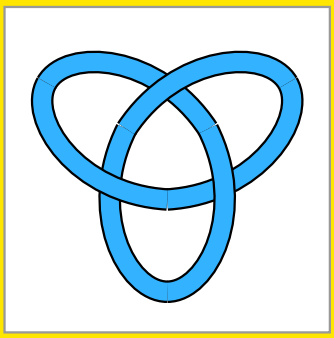
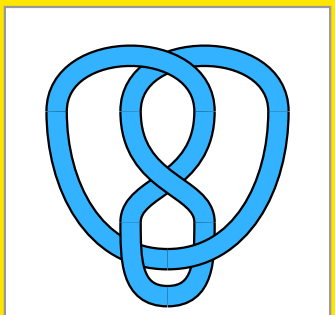
Mathematics 308 - Term Project

The Knot Theory





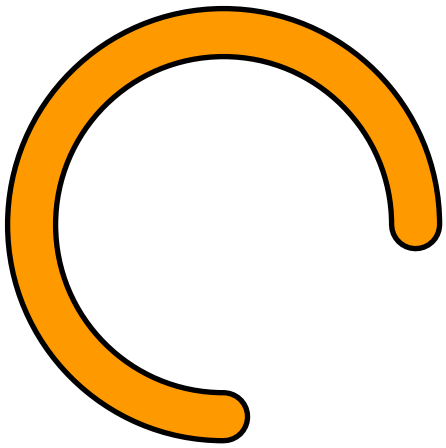
by Maggie Ying

Euclidean geometry - Professor Bill Casselman Fall 2003

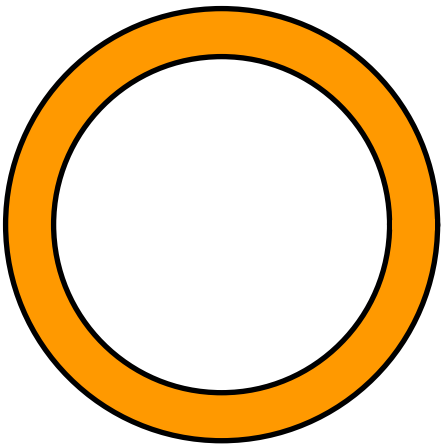
Maggie Ying
79342010

Mathematical Knots

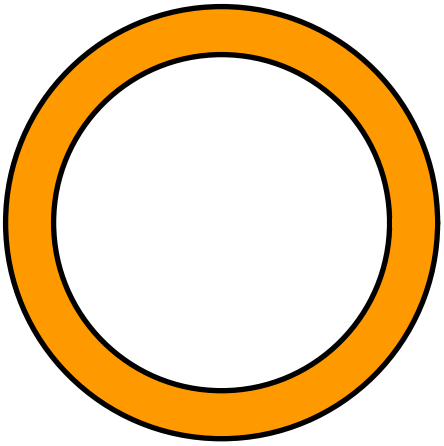
Mathematical Knots



Mathematical Knots



Mathematical Knots



loop - no loose ends

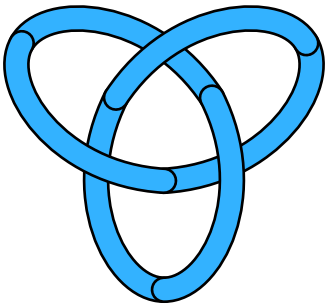
Central Problem

Central Problem

- Given

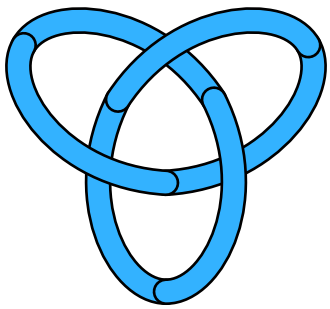
Central Problem

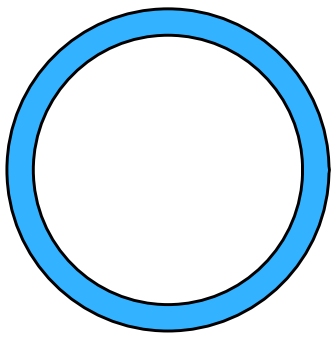
- Given



Central Problem

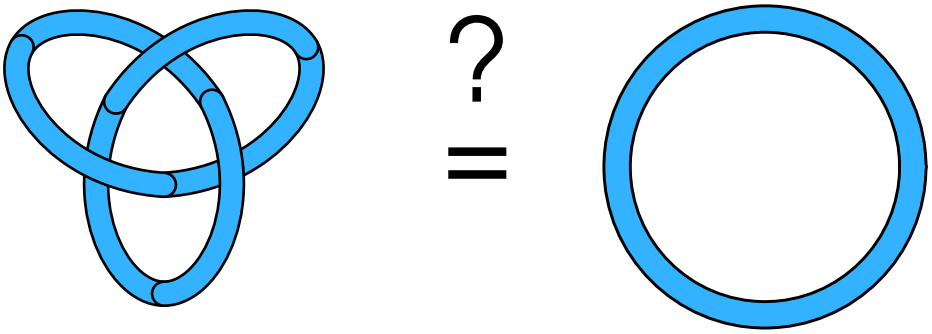
- Given





Central Problem

- Given



same projection?

Central Problem

Knot Invariant:

Central Problem

Knot Invariant:

- unchanged by Reidemeister Moves

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- eg. Jones Polynomial

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$$V(t) = (-t^{-3/4})^{-W(L)} \langle L \rangle$$

the same for all projections

Central Problem

Knot Invariant:

- unchanged by Reidemeister Moves

- eg. Jones Polynomial

$$V(t) = (-t^{-3/4})^{-W(L)} \langle L \rangle$$

<L> - Bracket Polynomial
W(L) - Writhe

the same for all projections

Reidemeister Moves

Reidemeister Moves

TYPE I :

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Reidemeister Moves

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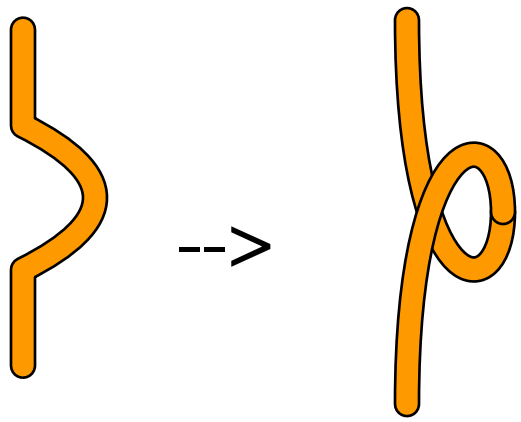
Reidemeister Moves

TYPE I :



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Reidemeister Moves

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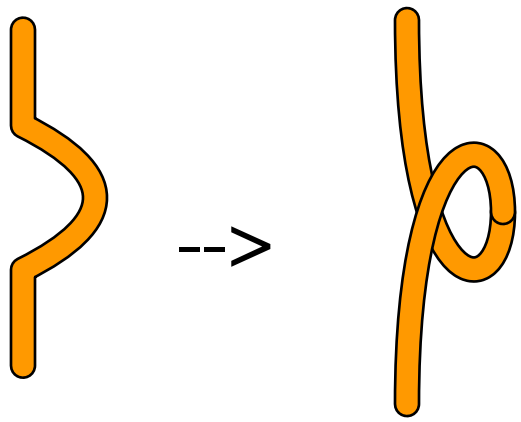

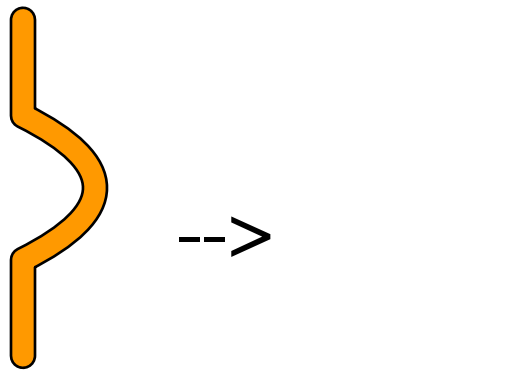

Reidemeister Moves

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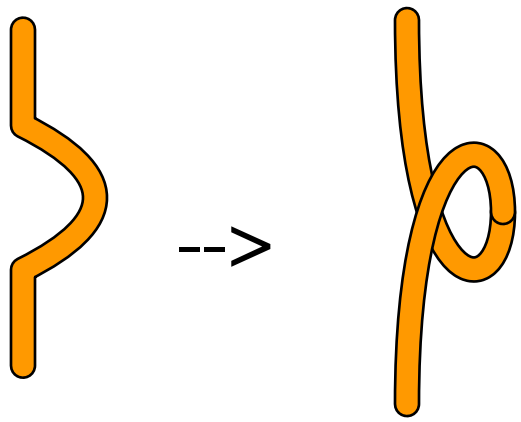

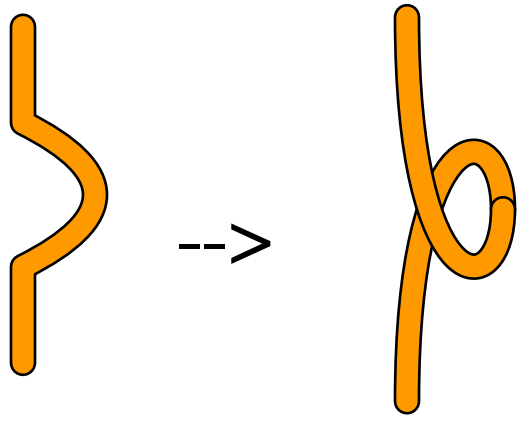

Reidemeister Moves

TYPE II :

Reidemeister Moves

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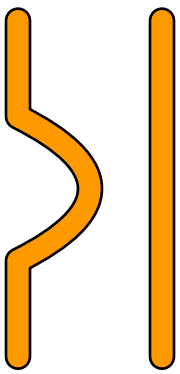

Reidemeister Moves

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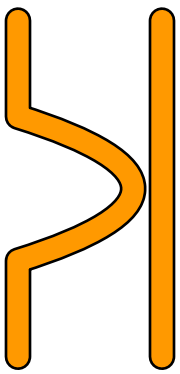

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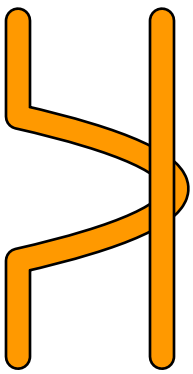

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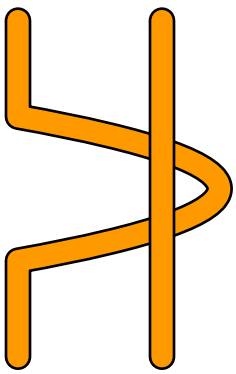

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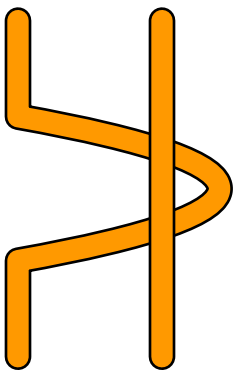

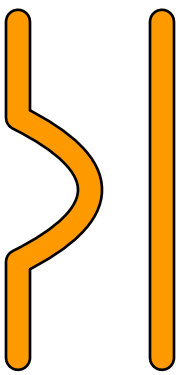

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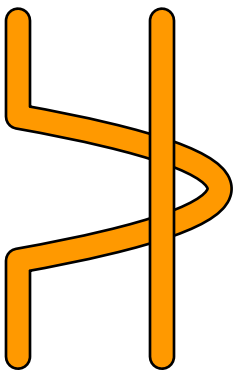

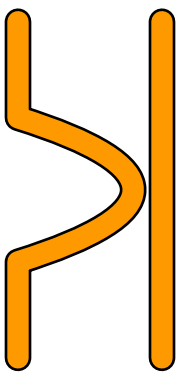

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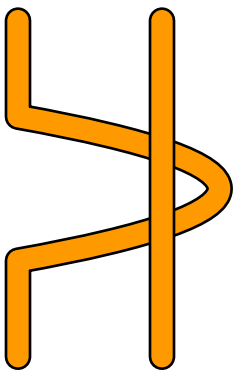

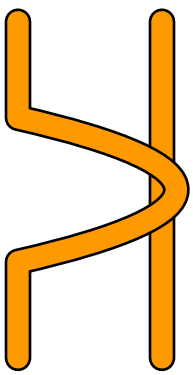

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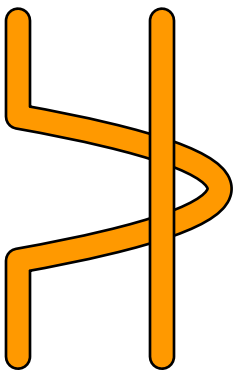

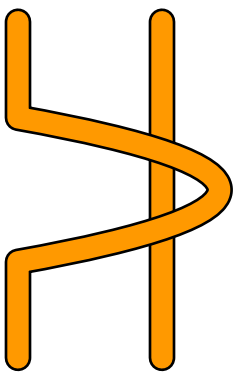

Reidemeister Moves

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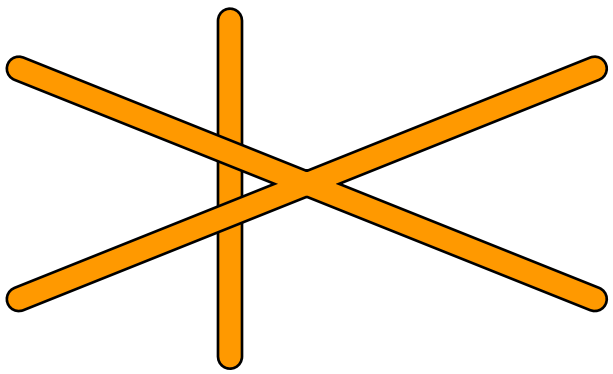

Reidemeister Moves

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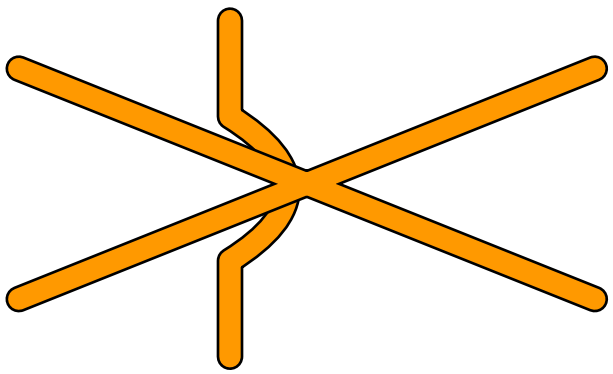

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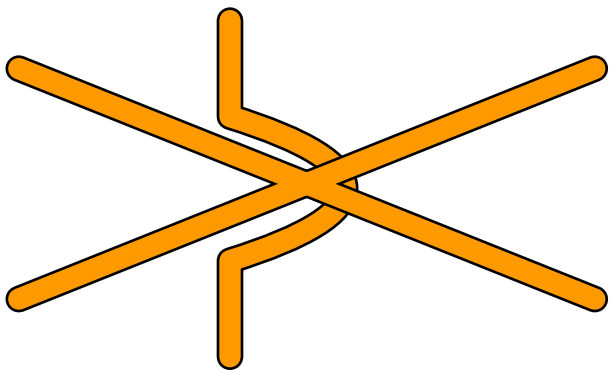

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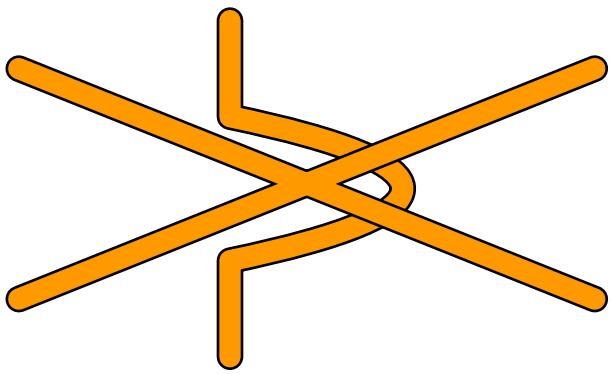

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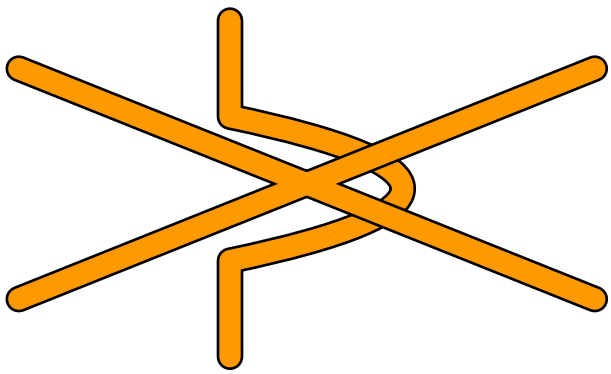

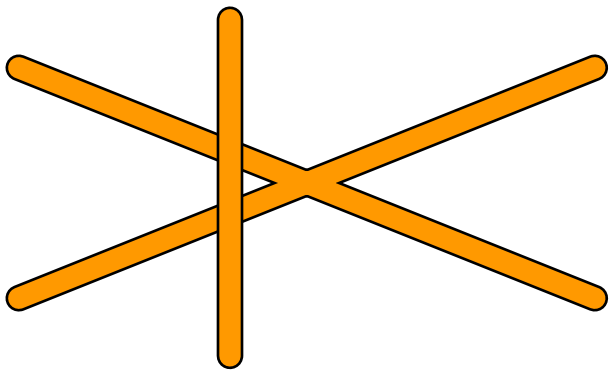

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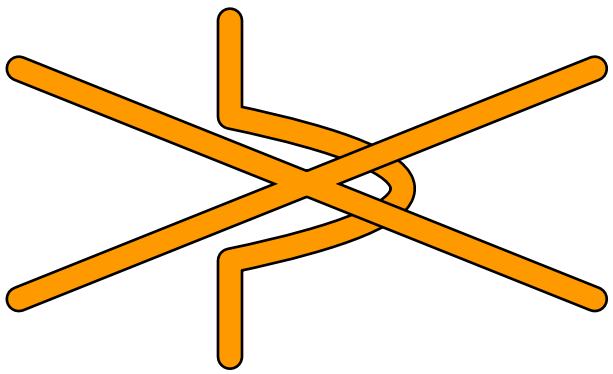

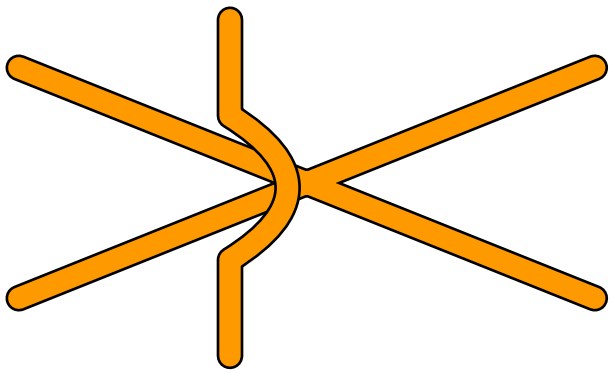

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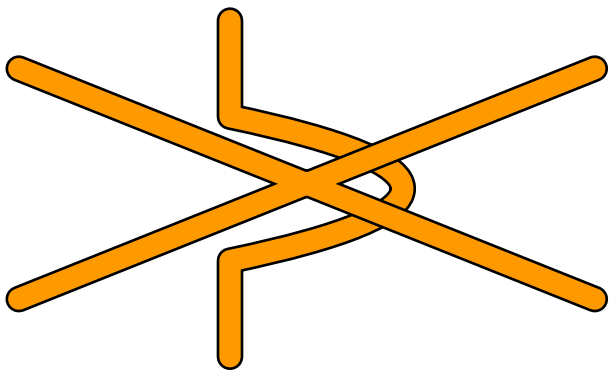

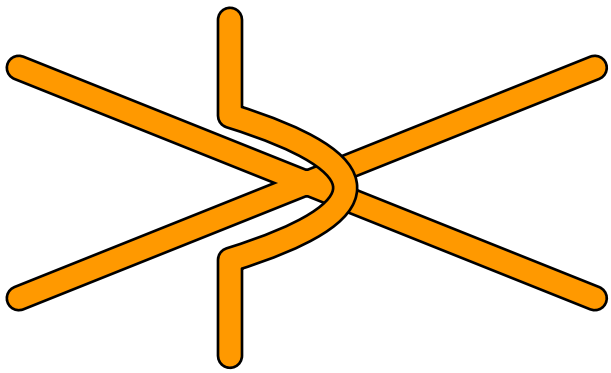

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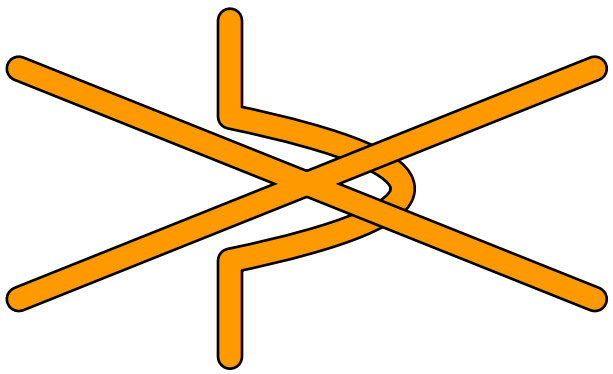
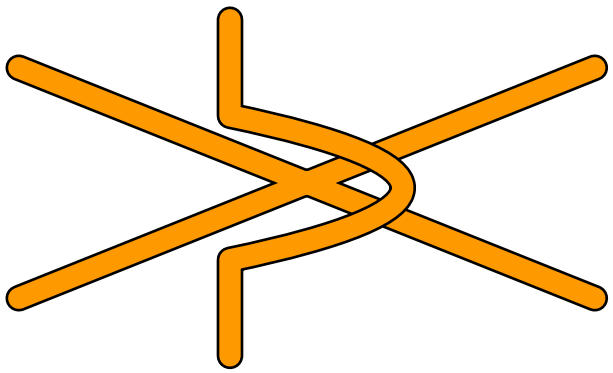




Bracket Polynomial

Bracket Polynomial

Rule I :

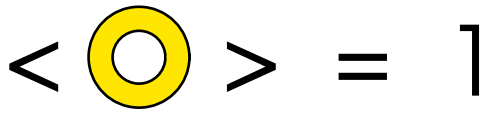
Bracket Polynomial

Rule I :

$< \bigcirc > = 1$

Bracket Polynomial

Rule I :



Rule II :

where $A = t^{-1/4}$

Bracket Polynomial

Rule I :

$$\langle \bigcirc \rangle = 1$$

Rule II :

$$\langle \times \rangle = A \langle) (\rangle + A^{-1} \langle \smile \frown \rangle$$

where $A = t^{-1/4}$

Bracket Polynomial

Rule I :

$$\langle \bigcirc \rangle = 1$$

Rule II :

$$\langle \times \rangle = A \langle) (\rangle + A^{-1} \langle \asymp \rangle$$

$$\langle \times \rangle = A \langle \asymp \rangle + A^{-1} \langle) (\rangle$$

where $A = t^{-1/4}$

Bracket Polynomial

Rule III :

Bracket Polynomial

Rule III :

$$< L \; U \; \bigcirc > \; =$$

$$(-A^{2} - A^{-2}) < L >$$

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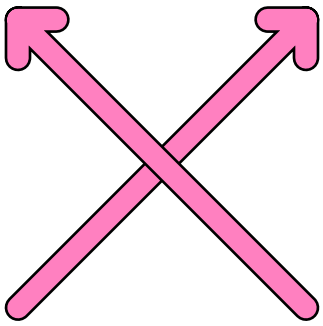

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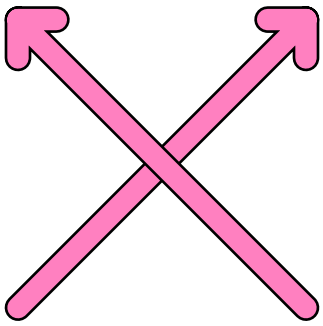

w(L) = +1

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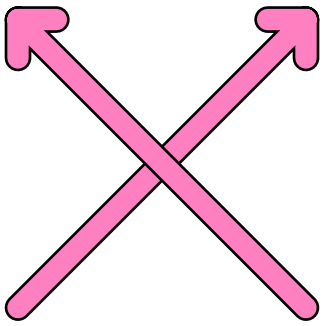

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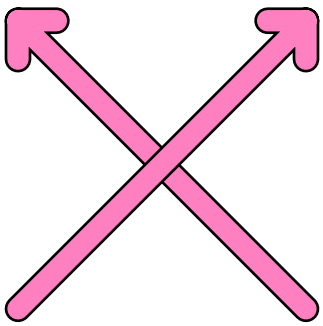

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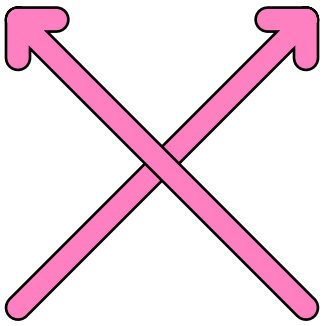

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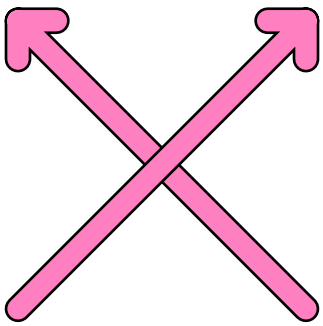
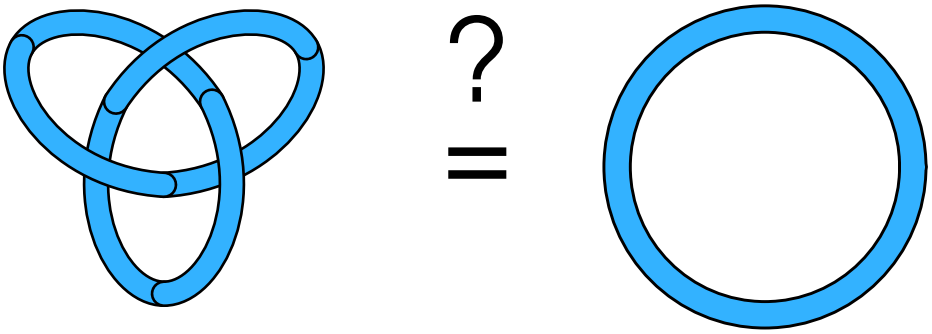


w(L) = -1

Conclusion

- Given



same Jones Polynomial is one

way to show same projection