Trisecting an angle

Mathematics 308

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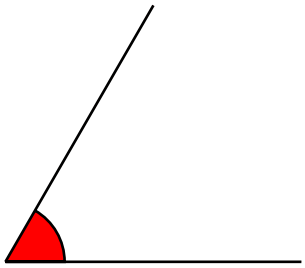
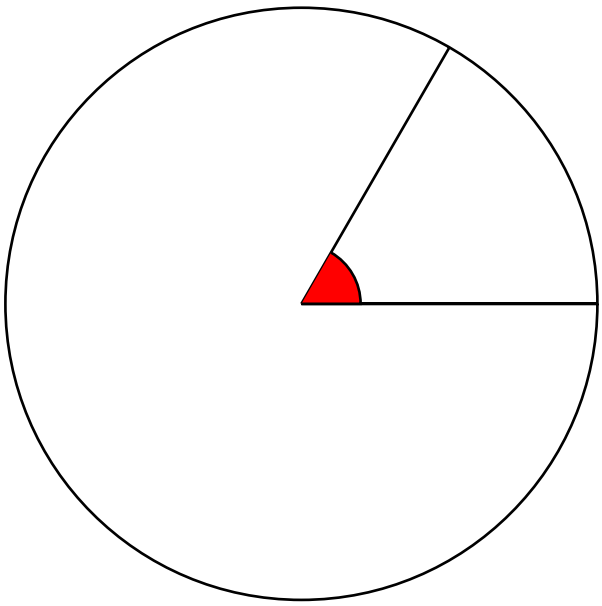
There are three classical problems in Greek mathematics that were extremely influential in the development of geometry. These problems were squaring the circle, doubling the cube and trisecting an angle.

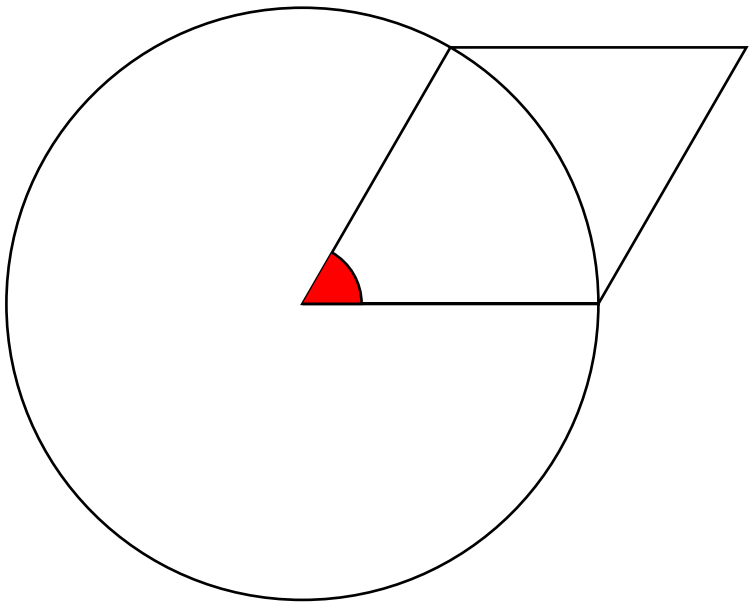
The problem of trisecing an arbitrary angle, which we examine here, only allows us to use a ruler and a compass.

First let’s see how we bisect an angle.

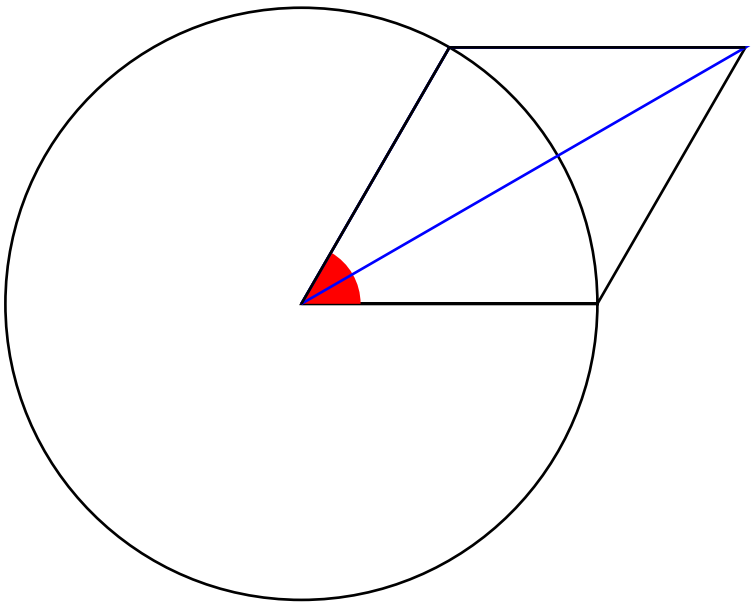


An arbitrary angle.





Complete the parallelogram.

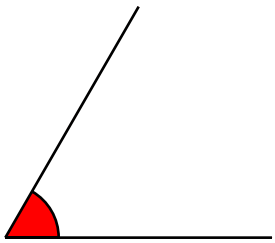


The diagonal bisects the given angle.

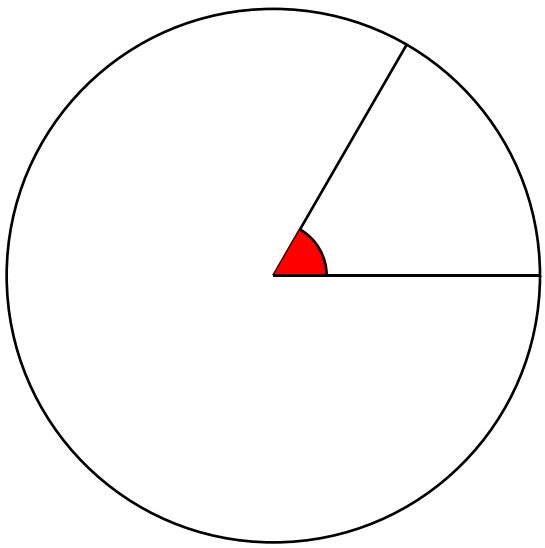
Now let’s see how we trisect an angle.

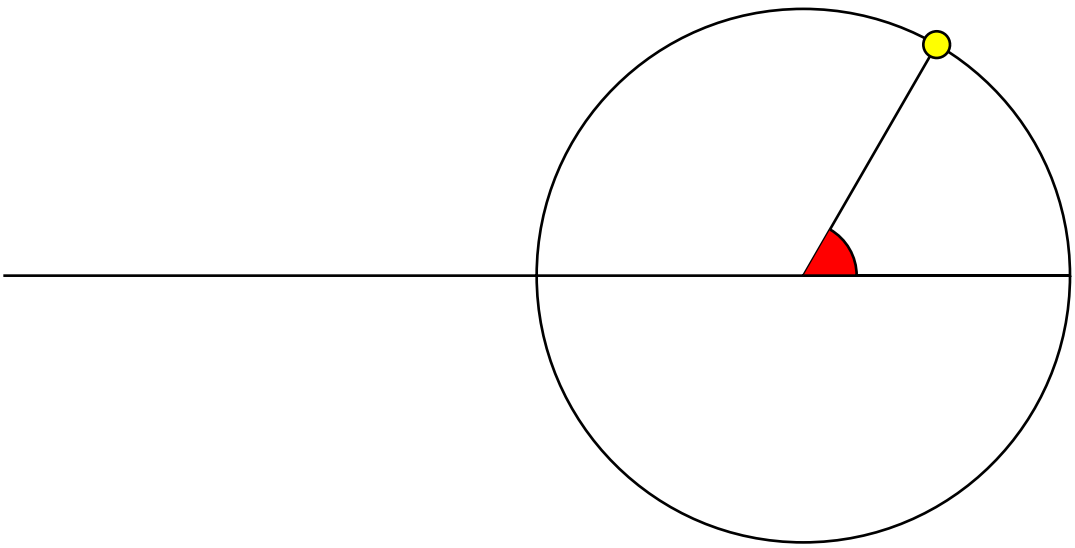
Method 1:

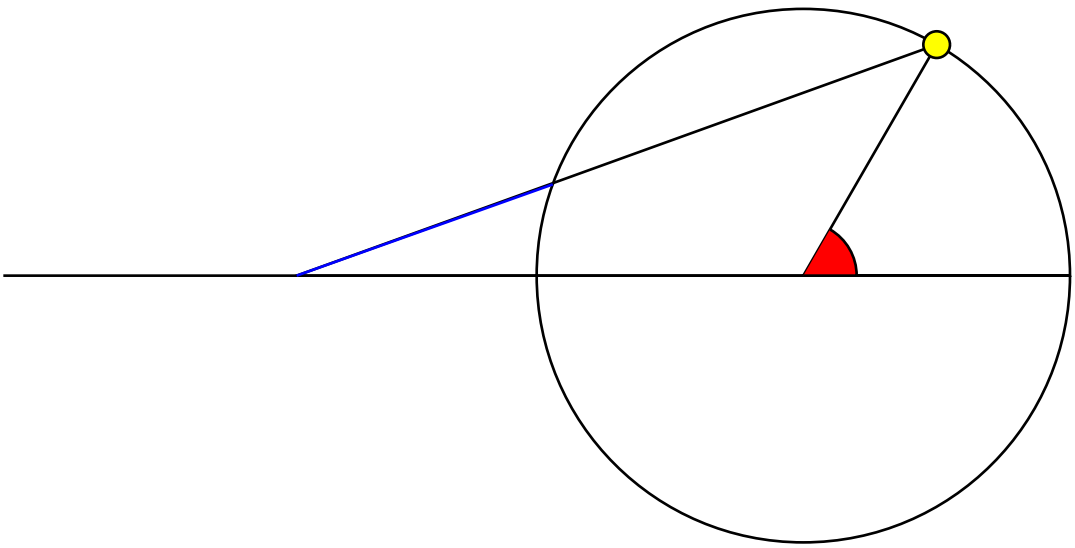
given by Archimedes



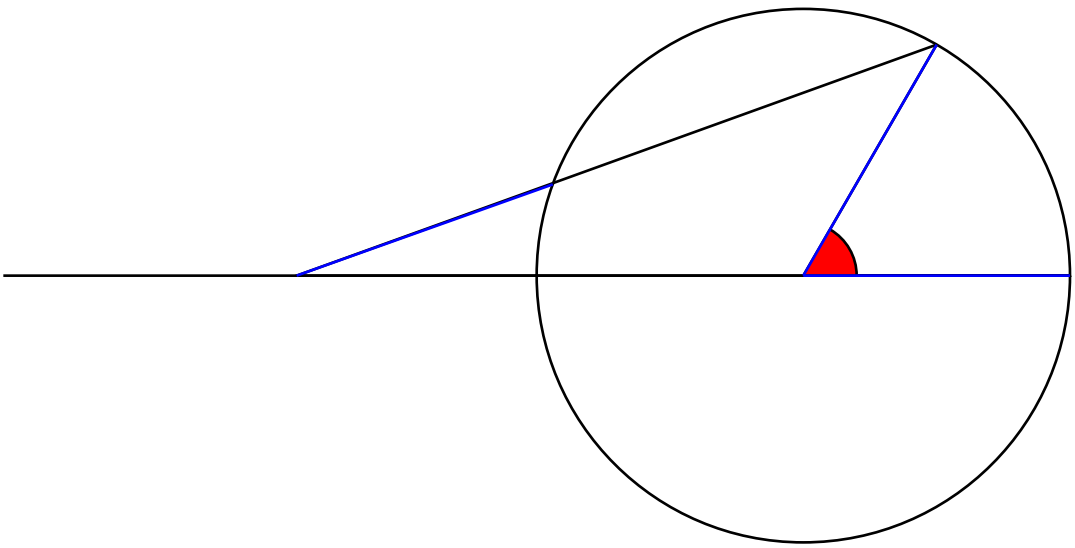
An arbitrary angle.

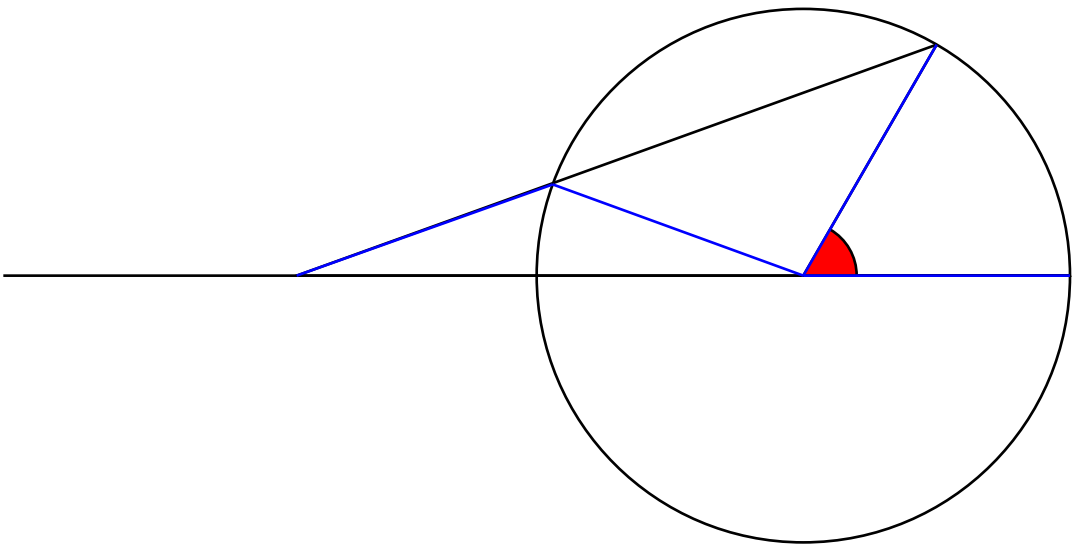


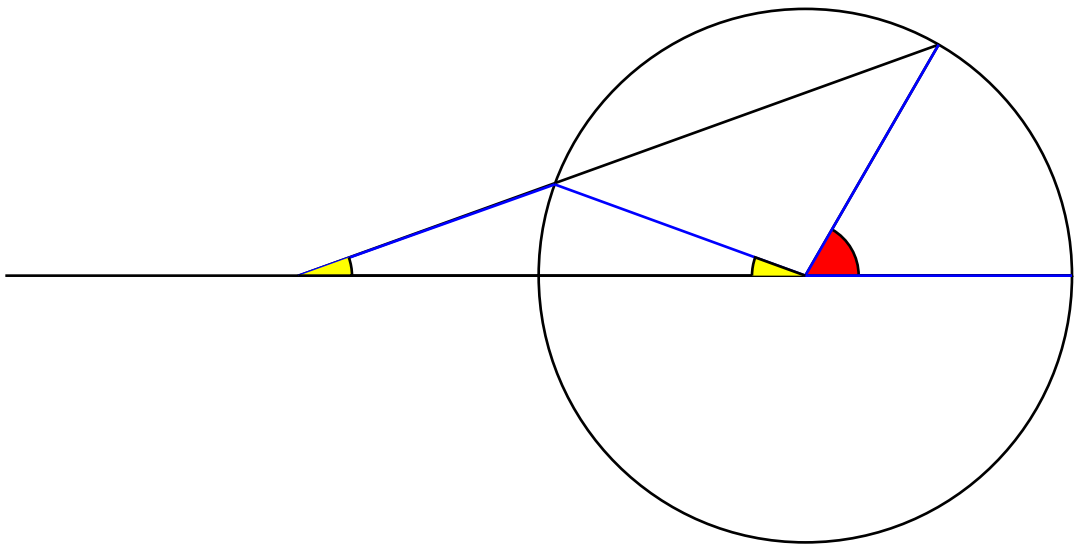


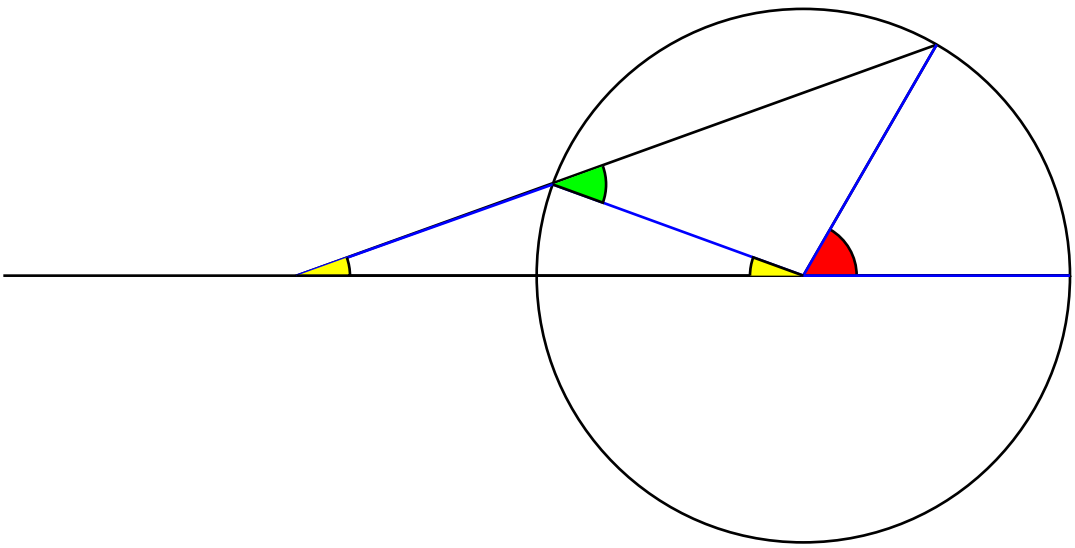


From the yellow point, draw a line that intersects with the horizontal line, such that the blue line segment equals the radius of the circle.



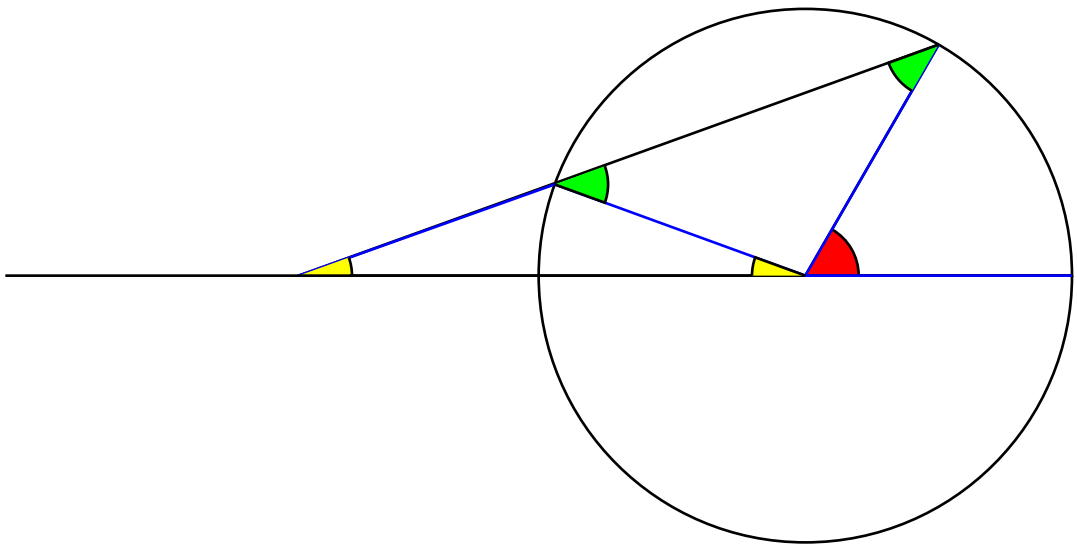


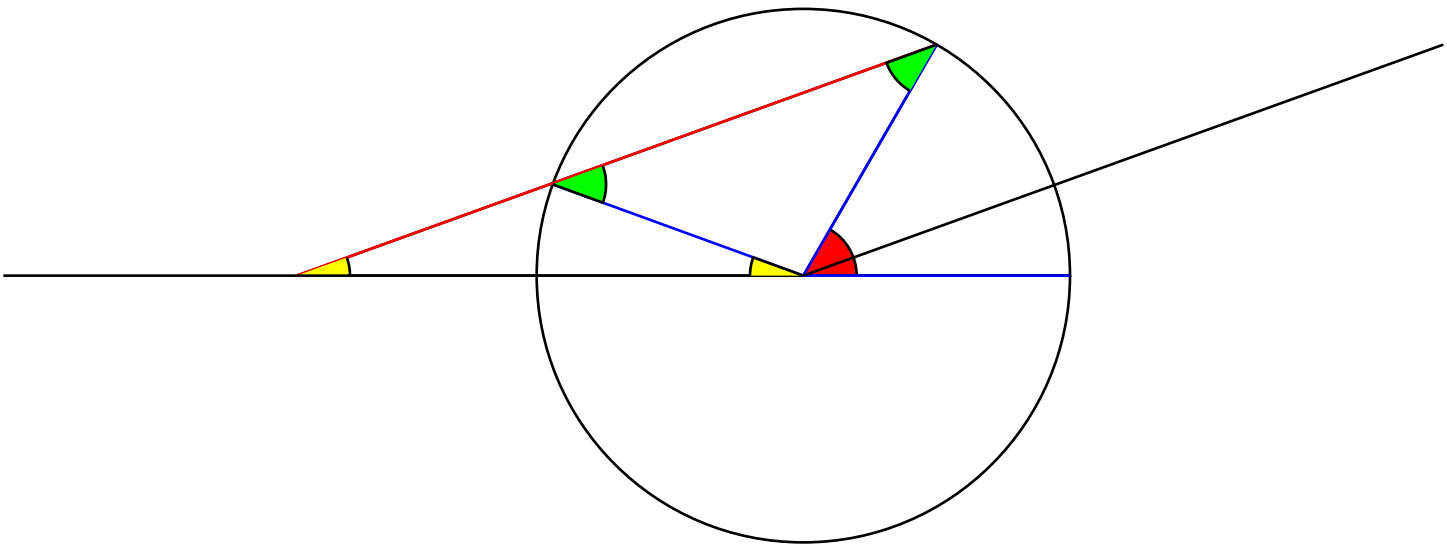




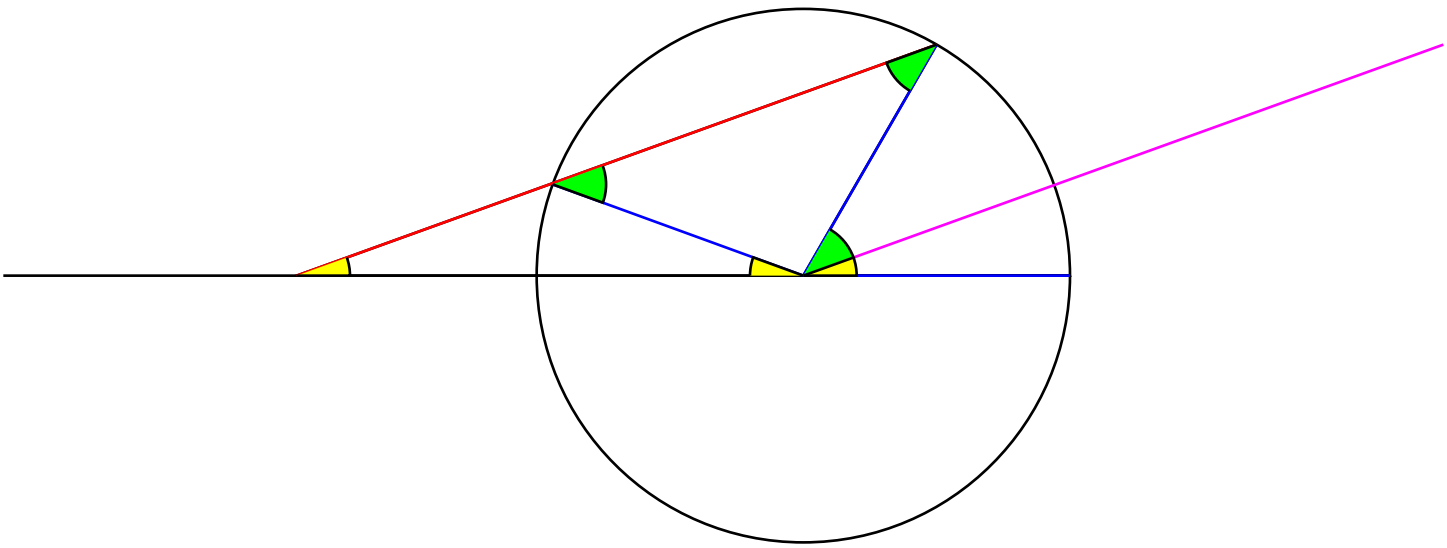
The green angle is the sum of the two yellow angles;

hence, the green angle is twice the yellow one.





From the origin, draw a line parallel to the red line.

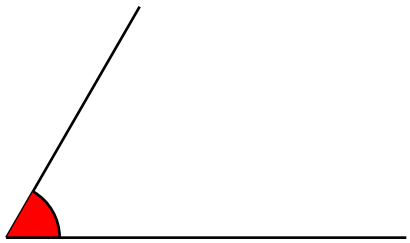


Because the green angle is twice the yellow angle,

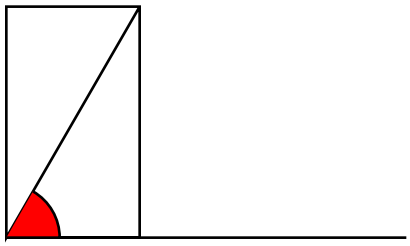
the purple line trisects the given angle.

Method 2:

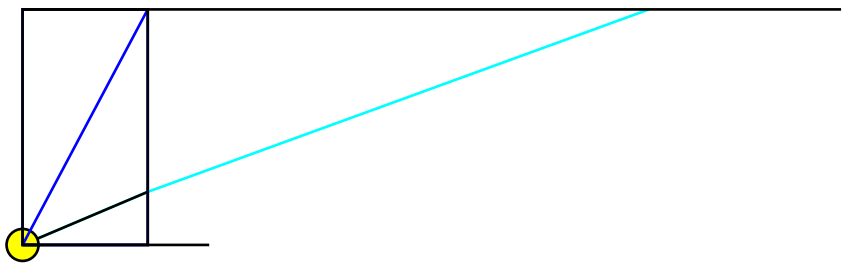
given by Hippocrates



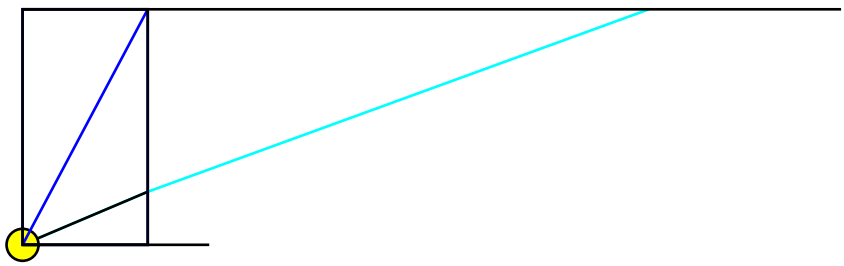
An arbitrary angle.

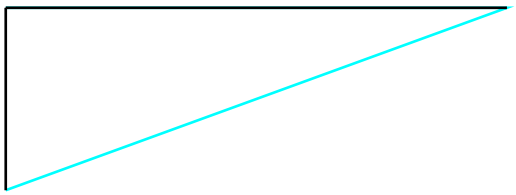


Complete the rectangle.

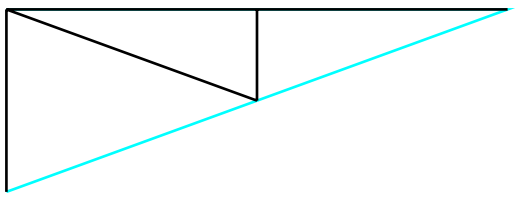


From the yellow point, draw a line that intersects the top horizontal line such that the light blue line is twice as long as the dark blue line segment.

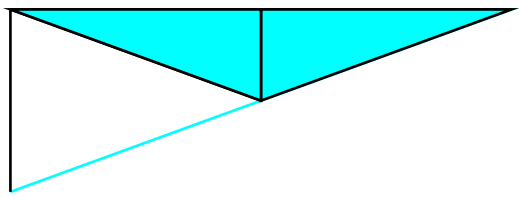




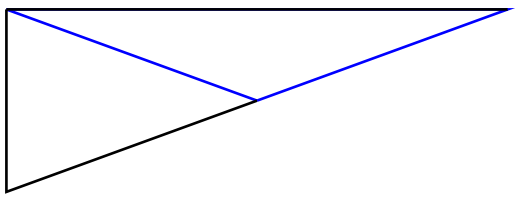
Isolate the triangle from the top graph.



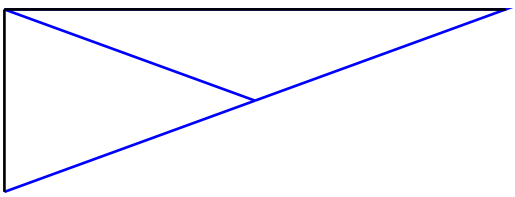
Take the middle point of the light blue line and connect it to the right-angle corner point. Also, from the middle point, draw a line that is perpendicular to the top horizontal line.



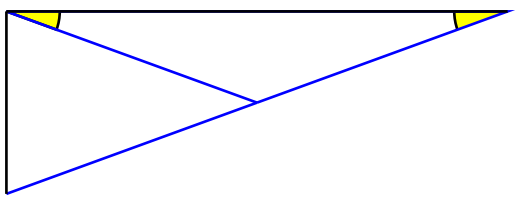
These two light blue rectangles are similar rectangles.

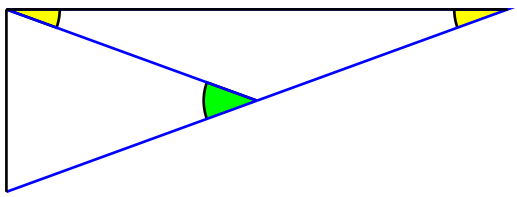


Therefore, these two blue lines are of the same length.



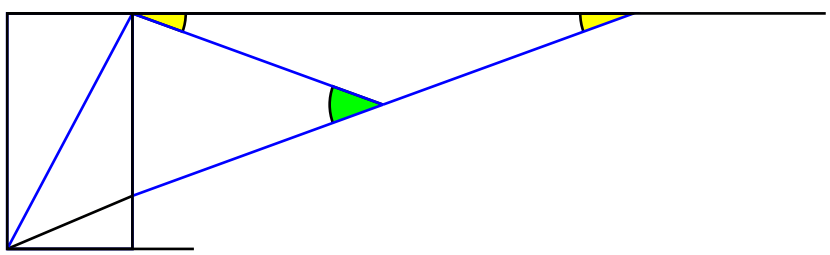
Hence, these three blue lines are of the same length.



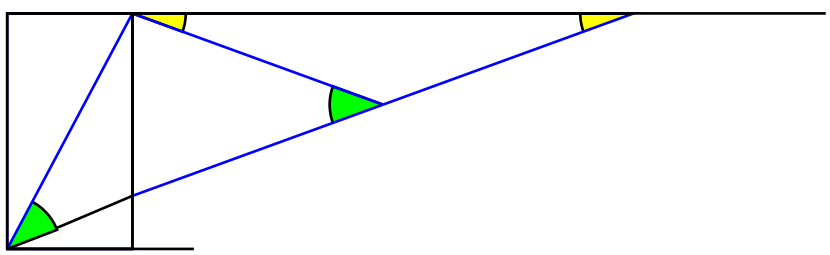


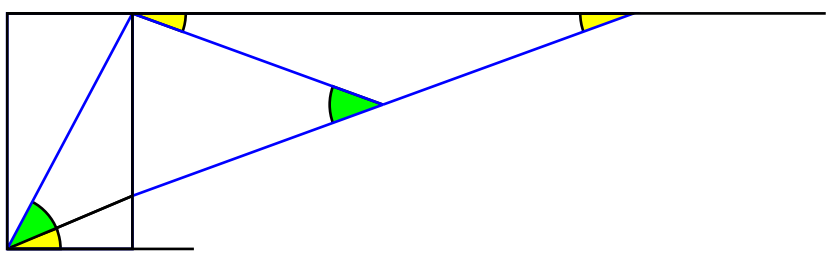
The green angle is the sum of the yellow angles.

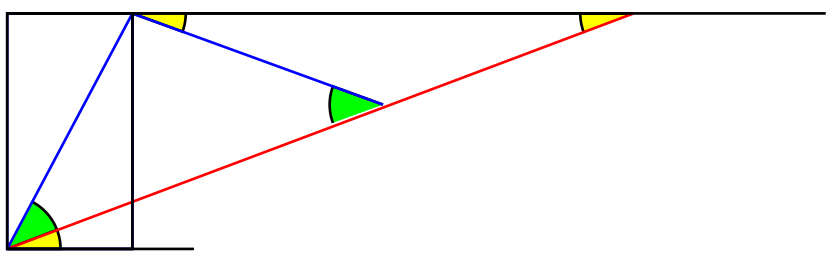
So, the green angle is twice as big as the yellow angle.



Add back the rectangle part.







Because the green angle is twice the yellow angle,

the red line trisects the given angle.

References

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