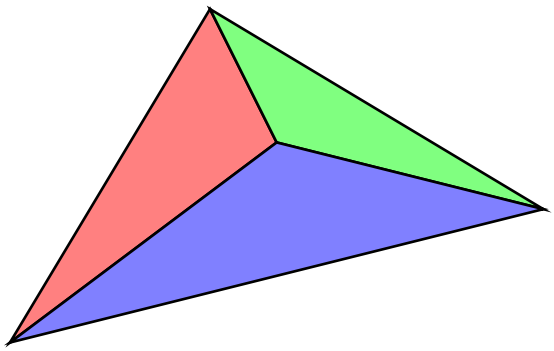
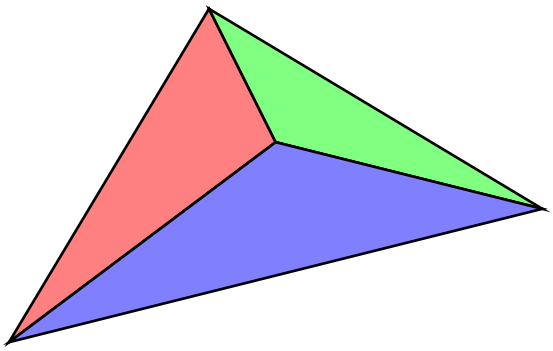
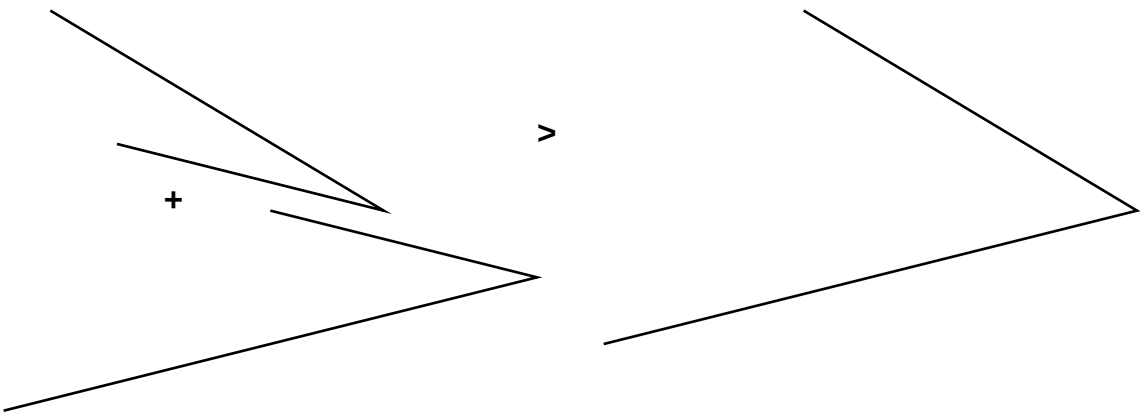
For any number of faces that touch at a vertex of a polygon existing in three dimensions a figure such as the following can be drawn, in two dimensions:

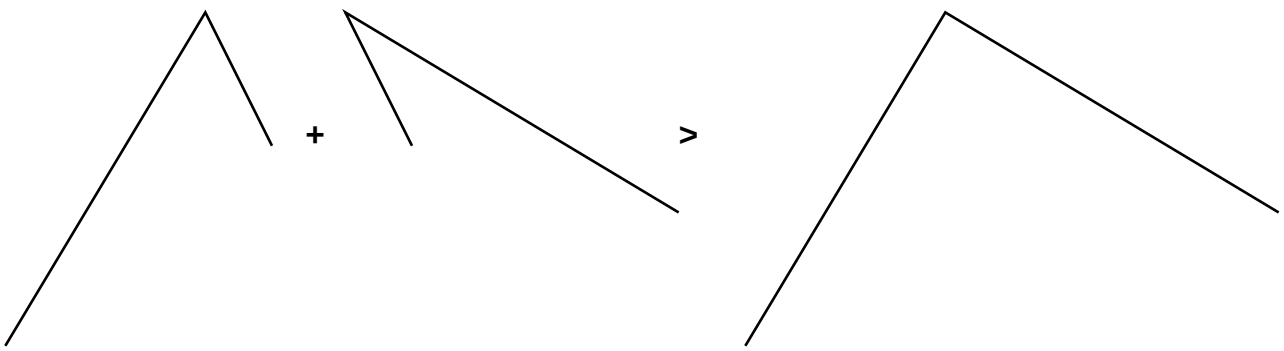


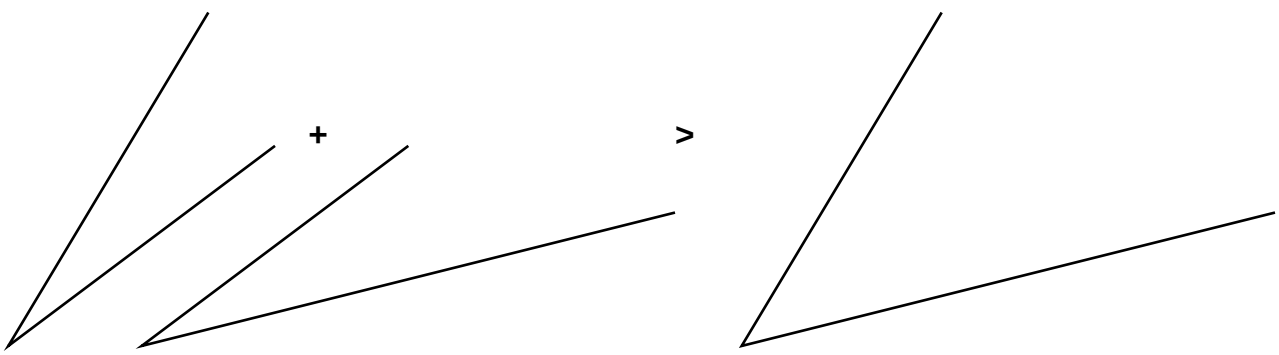
Since each angle at the edge of the larger triangle is contained by three different planer angles:

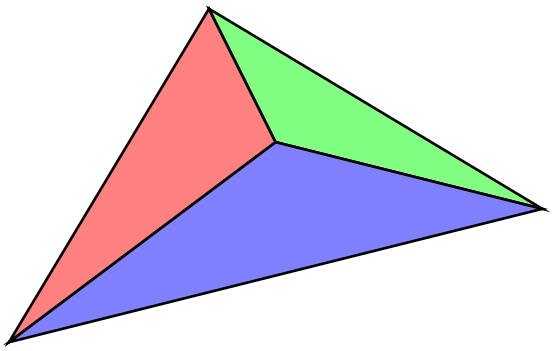


the sum of any two angles will be greater than the remaining one.
That is:

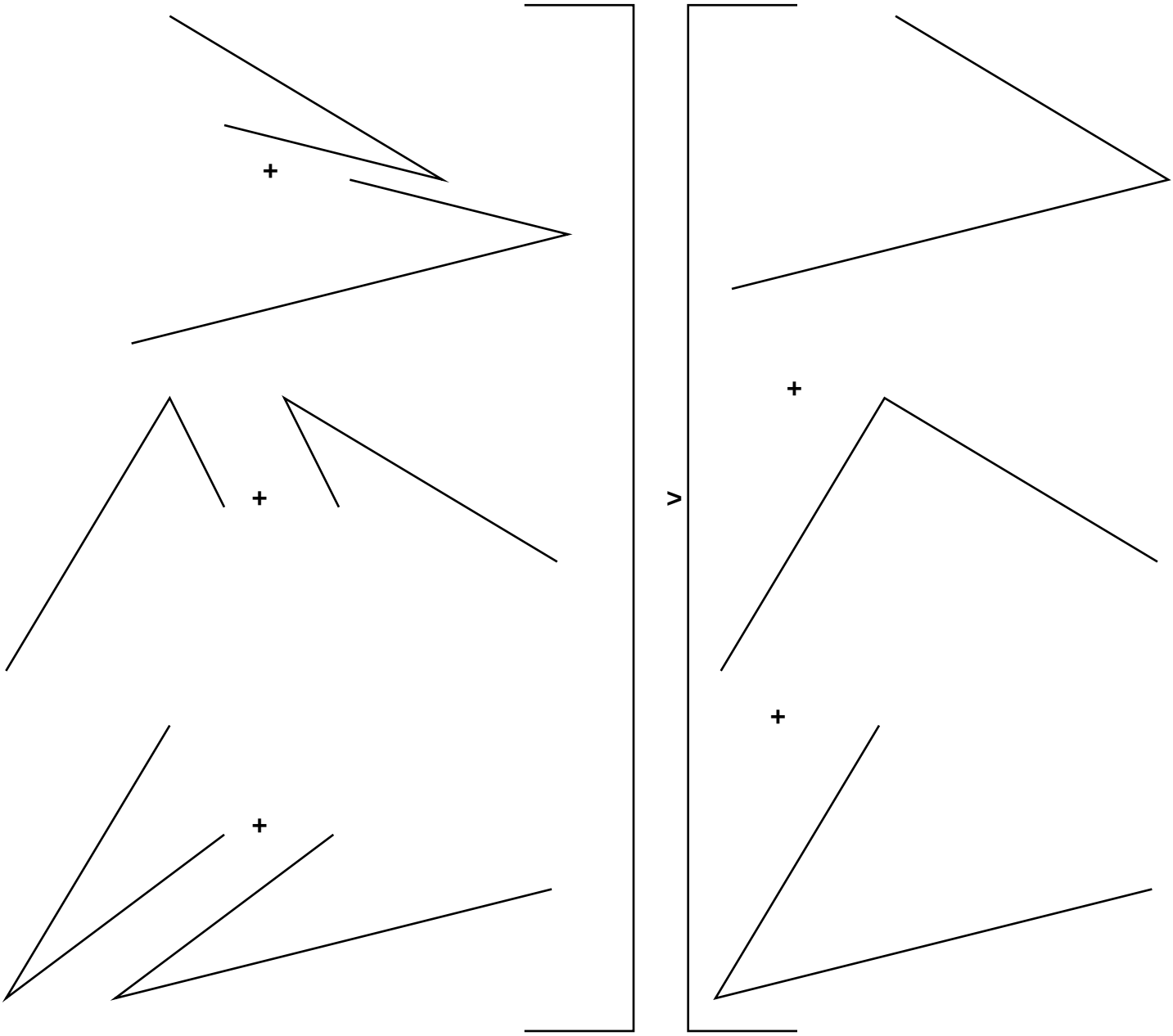


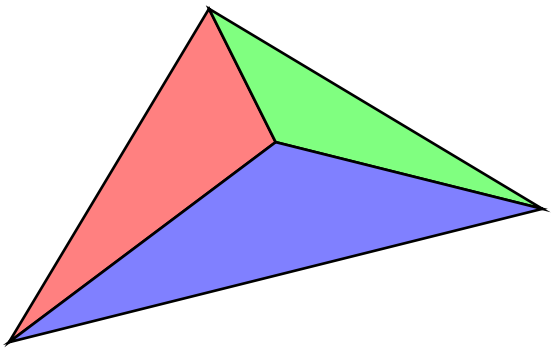






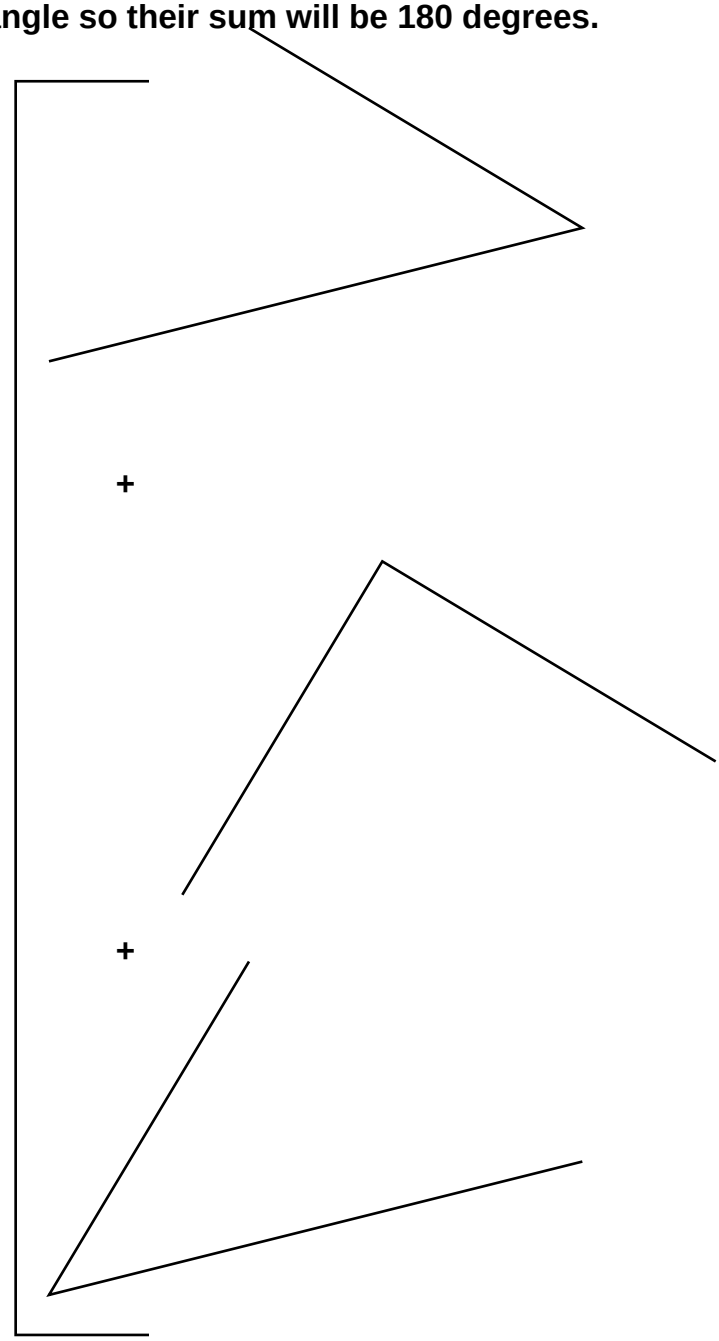
so the sum of all six smaller angles will be greater than the sum of the three larger ones. That is:

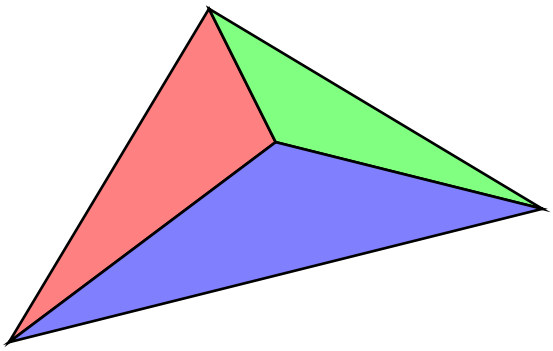




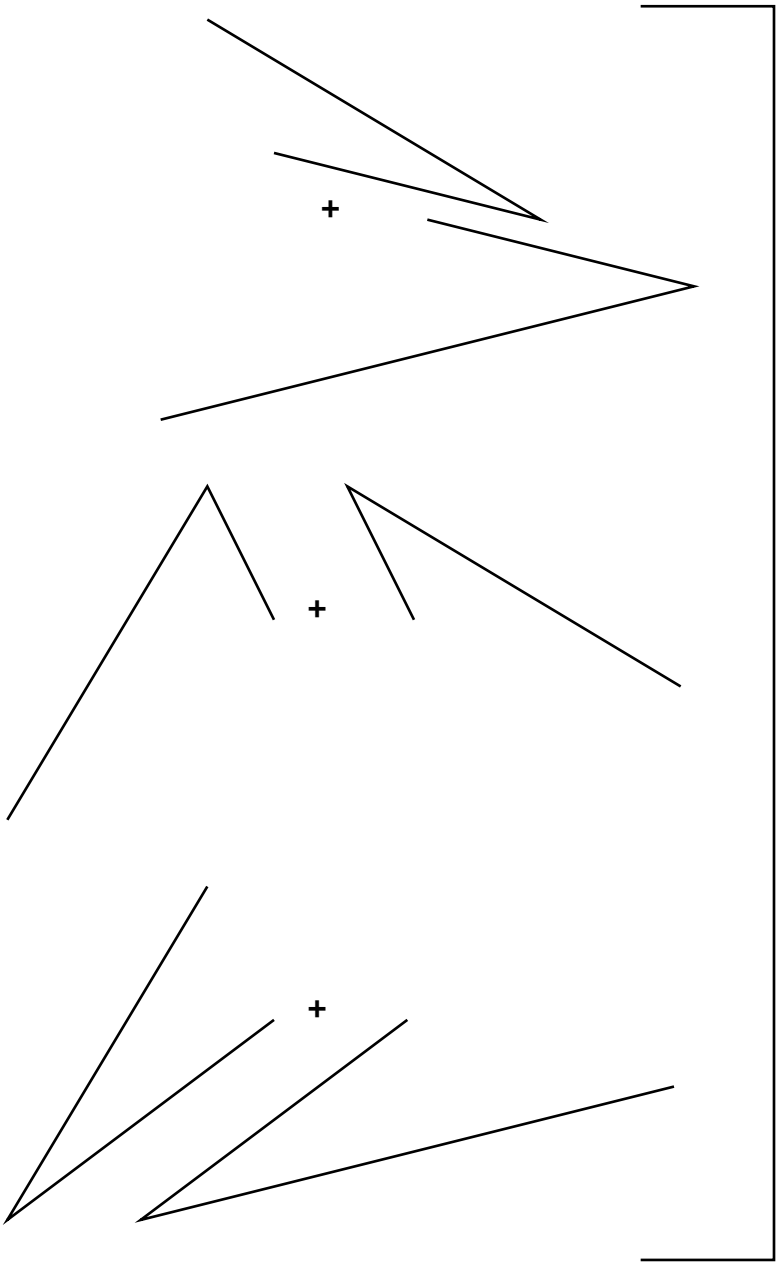
But the three larger angles form the outer triangle so their sum will be 180 degrees. That is:

2x90° = 180° =

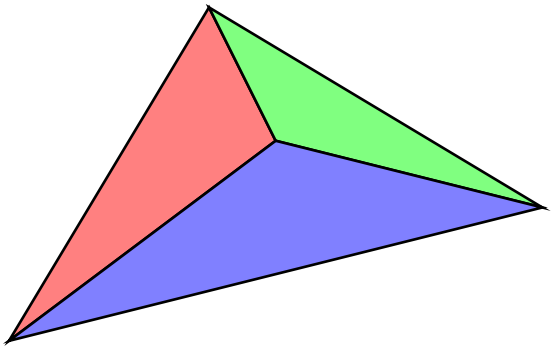




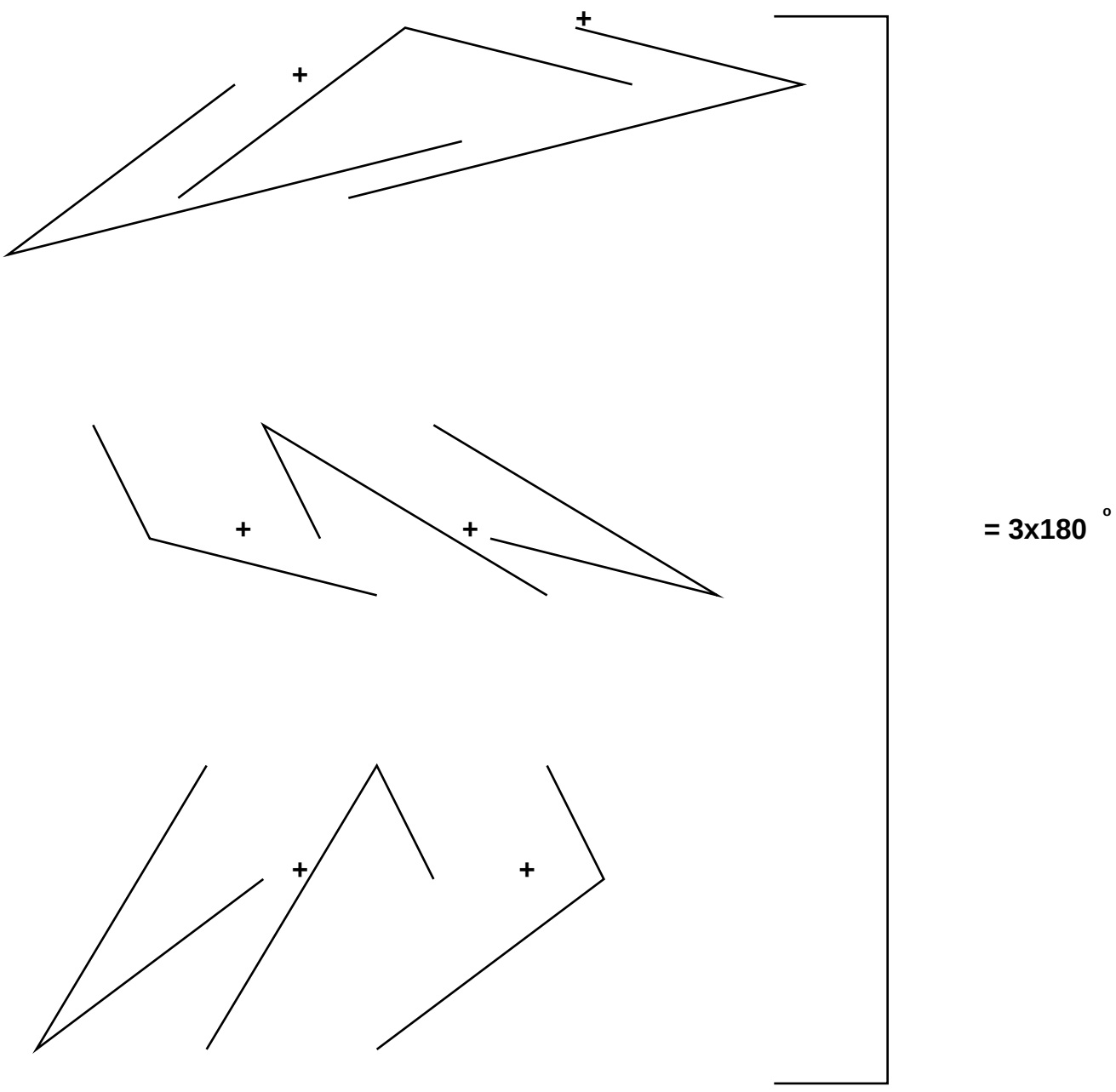
so the sum of all six smaller angles will be greater than 180 degrees.
That is:



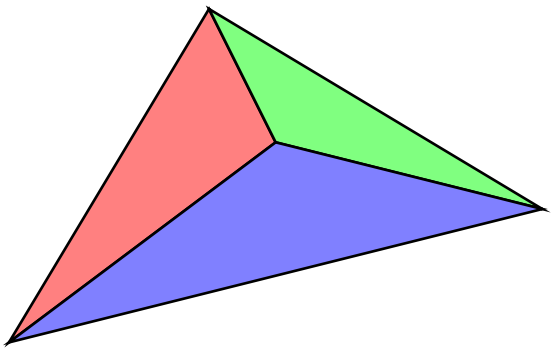
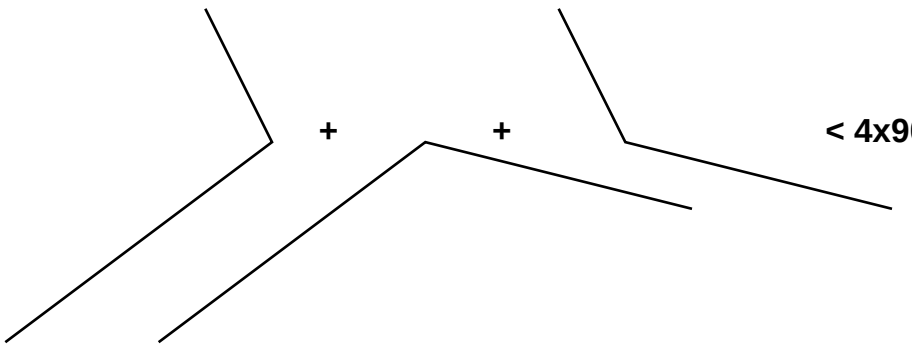
> 2x90° = 180°



But the sum of all nine angles in the whole diagram will be the sum of the three triangles combined. That is:



= 3x180 °



and sum of the angles around the vertex will be less then the difference which is sum of four right angles. That is: