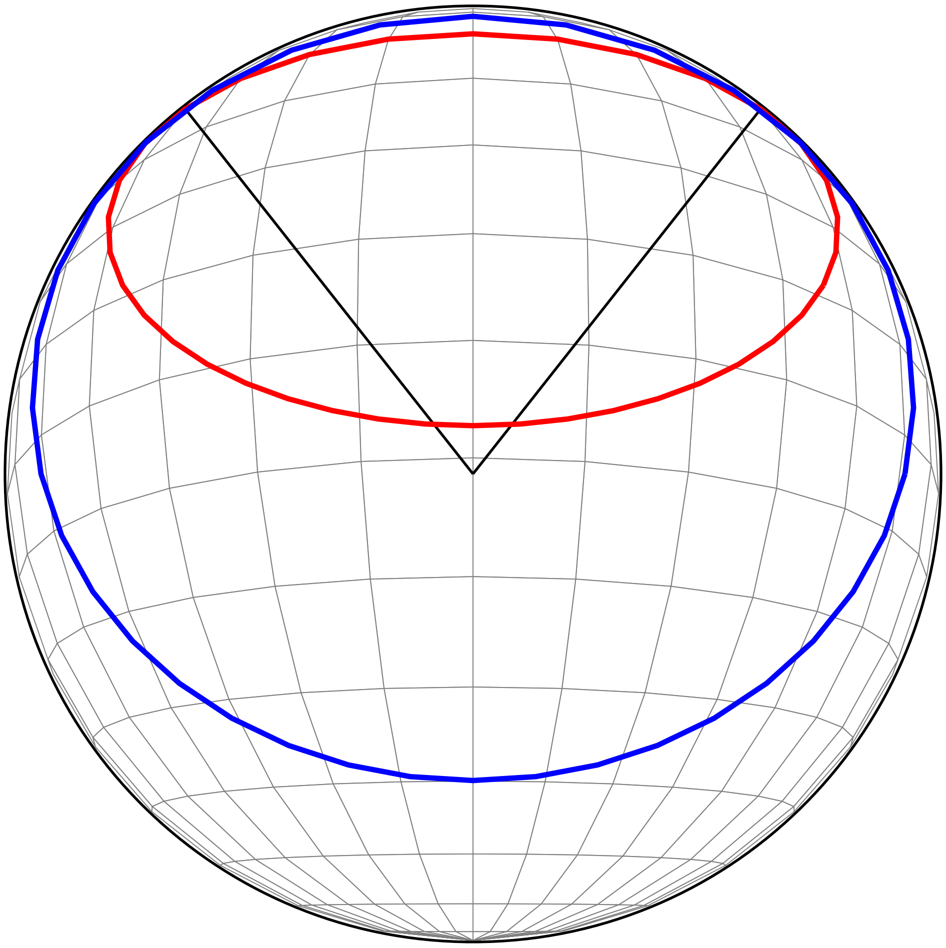

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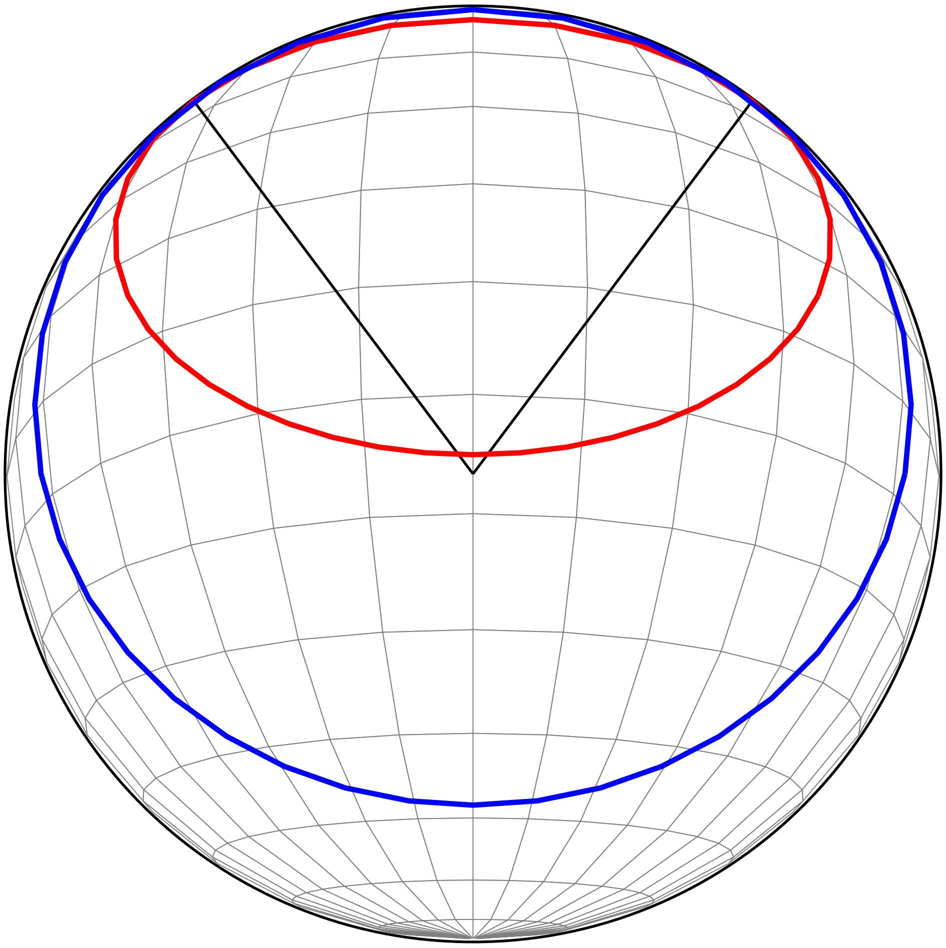

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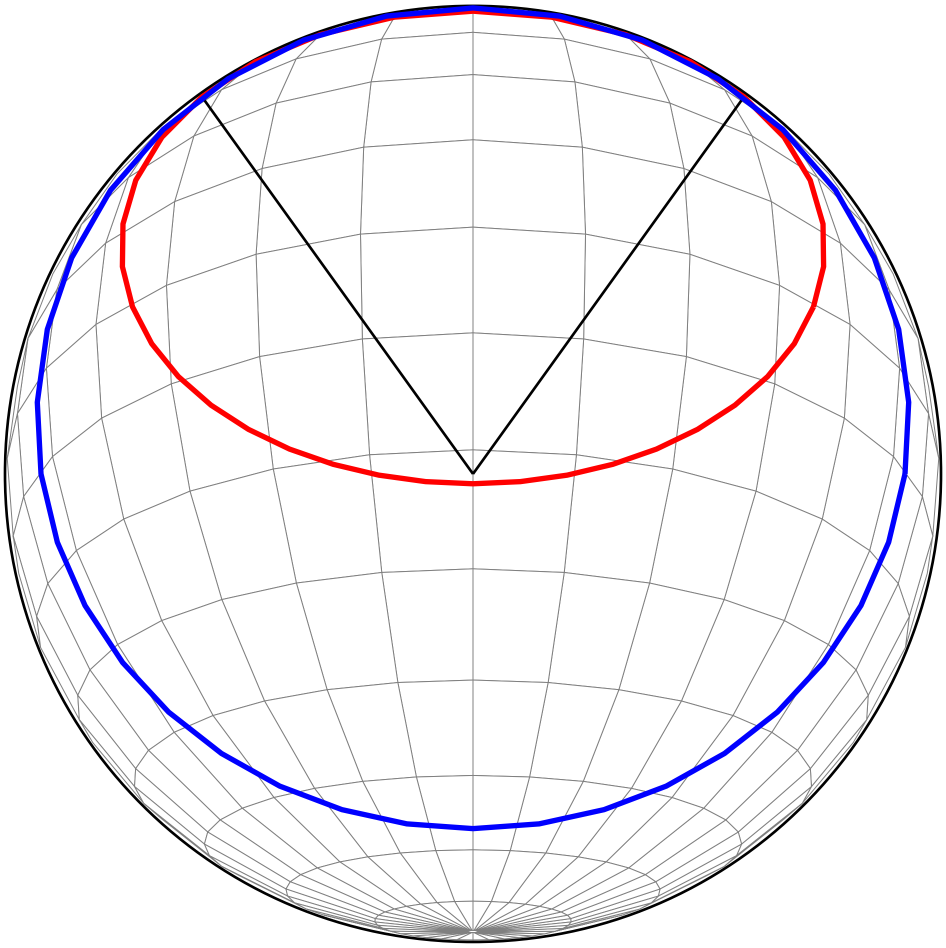

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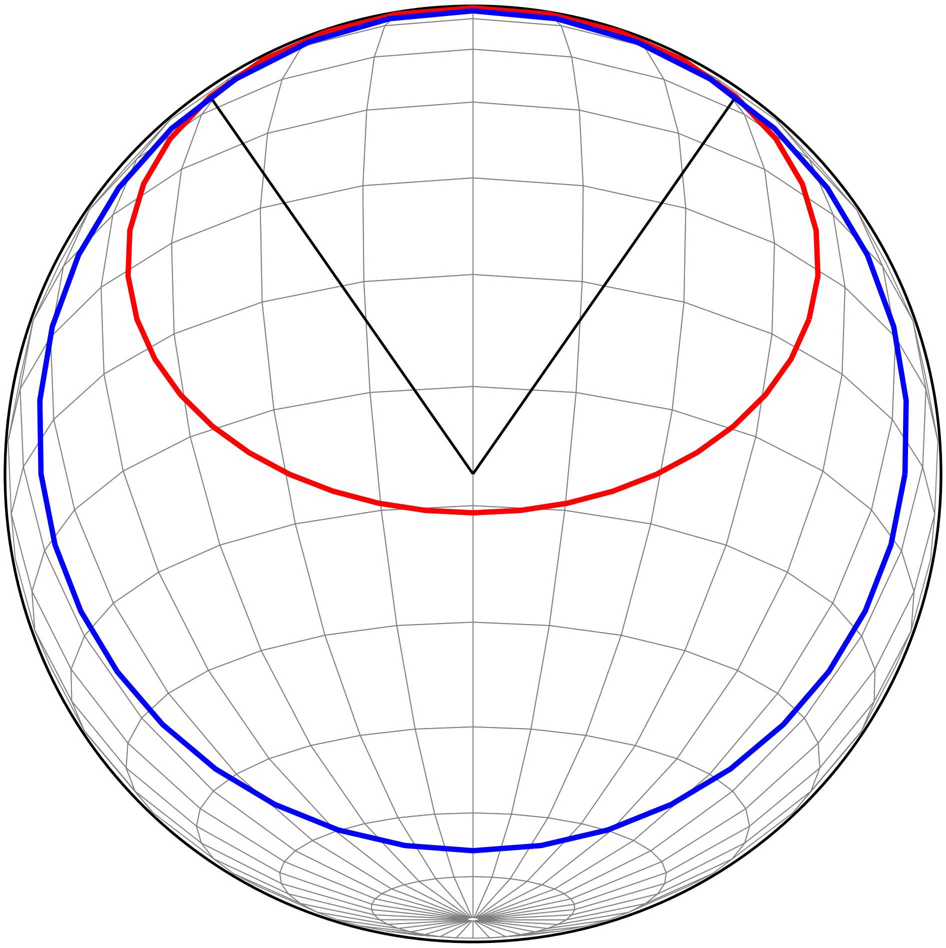

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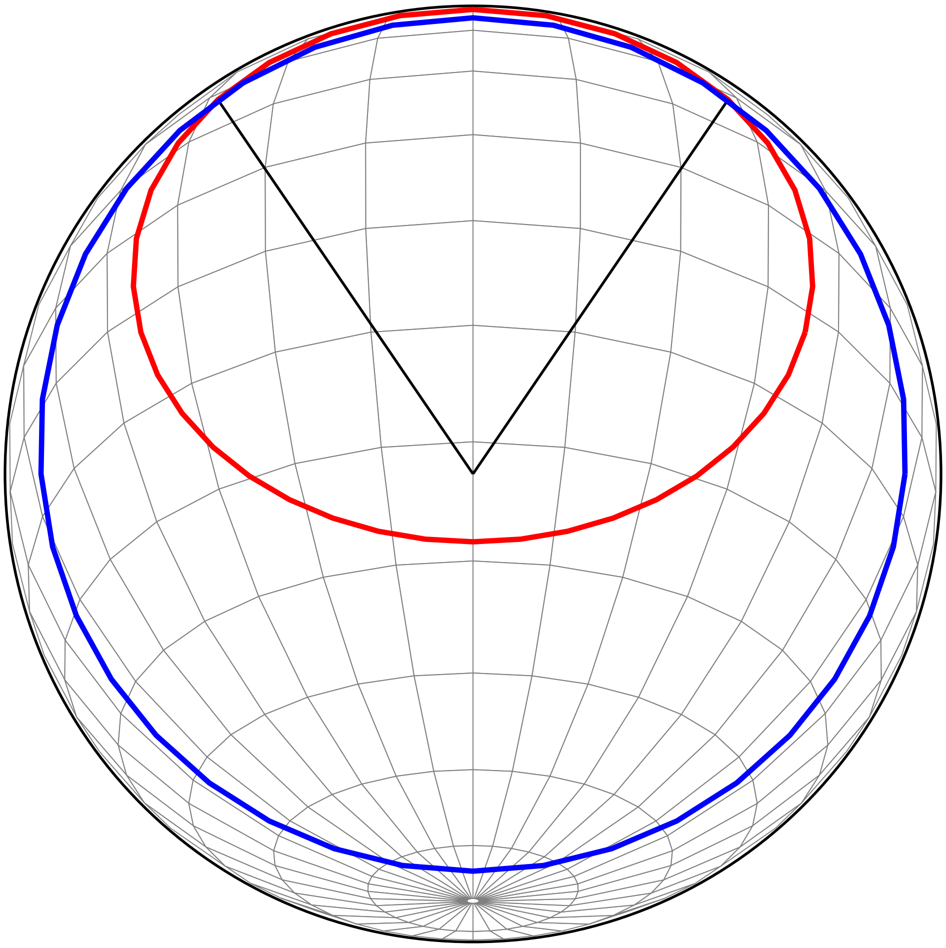

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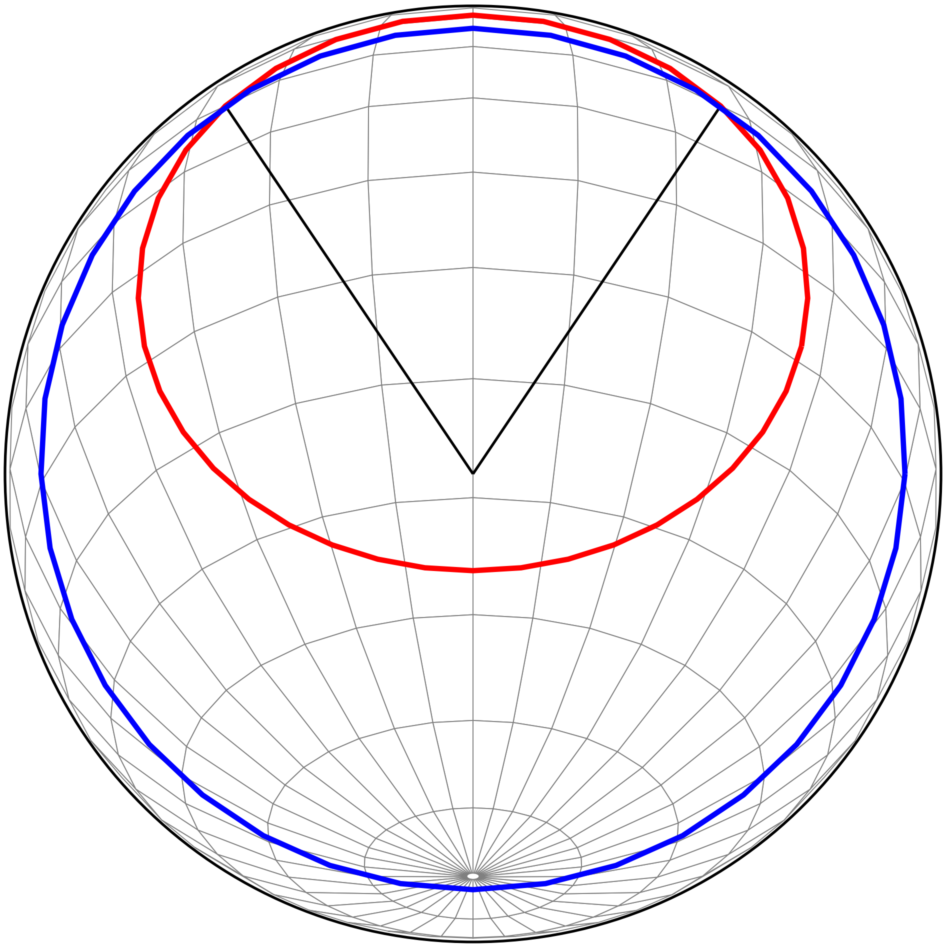
The black lines connect points on the surface of the sphere to the center of the circle. Let say we want to travel from one of these points to another. Let the red circle consists the compass course (the route in a Mercator's map) and the blue circle consists the great circle course. For the blue circle to consists the great circle course, the center of the blue circle must be the center of the sphere.



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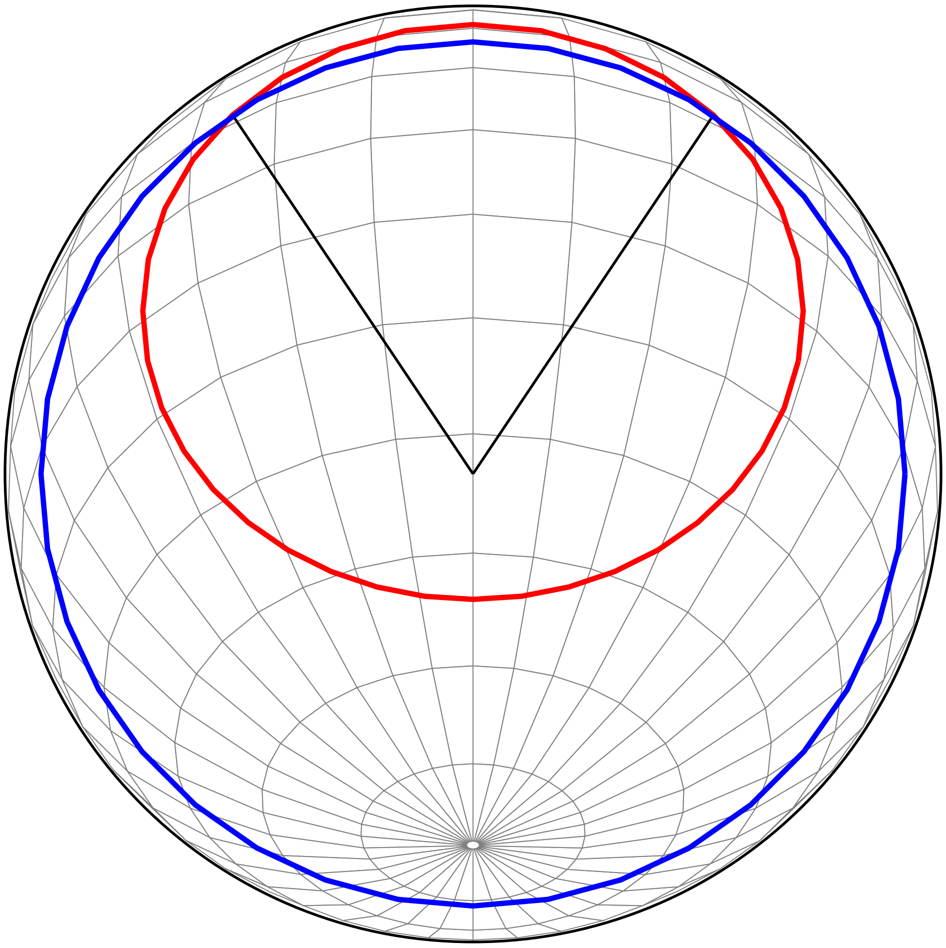

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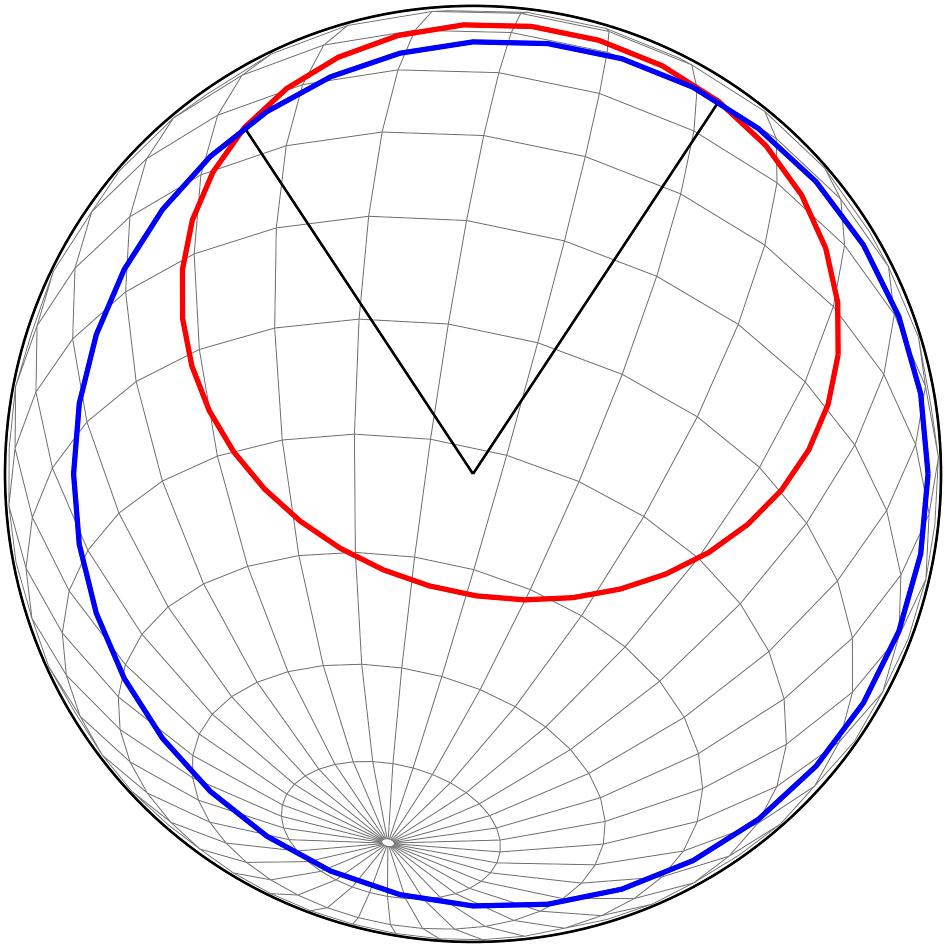

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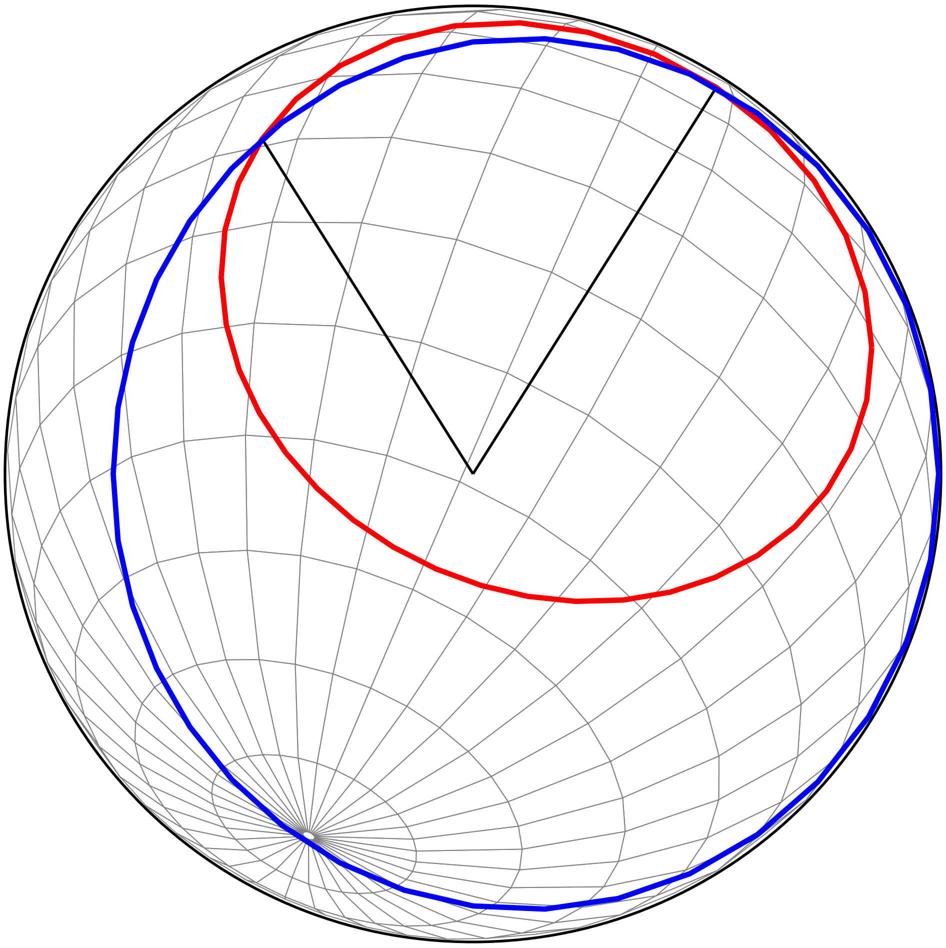

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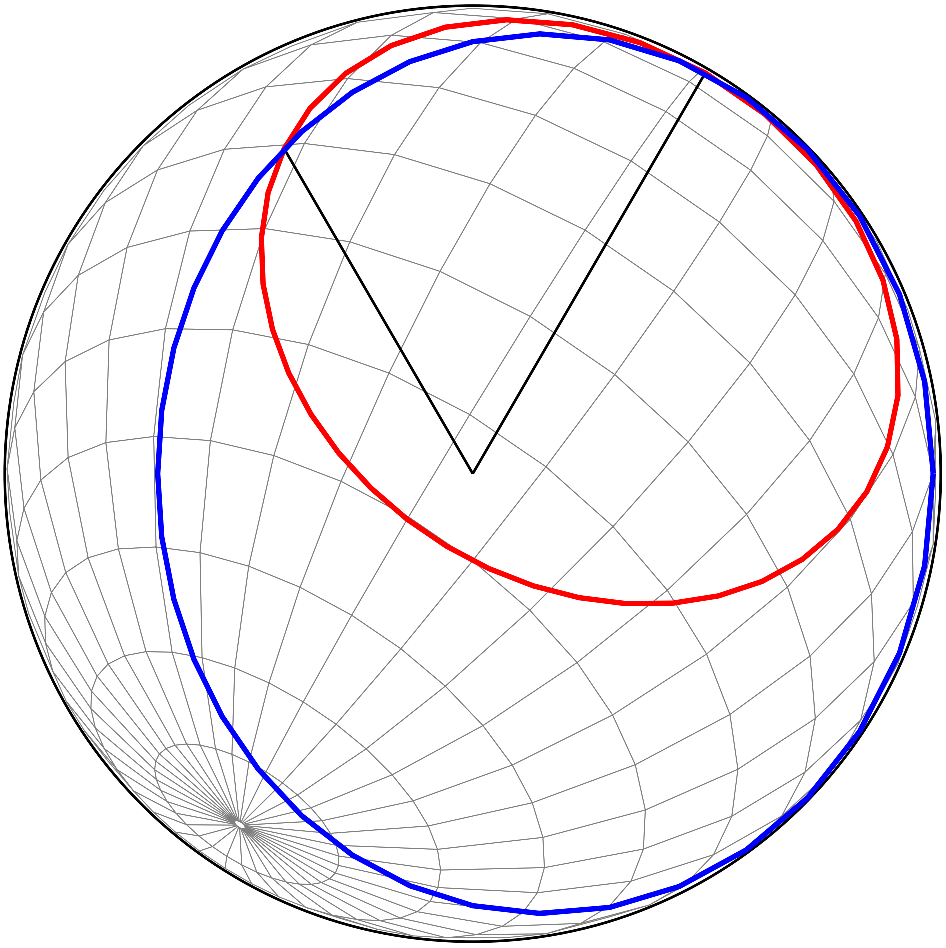

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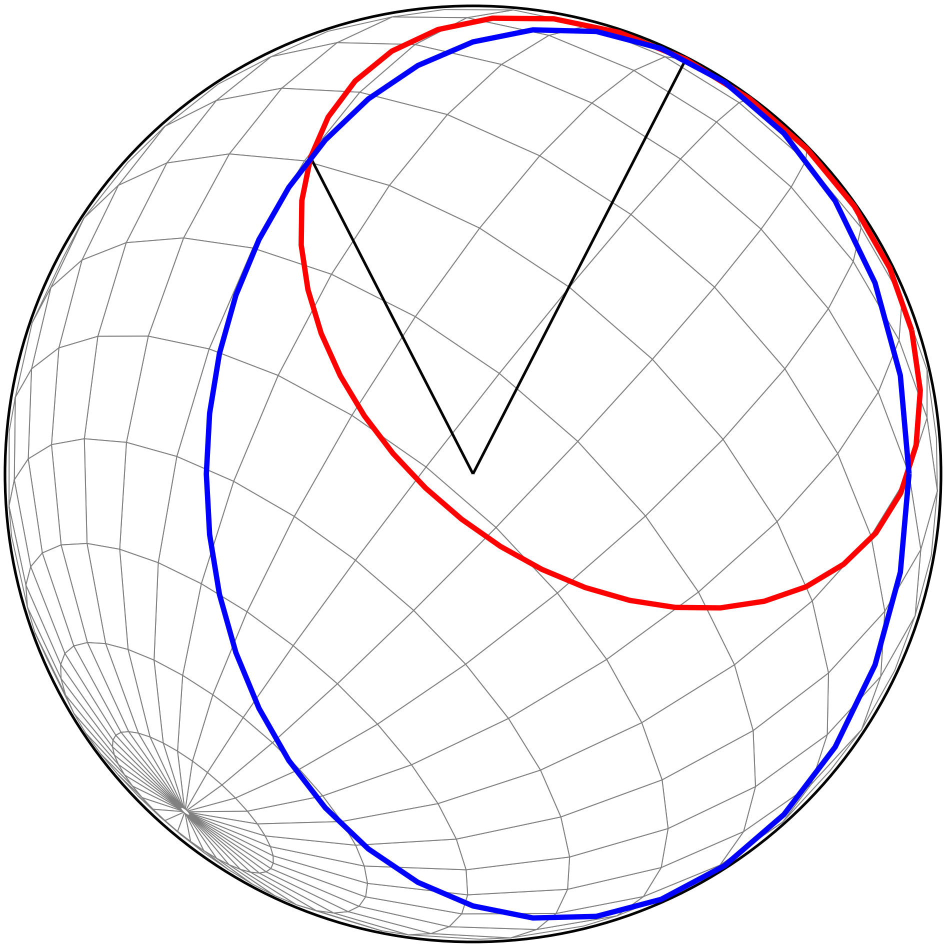

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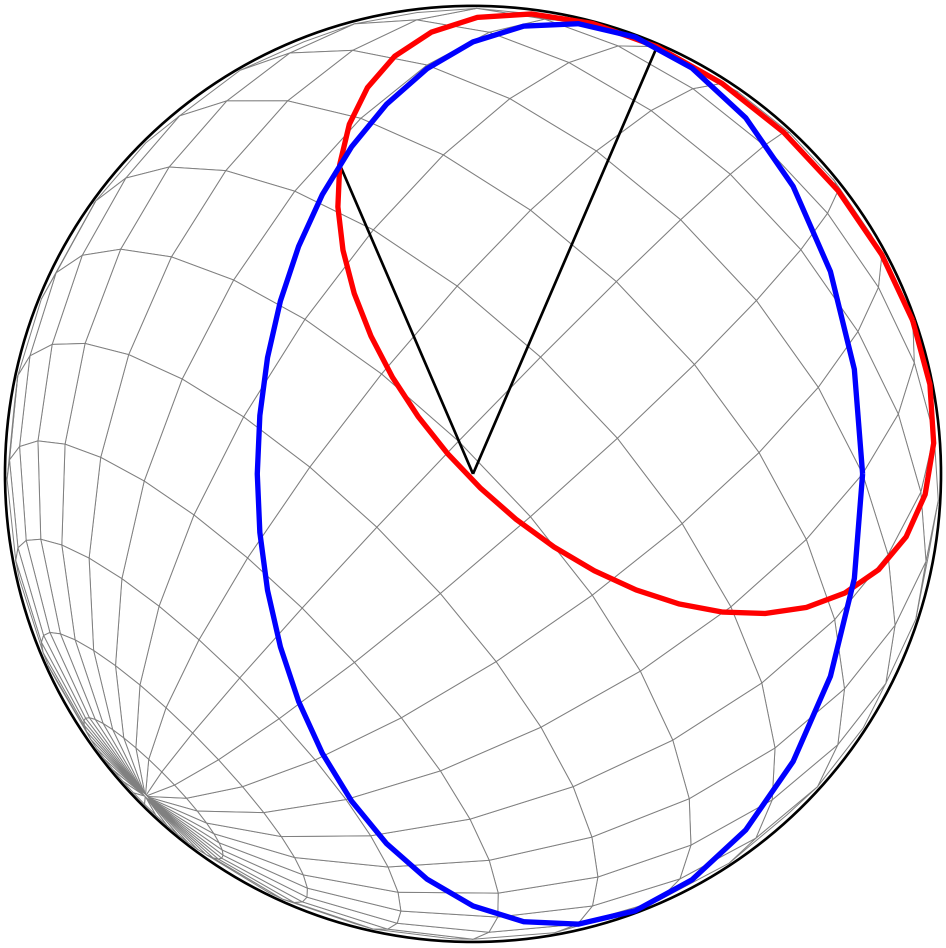

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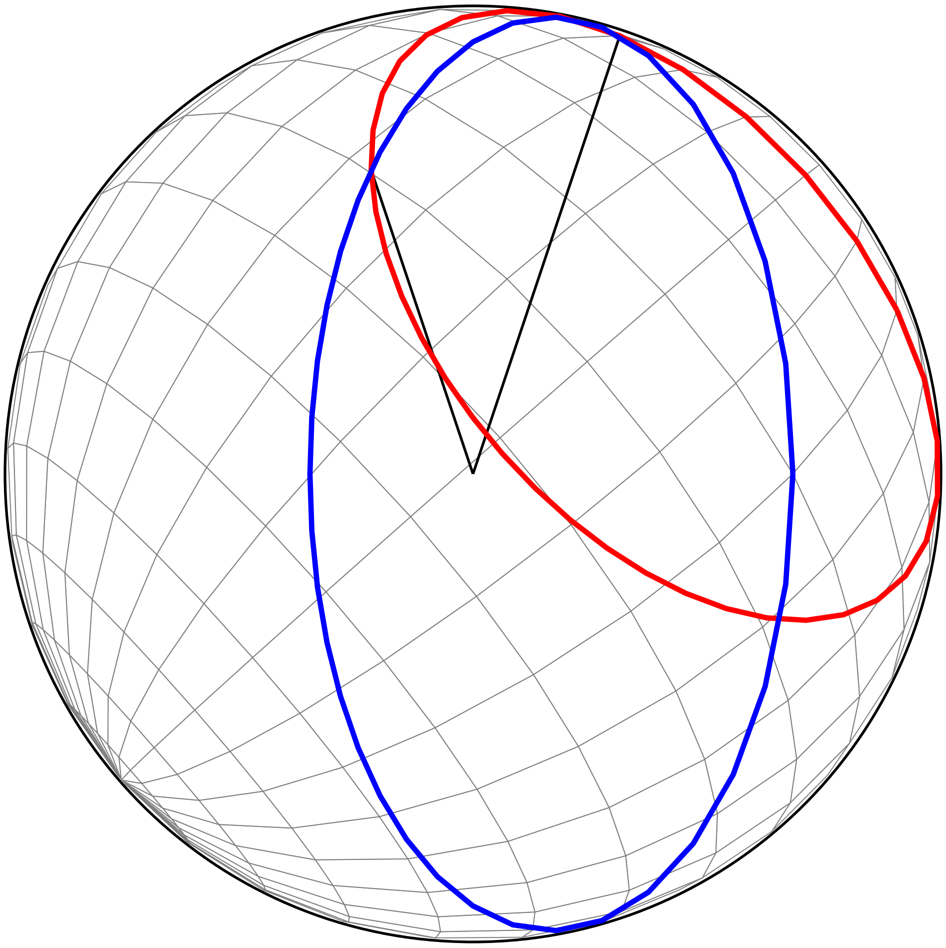

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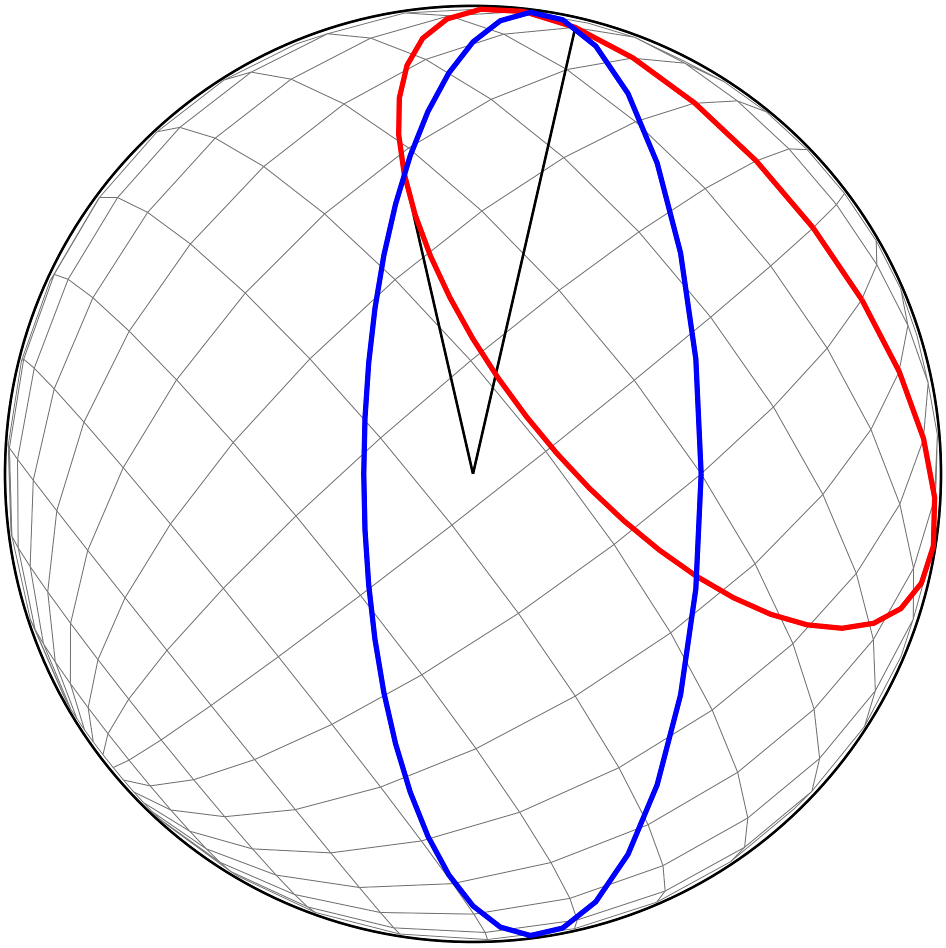

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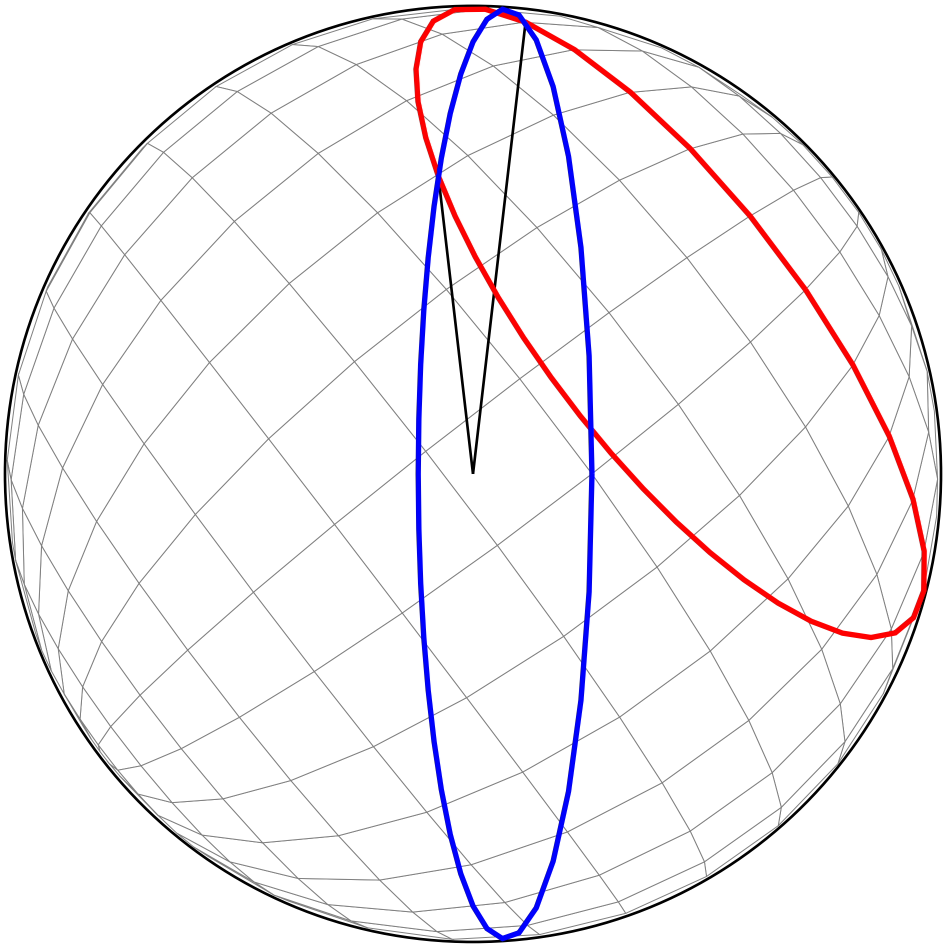

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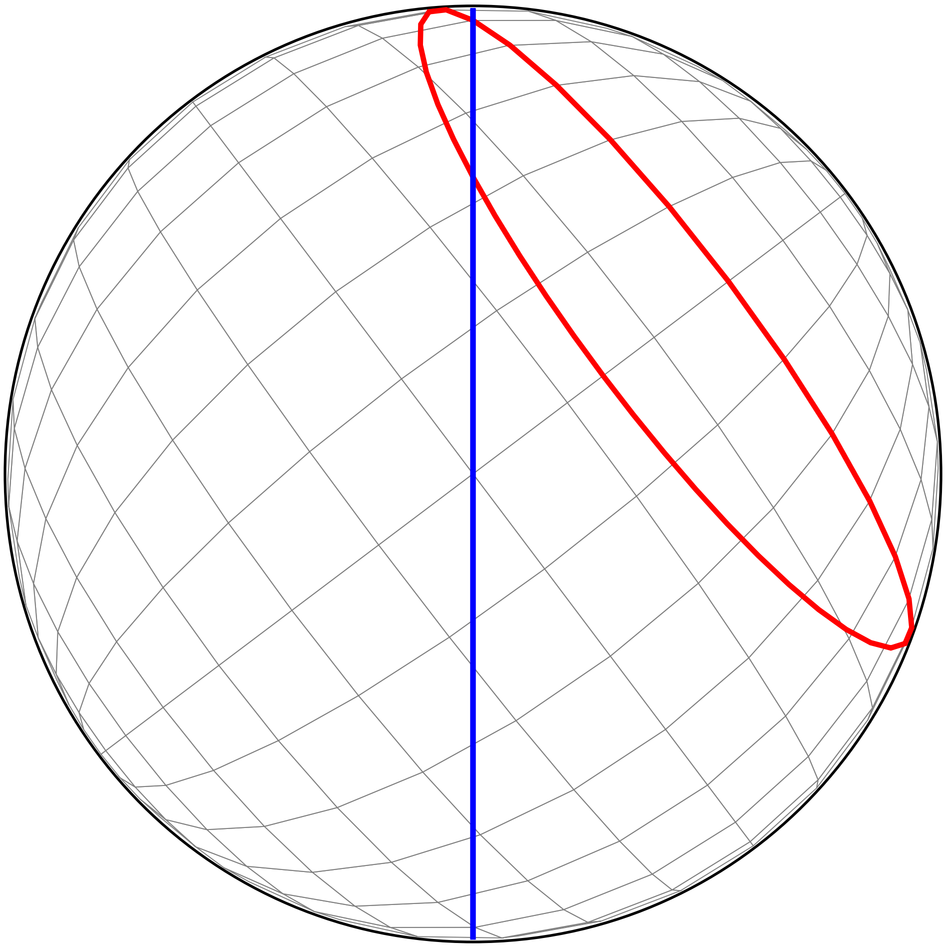
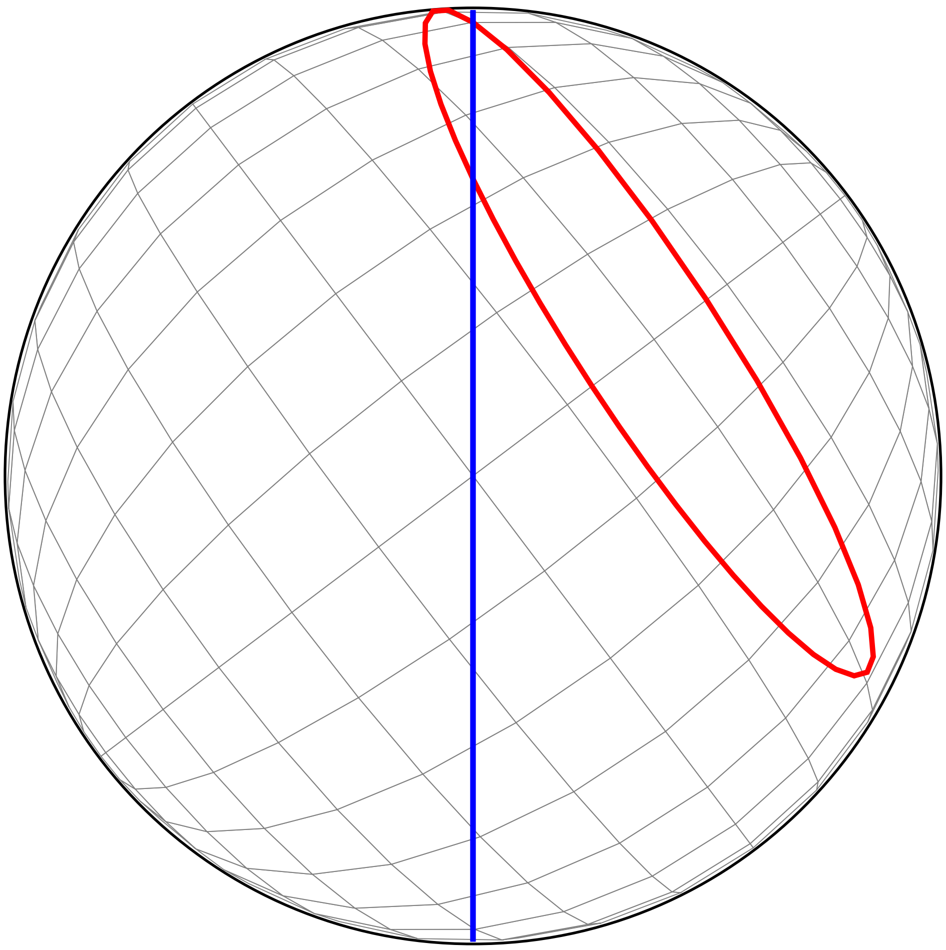


We will show that the great circle course is the shortest route through series of animation that involves rotations of the sphere and the red circle.

Now we rotate the red circle around the axis which consists both points on the sphere so that the red circle and the blue circle will lie on the same plane. Note that the red cirlce is no longer on the surface of the sphere. And he grey grids and the black lines are only there to help us to visualize the rotations.



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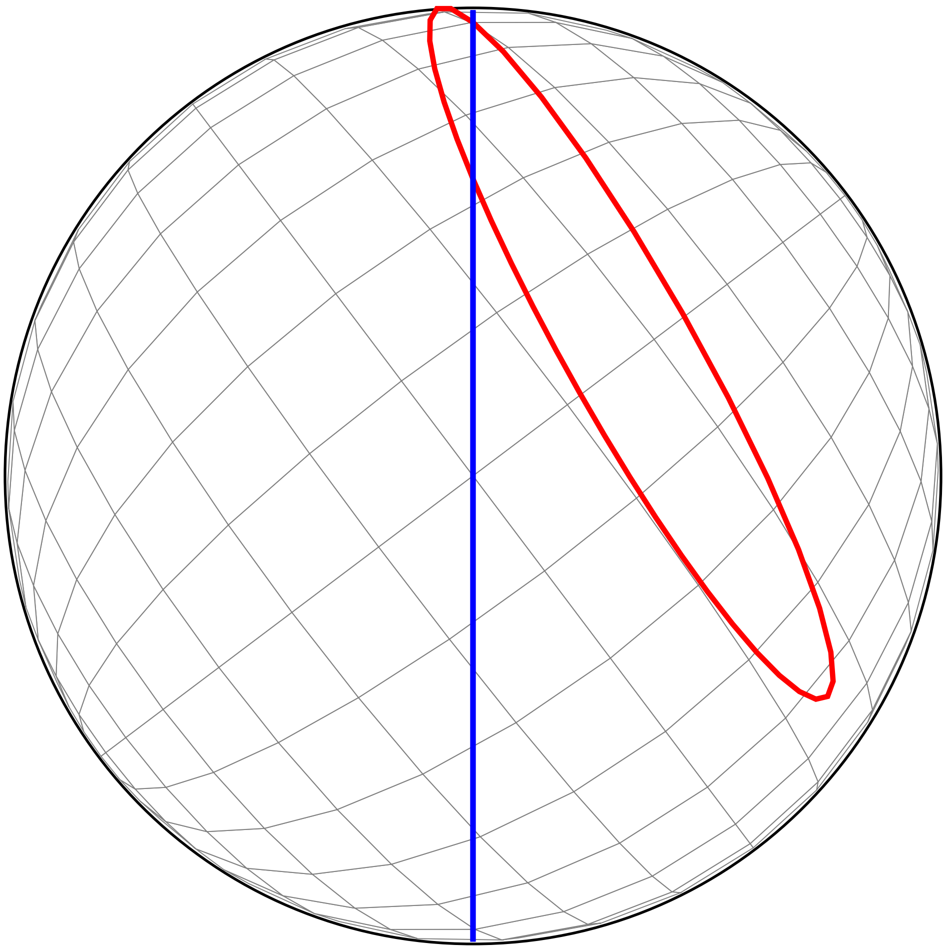

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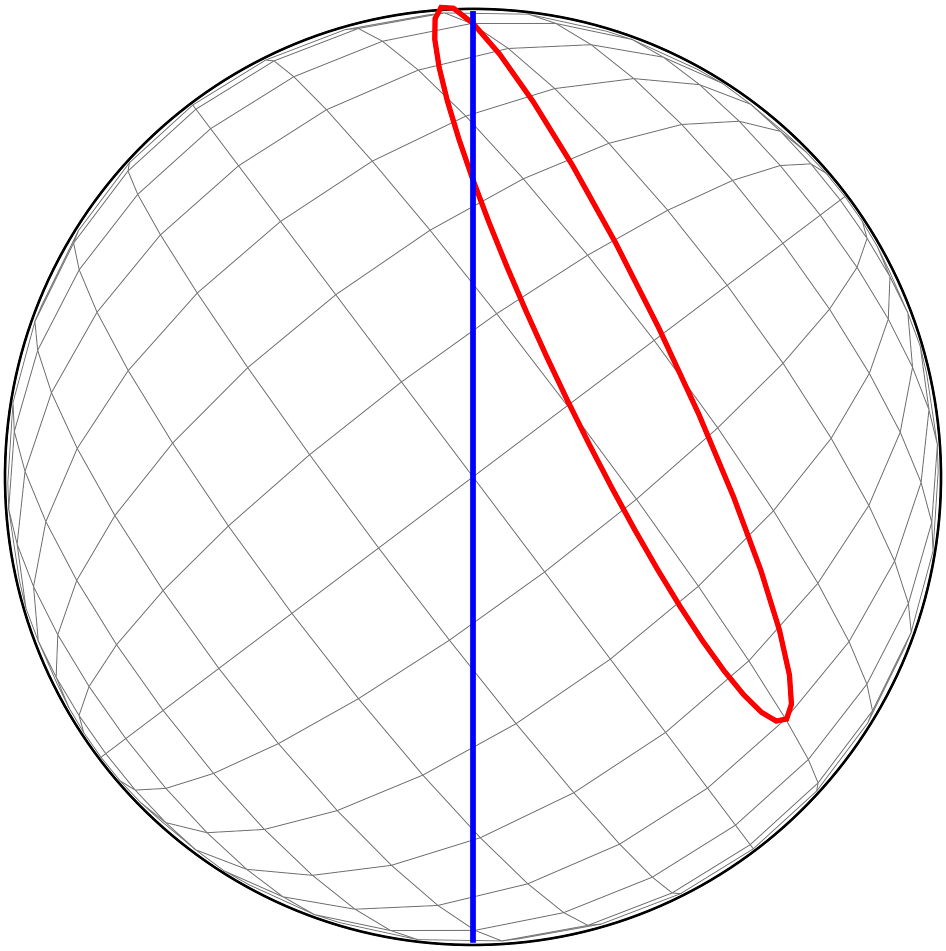

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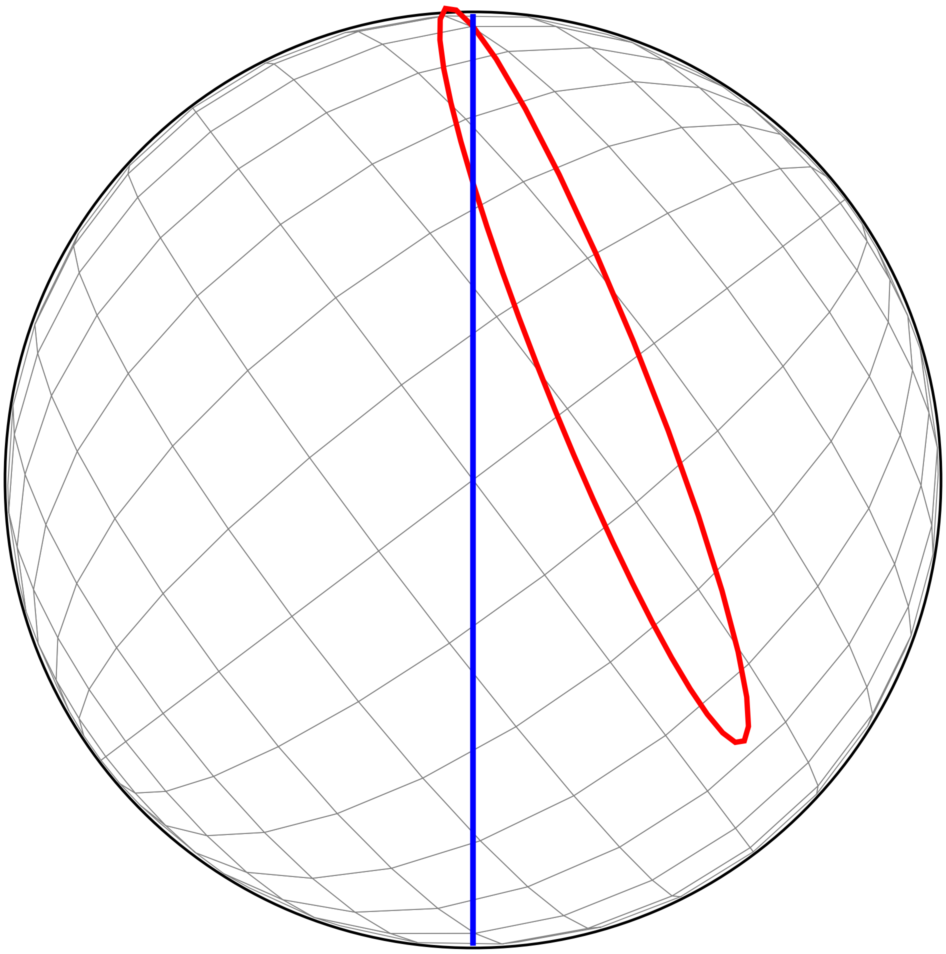

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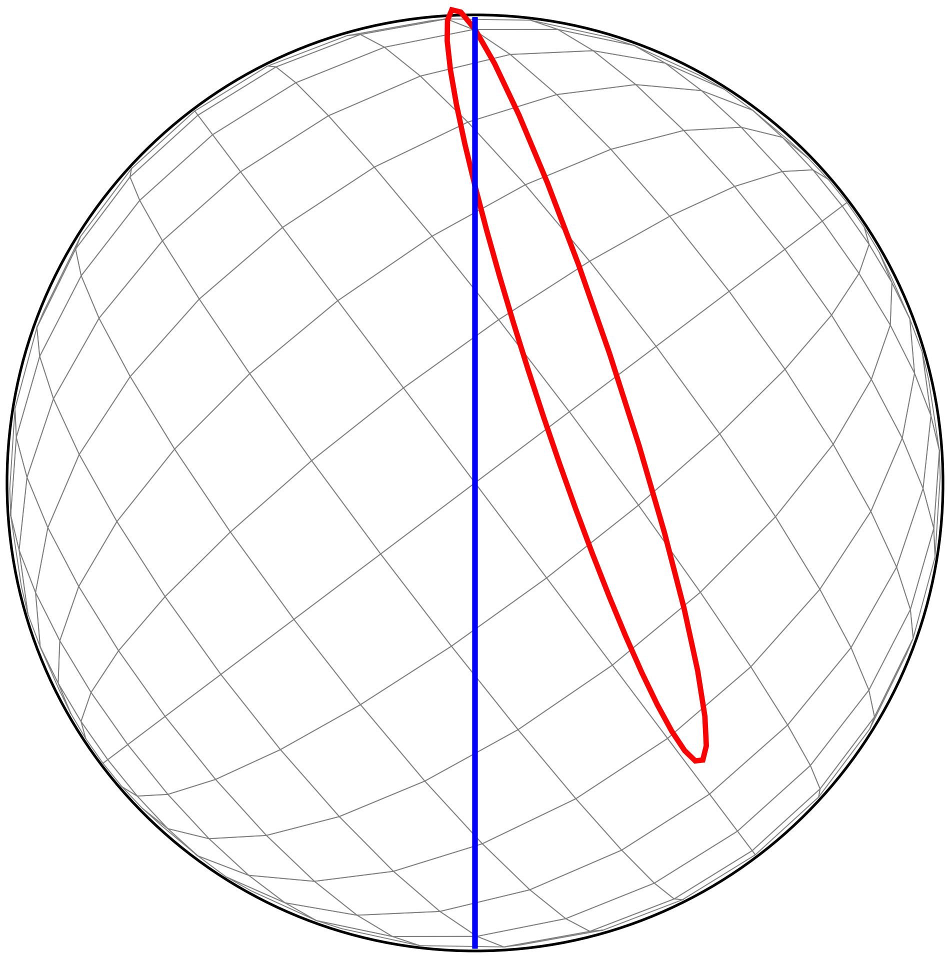

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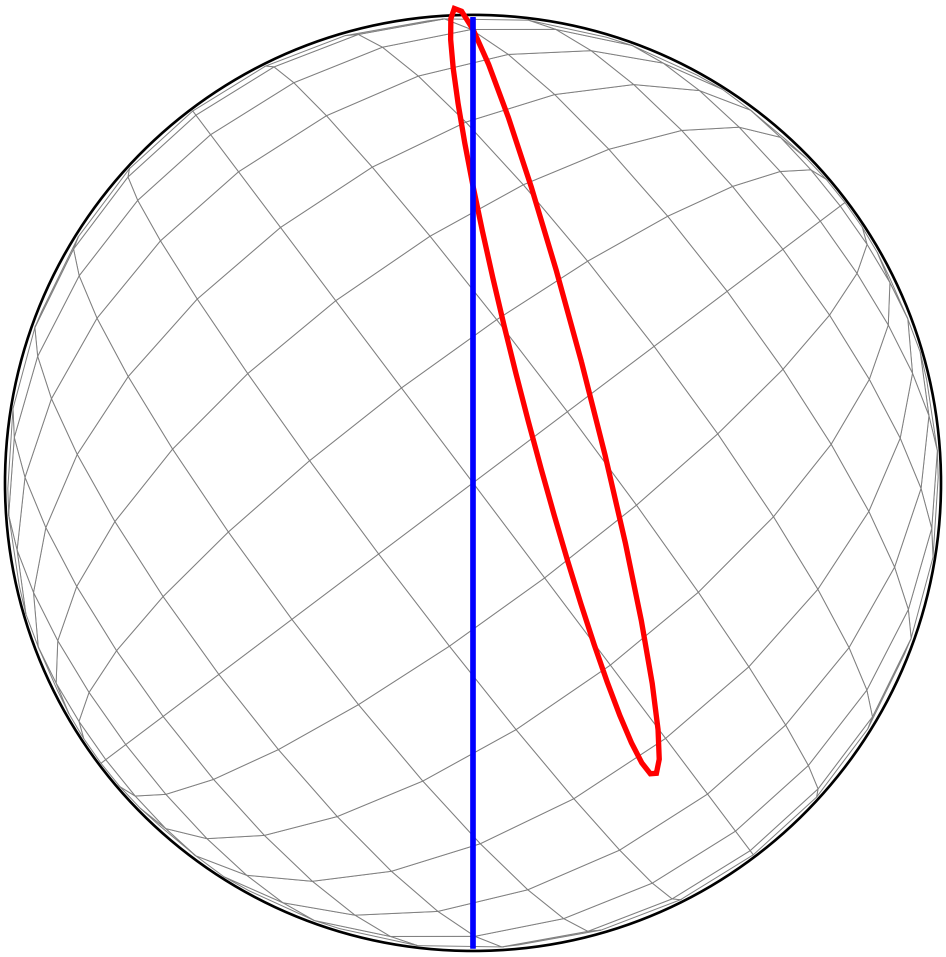

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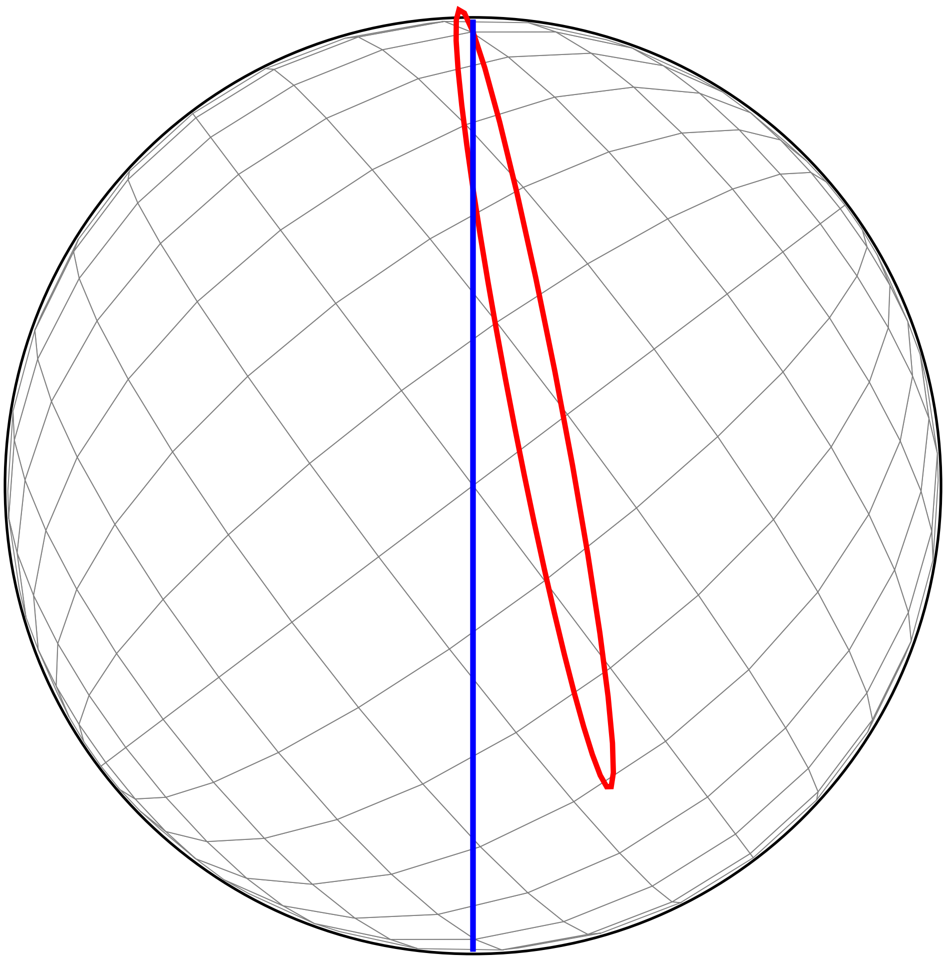

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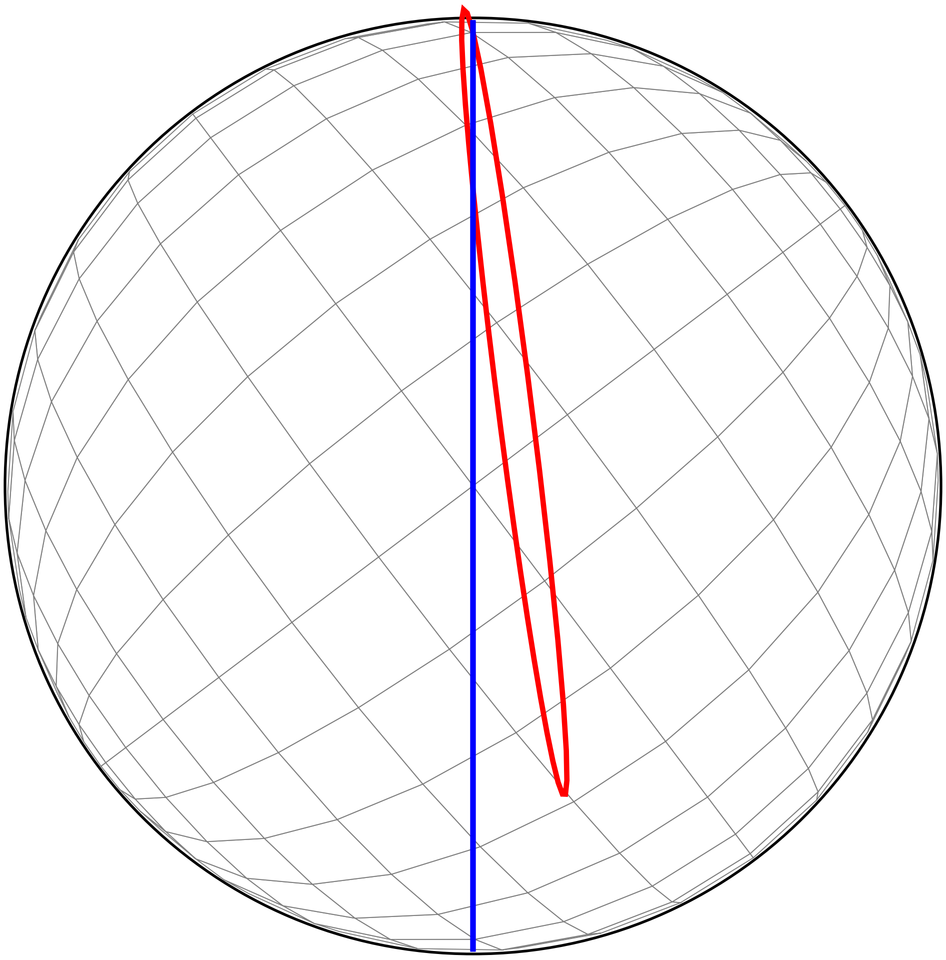

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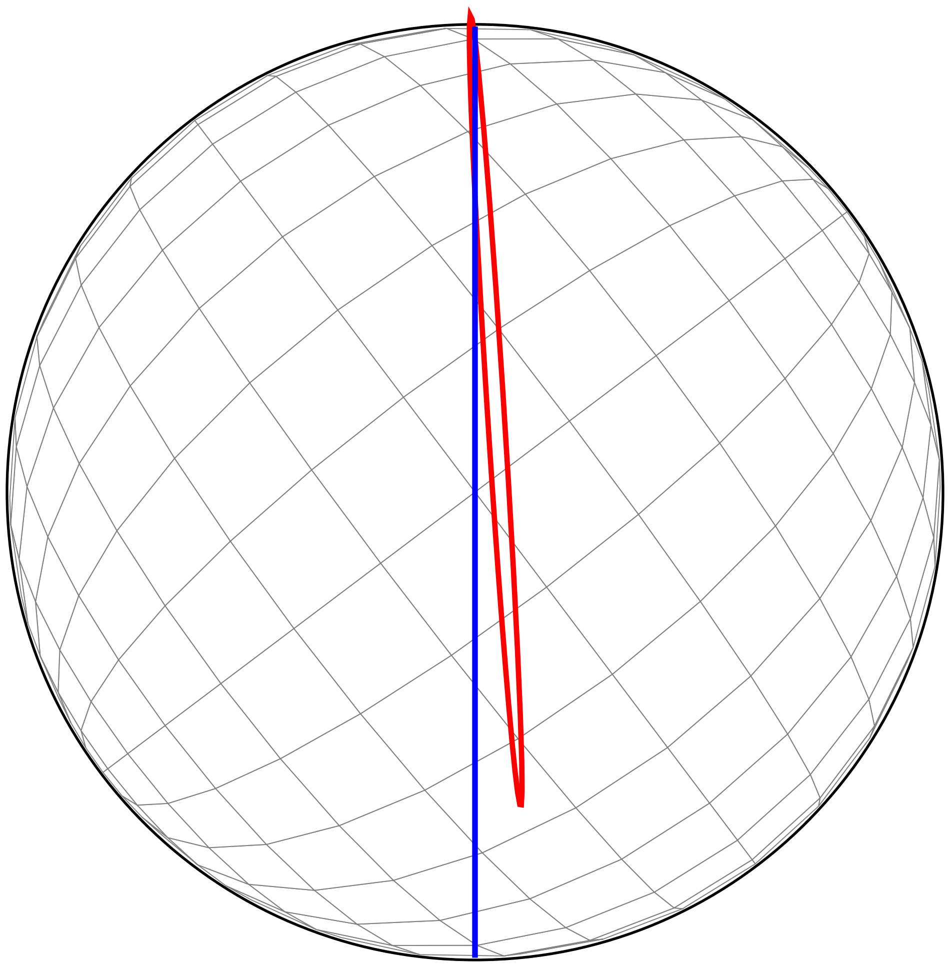

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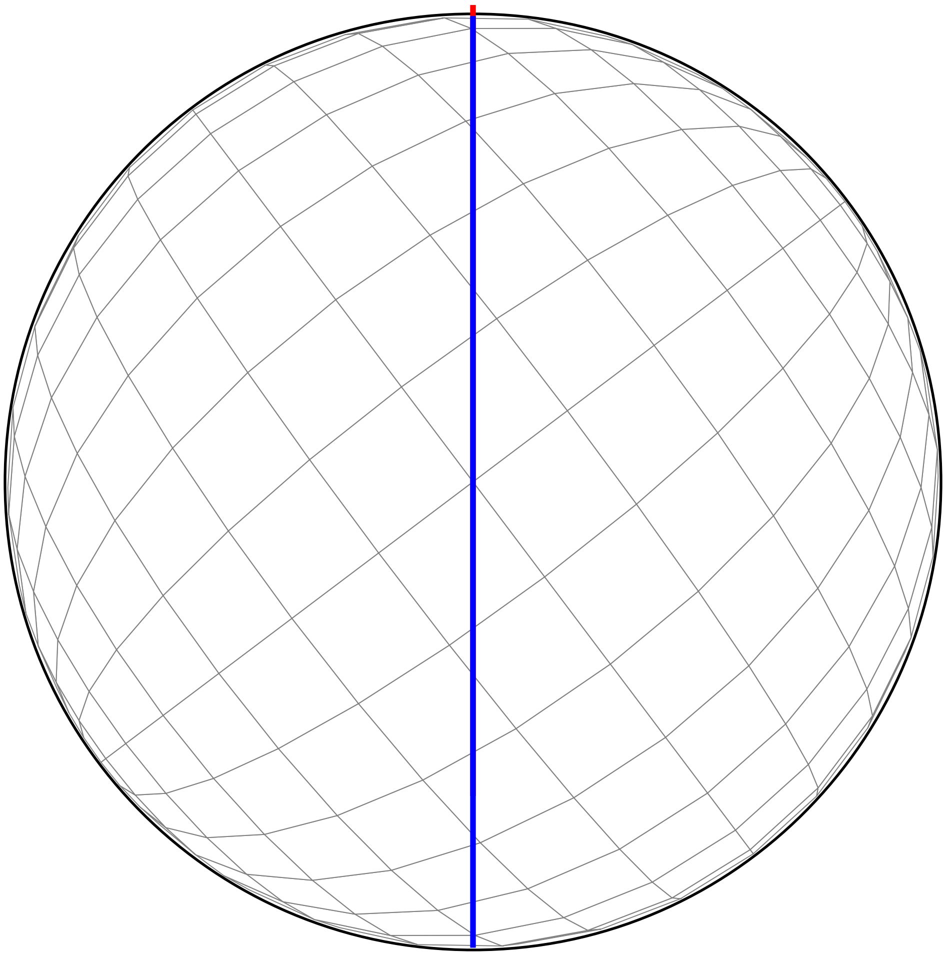

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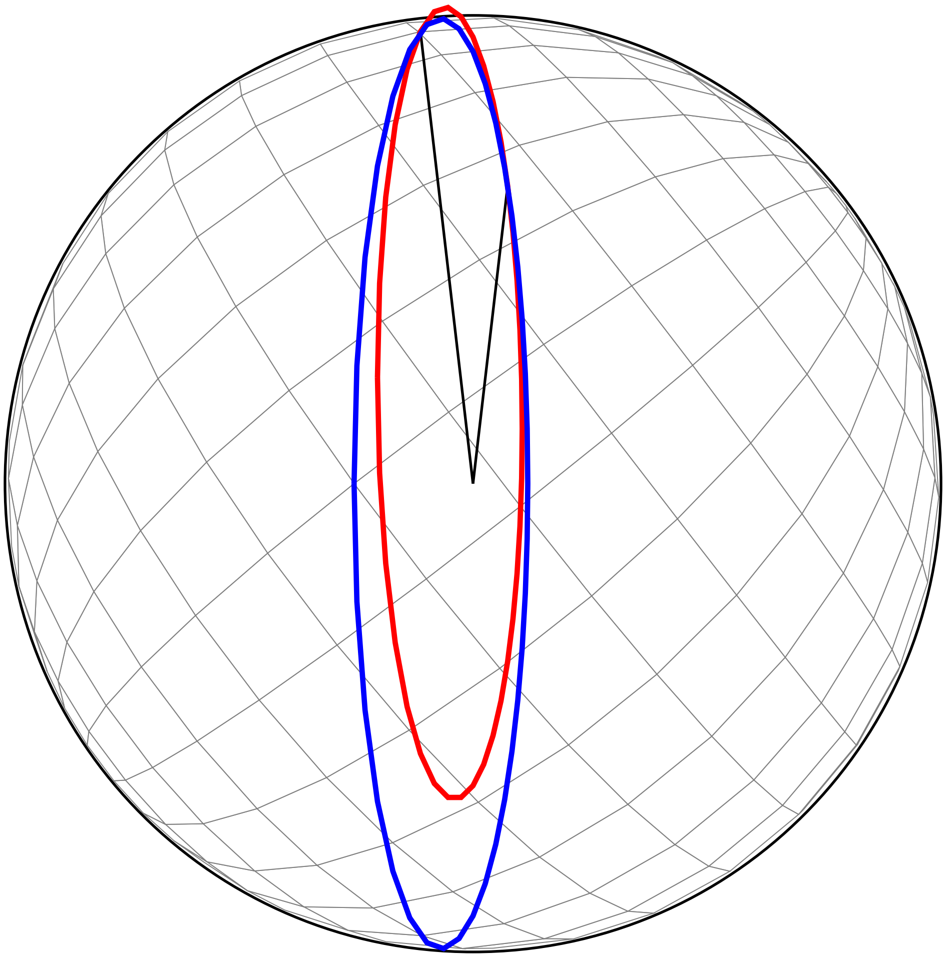

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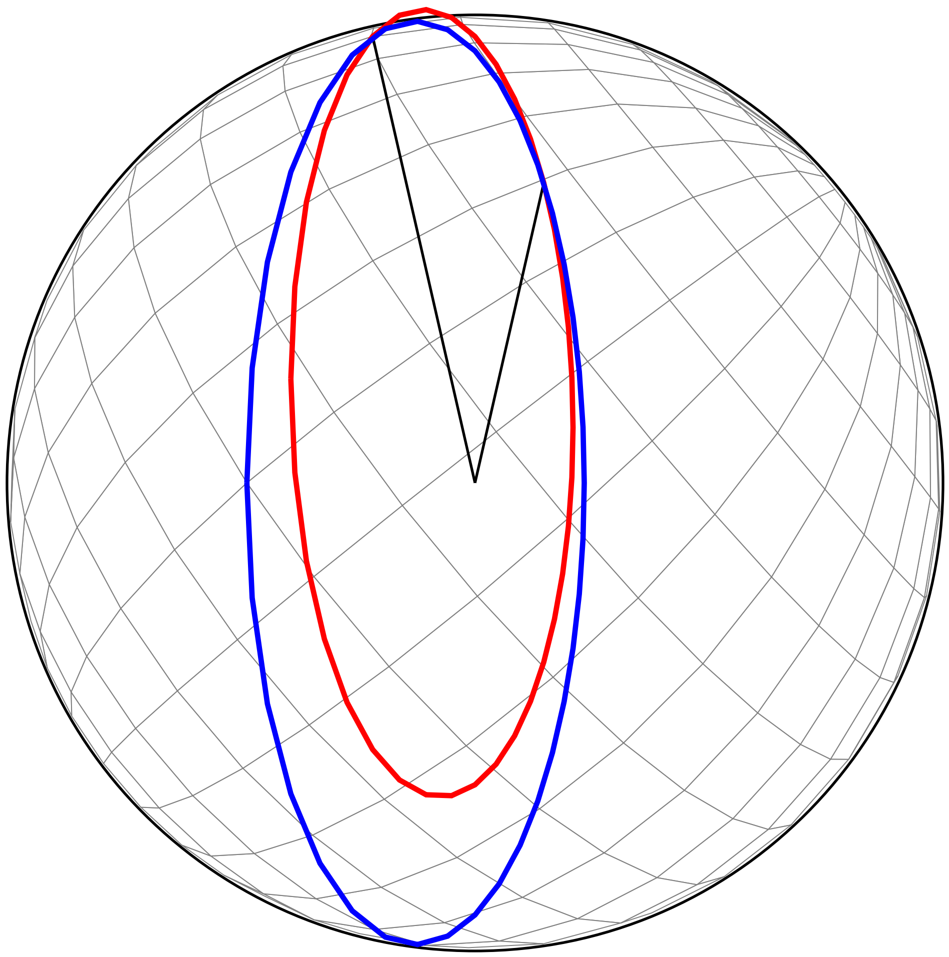

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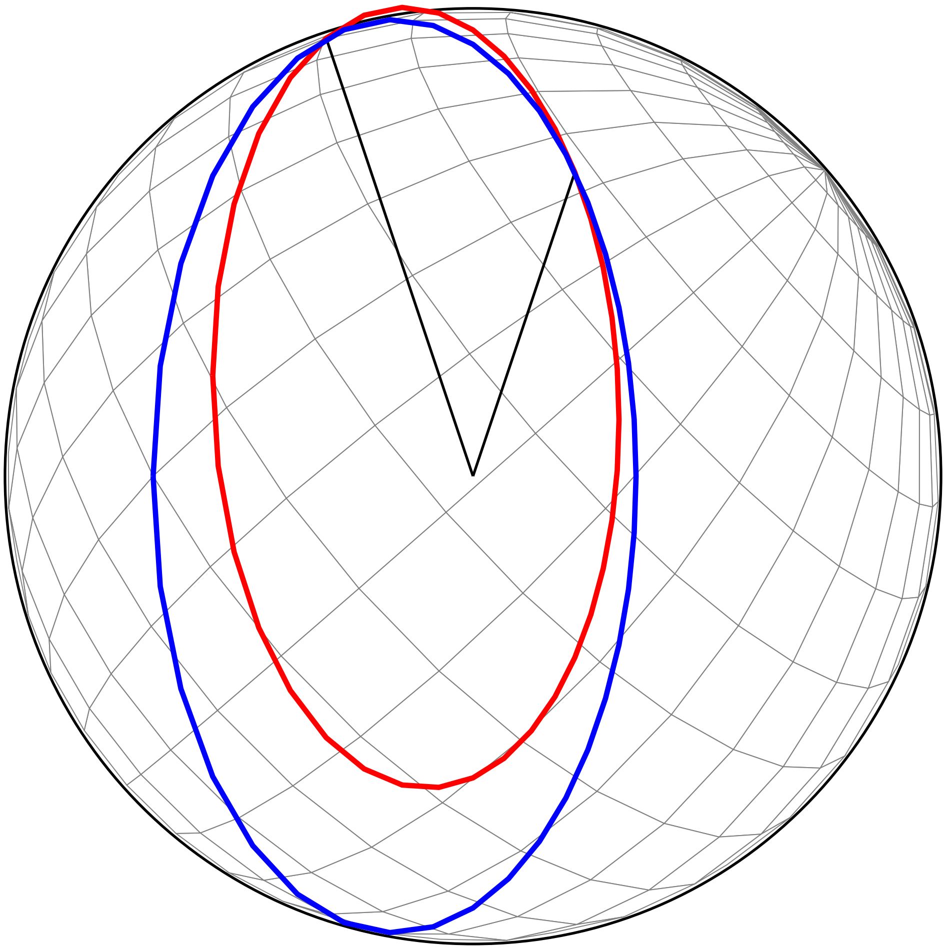

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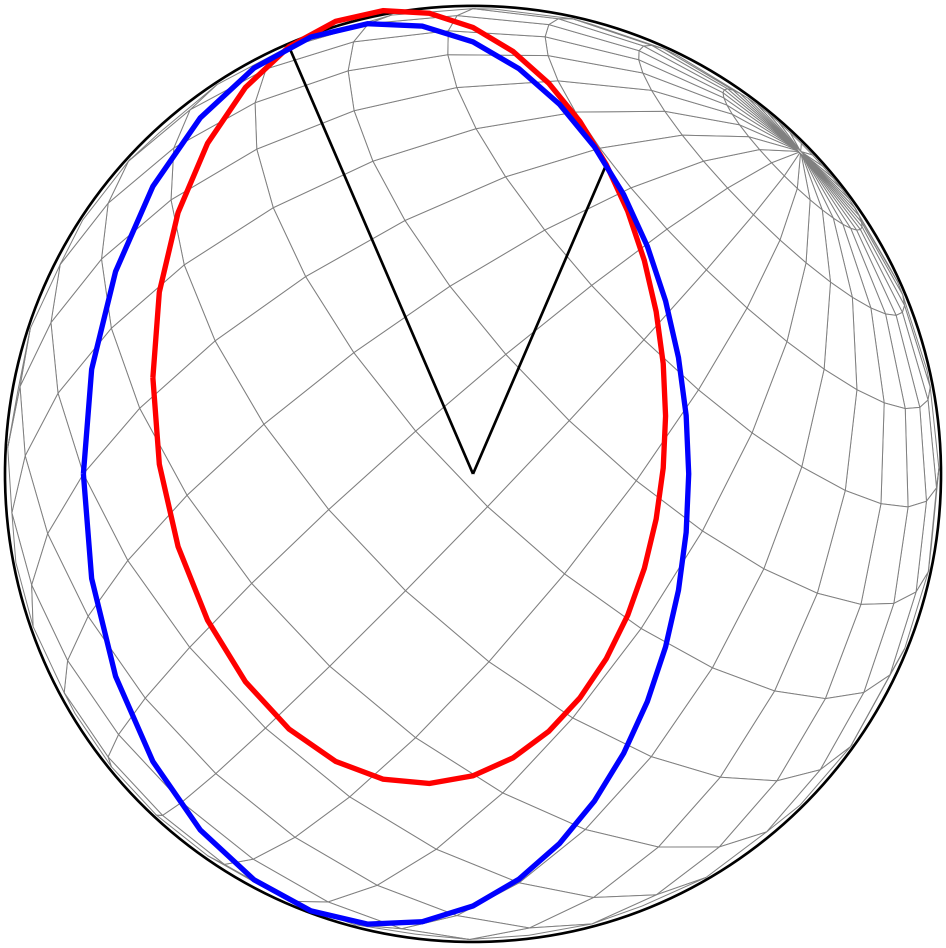

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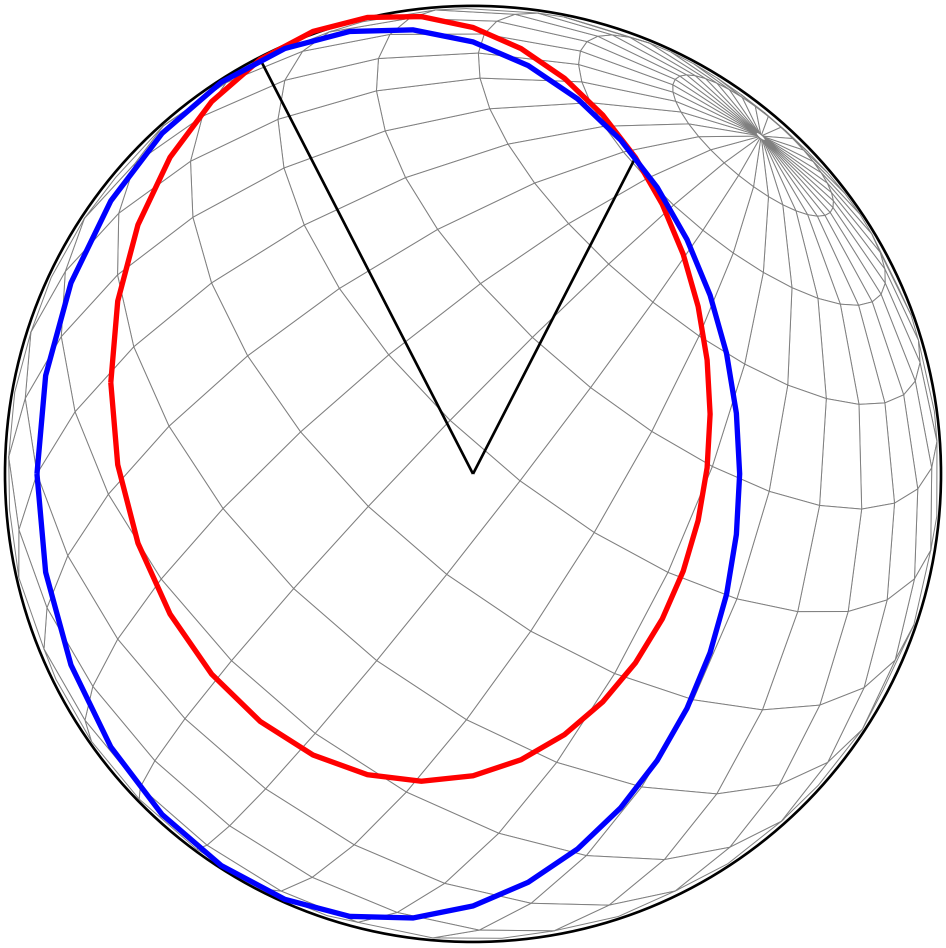

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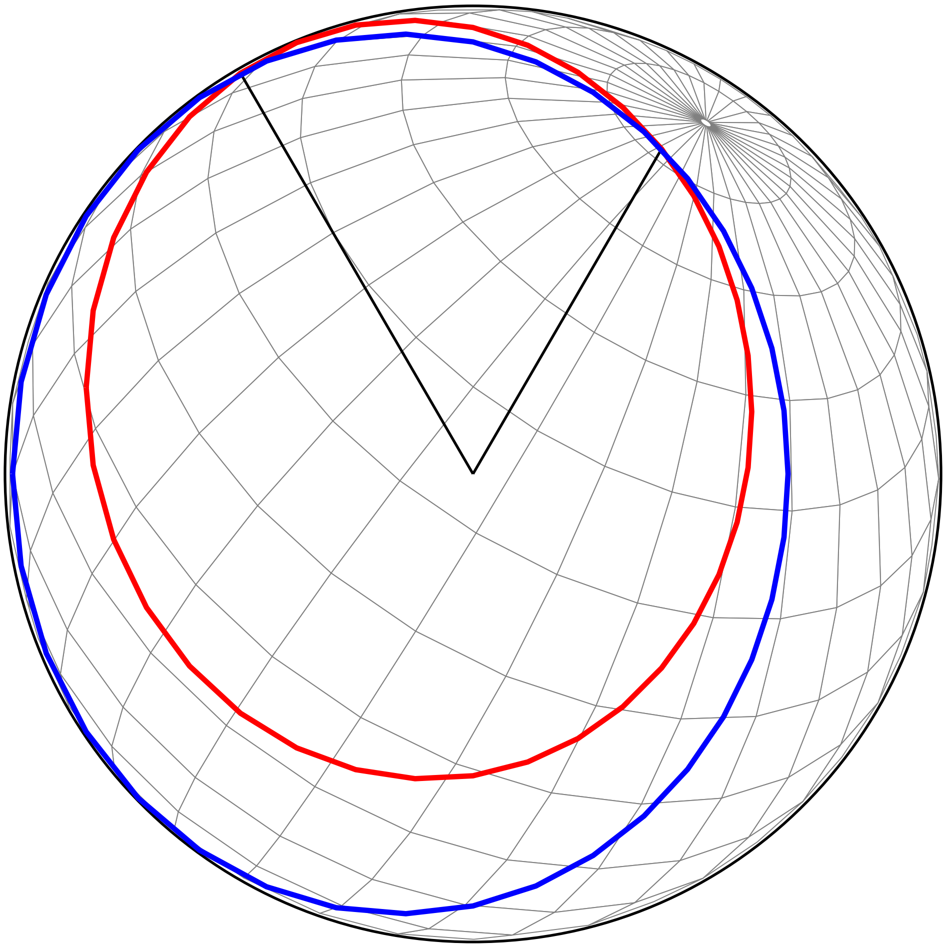

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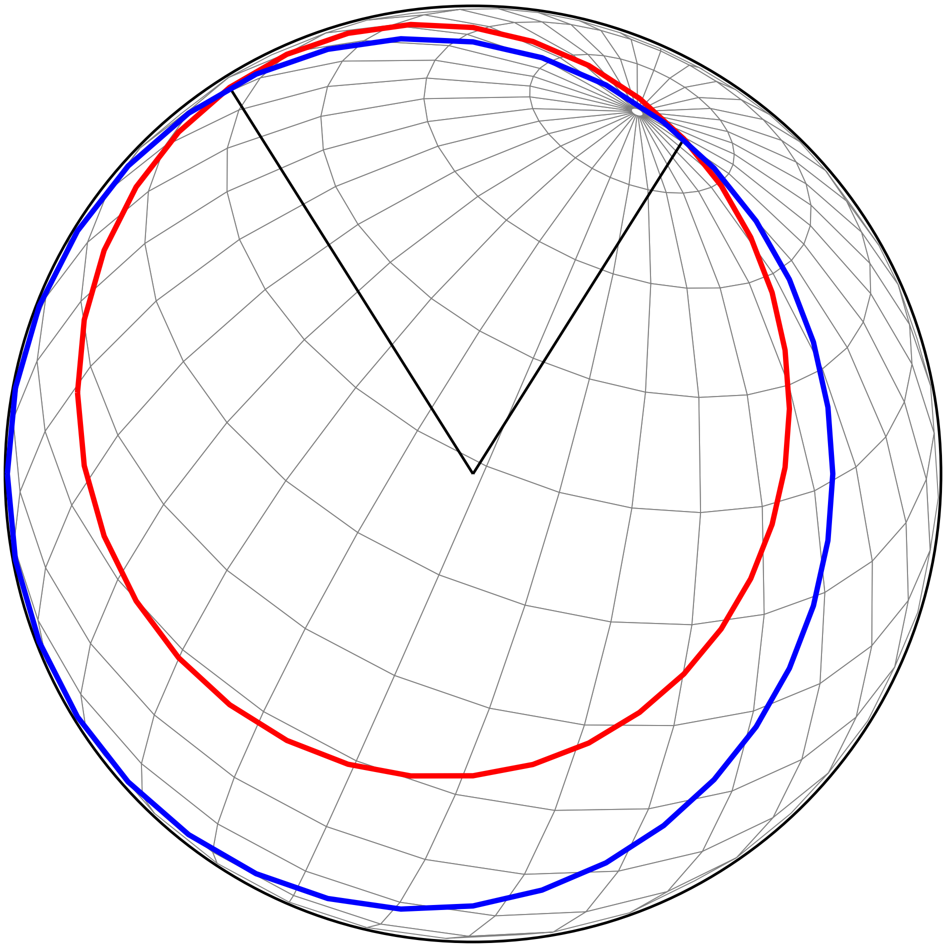

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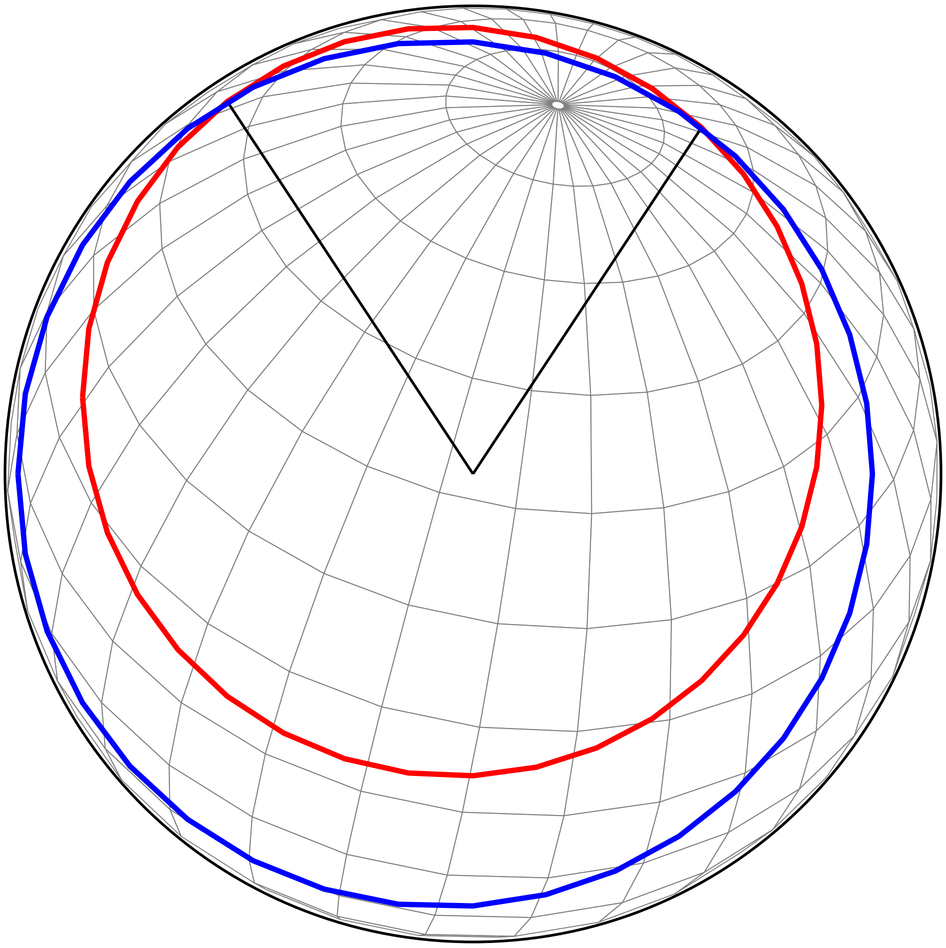

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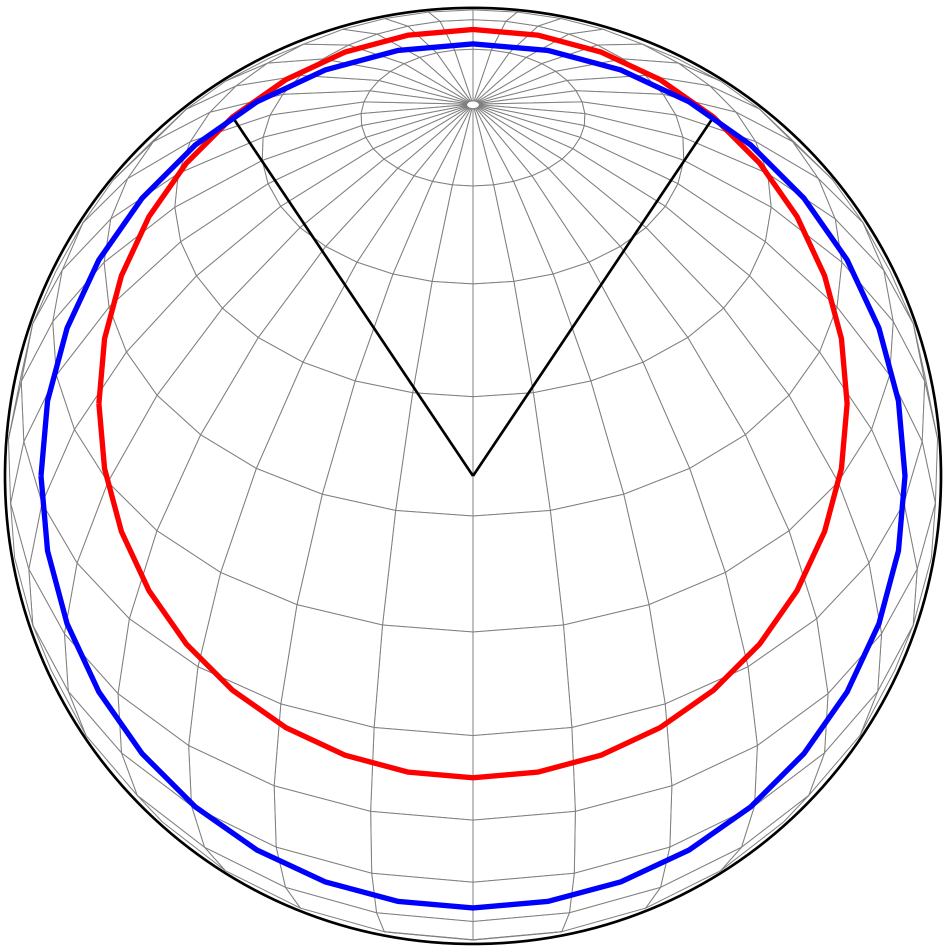
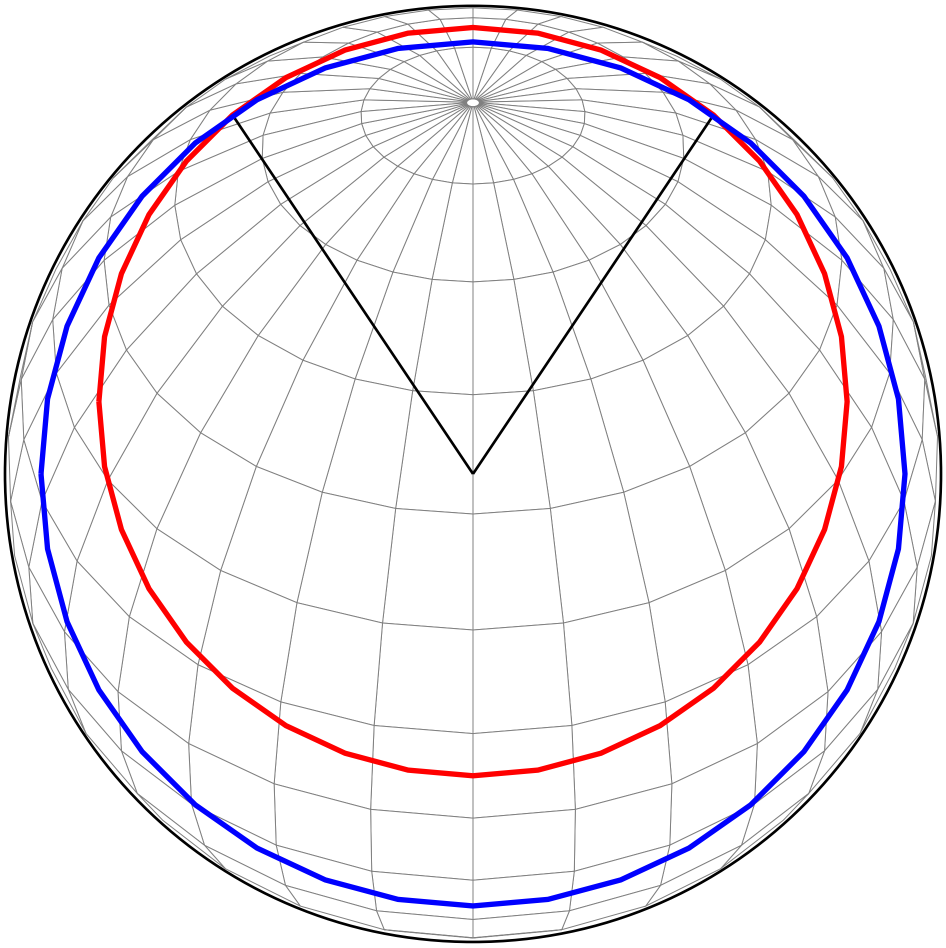


Since the circles are lain on the same plane, we can now compare them. For the courses between the points, the blue course is on the inside and the red course is on the outside. From this picture, it is obvious that the red course is longer than the blue's. As a result, the compass course is longer than the great circle course. In this animation, we have the case that two points are on the same



latitude. But this is also true for points with different latitudes. Just imagine that the grey grids are rotated around the center of the sphere so that the two points are on the same latitude. If the two points are not on the rotated equator, then the great circle course is still the shortest route. This is because the red circle is always smaller than the blue circle. So the red course between the points is always on the outside and longer the the blue course.

The End