Hyperbolic Geometry

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Math 308

Parallel Postulate

In geometry, the parallel postulate, also Euclid’s fifth postulate in Euclid’s Elements, is a distinctive axiom in what is now called Euclidean geometry. The parallel postulate states the following:

If a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles

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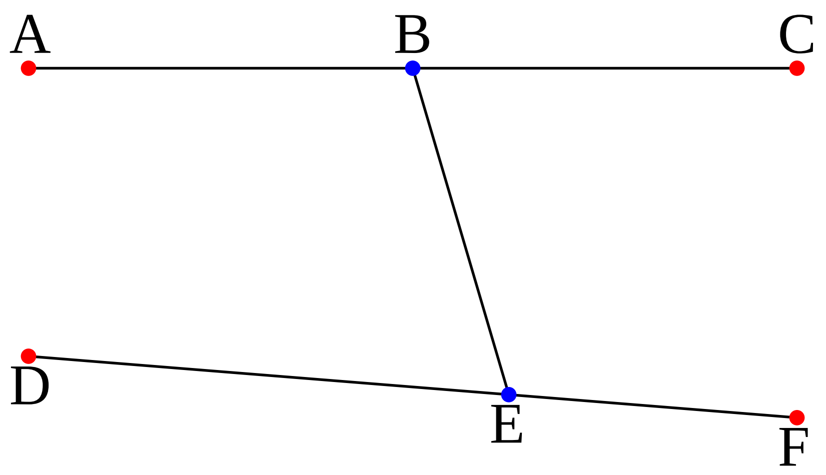

In the diagram, if angle ABE plus angle BED is less than two right angles (180 degrees), then lines AC and DF will meet when extended in the direction of A and D

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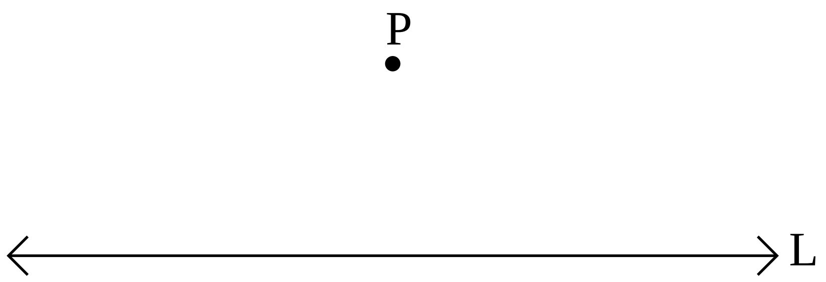

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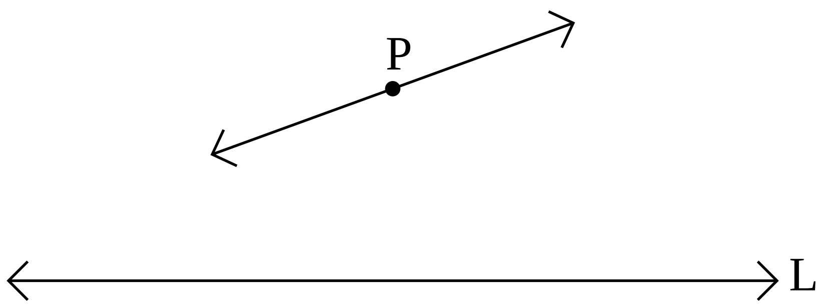

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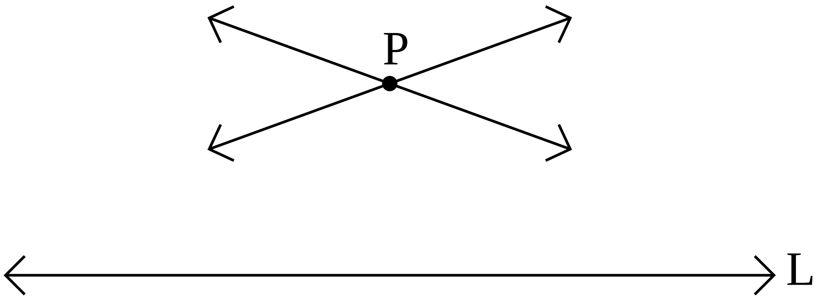

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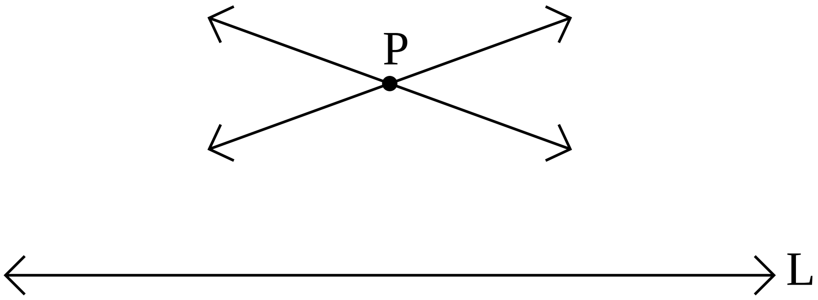

In this case parallel means the lines do not intersect L. Even when the lines are extended they remain a constant distance from L.
In hyperbolic geometry, the term parallel only applies to lines that don't intersect in the hyperbolic plane but intersect at the circle at infinity.

Modeling the Hyperbolic Plane

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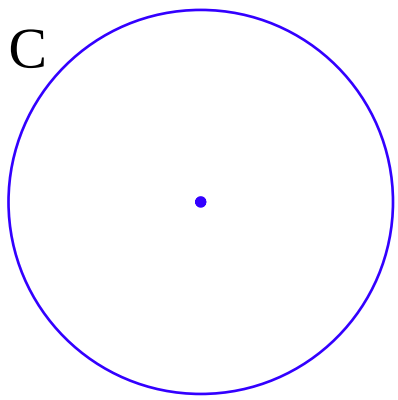

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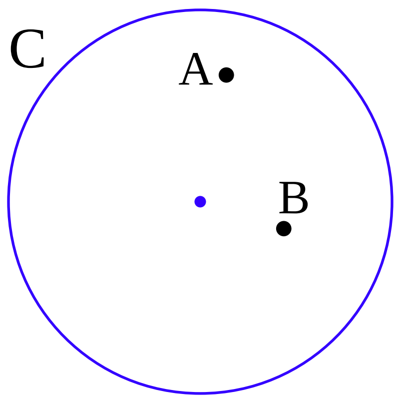

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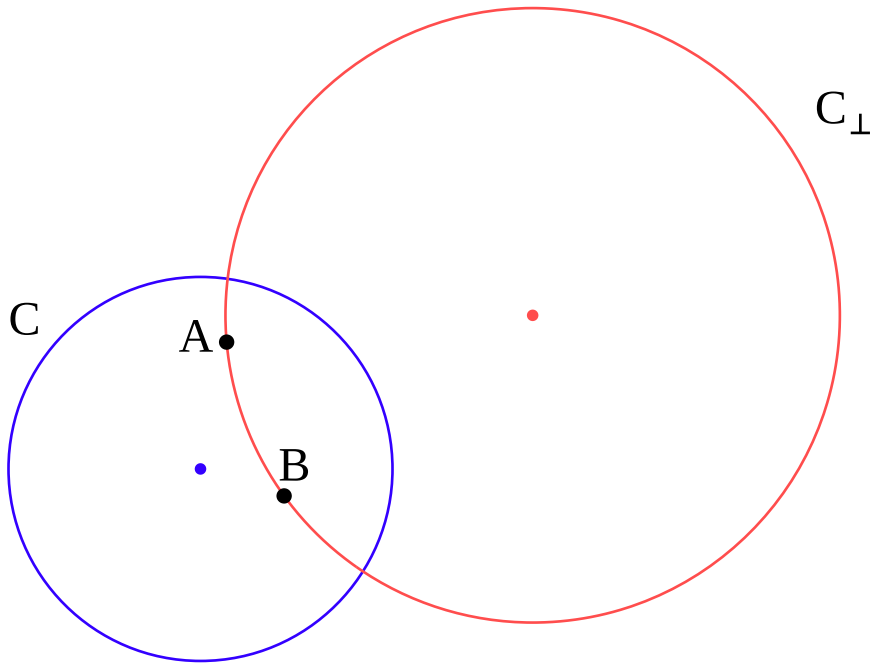

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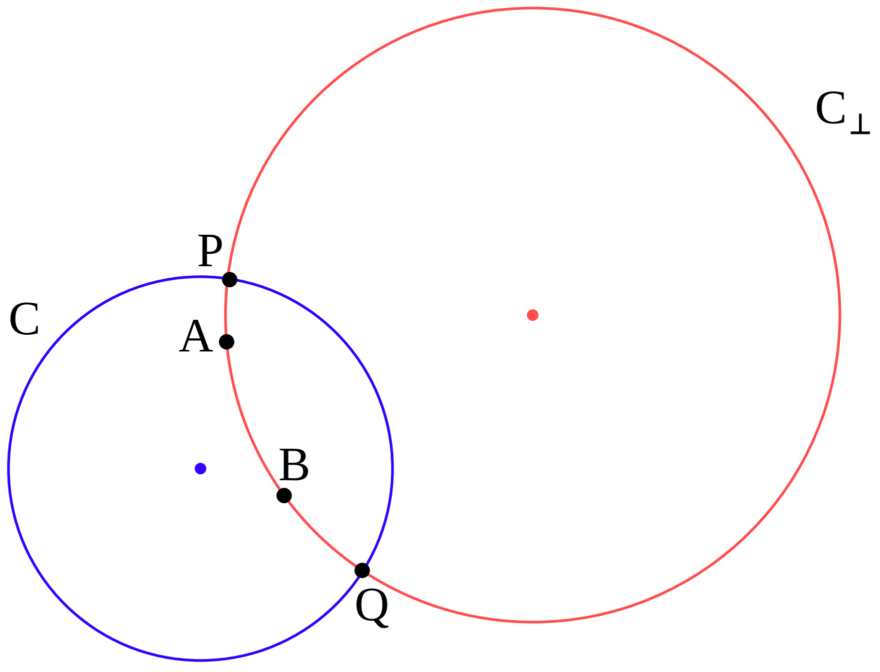

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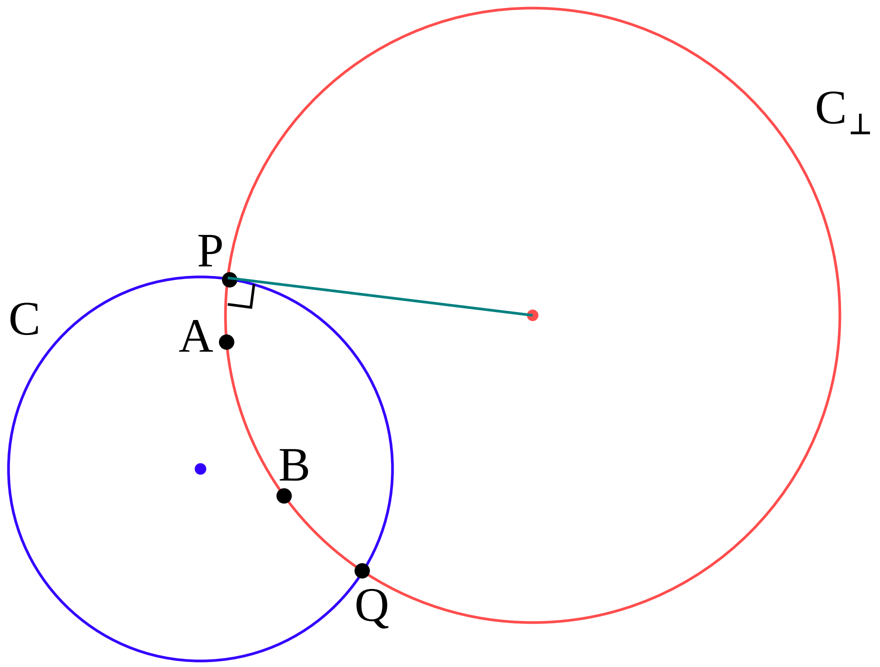

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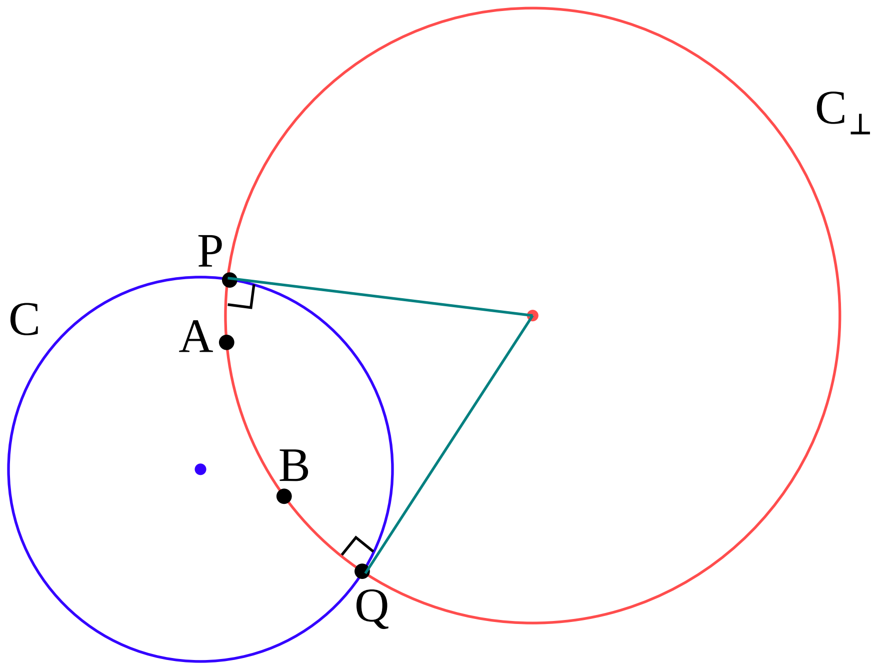

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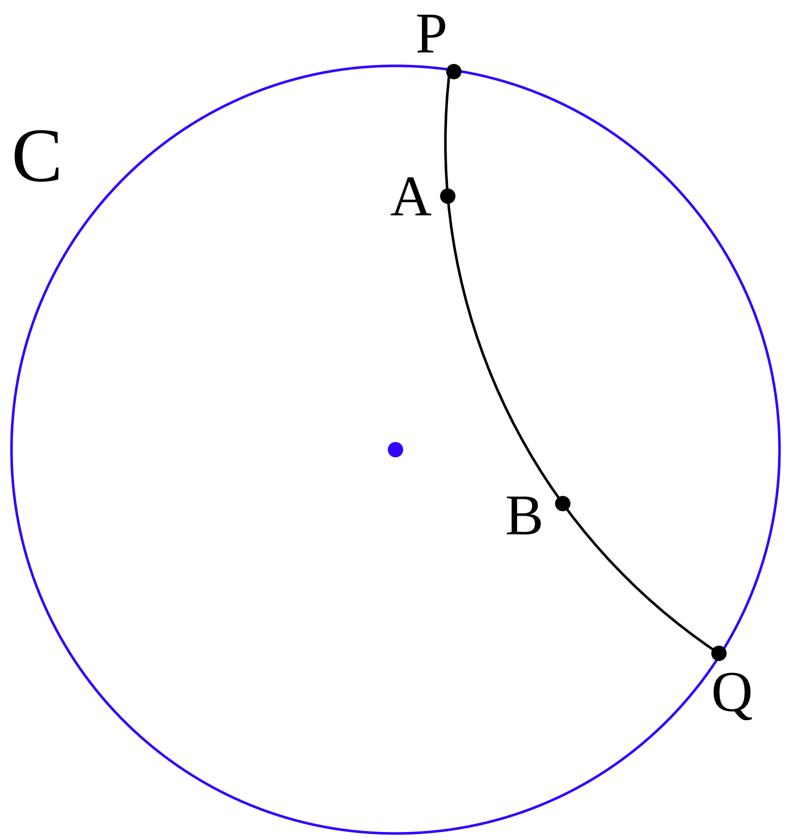
Cross Ratio

In the Poincare Disk model, the cross-ratio (AB,QP) is defined as:

$(\overline{AQ}/\overline{BQ})(\overline{BP}/\overline{AP})$, where A and B are two points inside the circle C, and P and Q are the ends of the P-line passing through A and B.



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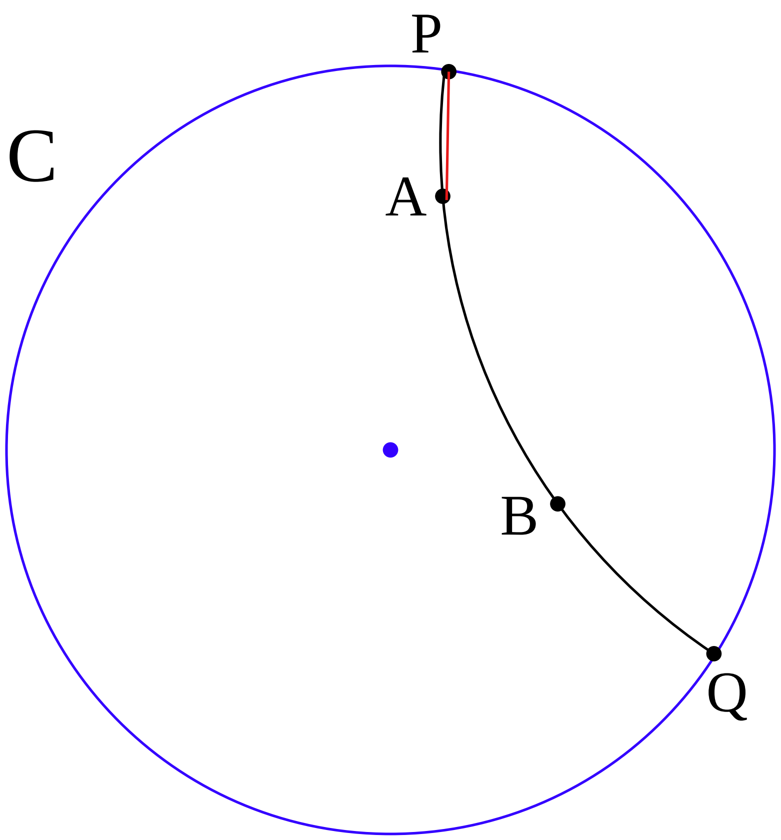

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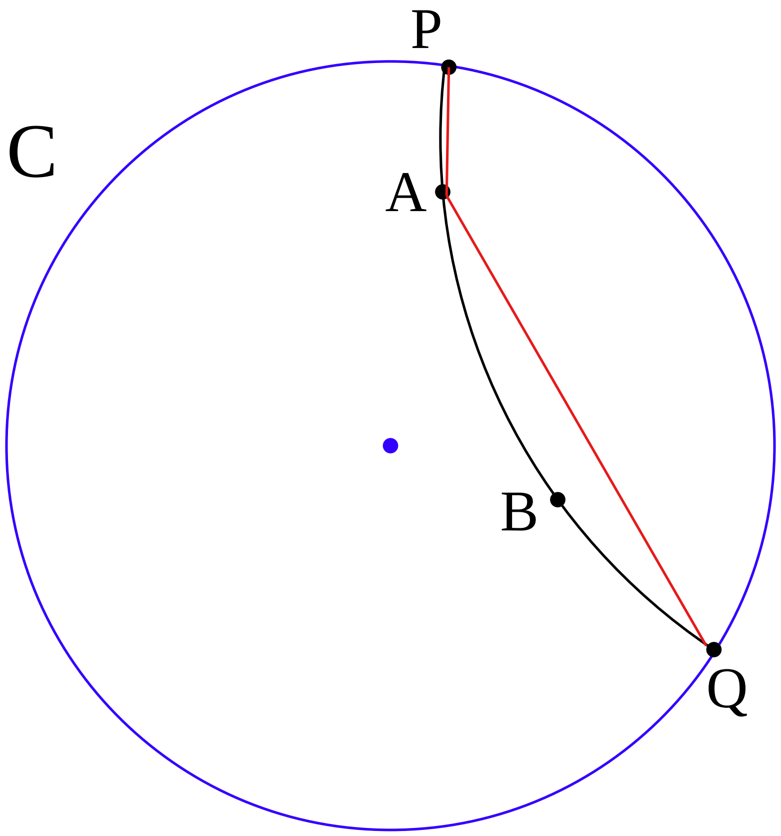

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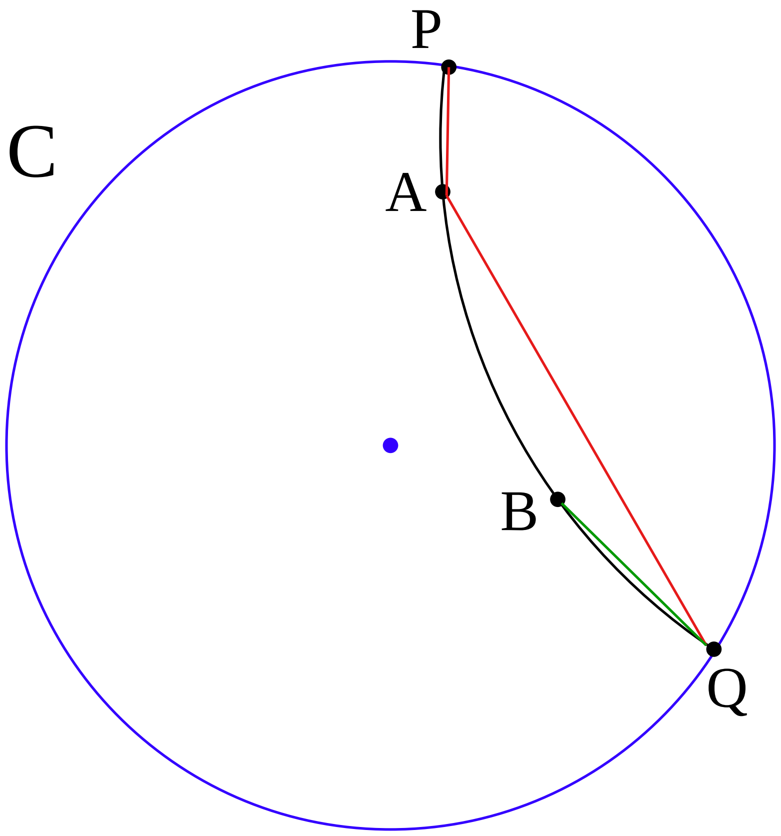

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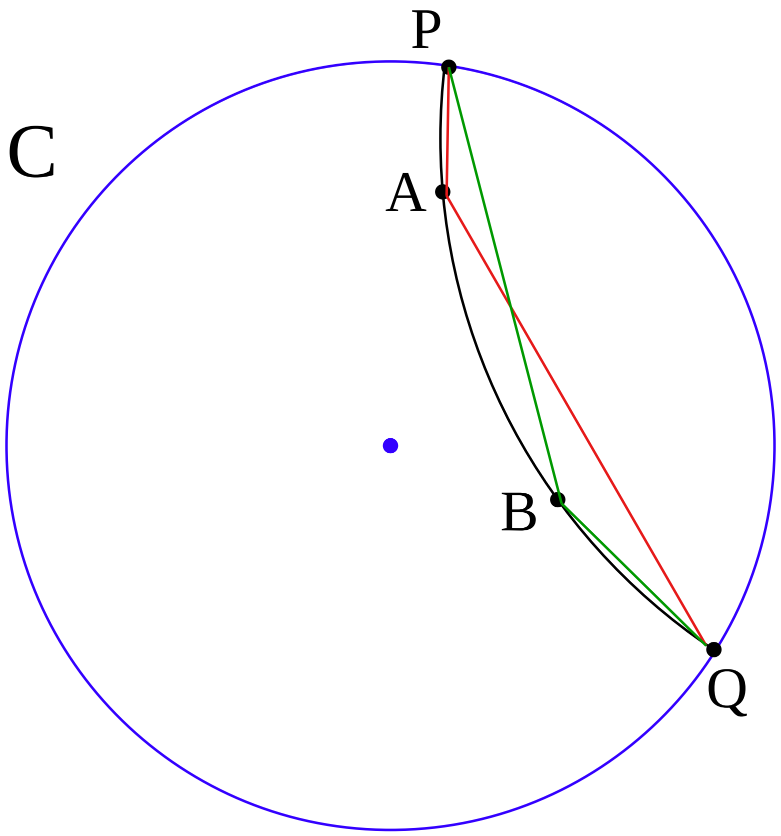

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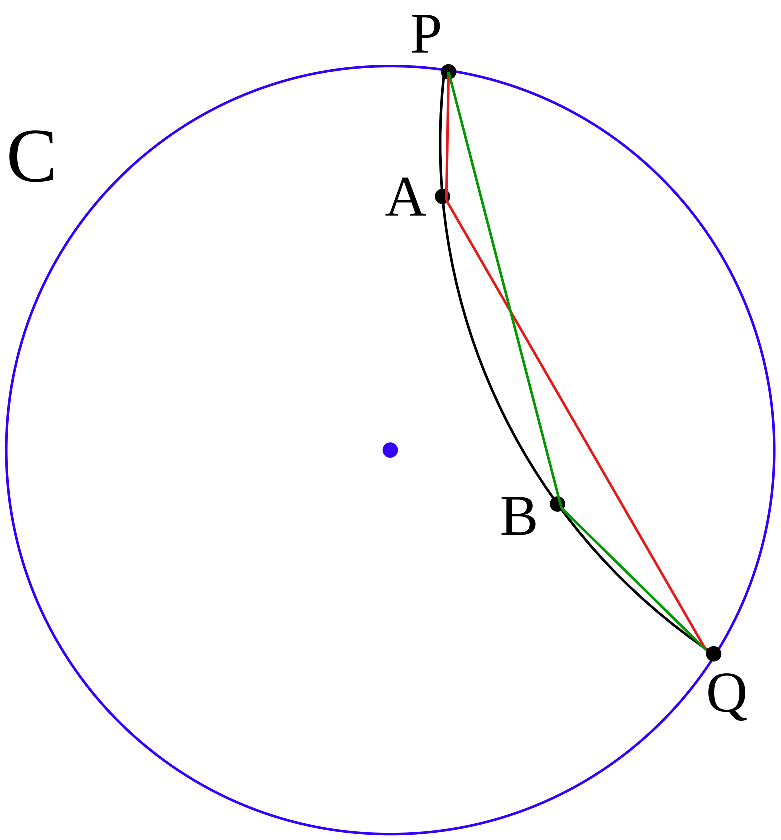
Length in the Poincare Disk

In the Poincare Disk model, the hyperbolic distance, or Poincare-length, of $\overline{AB}$ is defined as:

$$d(AB)=\log\left|(AB,QP)\right|$$



Properties of Poincare-length:

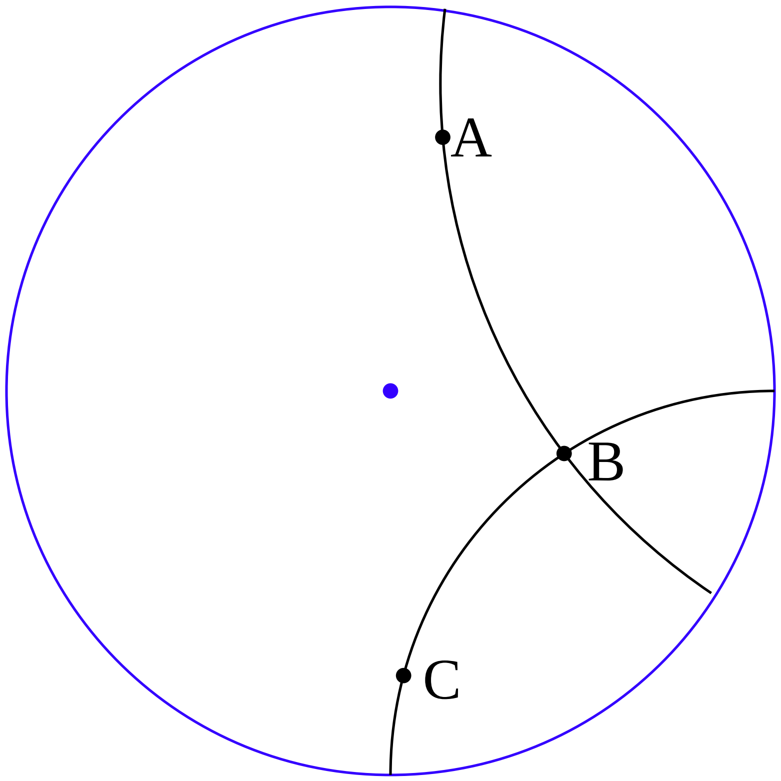
1.d(AB)=d(BA), d(AB)>0 if A and B are distinct points.

2.Let O be the center of the circle C, with radius r, and A be a point inside C, distance x from O. Then d(OA)=log(x+r)-log(r-x).

3.Two segments, $\overline{AB}$ and $\overline{DE}$ for example, are said to be congruent if d(AB)=d(DE)

Measuring Angles

Given three points, A, B, and C, construct the P-lines BA and BC.



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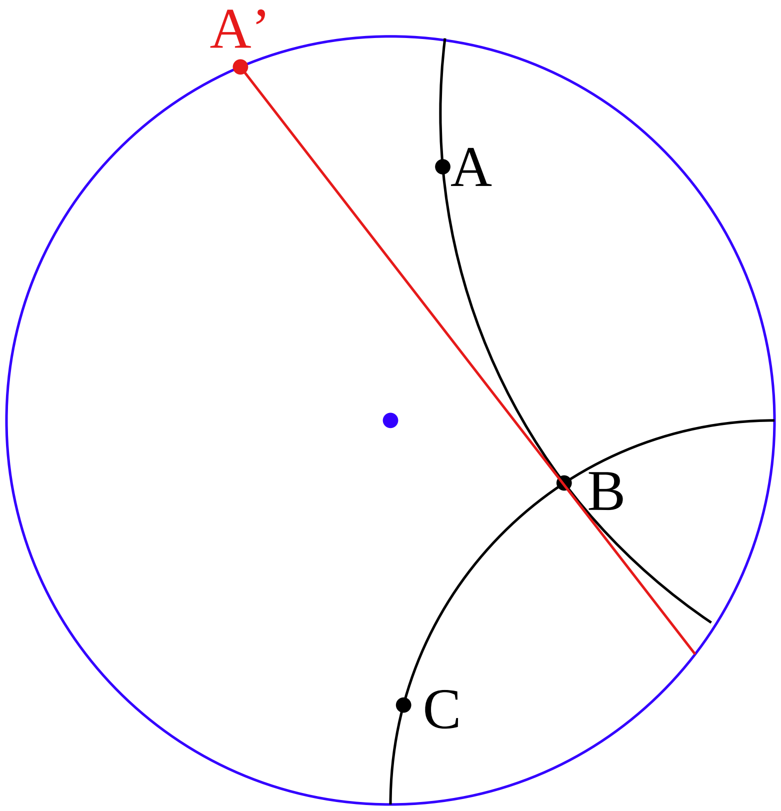

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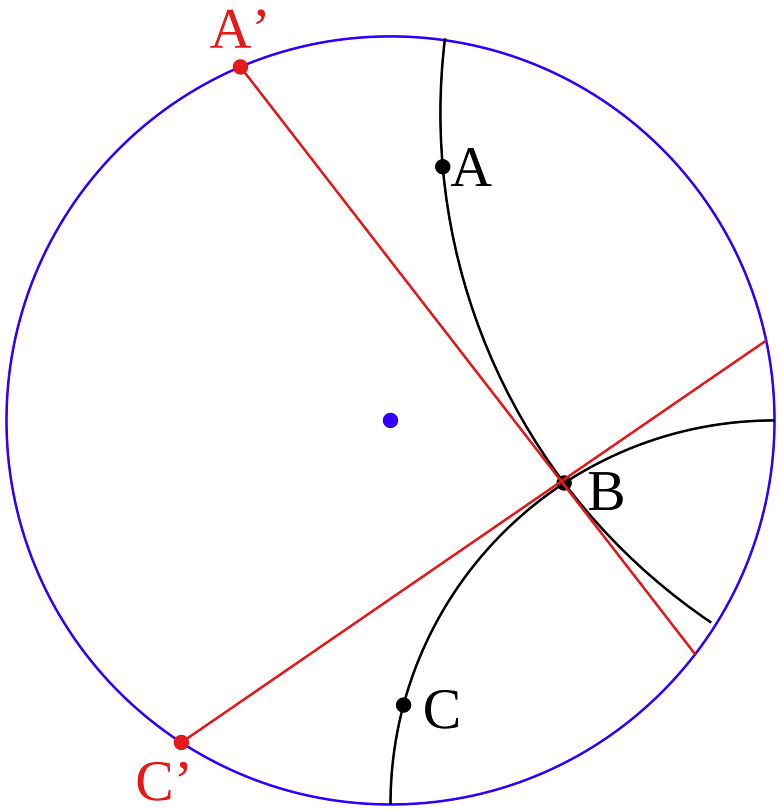

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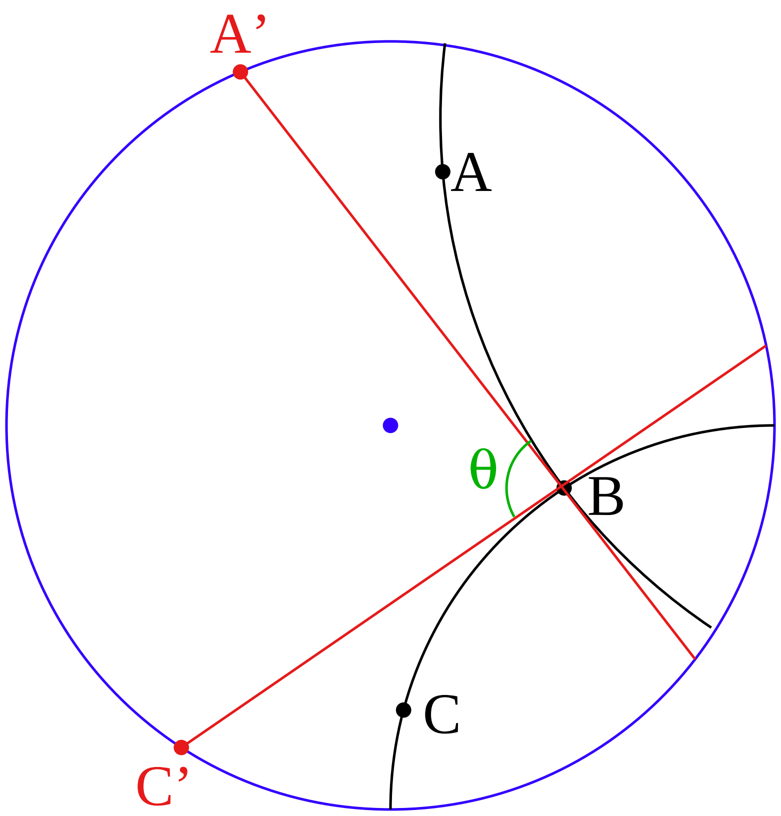
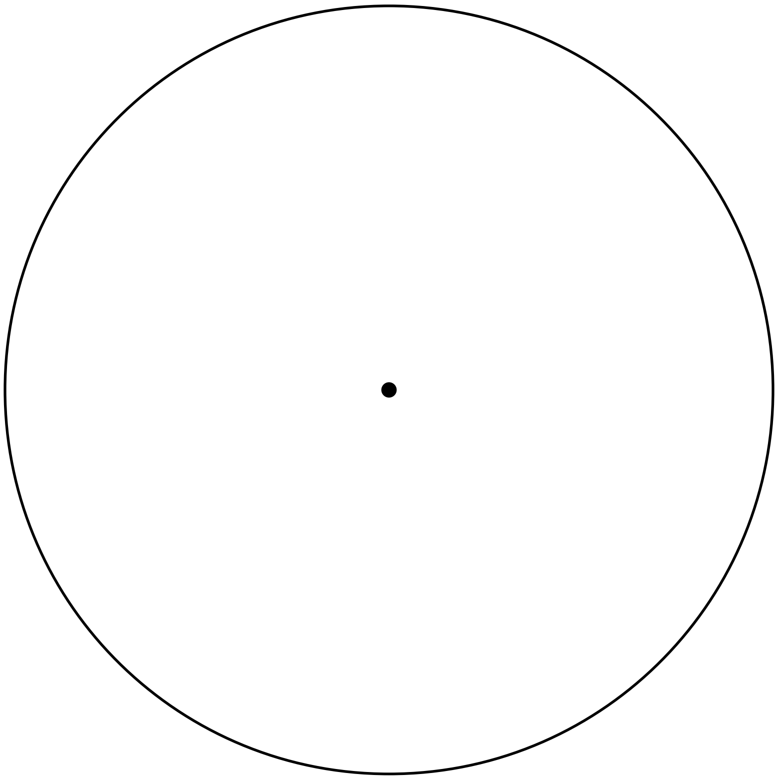
The measure of the angle ABC is defined to be equal to the measure of the Euclidean angle A'BC'.



Hyperbolic Triangle

The interior angles of a triangle sum to less than 180 degrees in Hyperbolic Geometry since the parallel postulate does not hold.

Proof:



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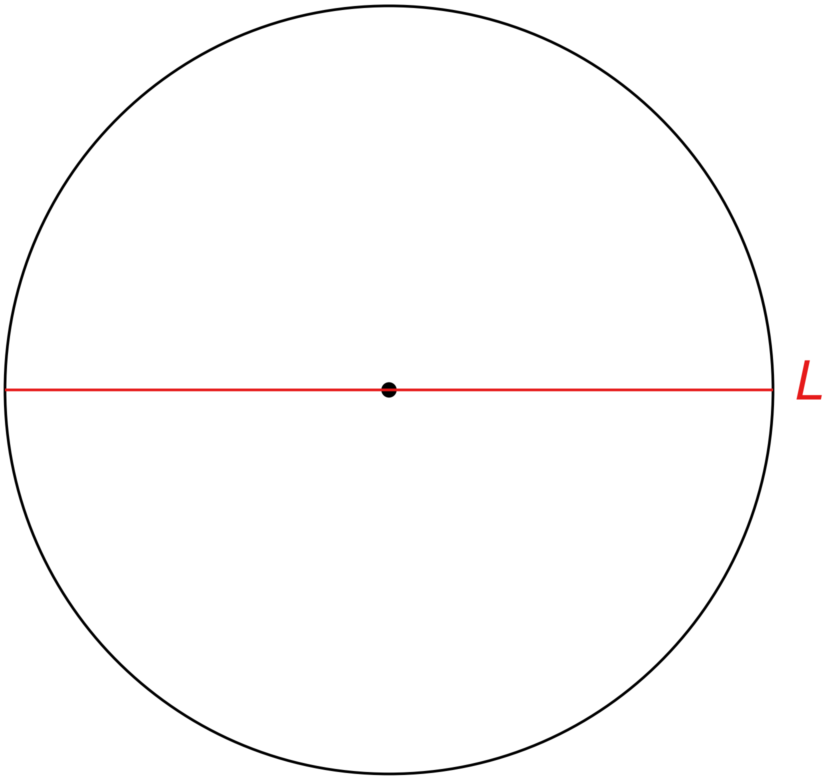

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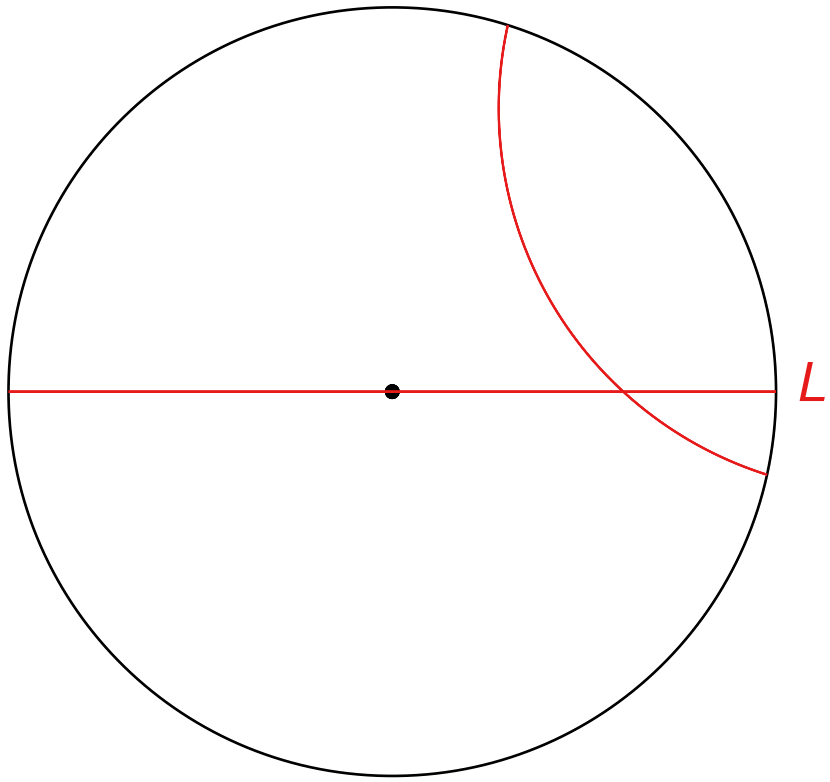

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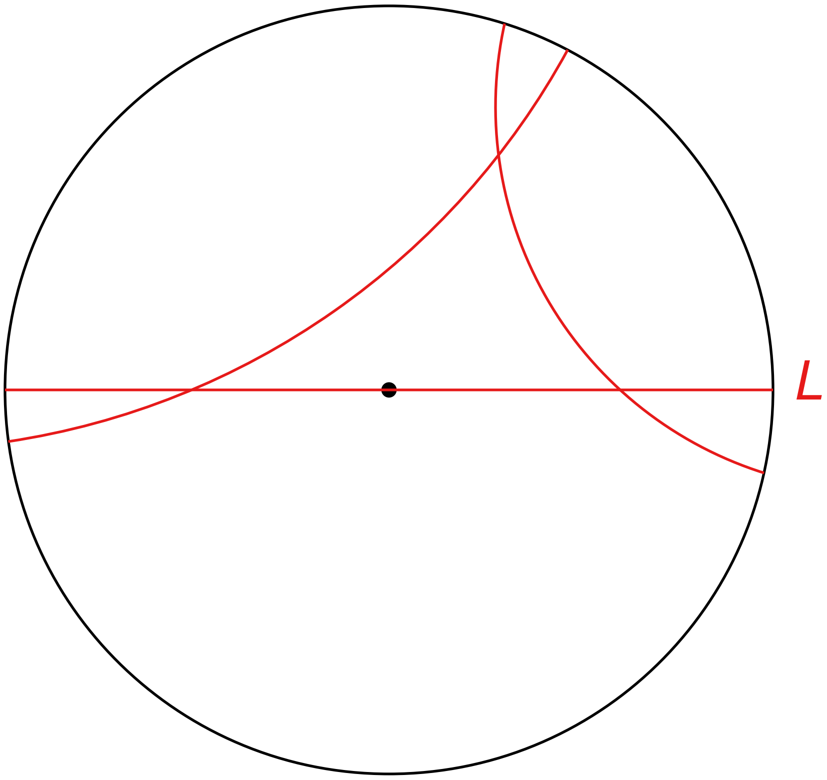

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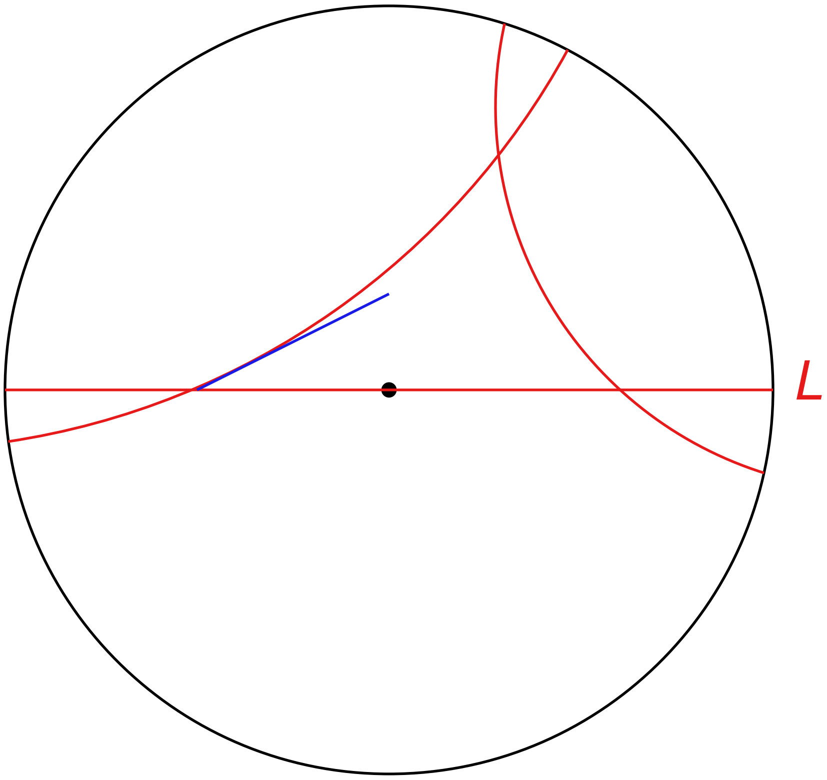

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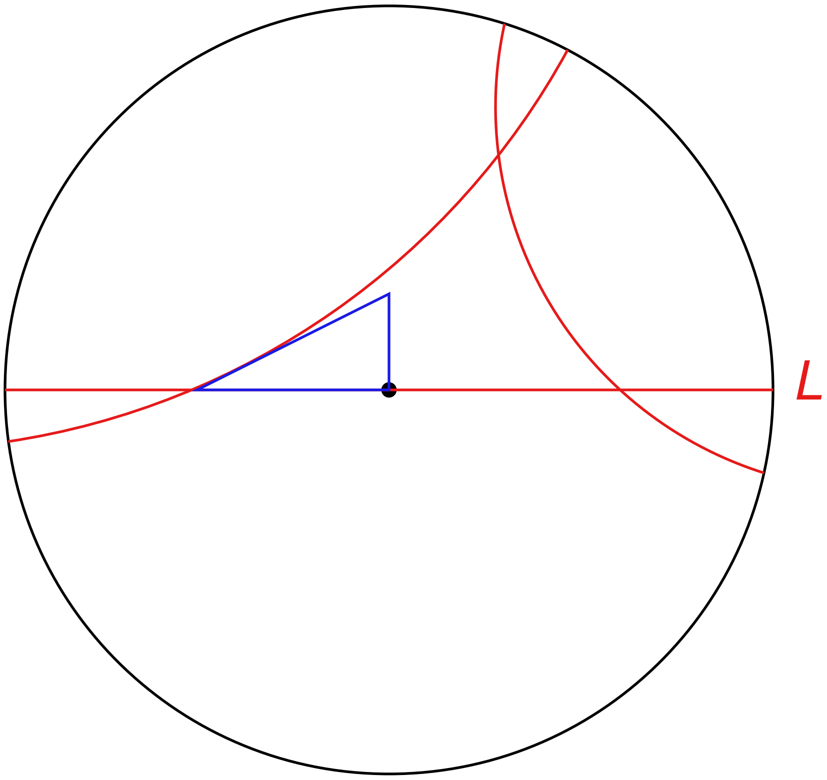

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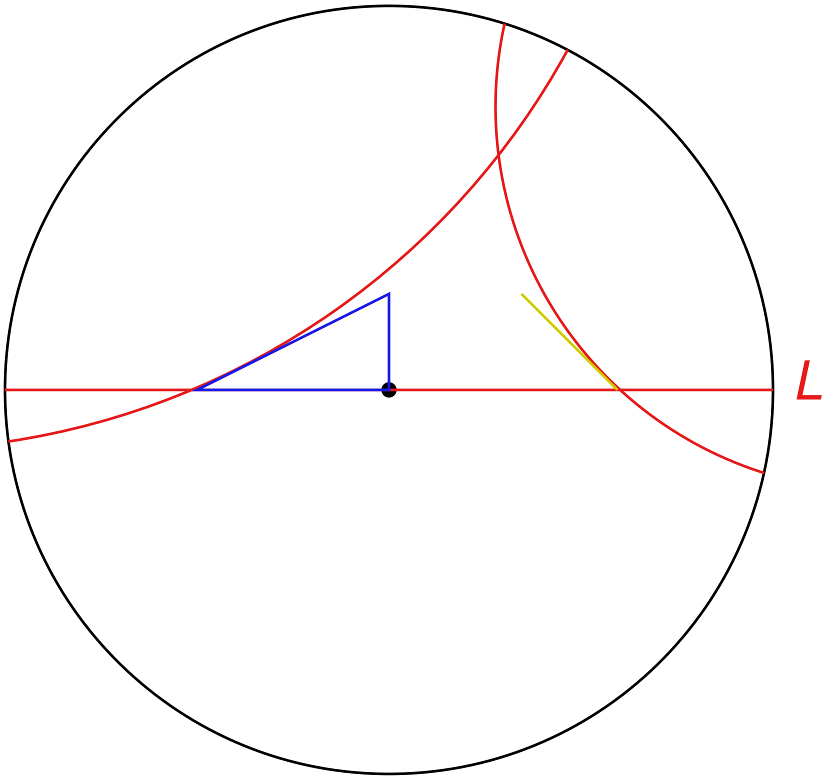

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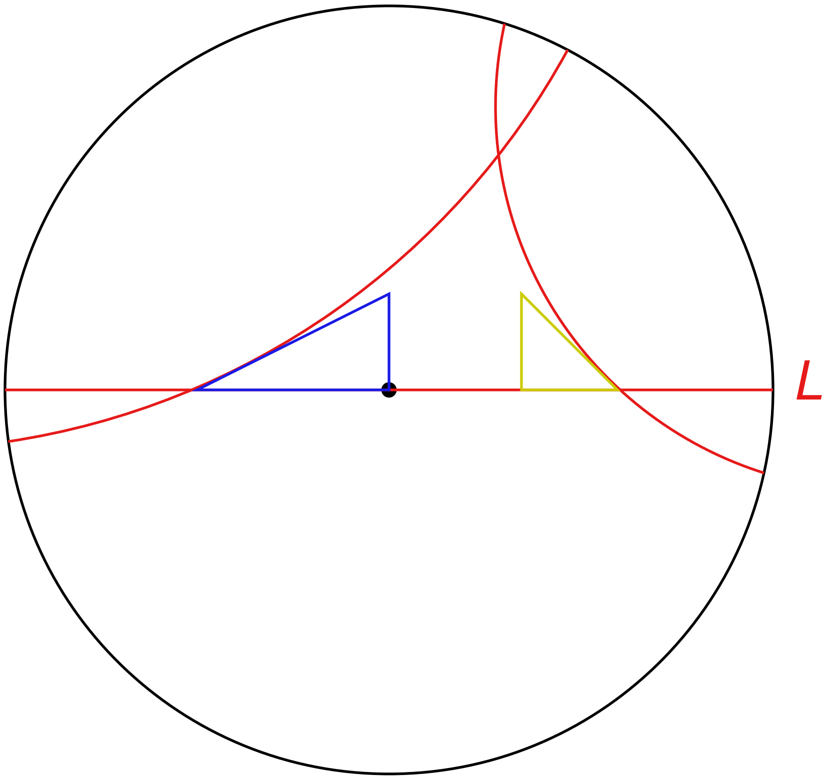

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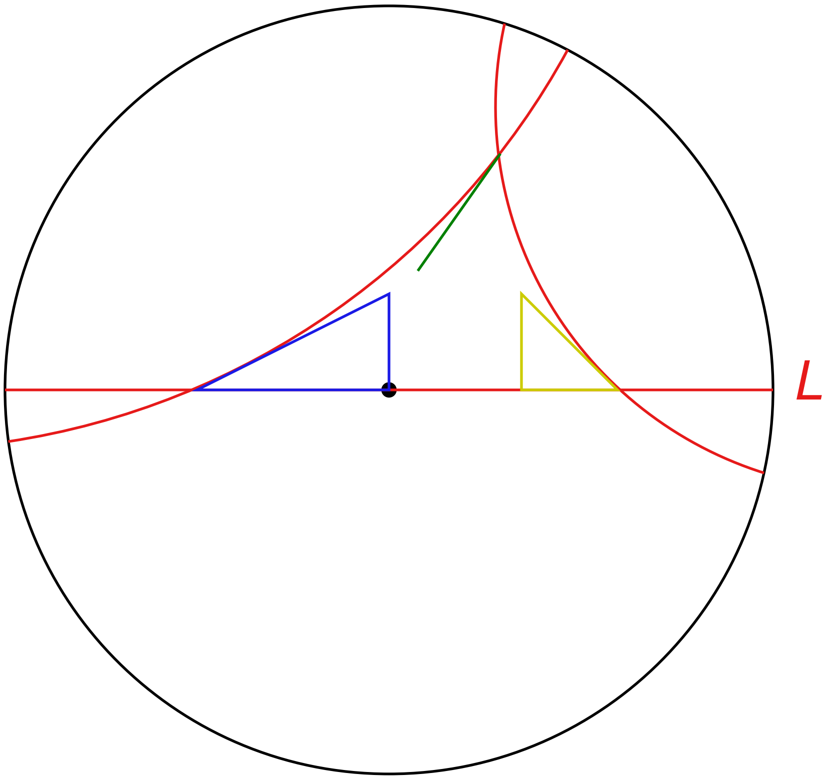

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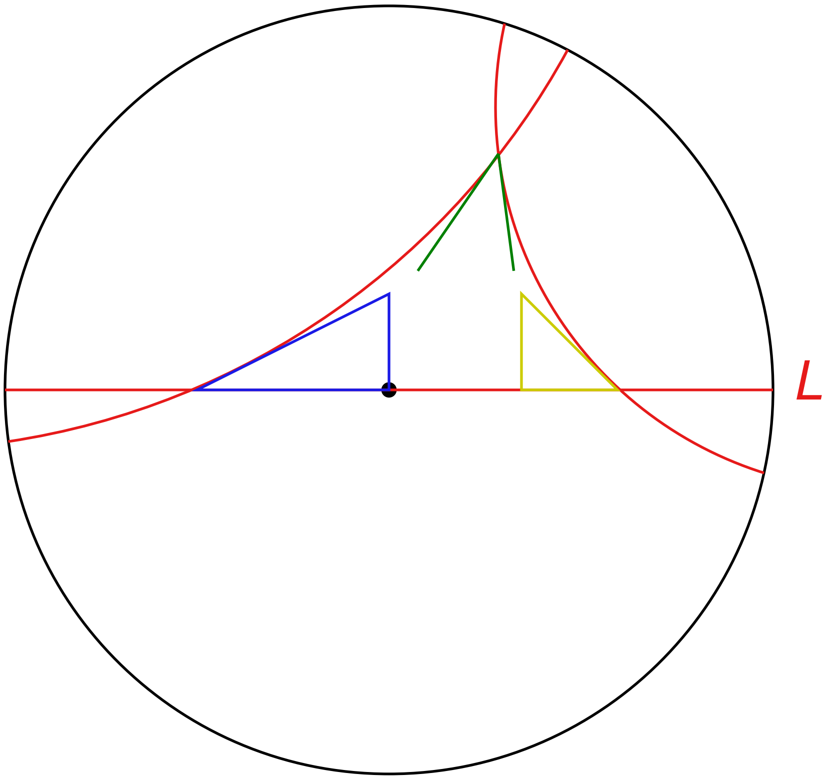

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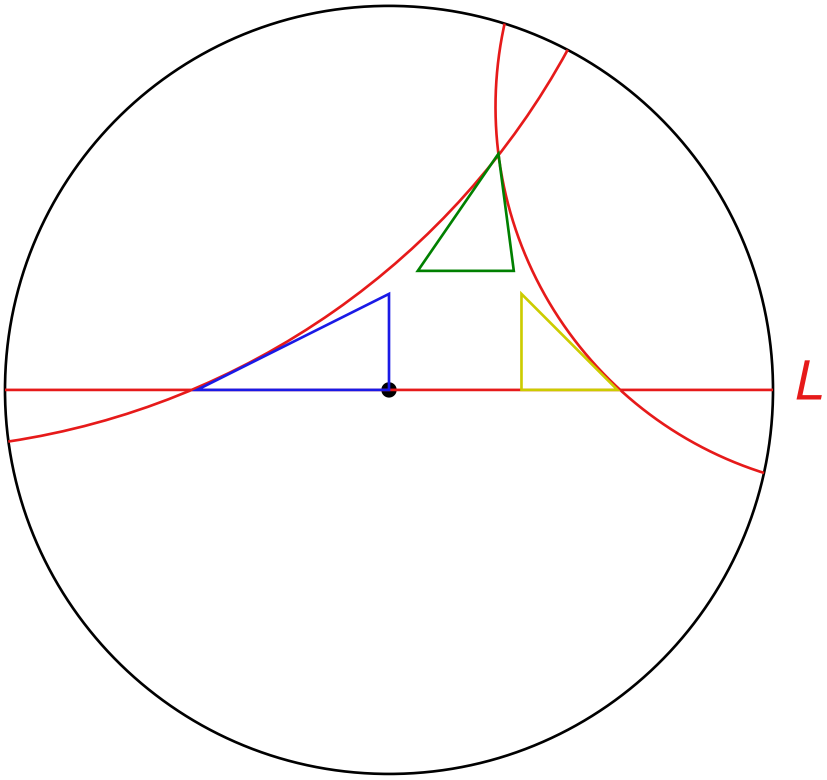

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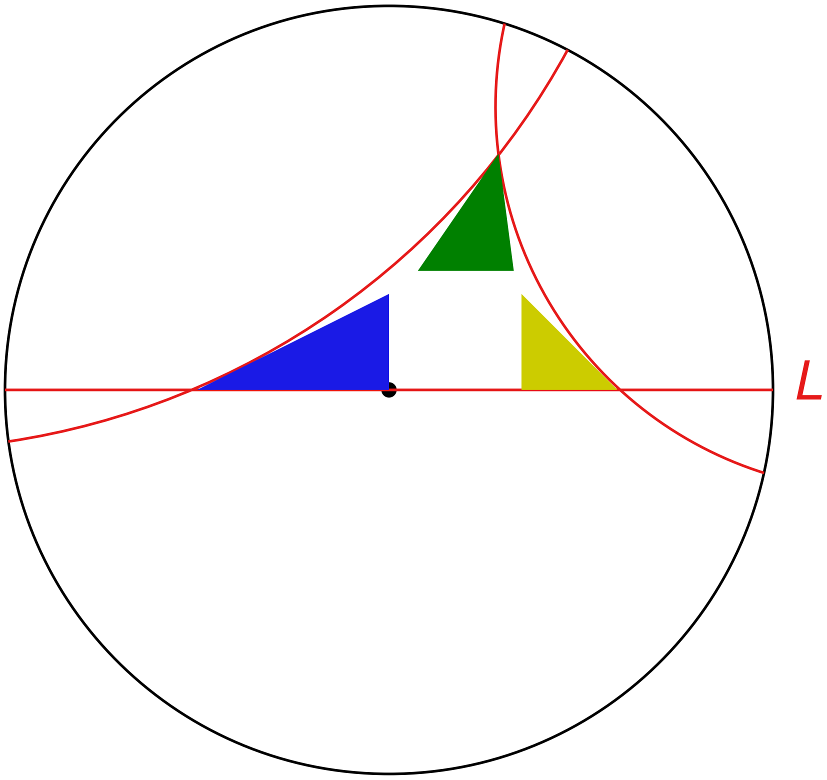

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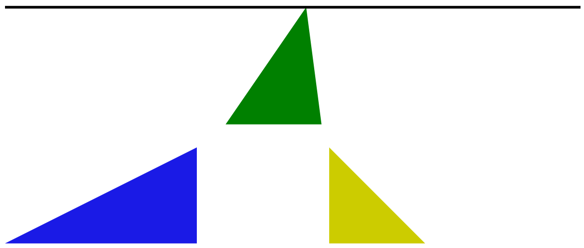

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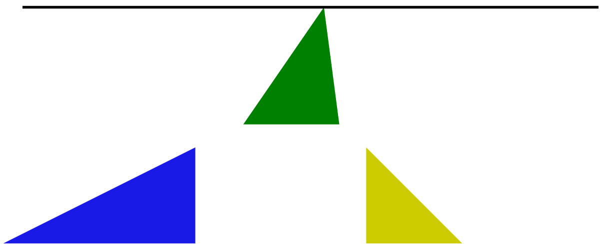

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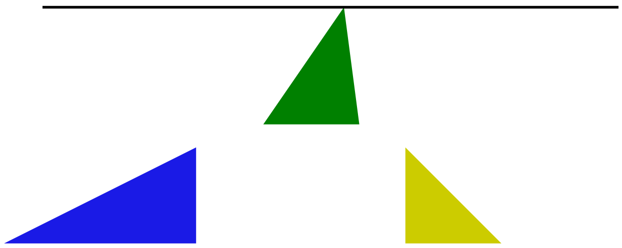

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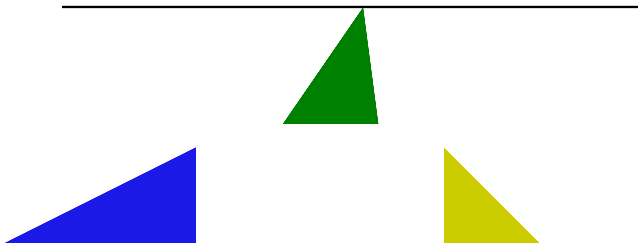

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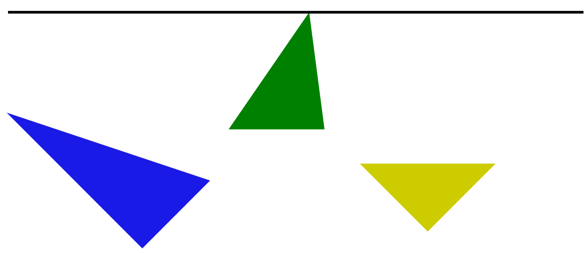

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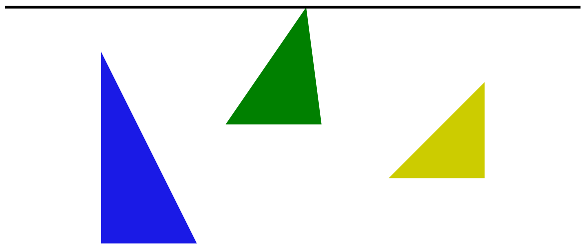

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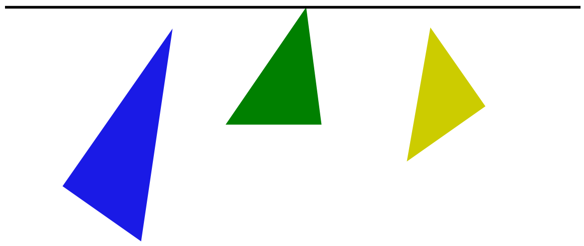

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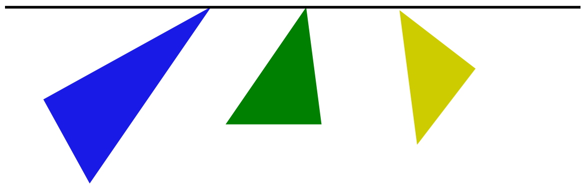

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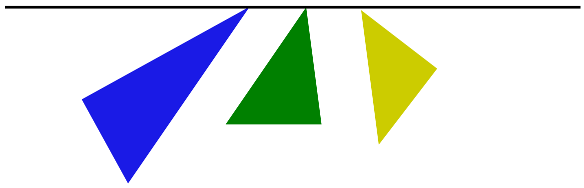

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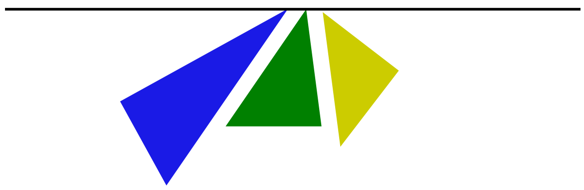

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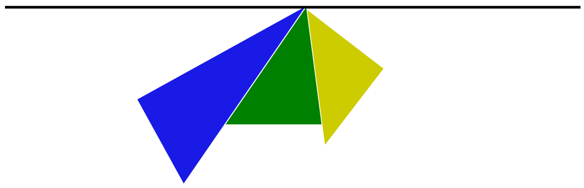
Proof: It is easily seen that these 3 angles sum to less than 180 degrees.



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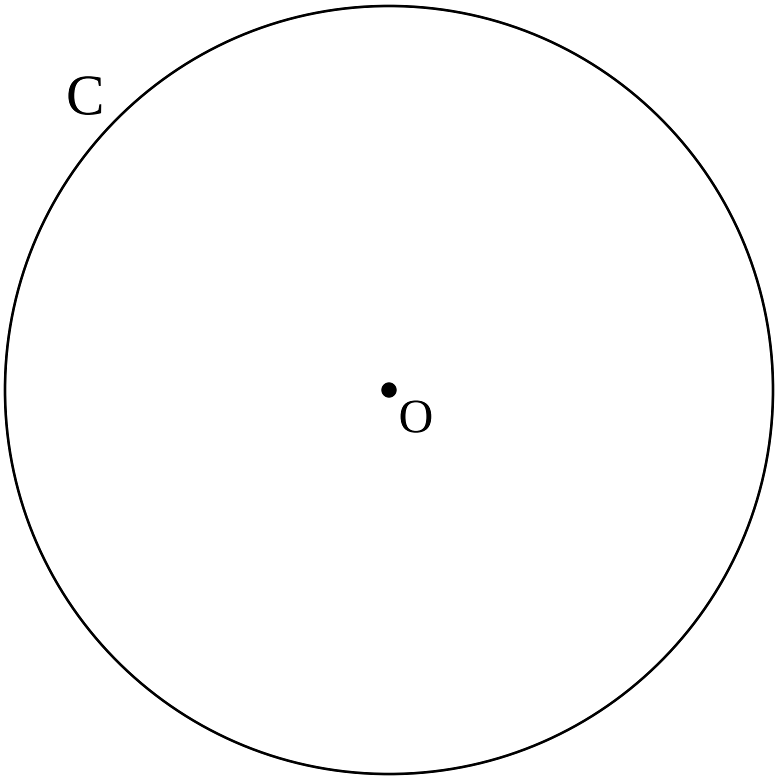
Using the Poincare Disk model to prove.



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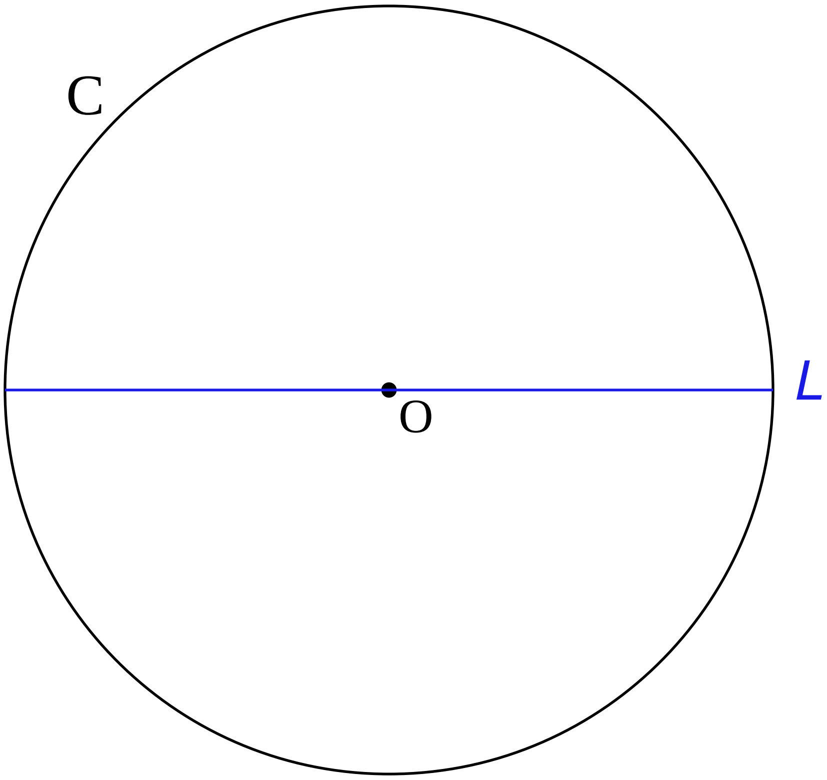

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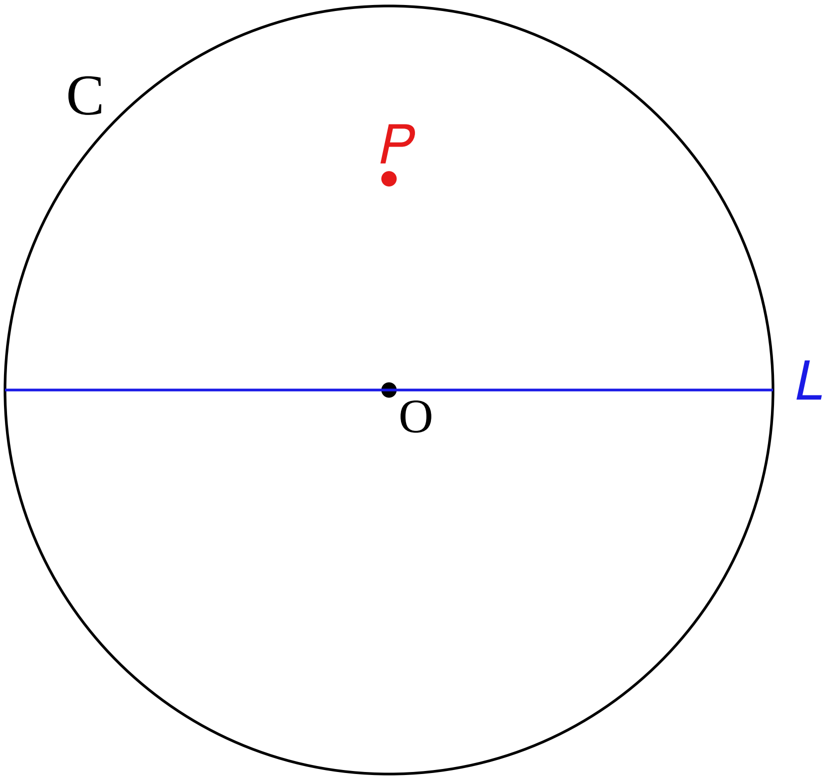

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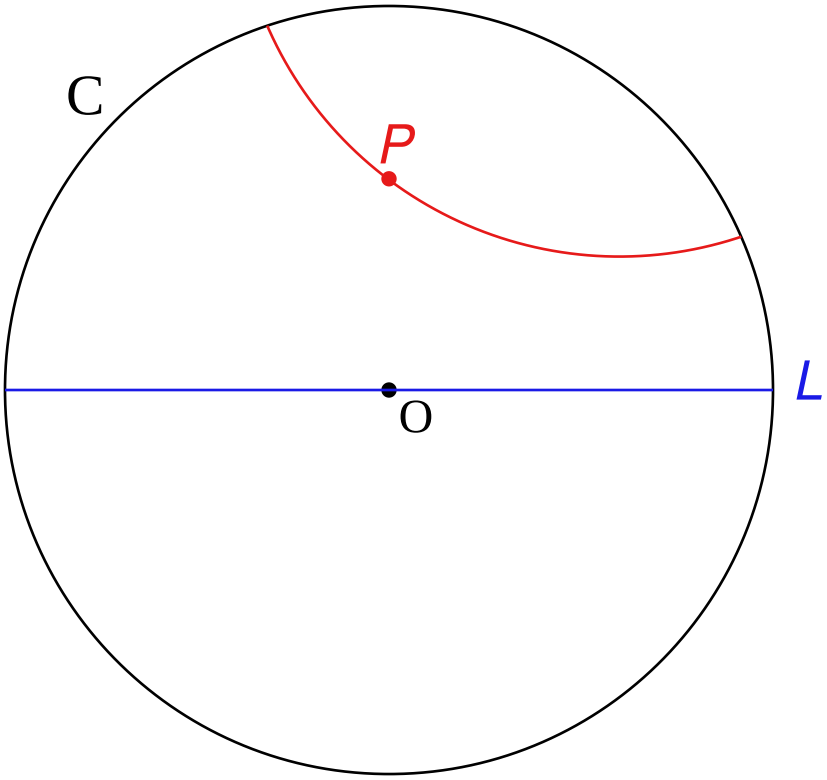

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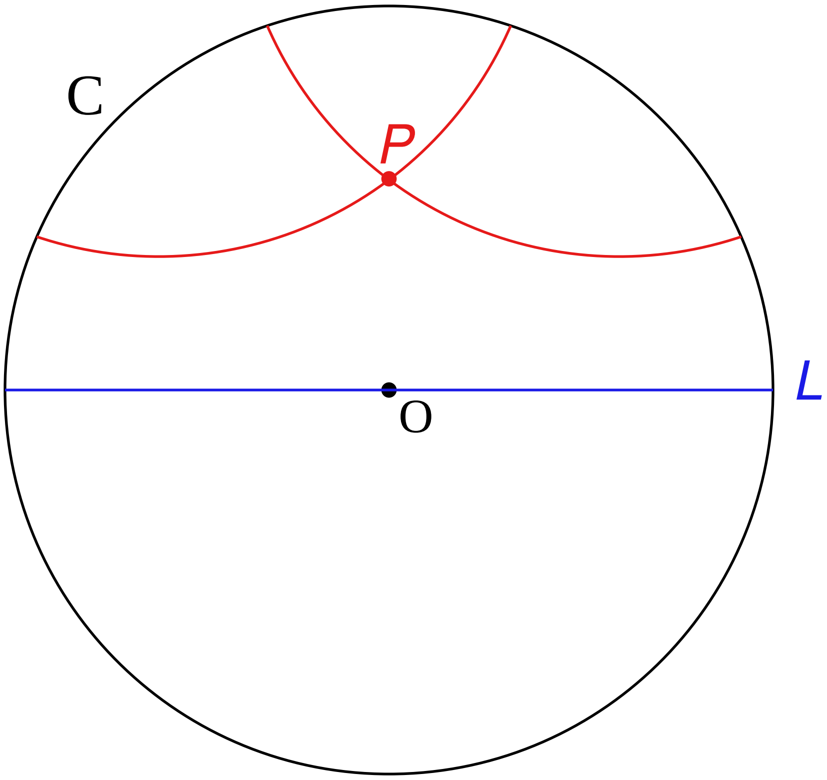

Proof:

The two P-lines passing through P do not intersect the P-line, L, inside the circle, therefore , they are parallel to L.

In the Poincare Disk model, it is of no concern what the lines do outside of the circle.

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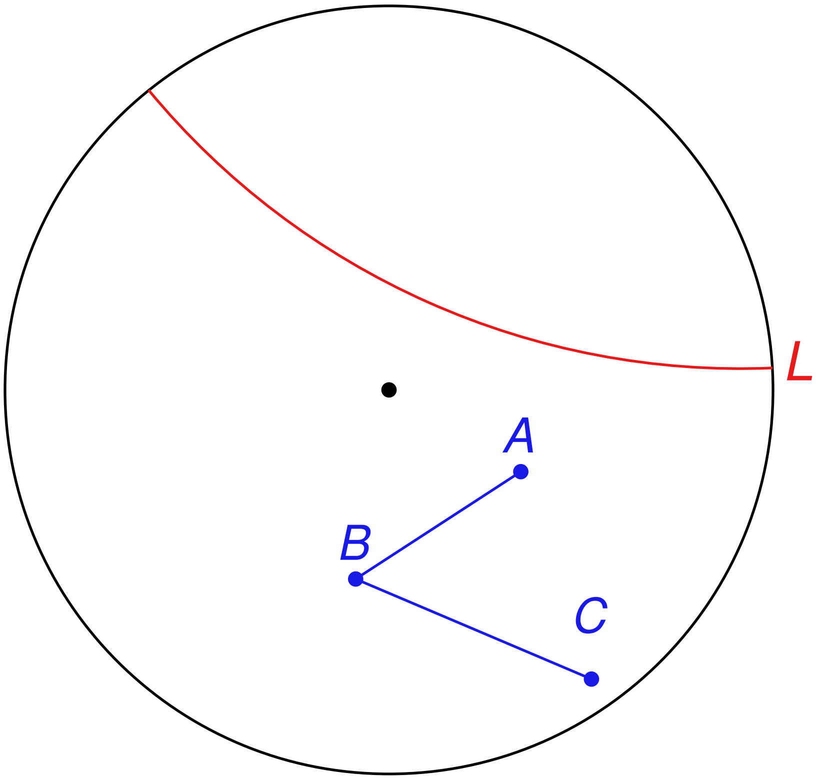

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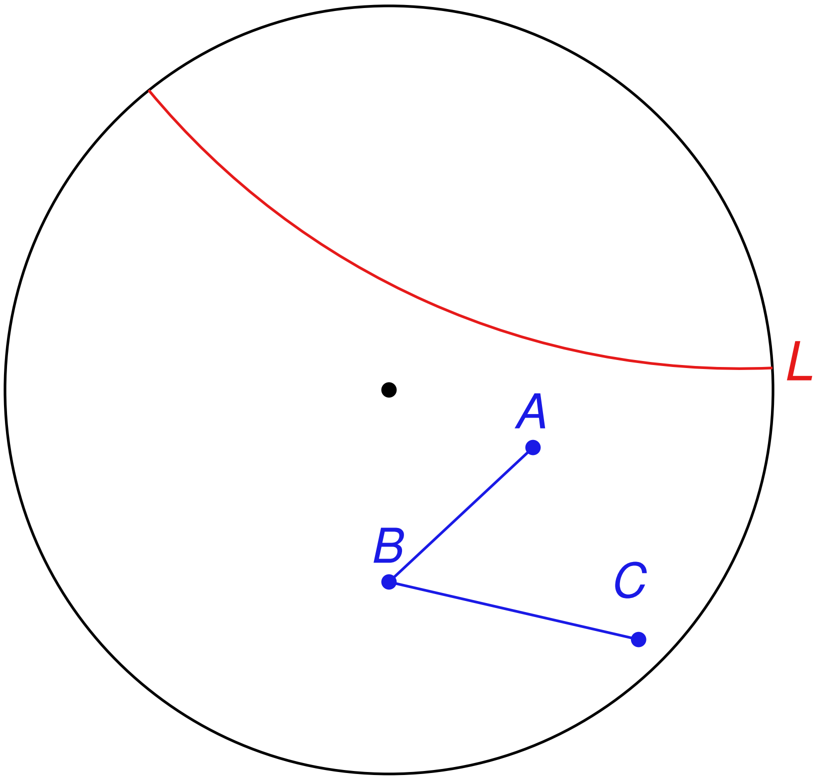

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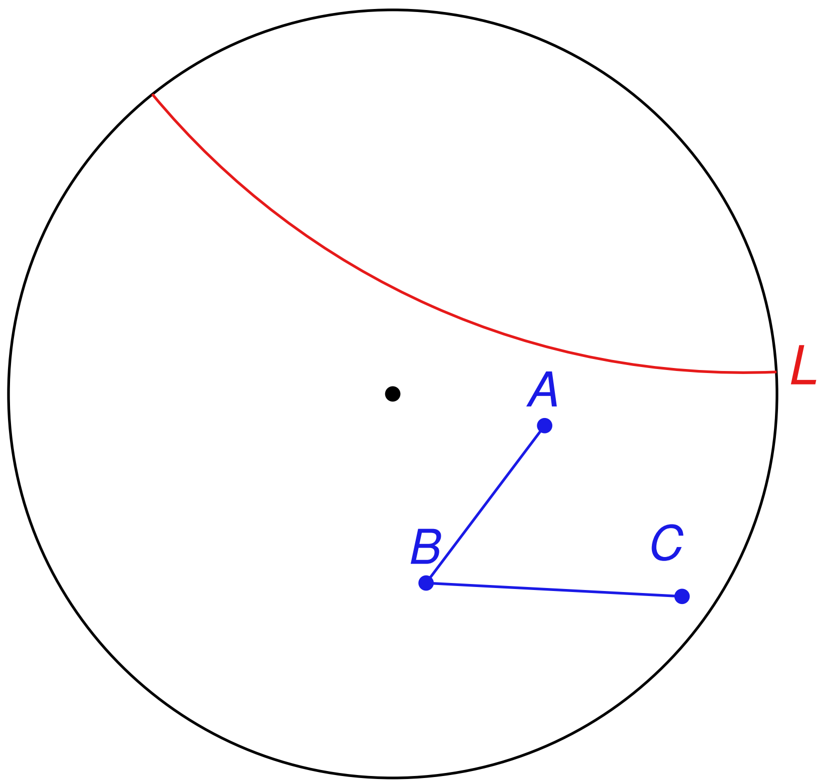

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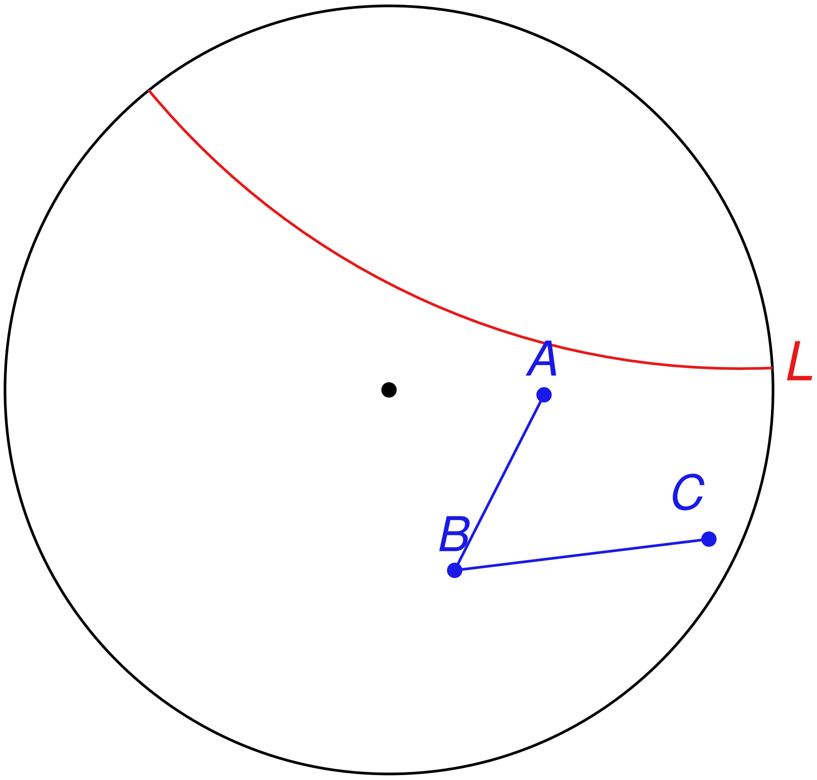

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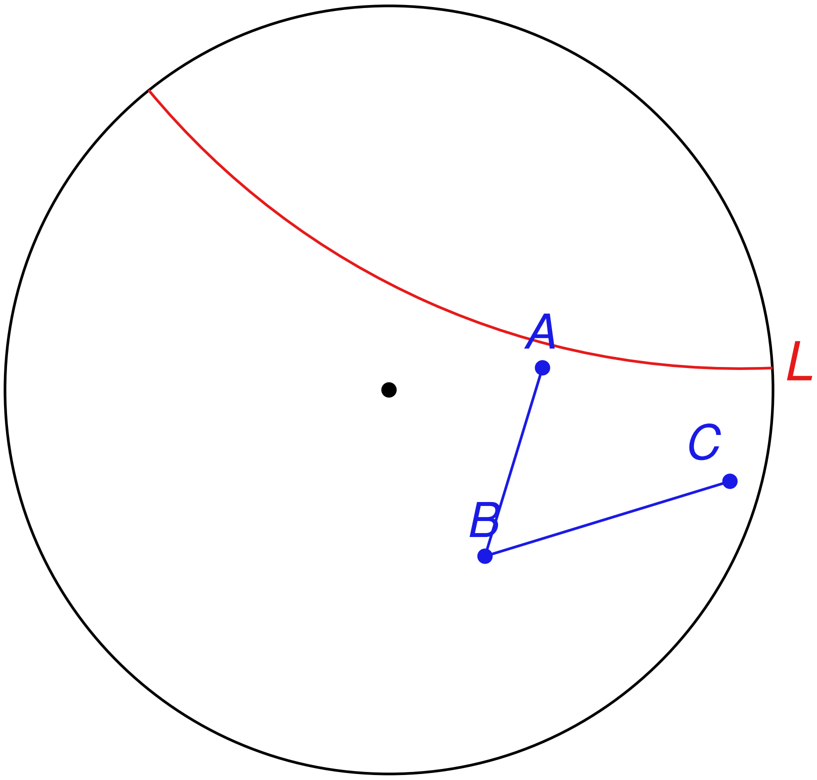

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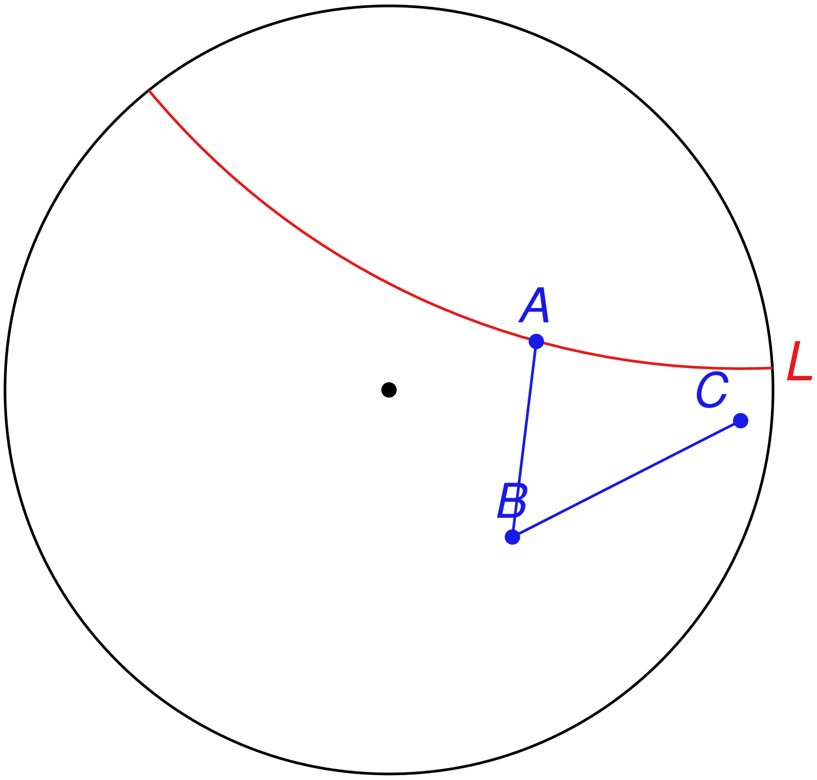

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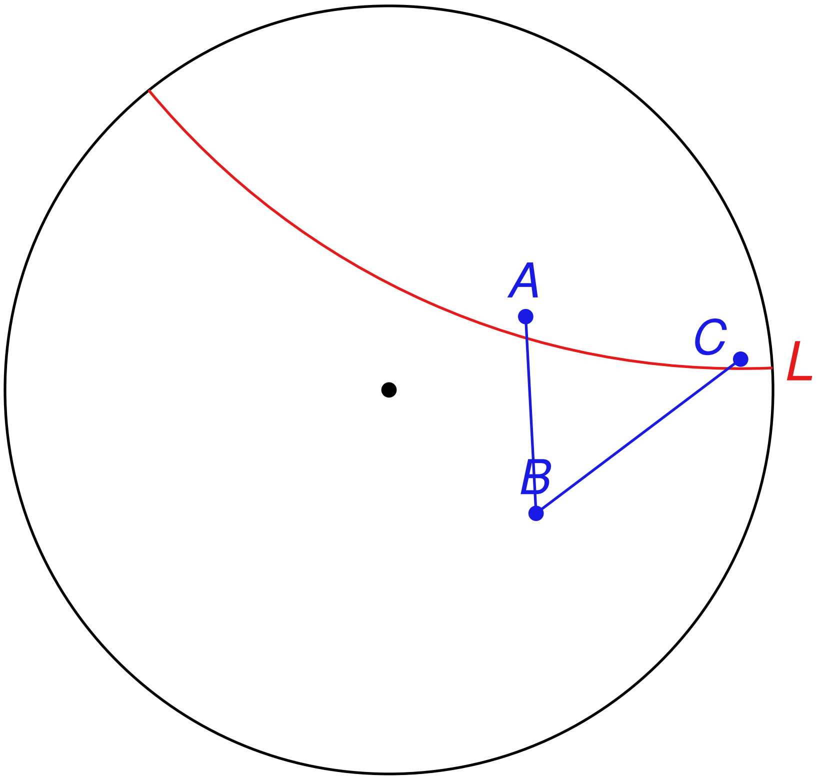

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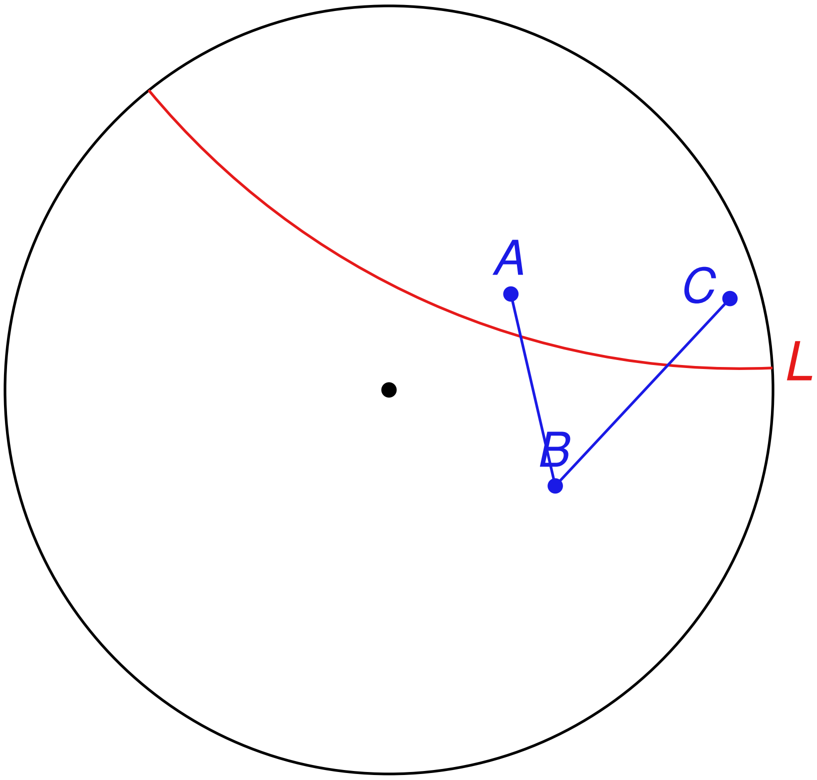

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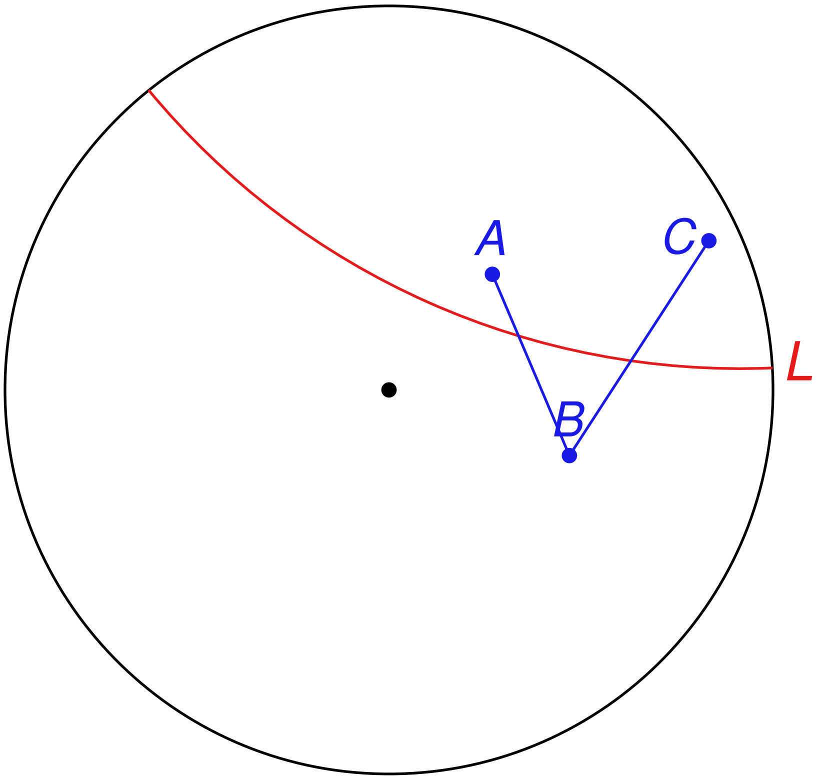

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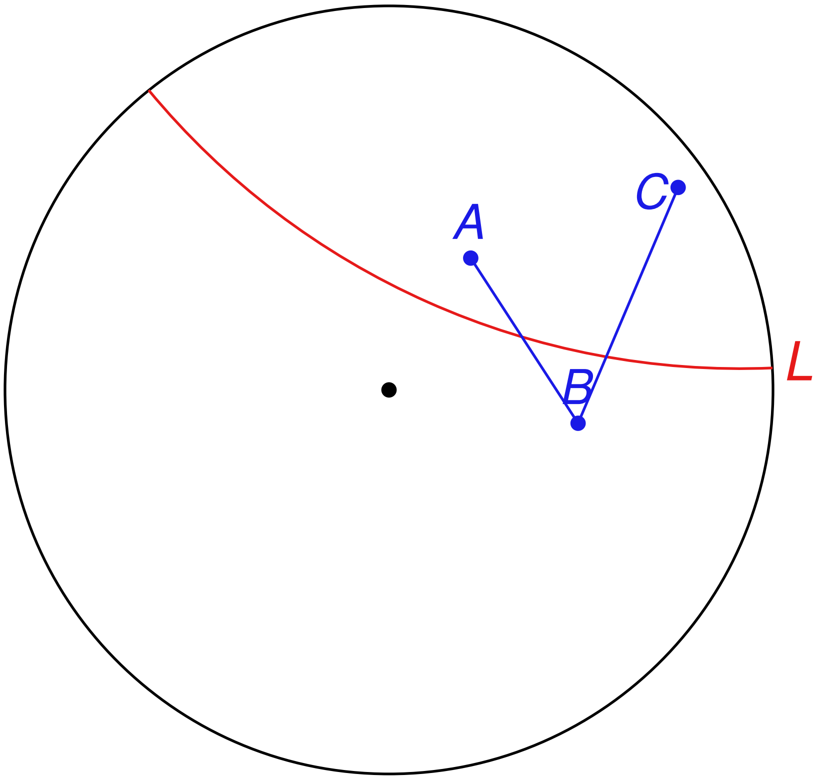

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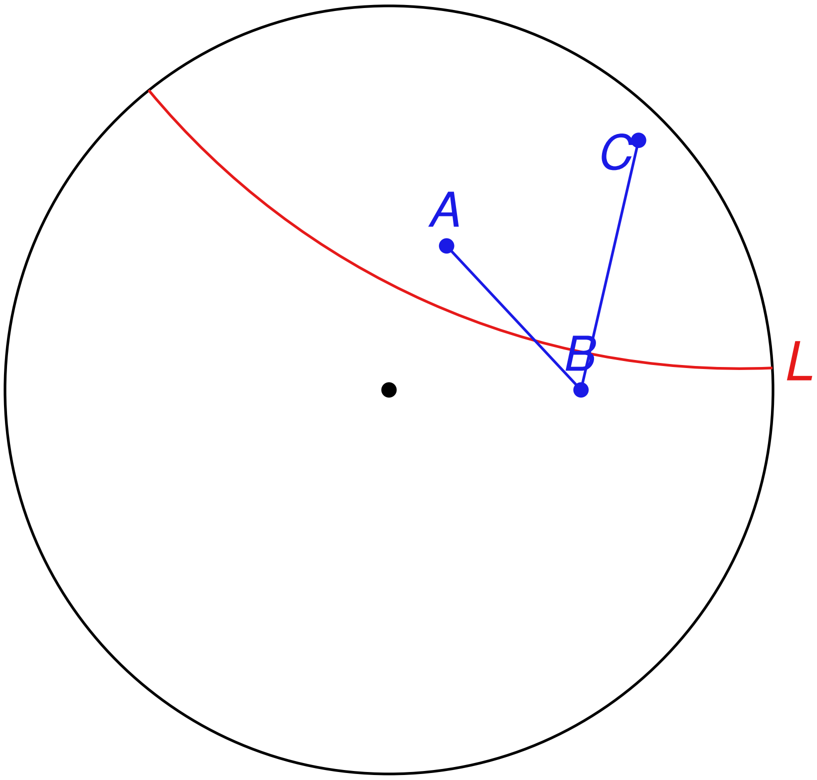

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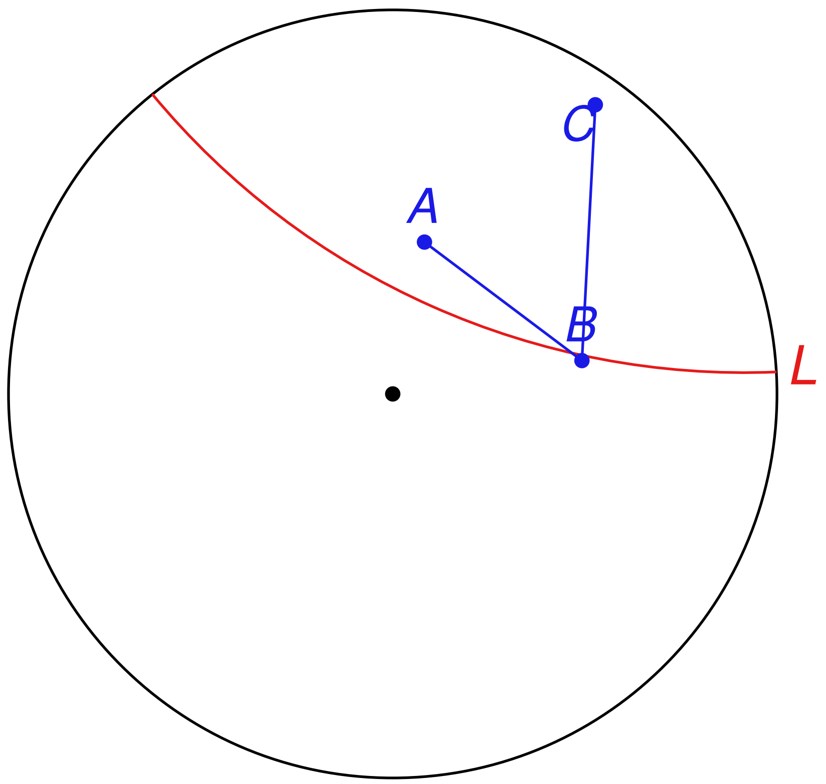

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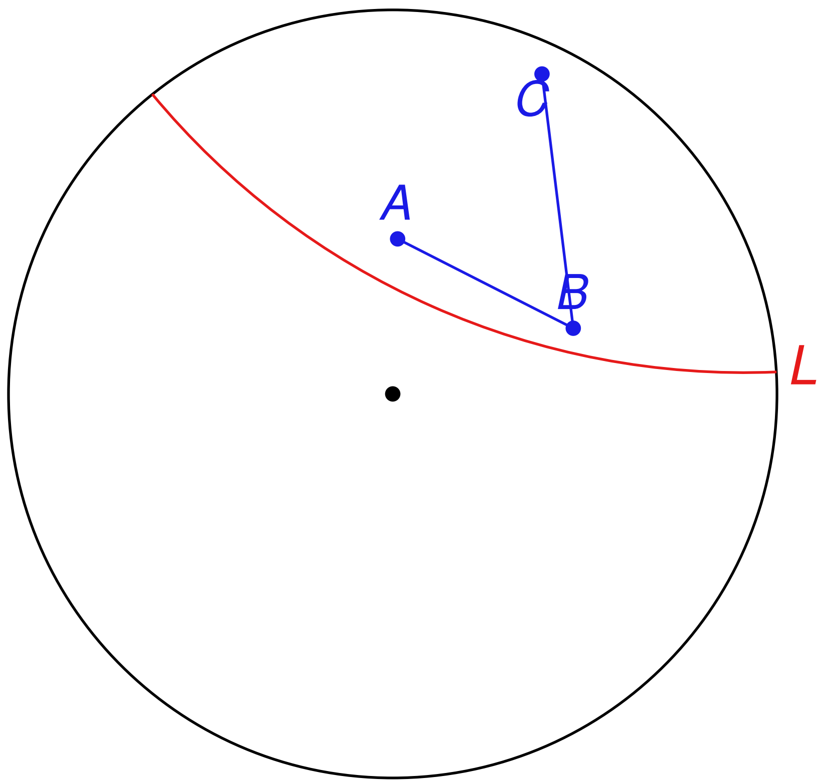

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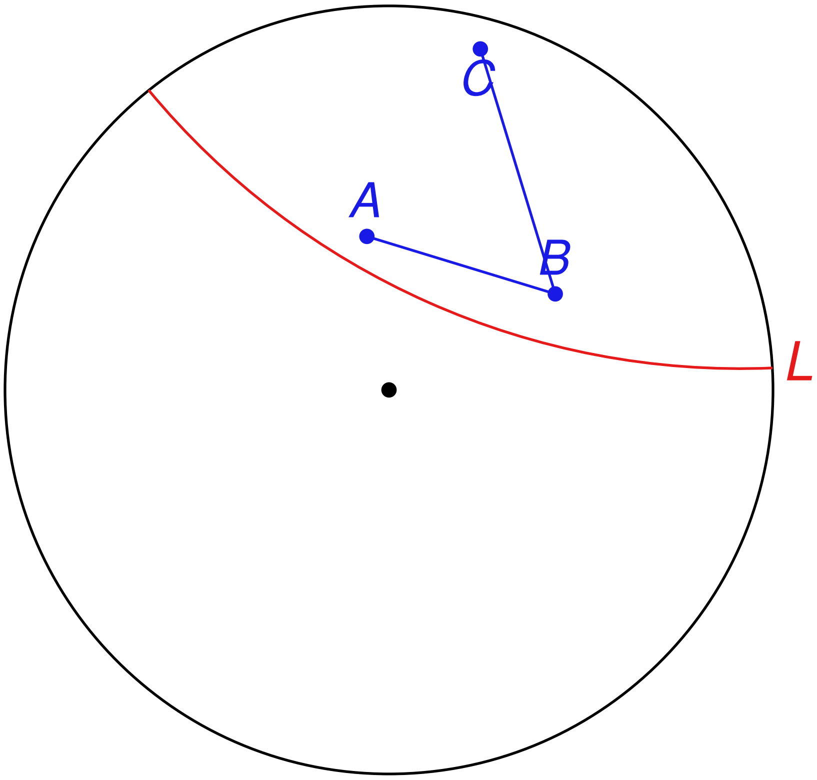

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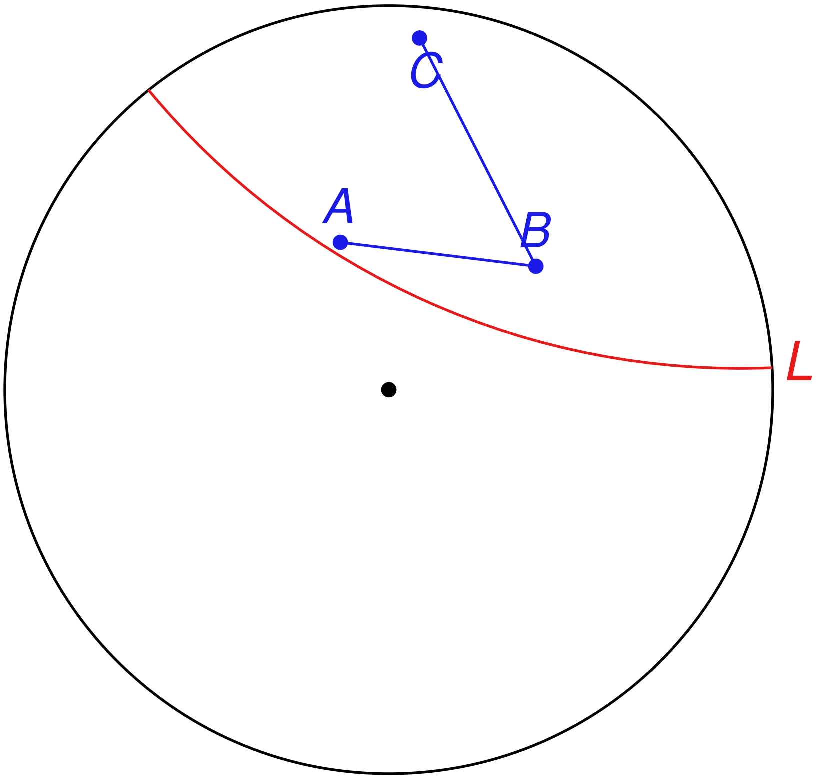

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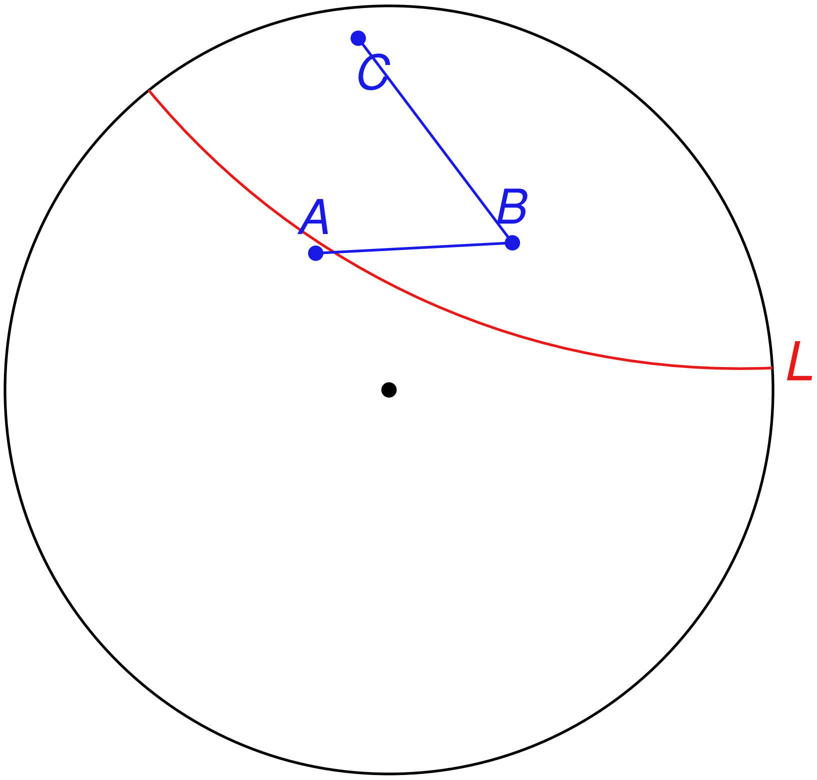

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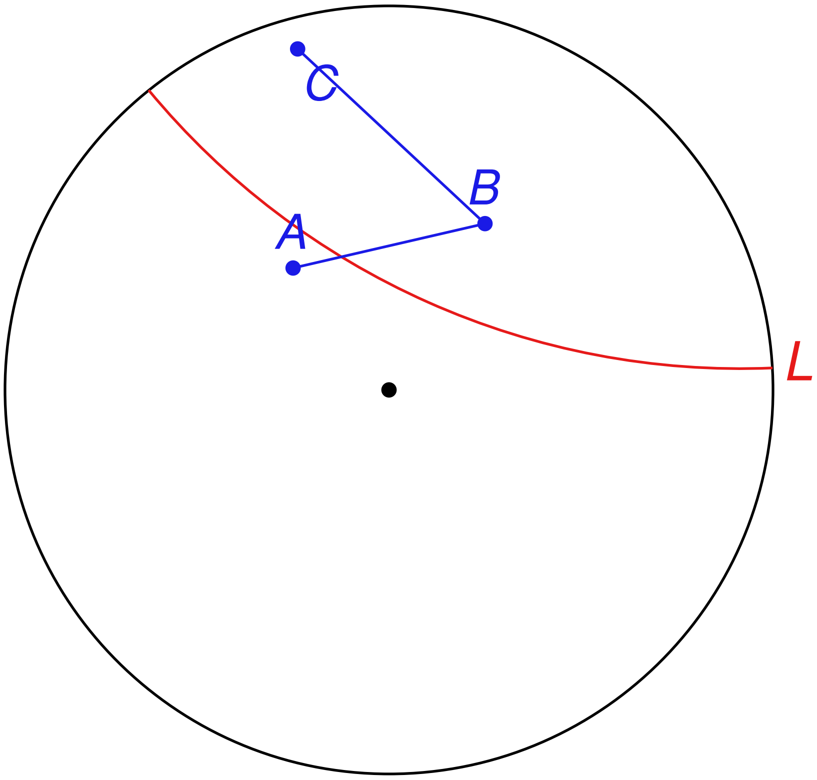

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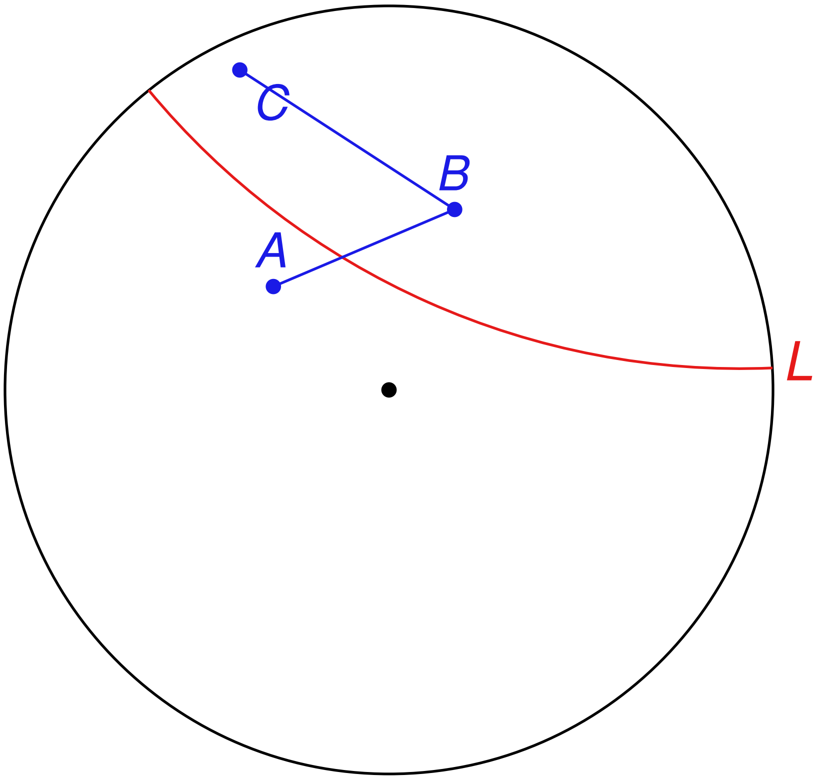

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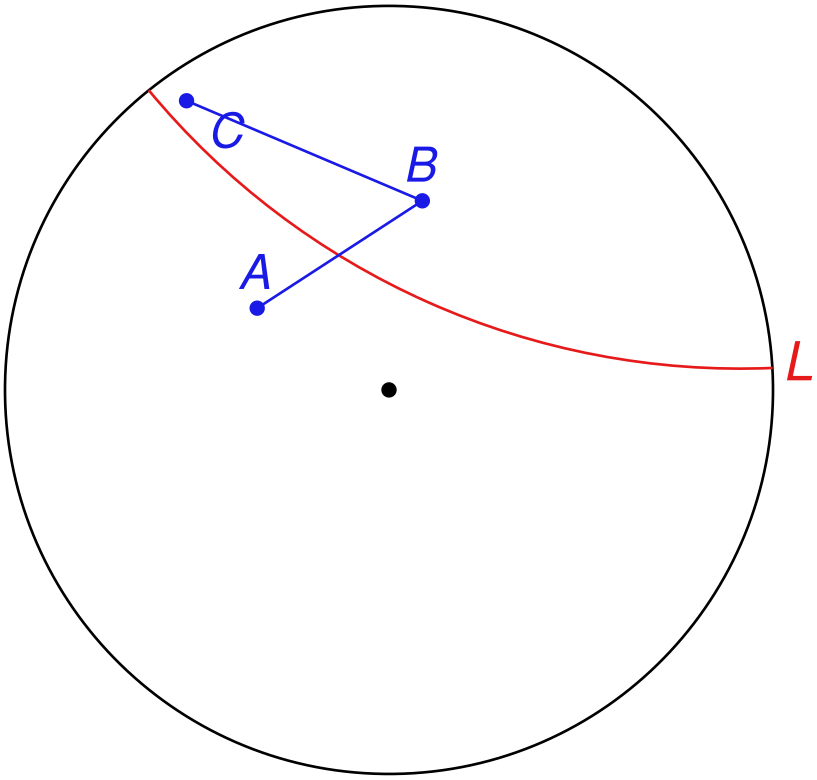

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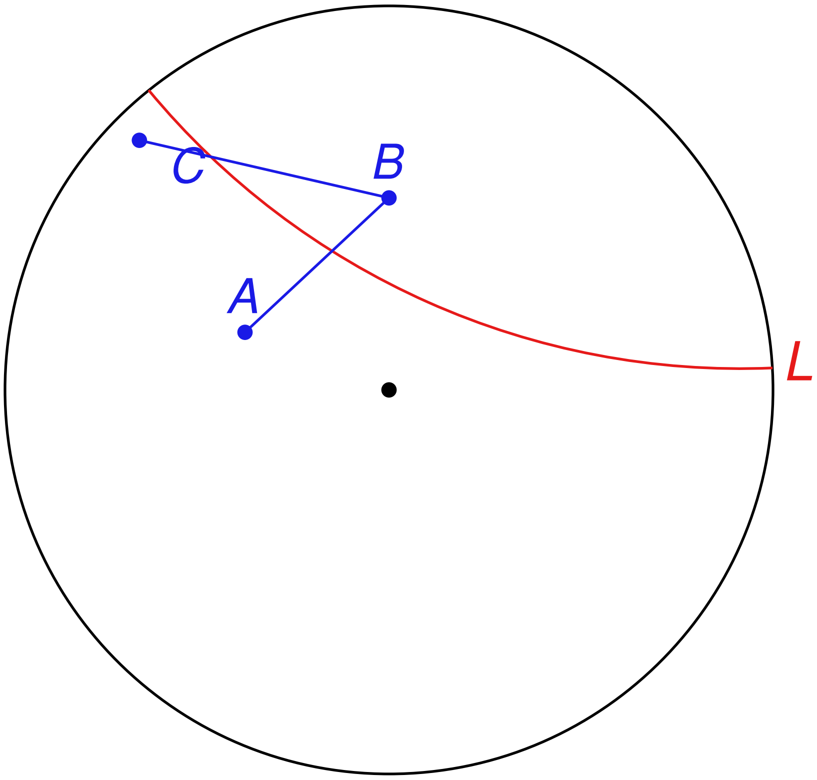

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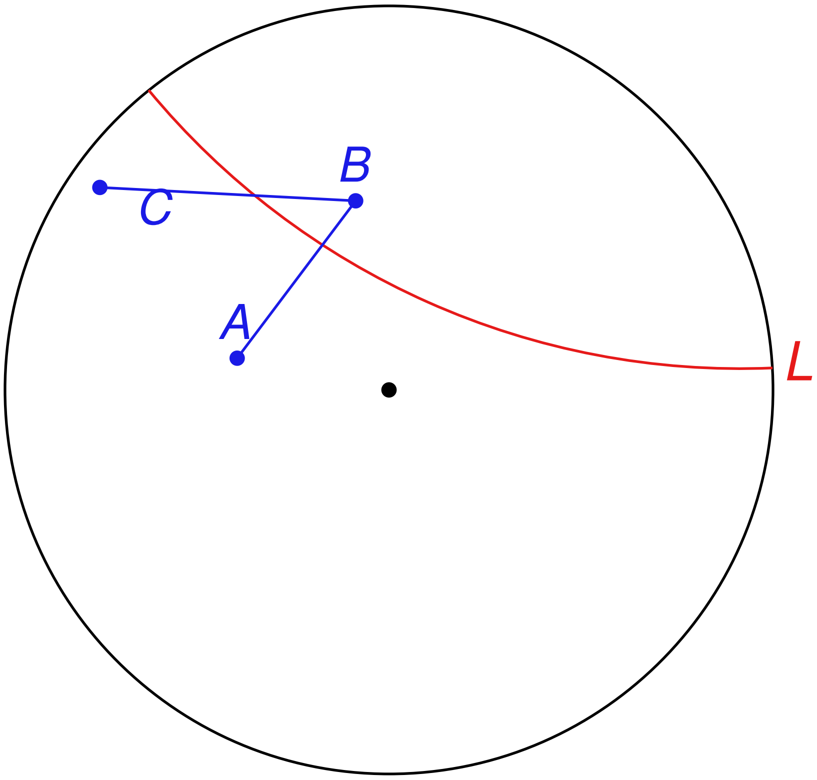

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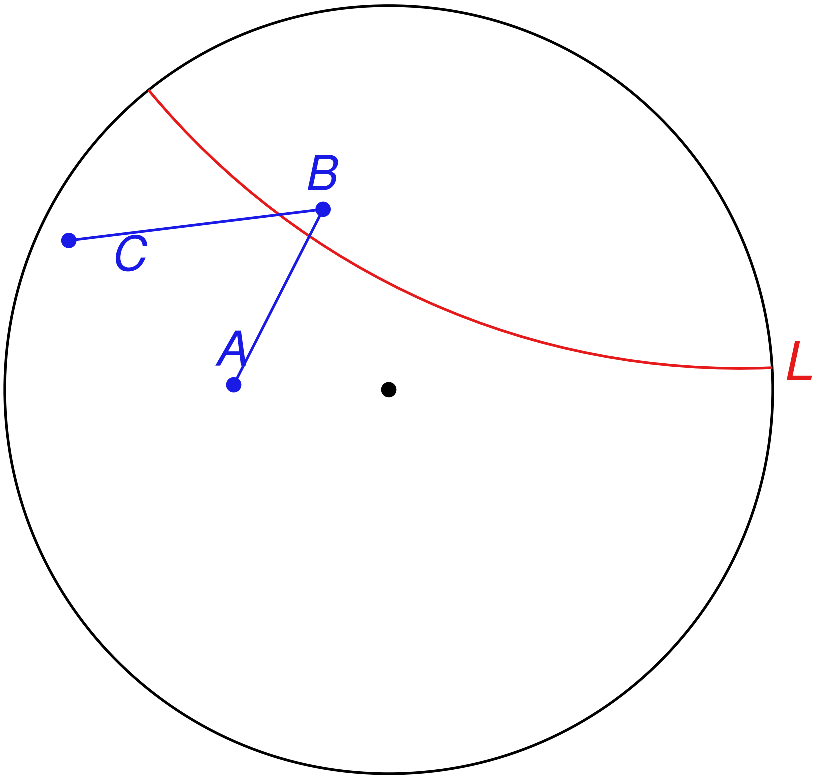

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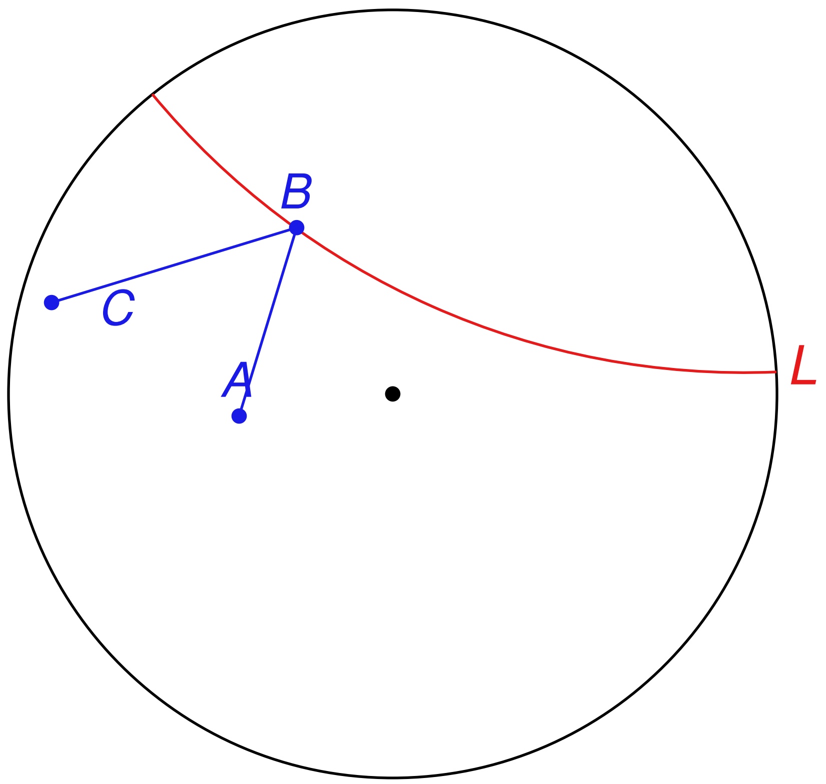

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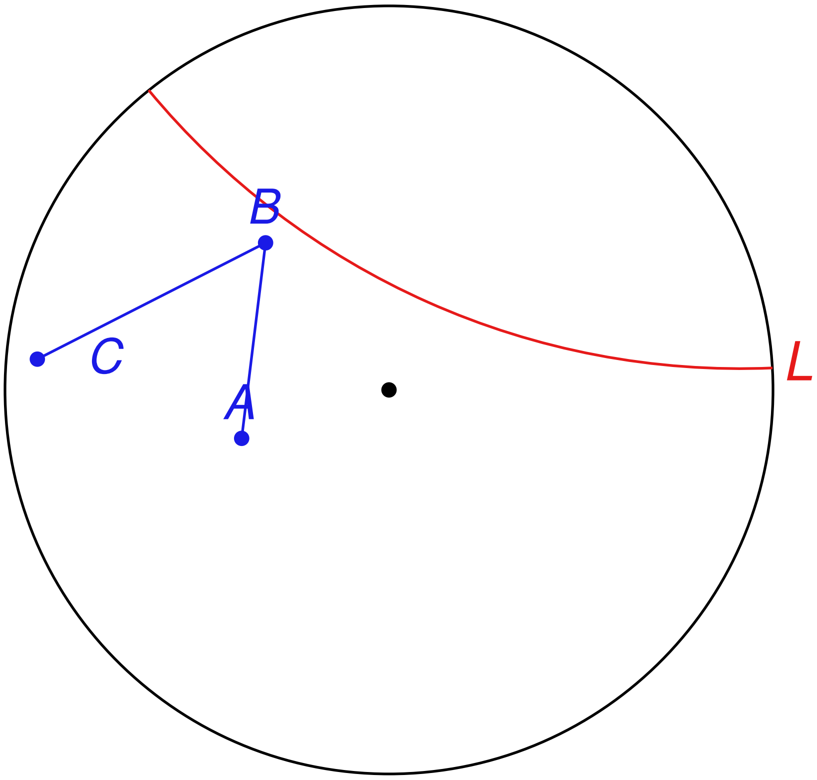

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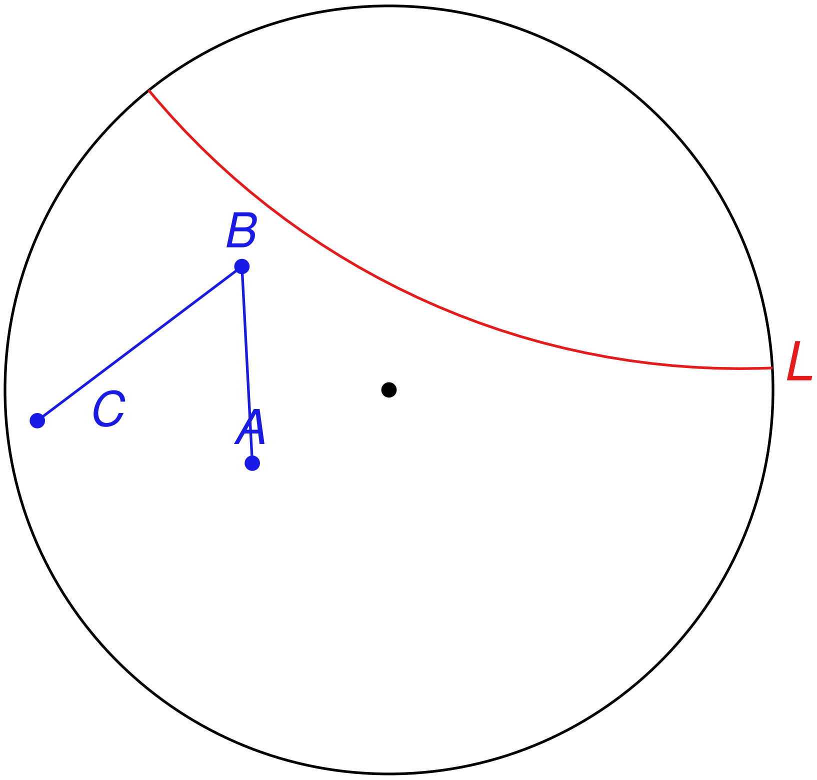
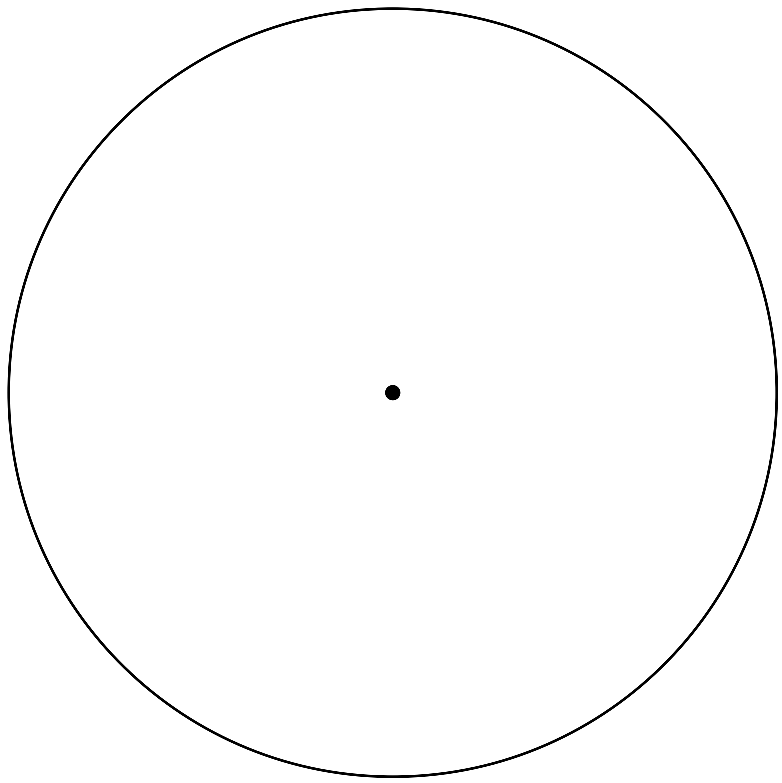


Visualizing the Hyperbolic Plane

The figure below, a circle in the Euclidean plane, will be used to visualize a coordinate system in the Poincare Disk for Hyperbolic Geometry.



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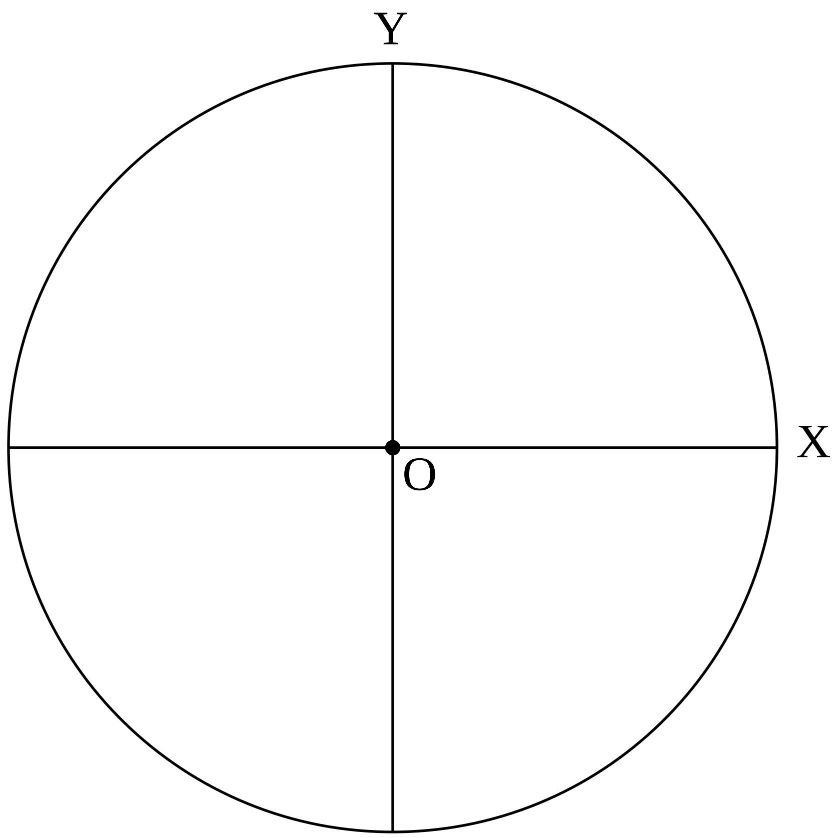

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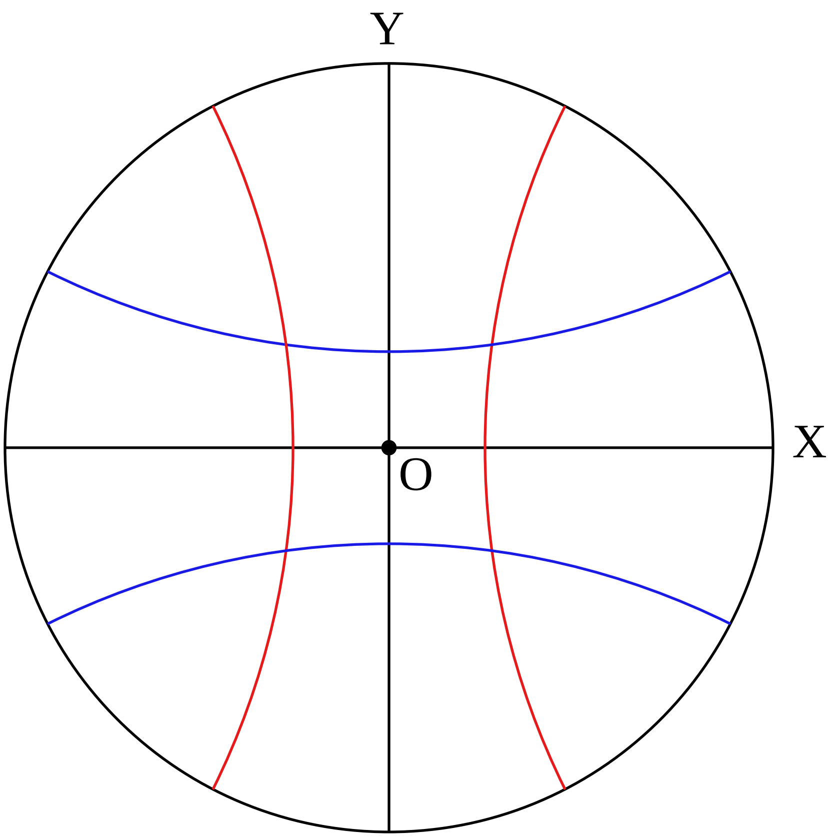
We will now draw 24 infinite lines in the disk used to define the coordinate system.



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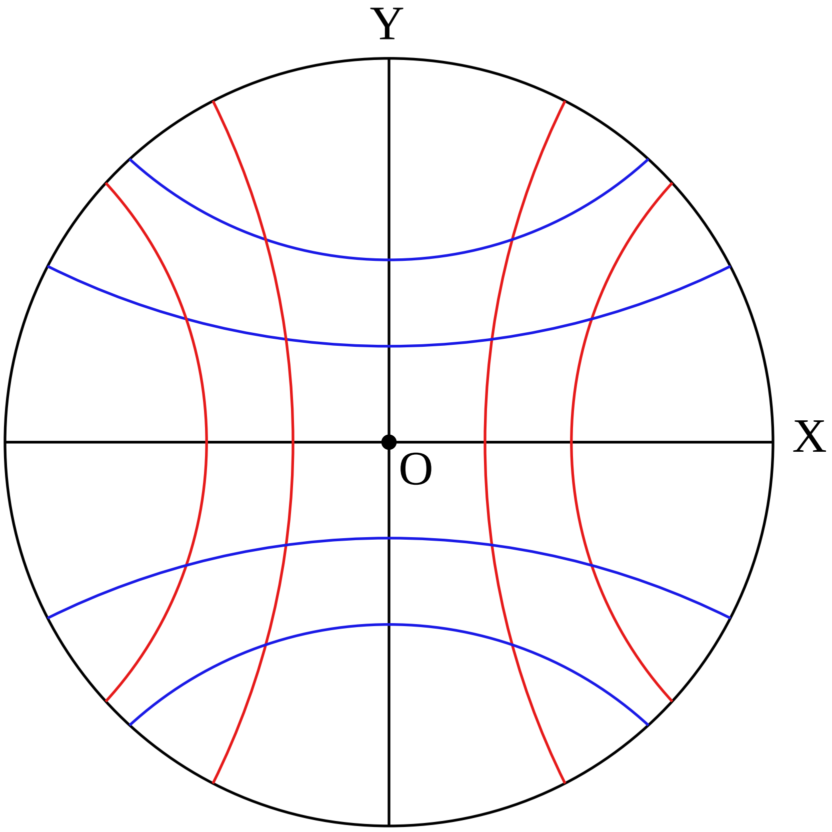

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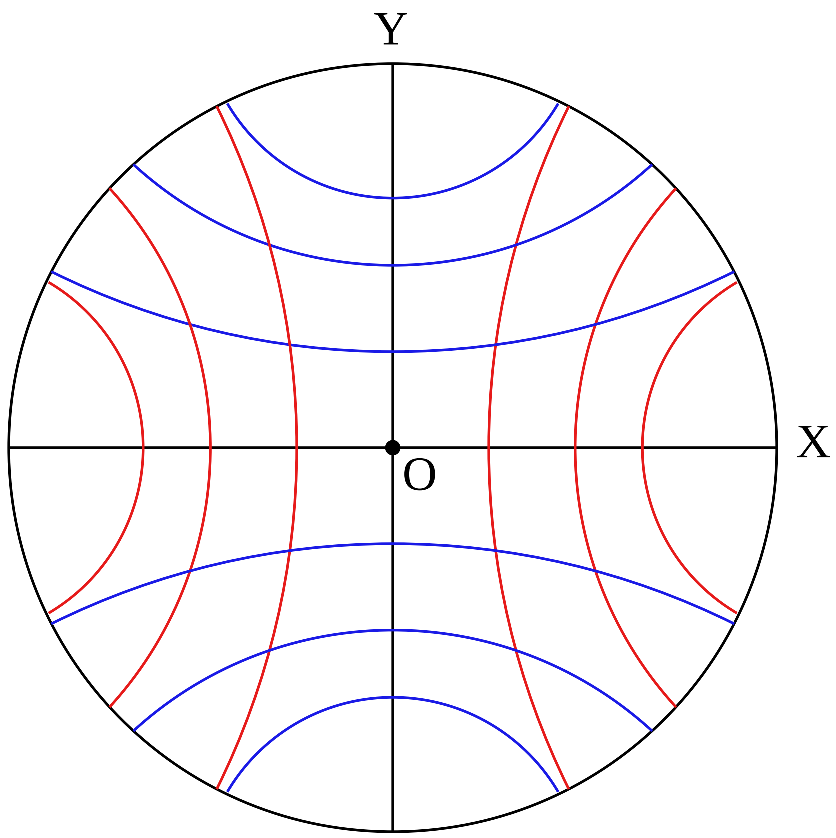

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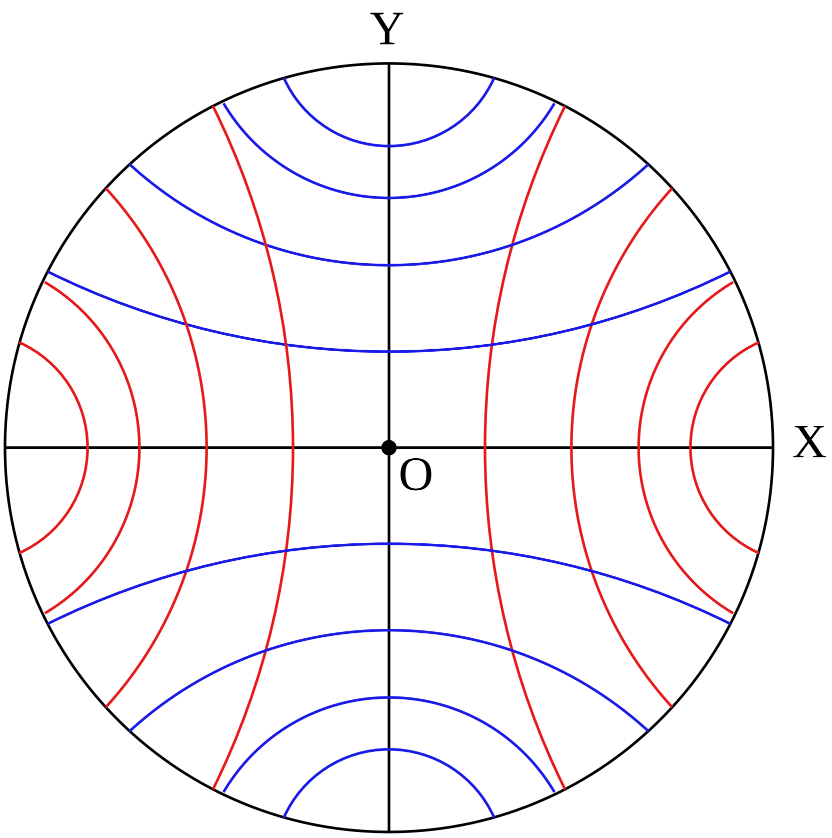

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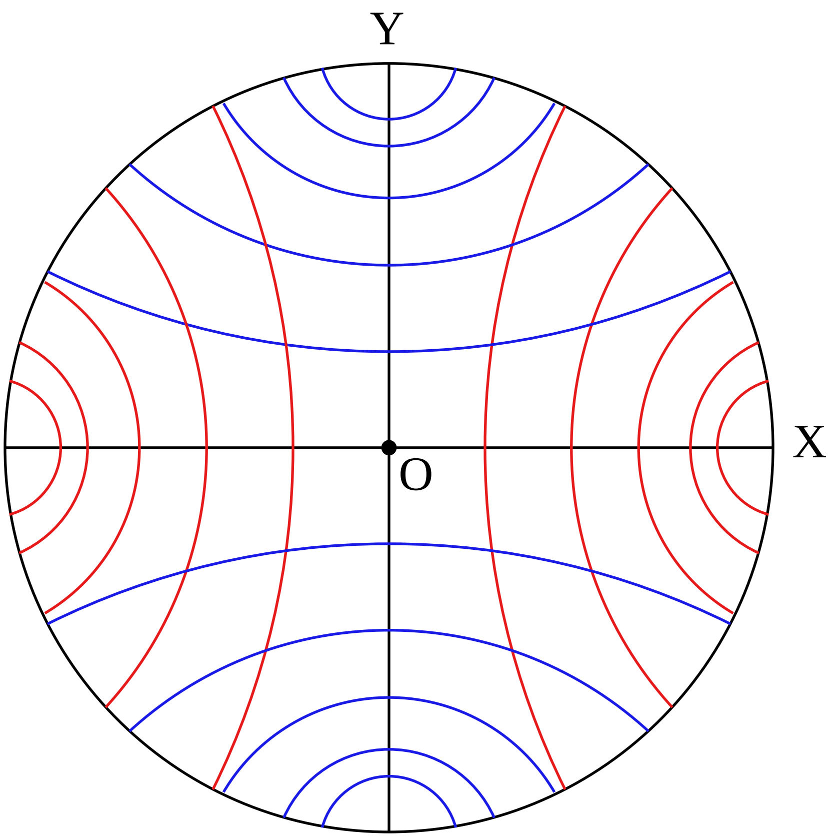

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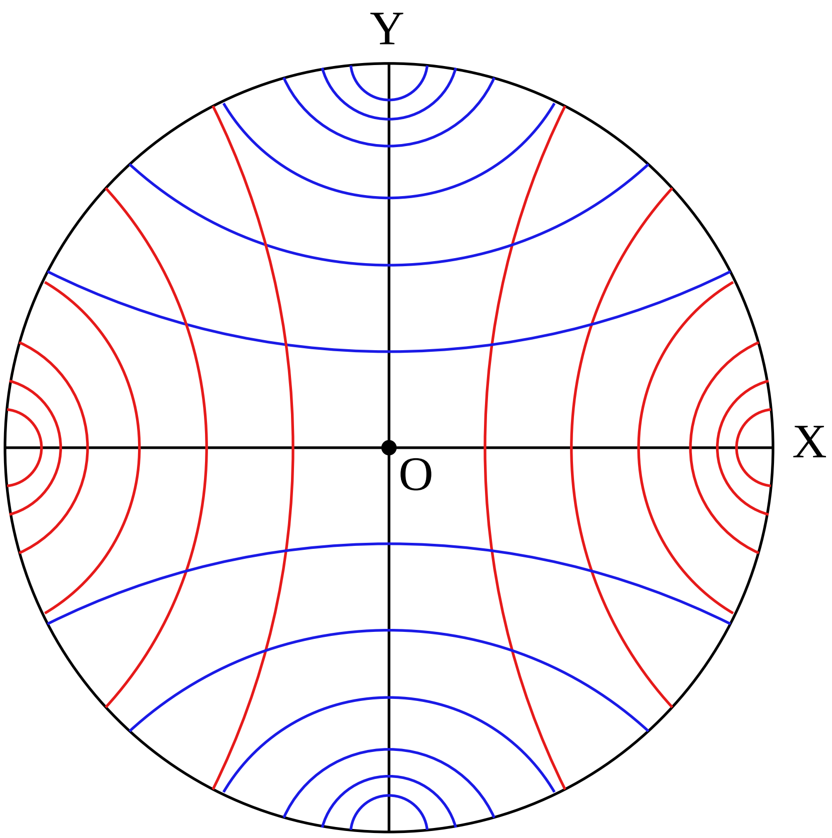

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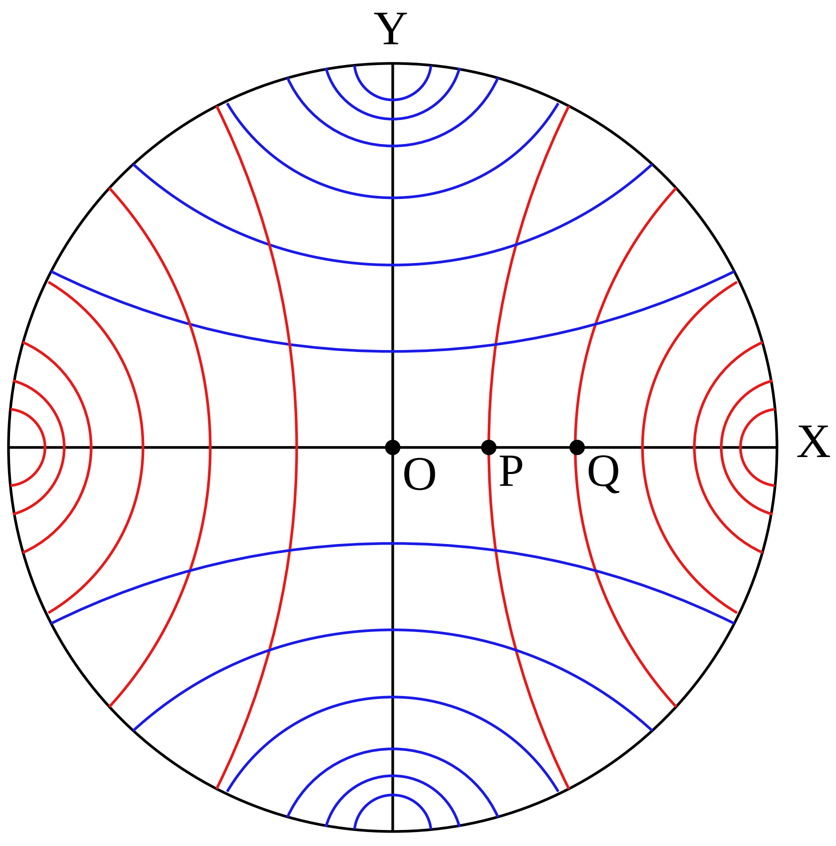

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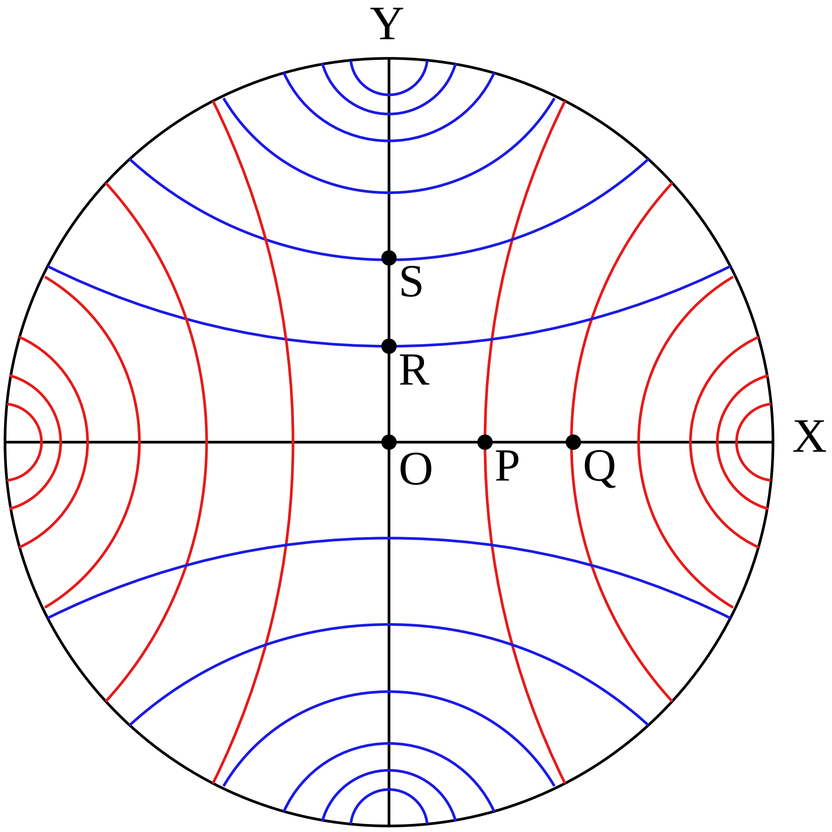
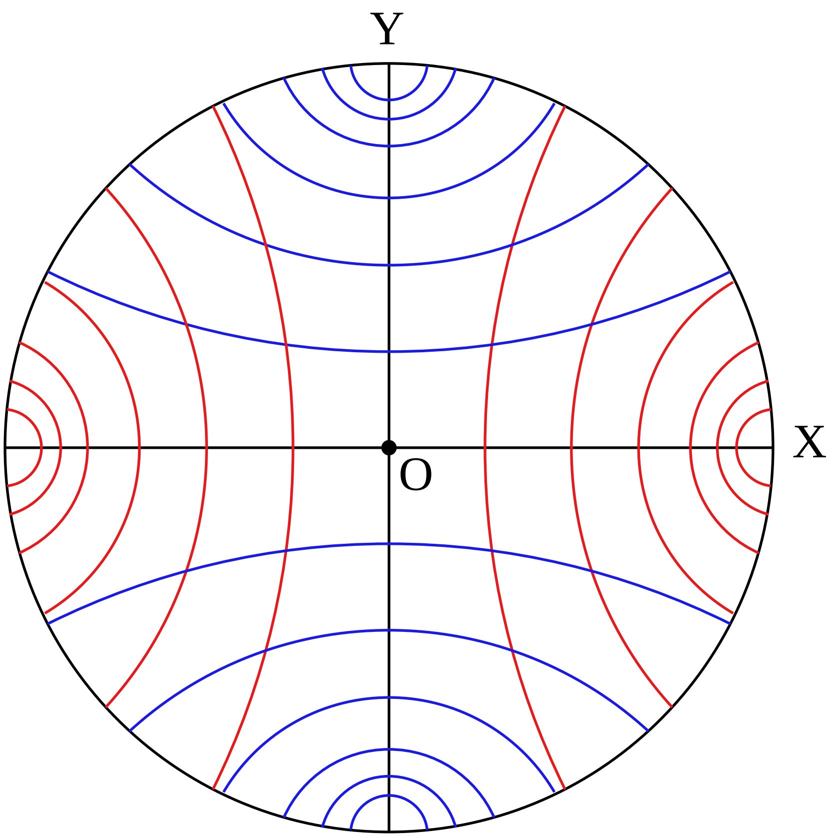
Each axis is intersected by perpendicular lines at intervals of 0.5 units. Therefore, the length of the segment OP=0.5 and the segment PQ=0.5 as well.

It follows that OR=RS=0.5 as well on the y-axis.



Visualizing the Hyperbolic Plane

In Euclidean geometry, the coordinates of a point (x,y) are where x is the perpendicular distance from the point to the x-axis and y is the perpendicular distance from the point to the y-axis. This same definition can be applied to Hyperbolic plane as well.



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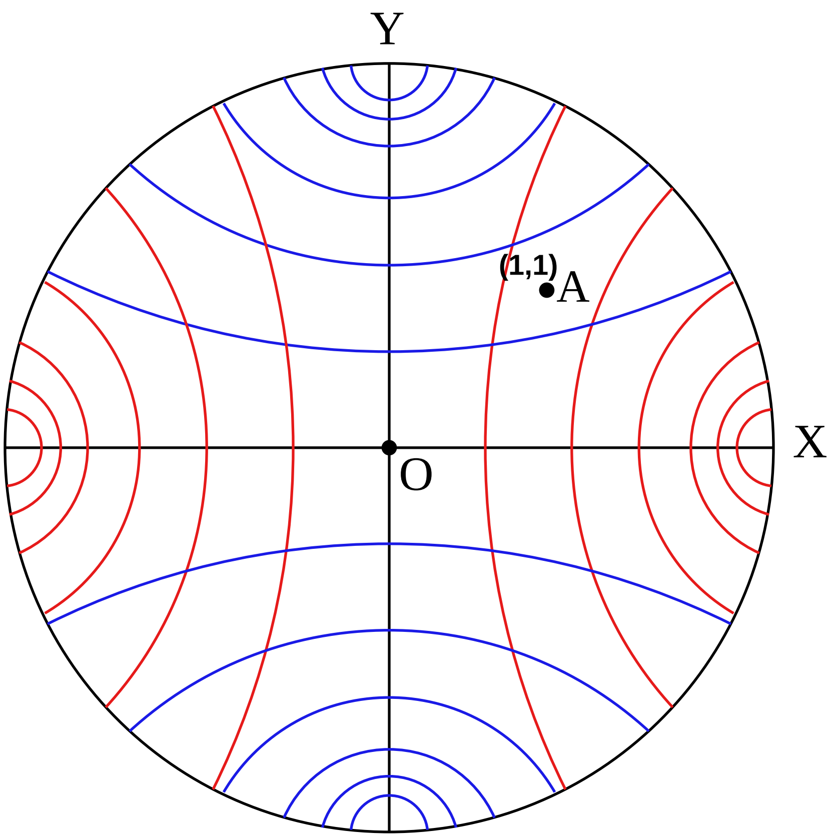

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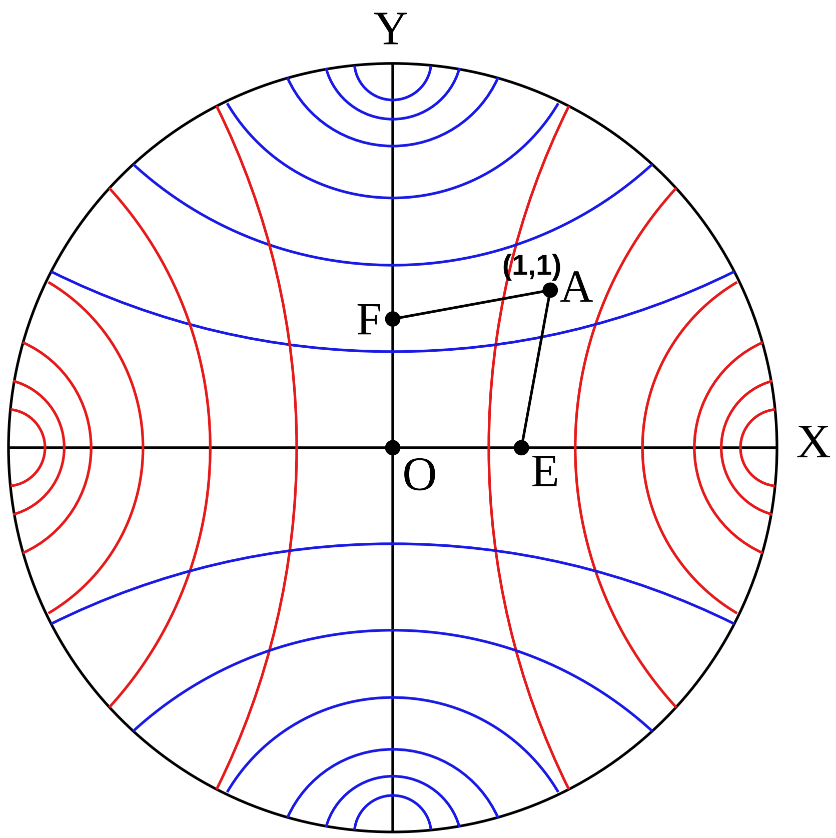
The length of the perpindicular from A to the x-axis is 1.0 units and the length of the perpindicular from A to the y-axis is 1.0 units as well.



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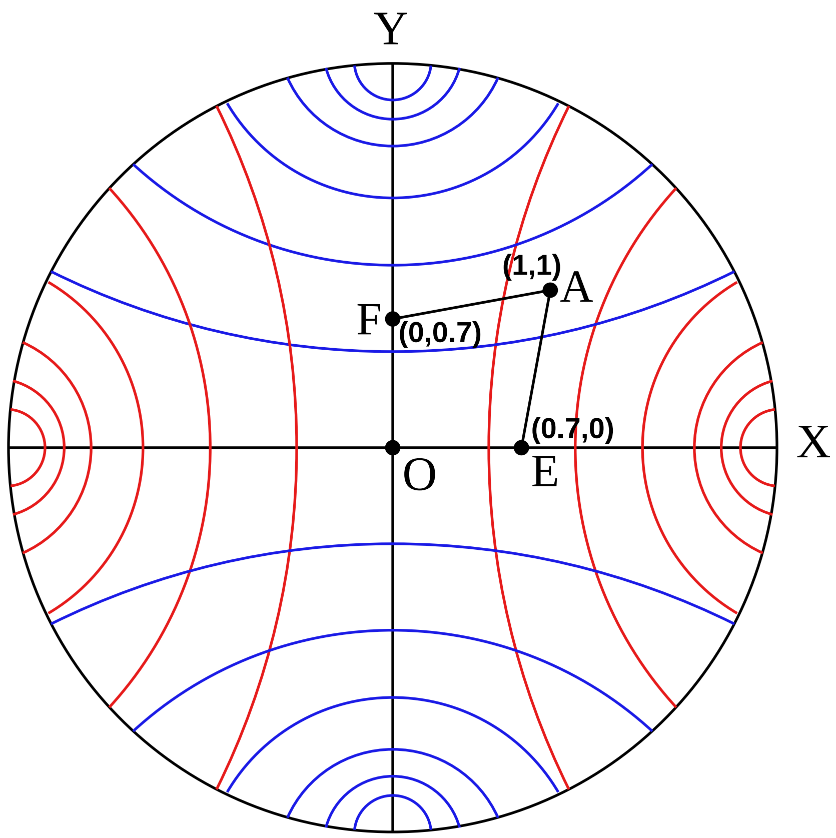
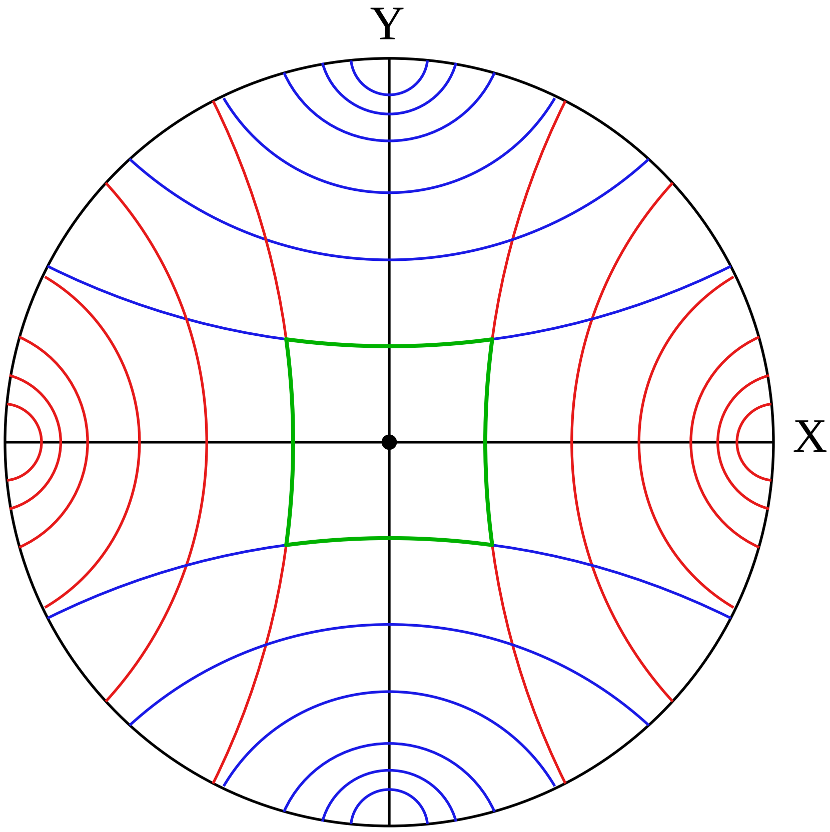


However, the distance from the origin, O, where the perpendicular intersects the x-axis, Point E, is only 0.7 units, as well as for the y-axis at Point F.

Hyperbolic Square

An example of a square in Hyperbolic Geometry.



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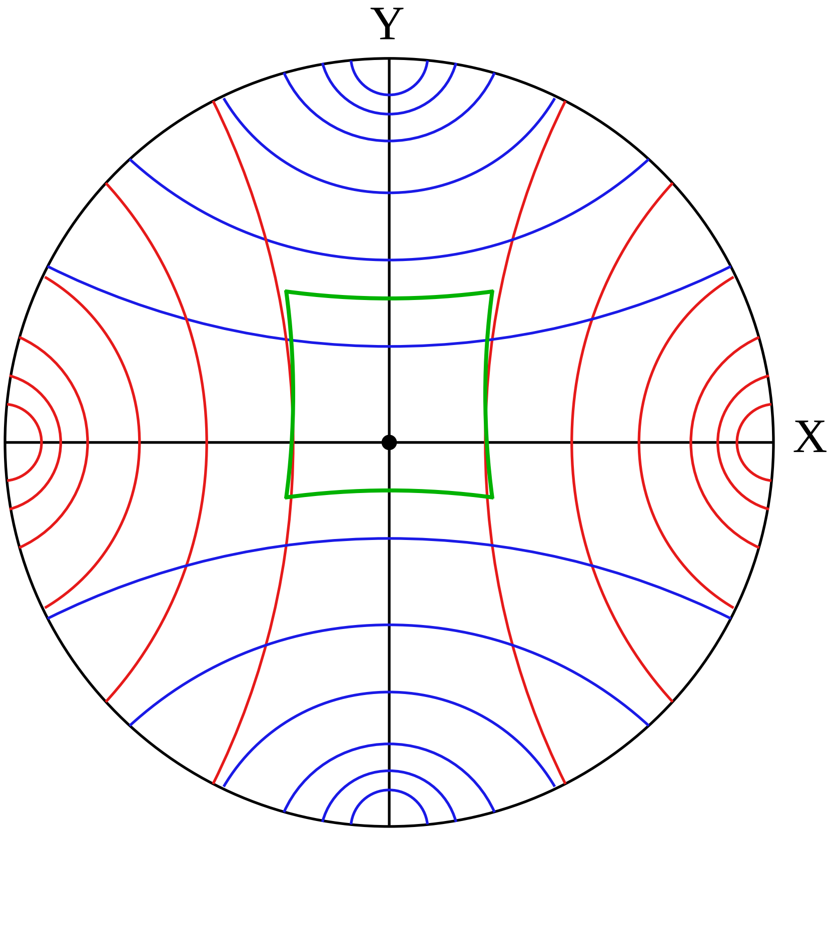

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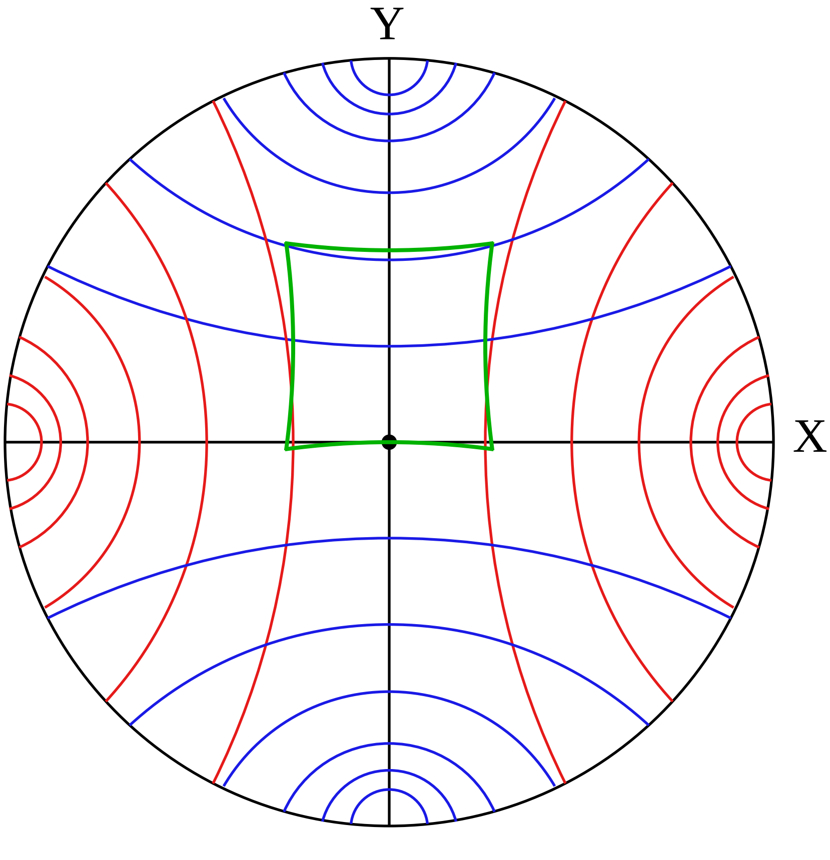

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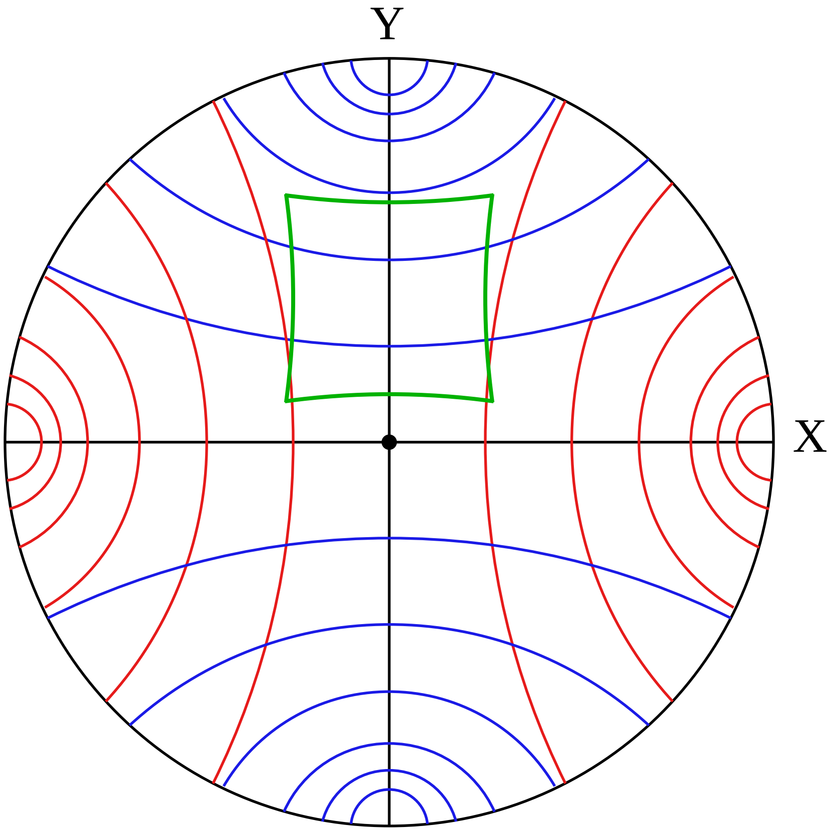

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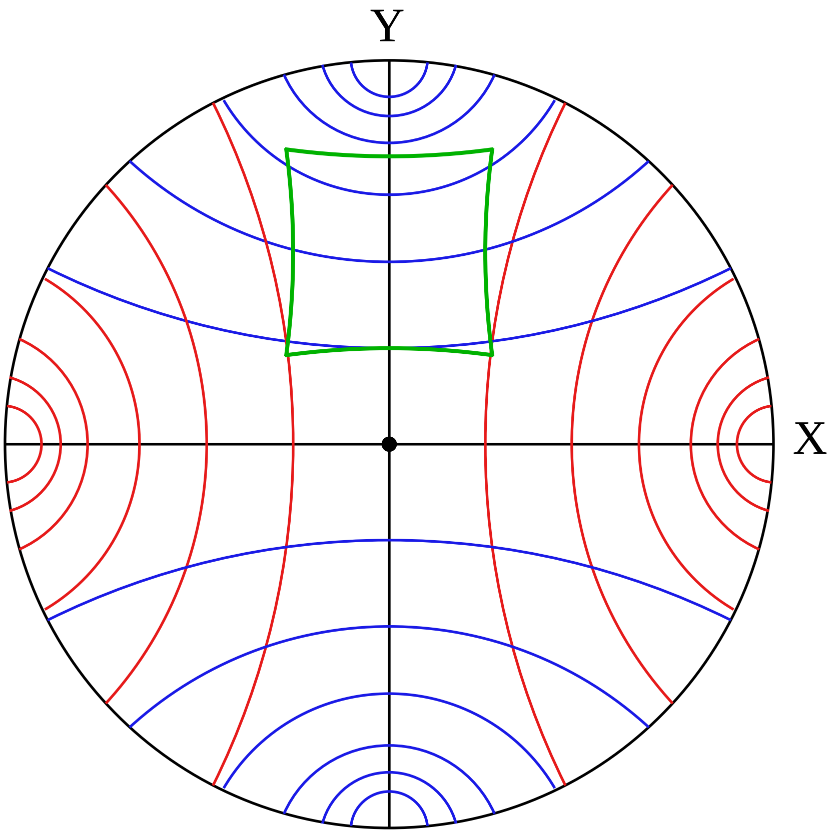

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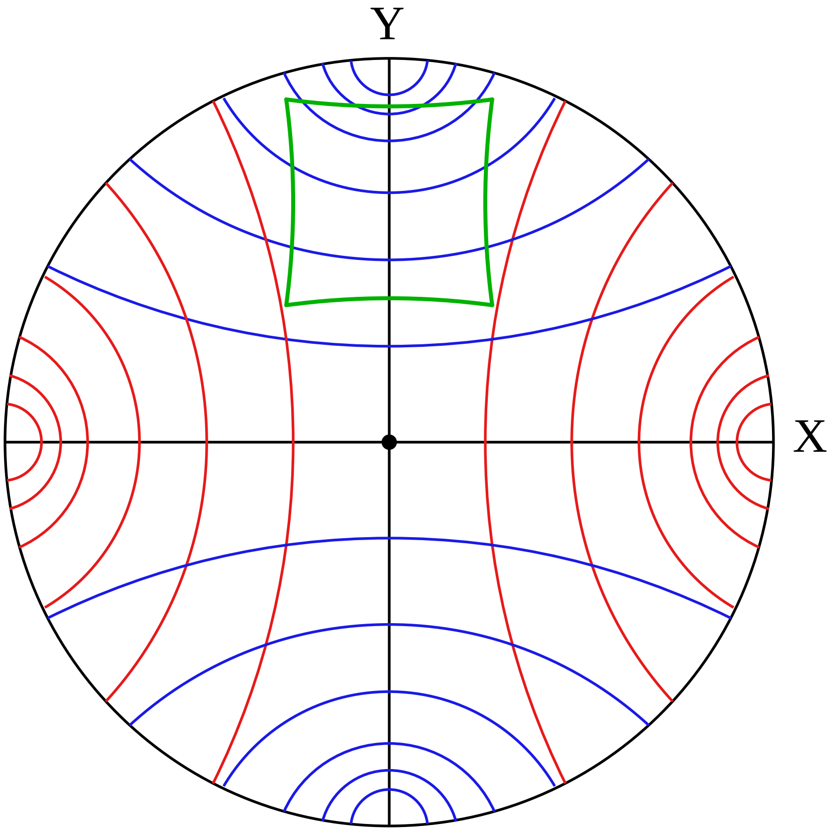

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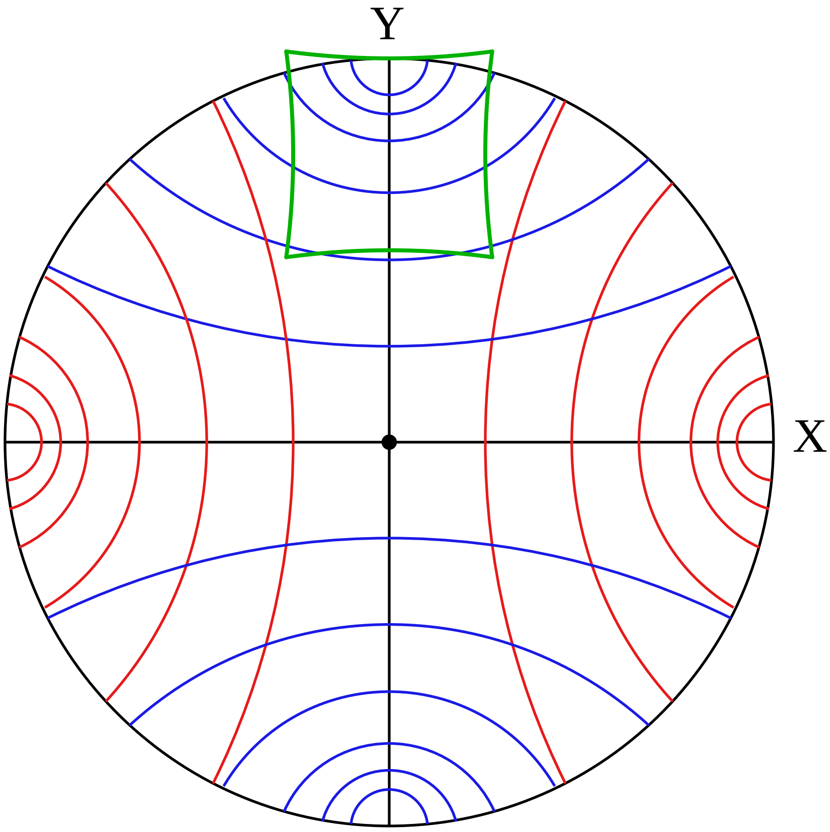

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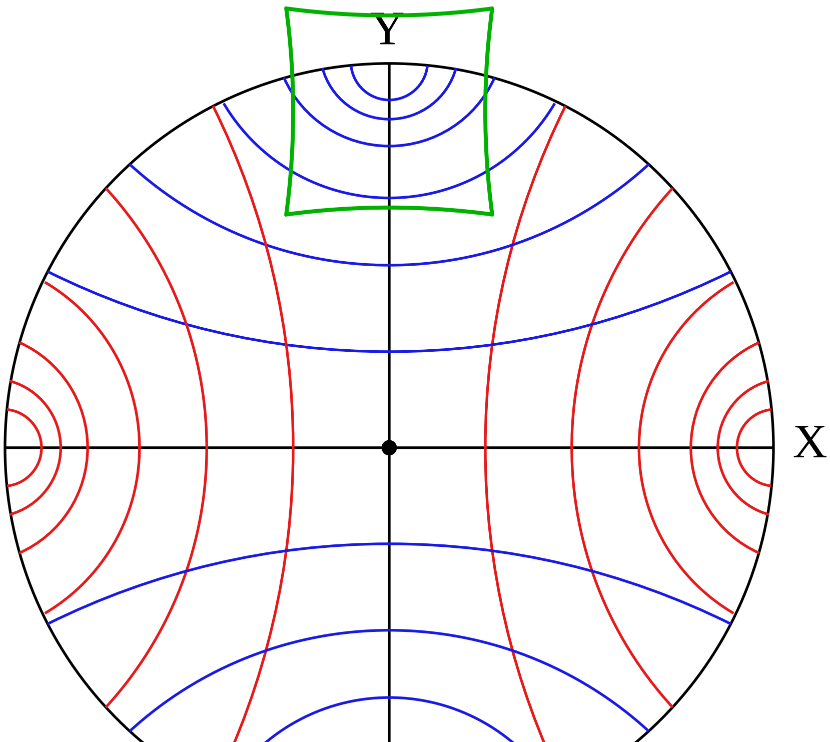

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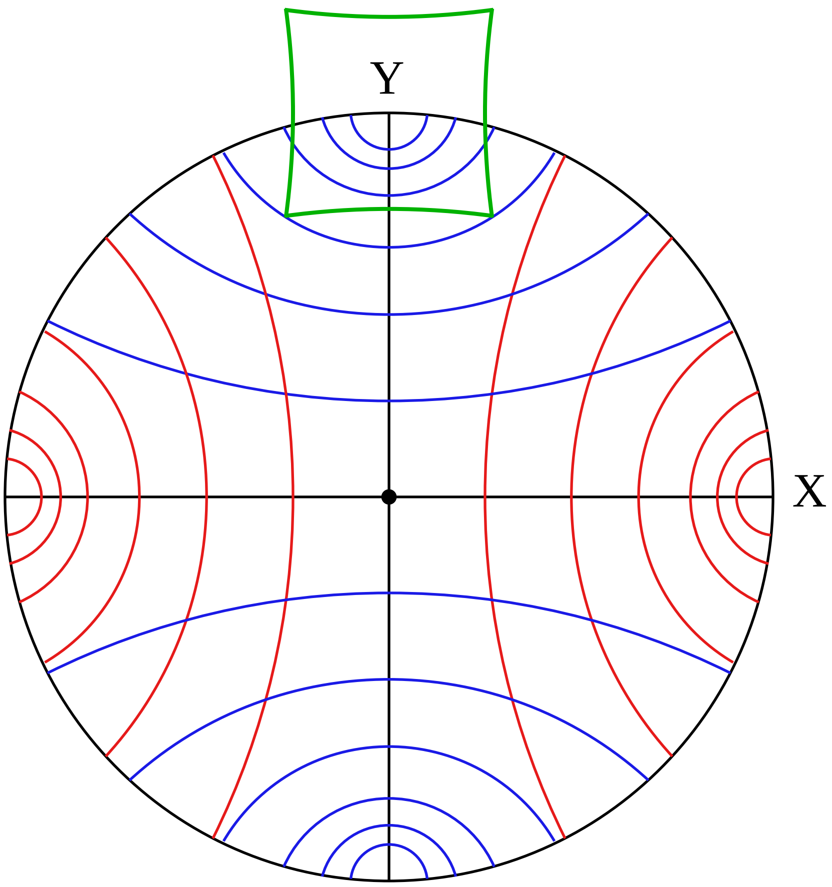

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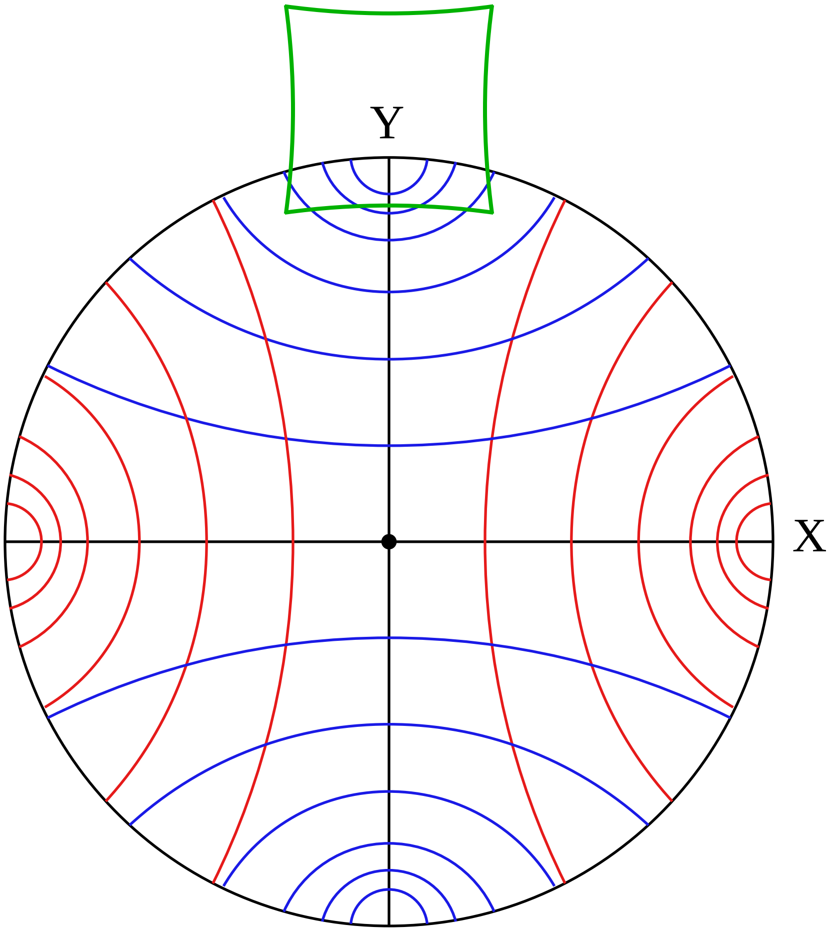

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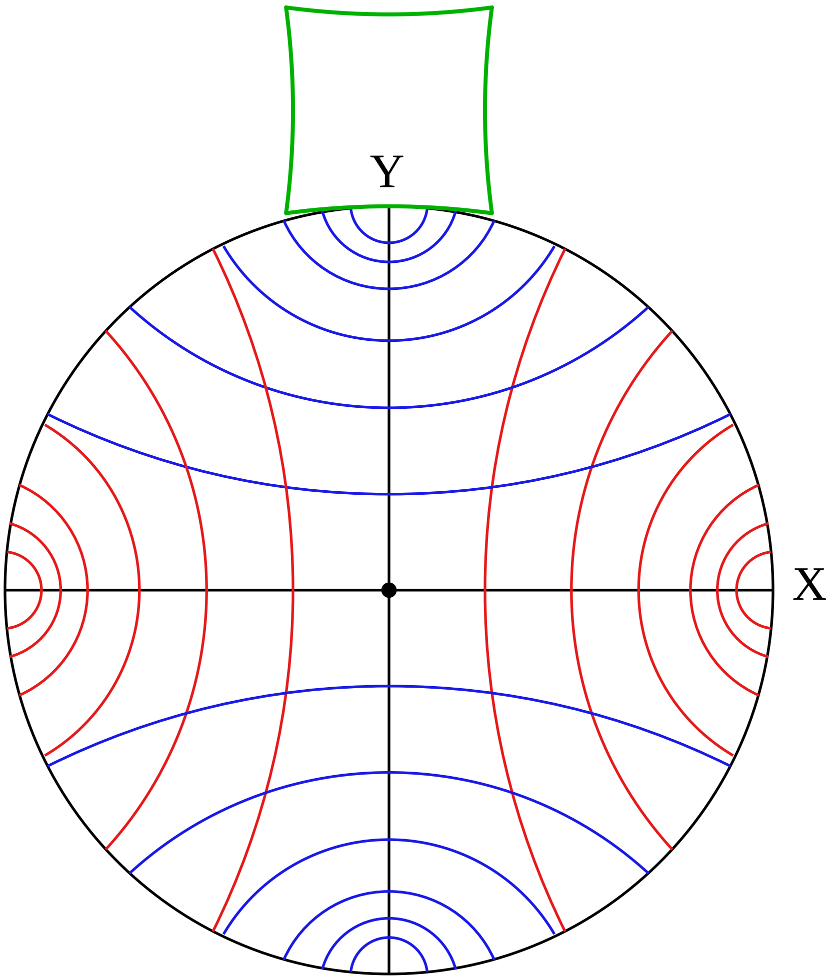

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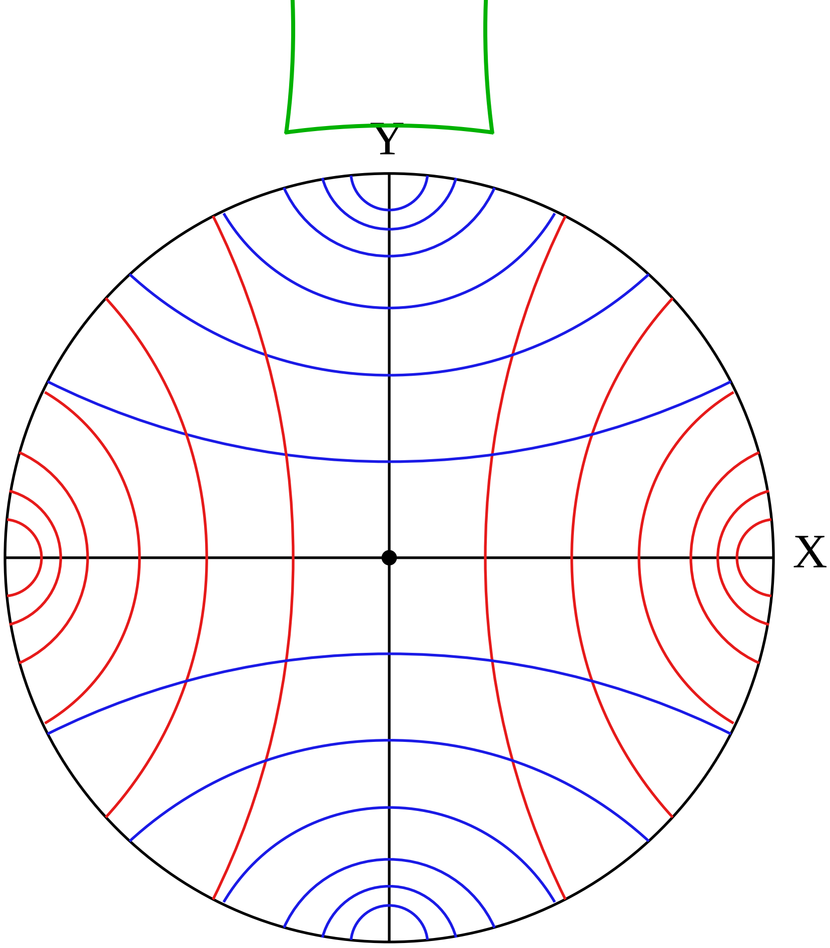

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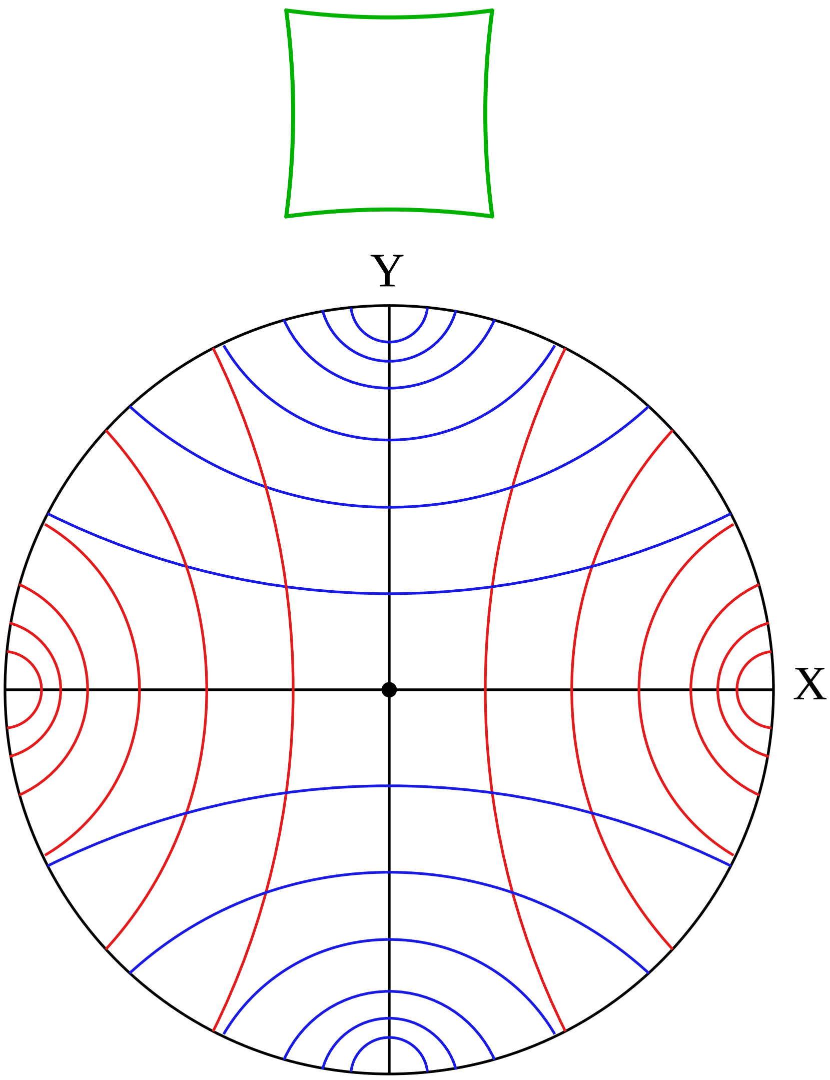

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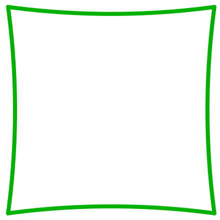

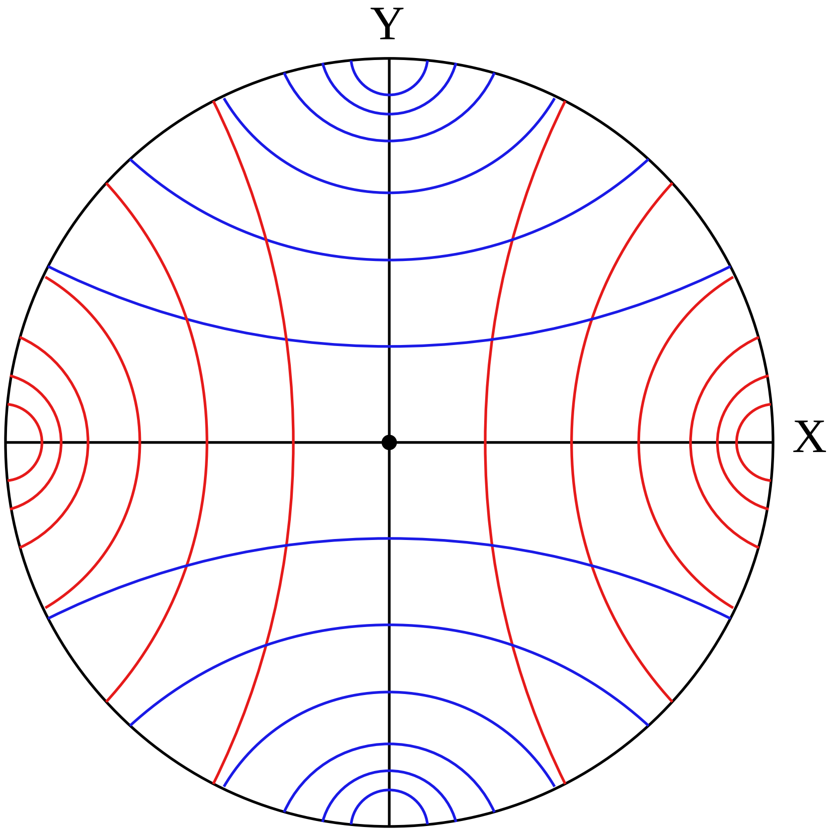

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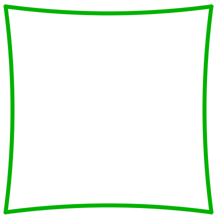

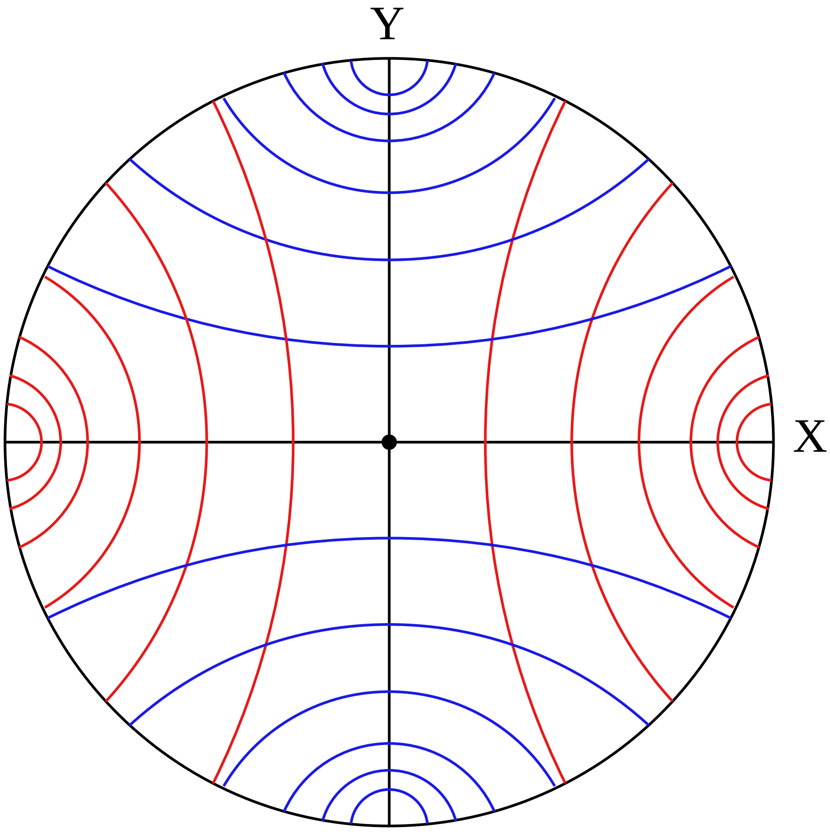

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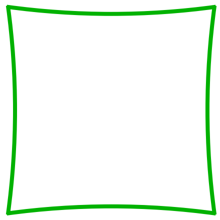

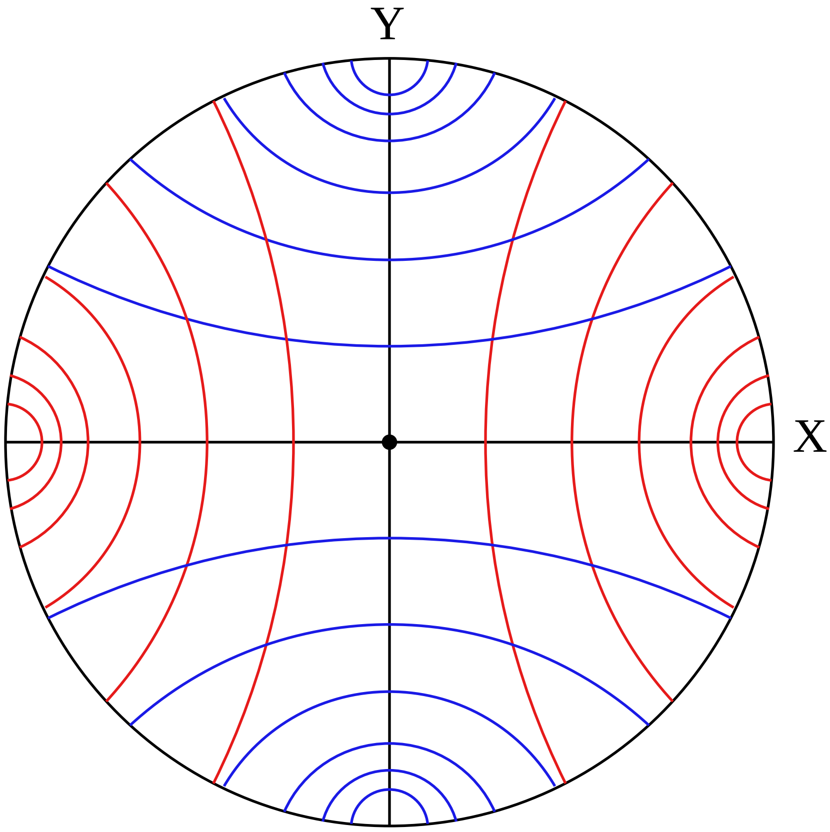

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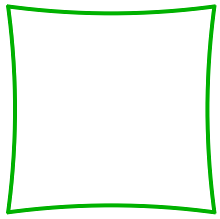

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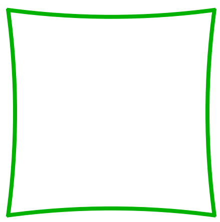

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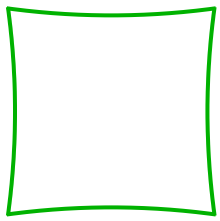

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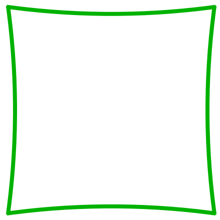

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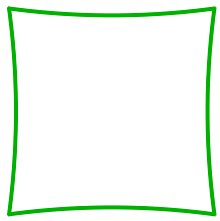

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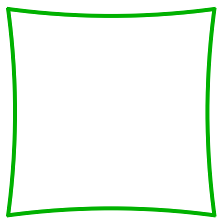

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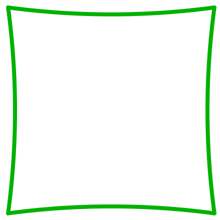

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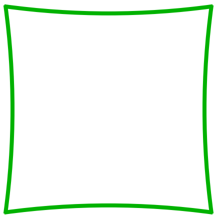

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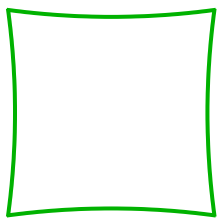

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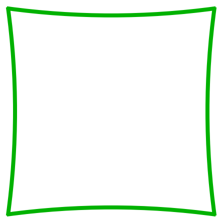

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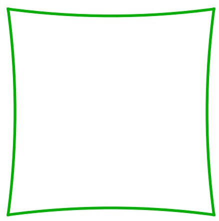

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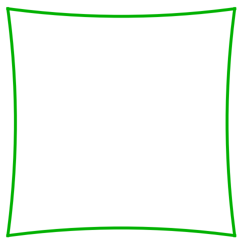

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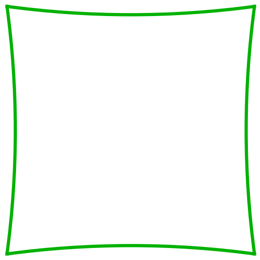

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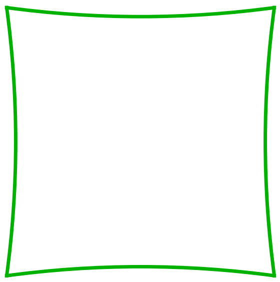

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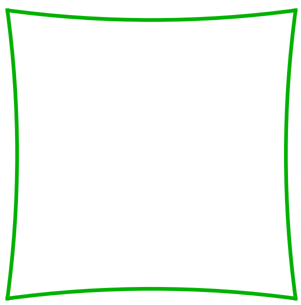

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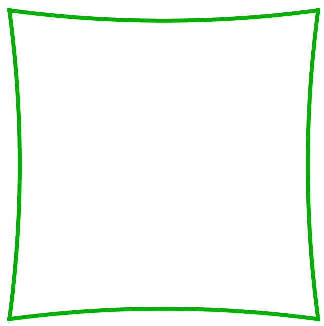

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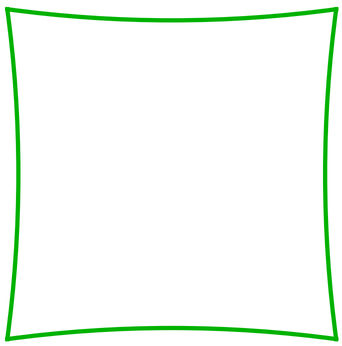

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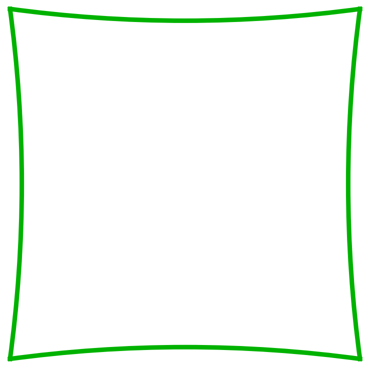

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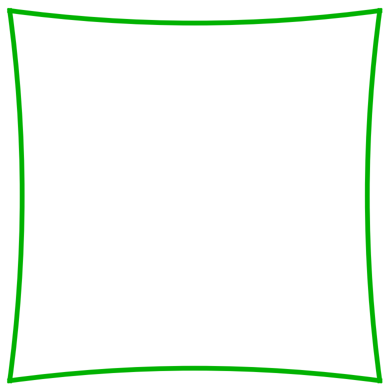

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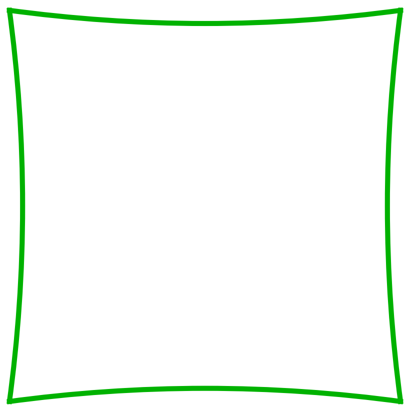

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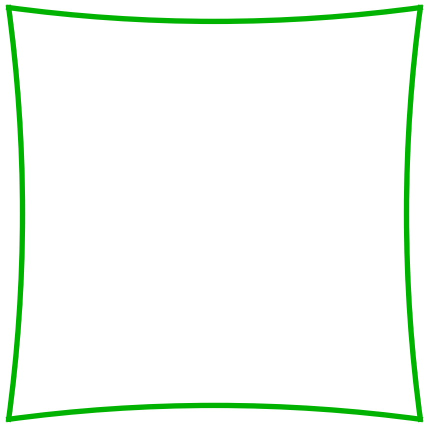

Hyperbolic Cube

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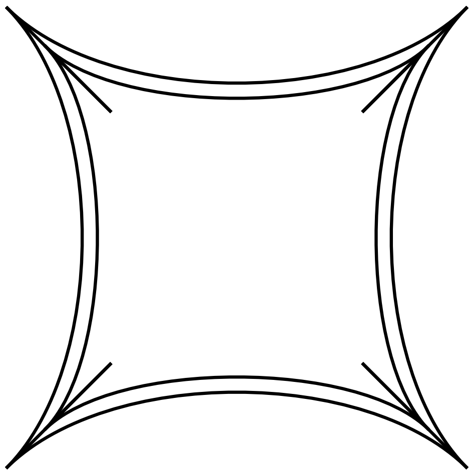

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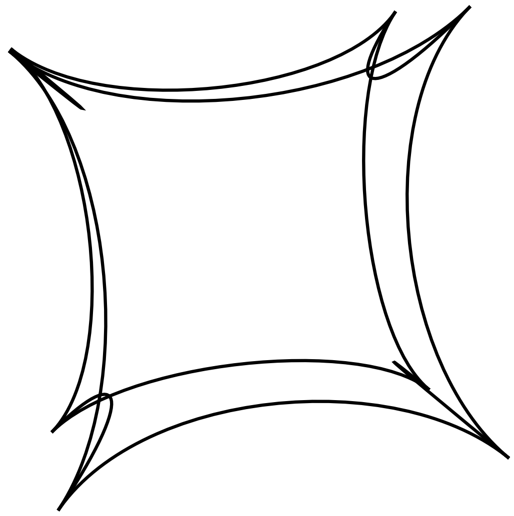

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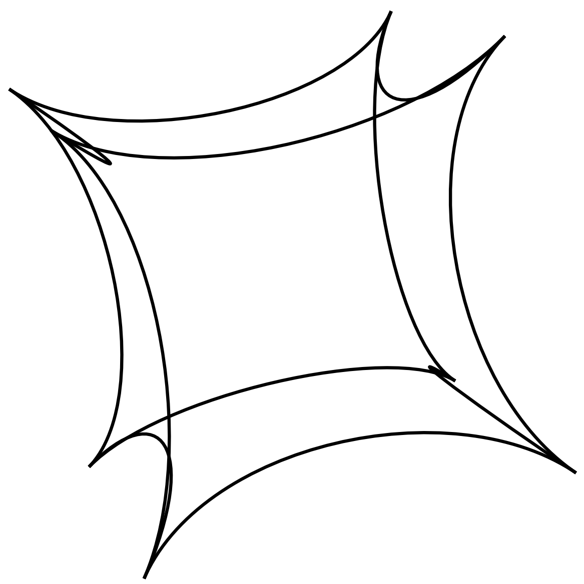

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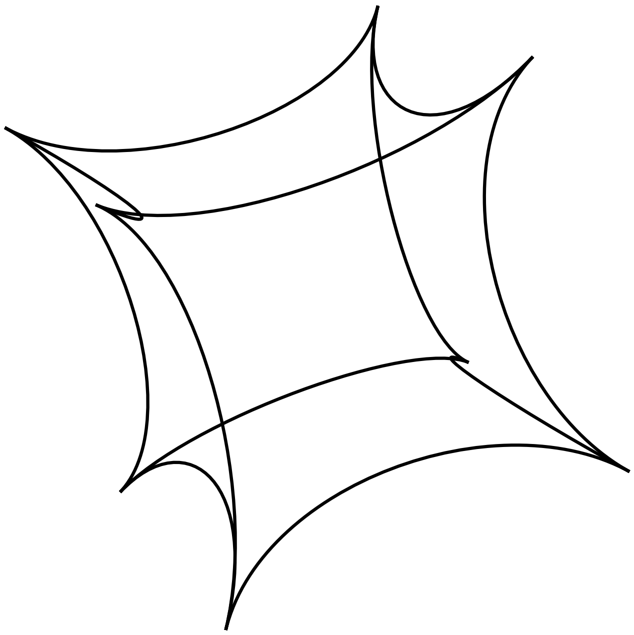

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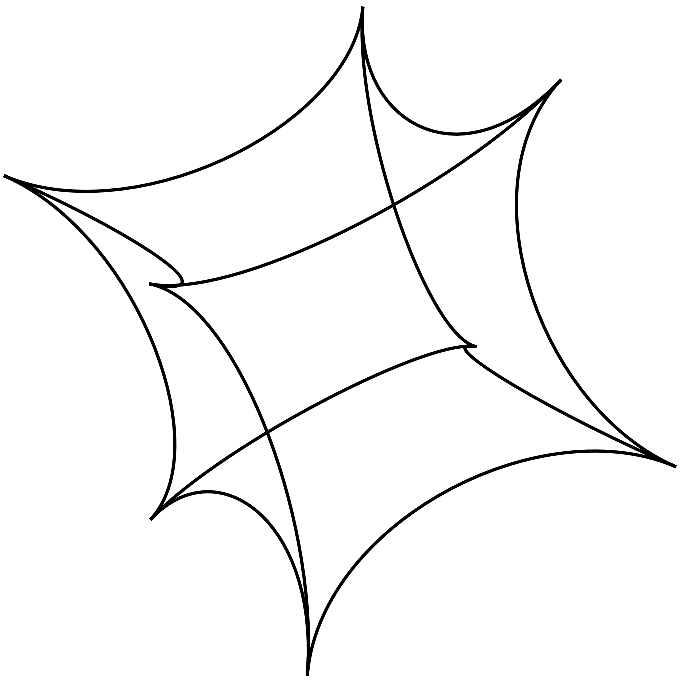

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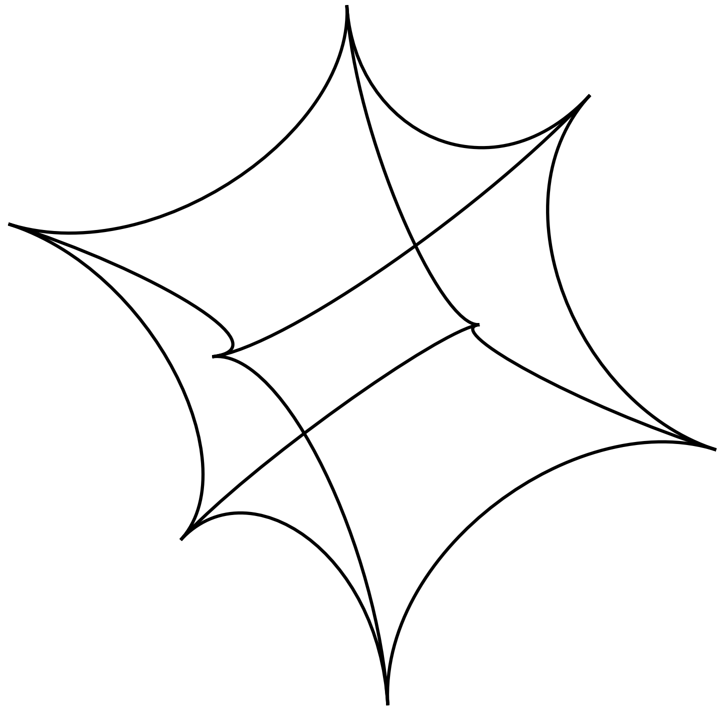

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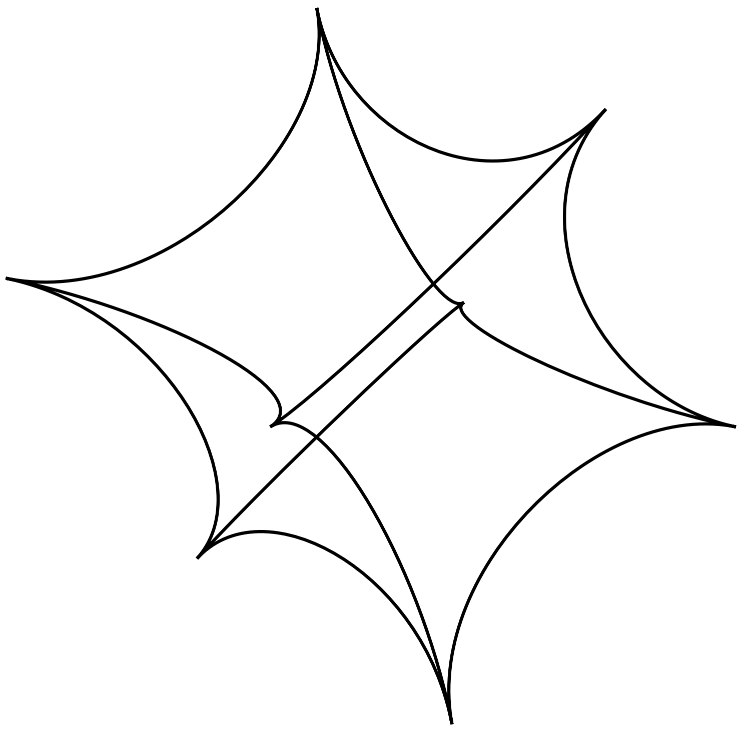

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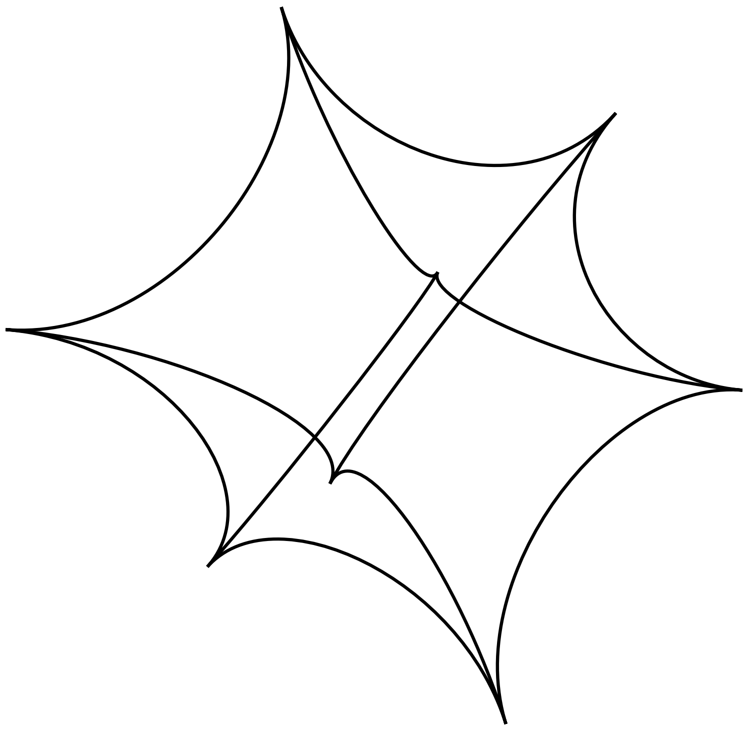

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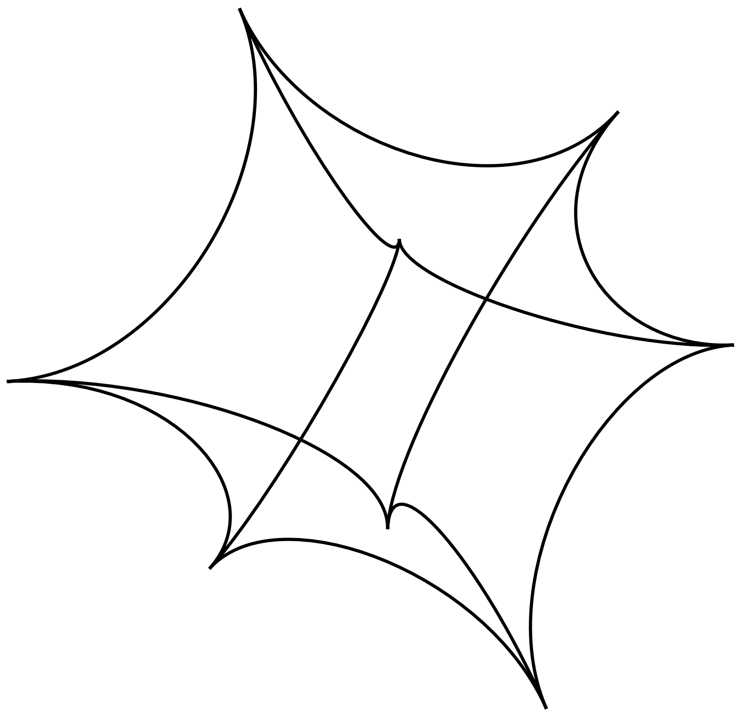

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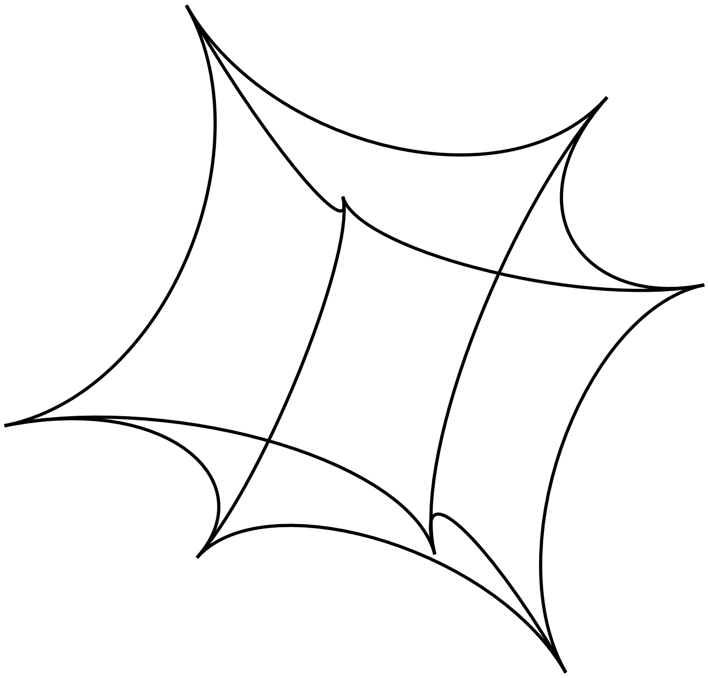

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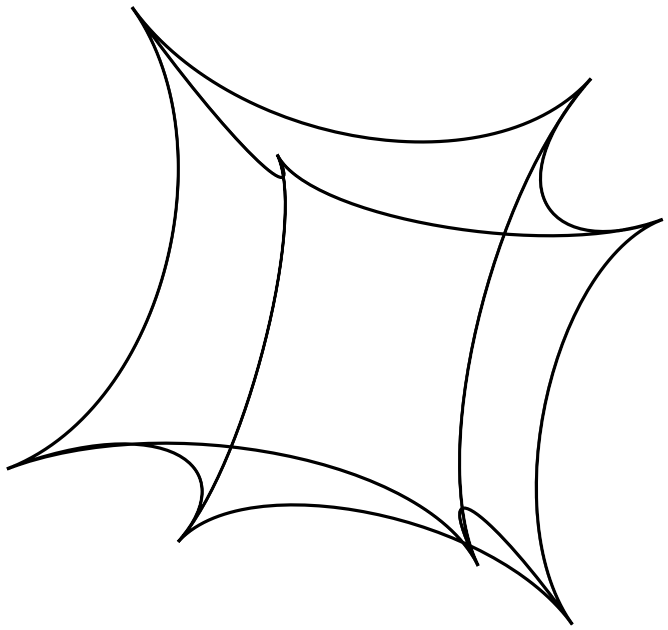

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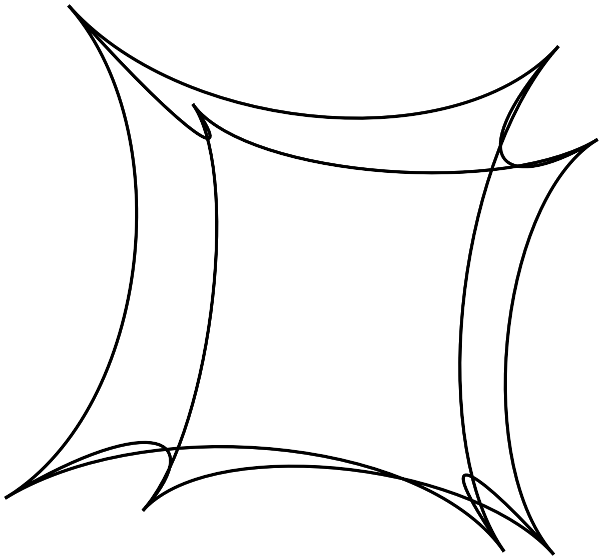

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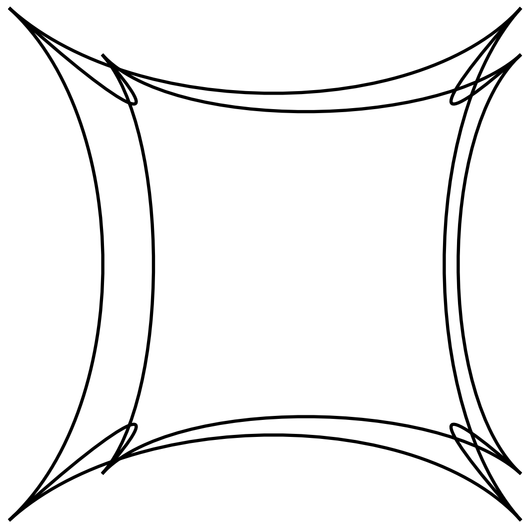

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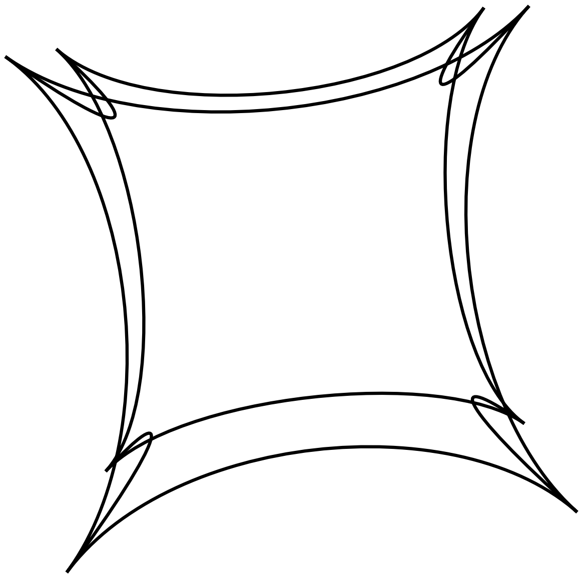

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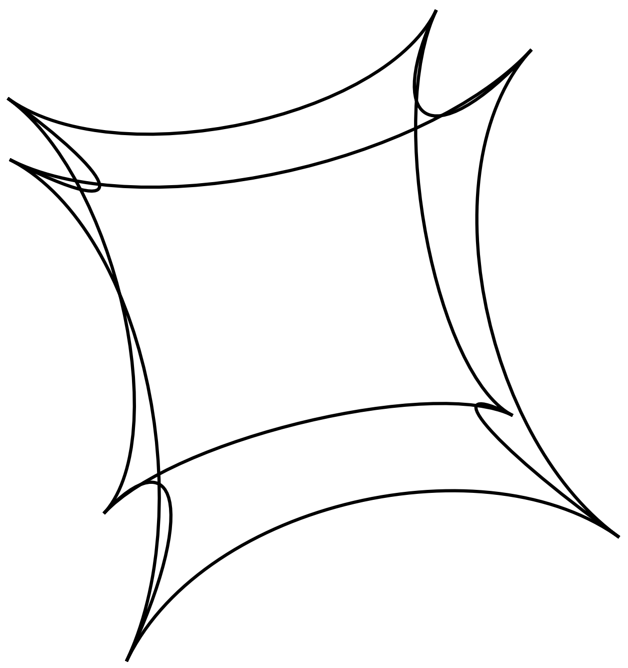

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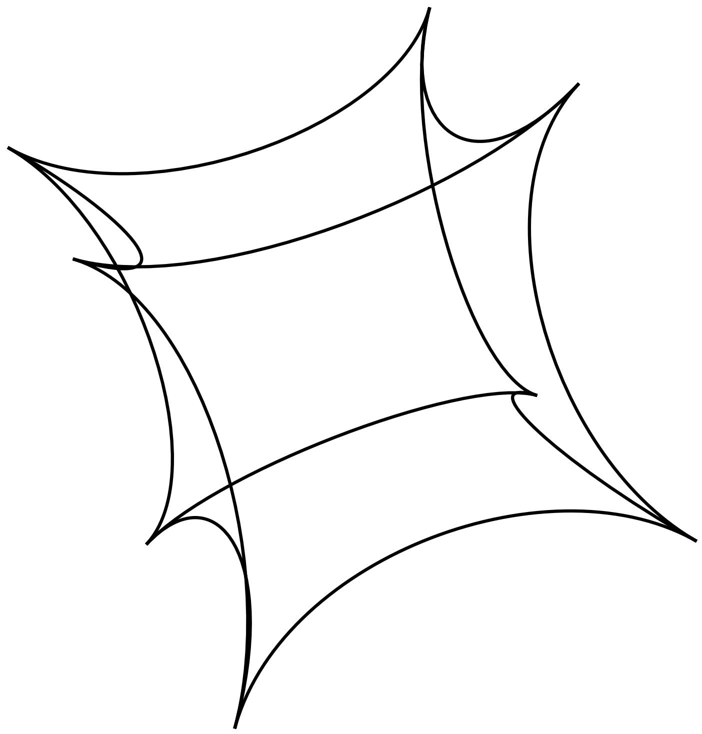

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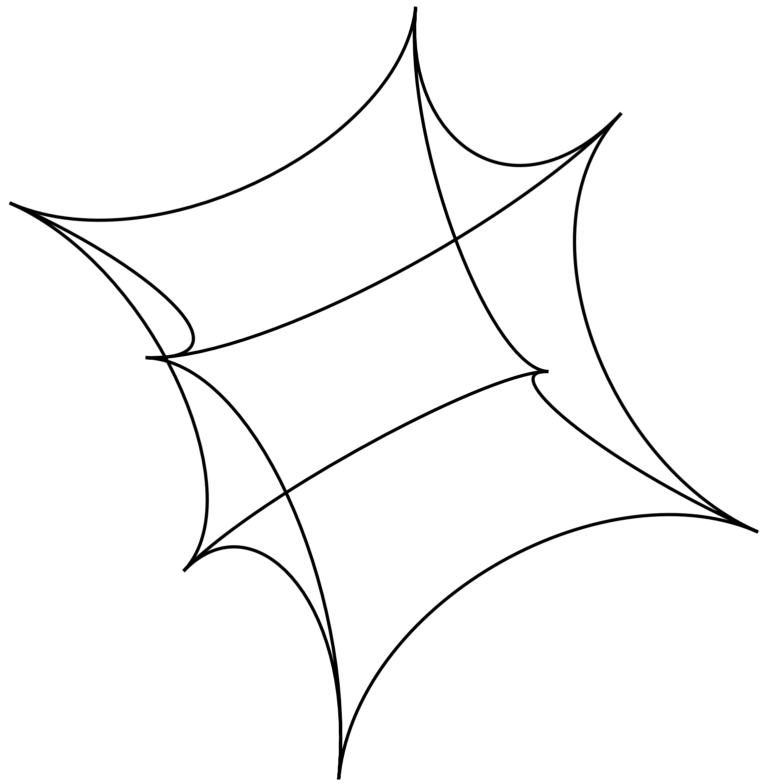

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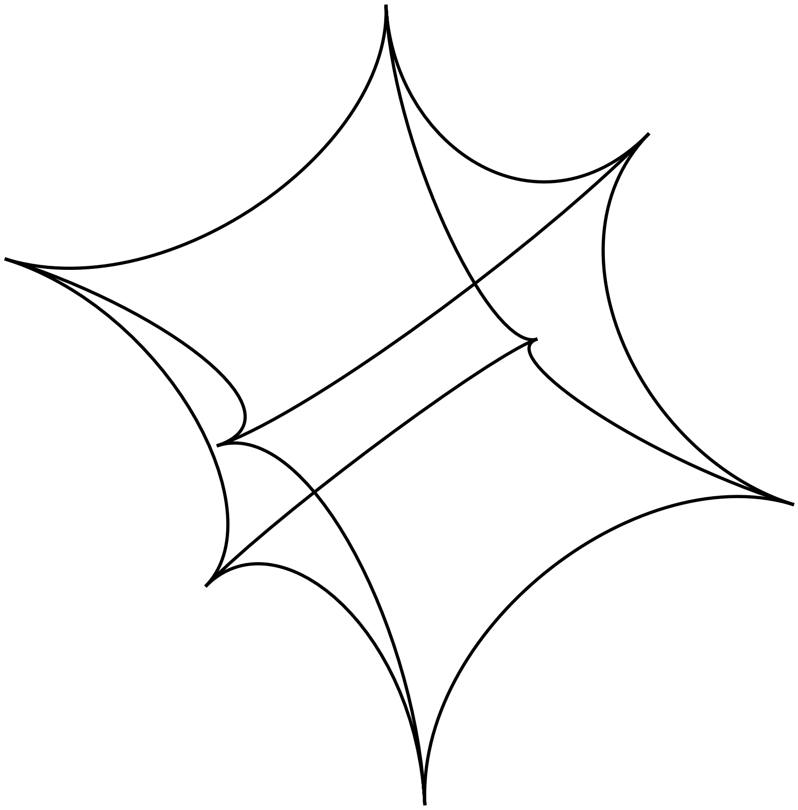

Hyperbolic Cube

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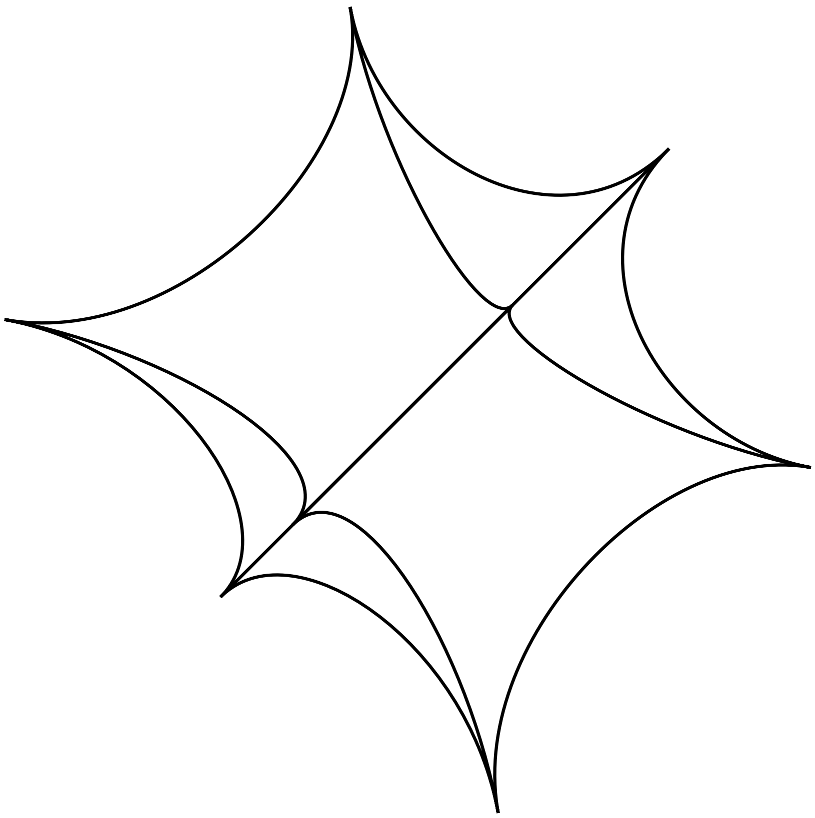

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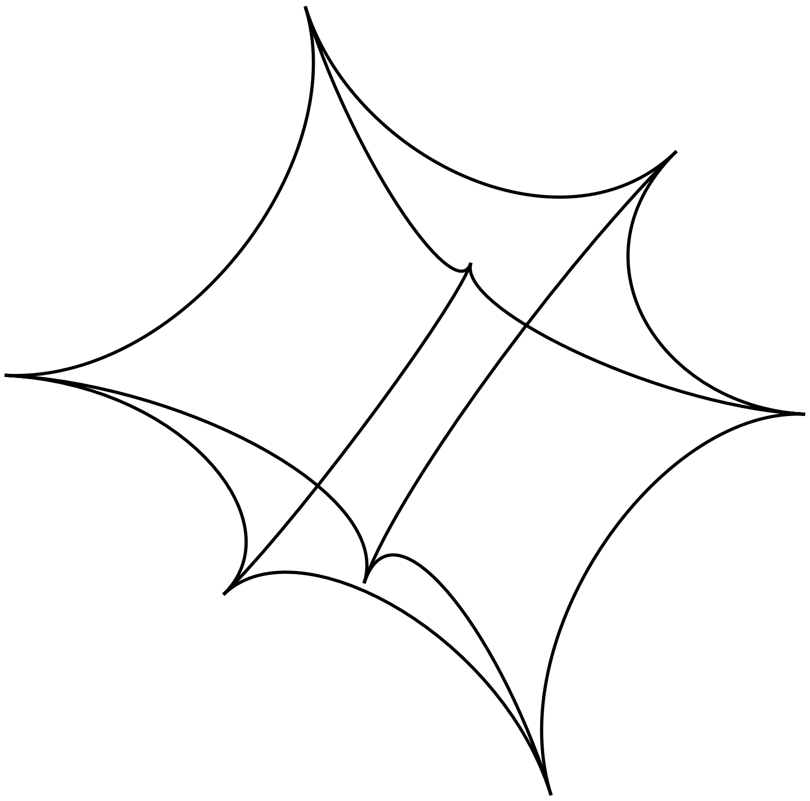

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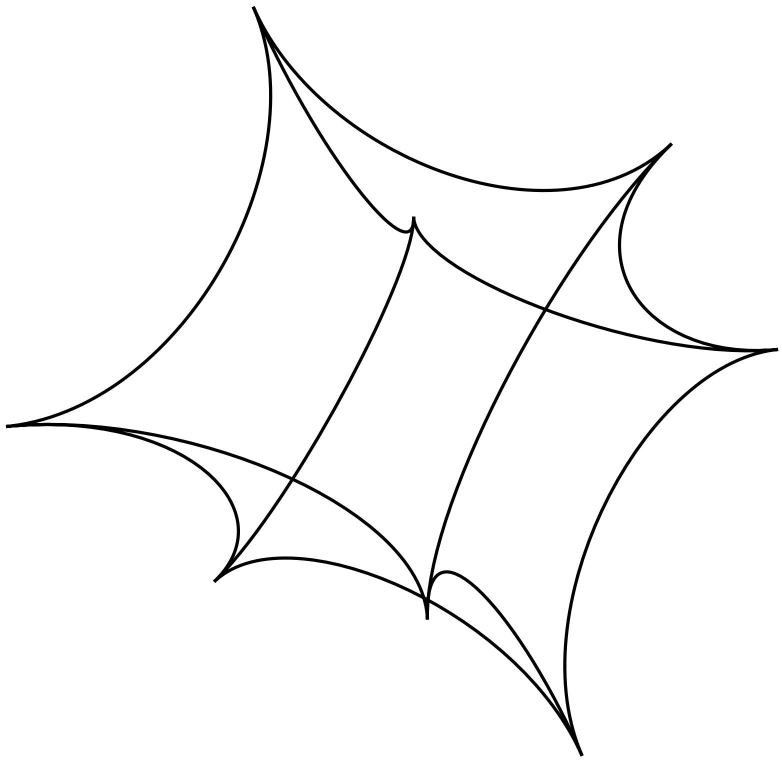

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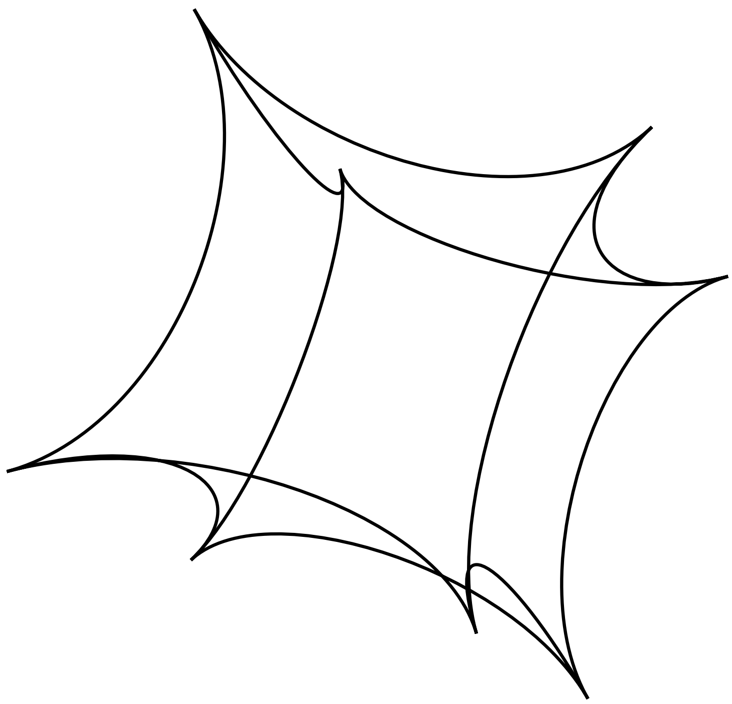

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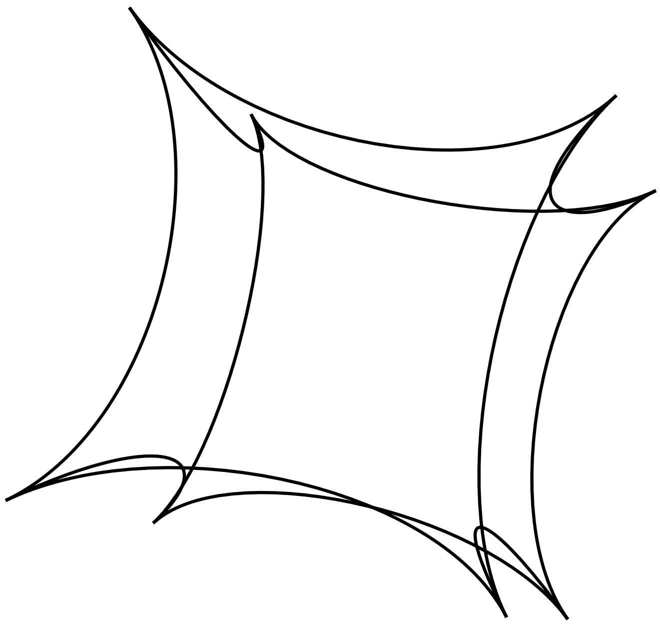

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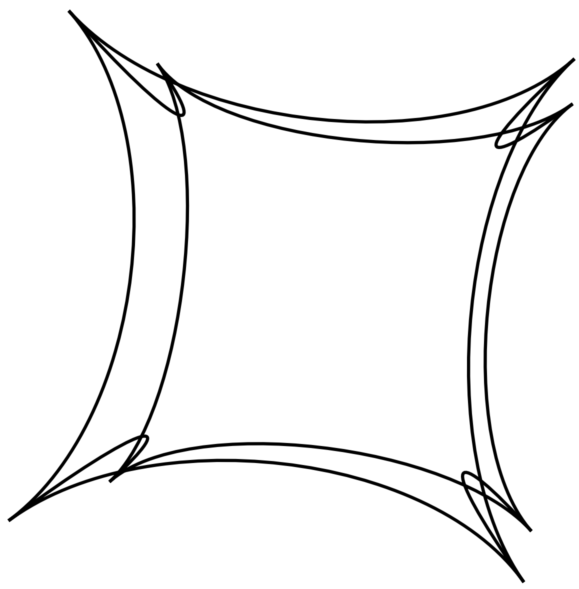

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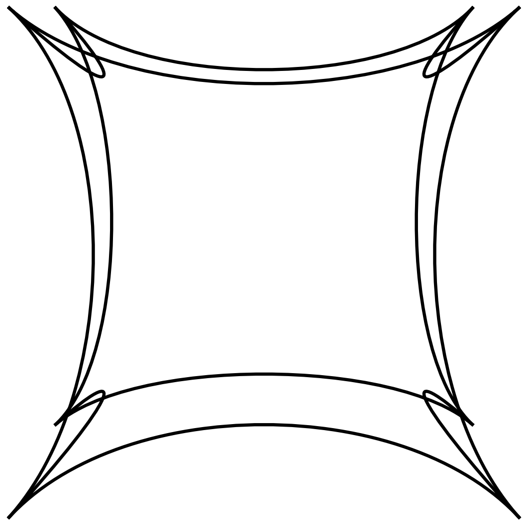

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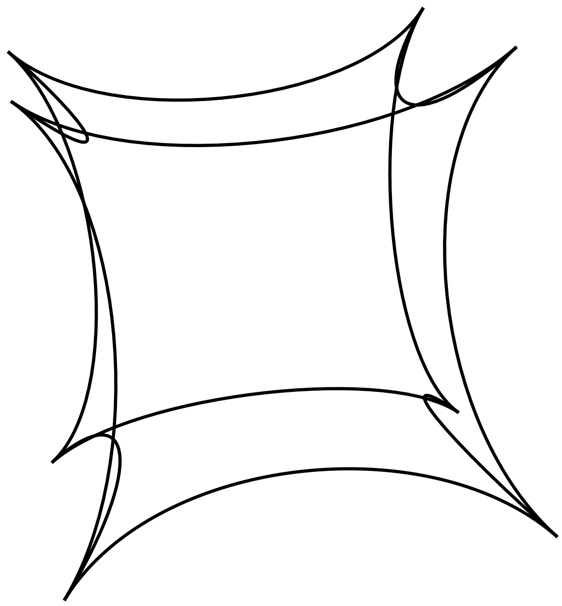

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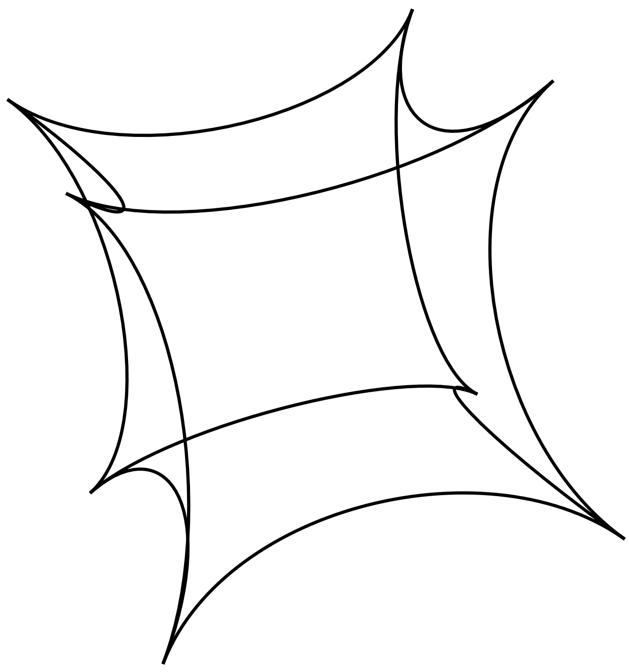

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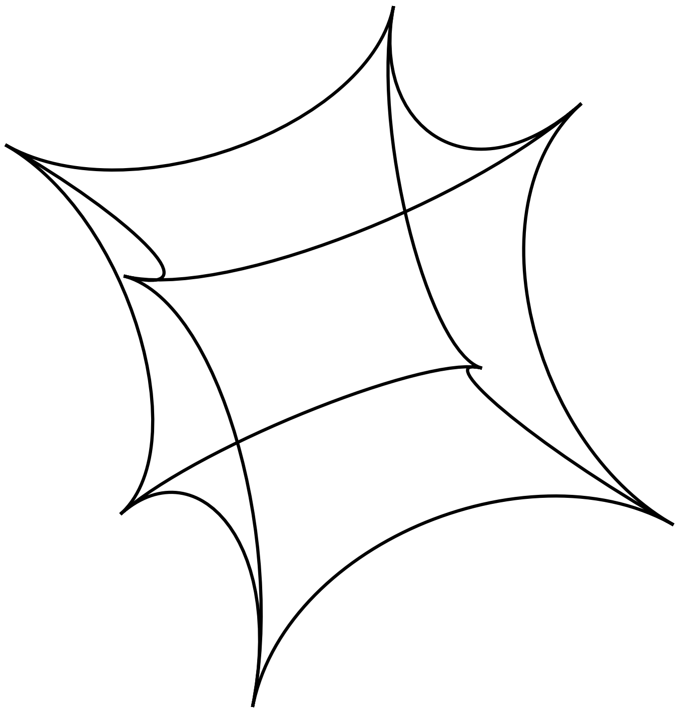

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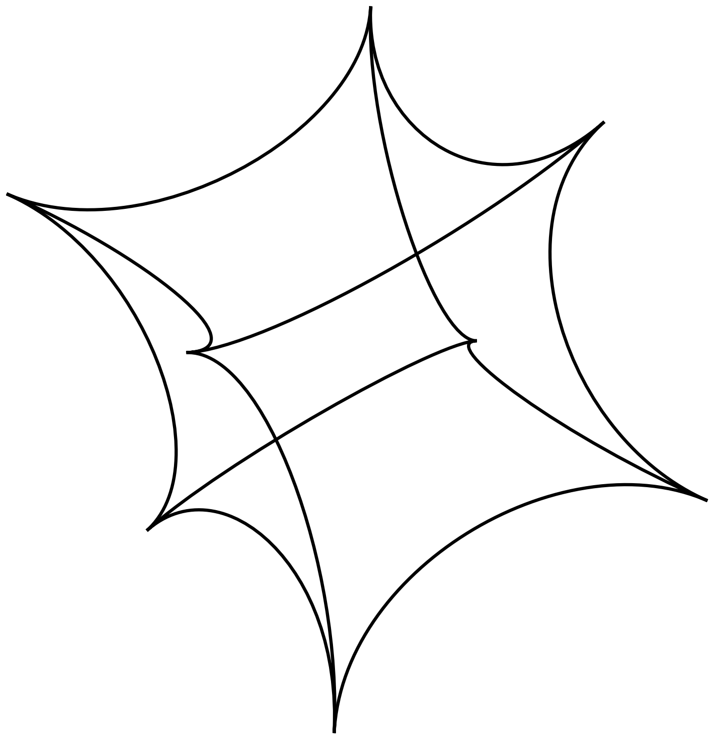

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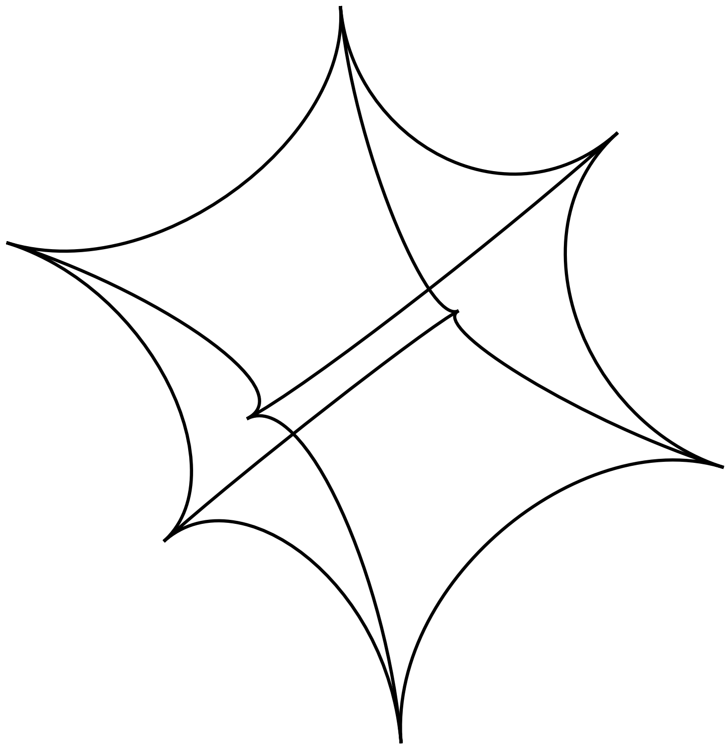

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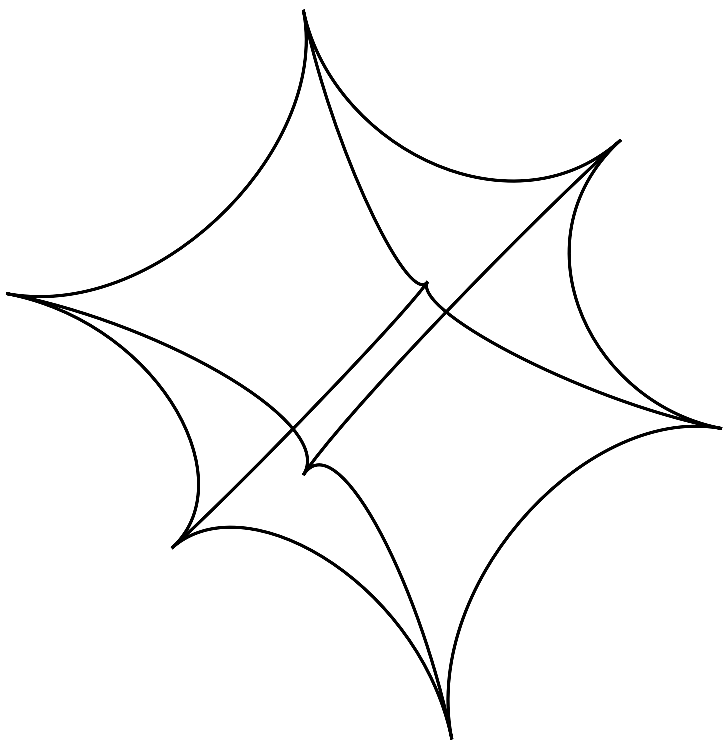

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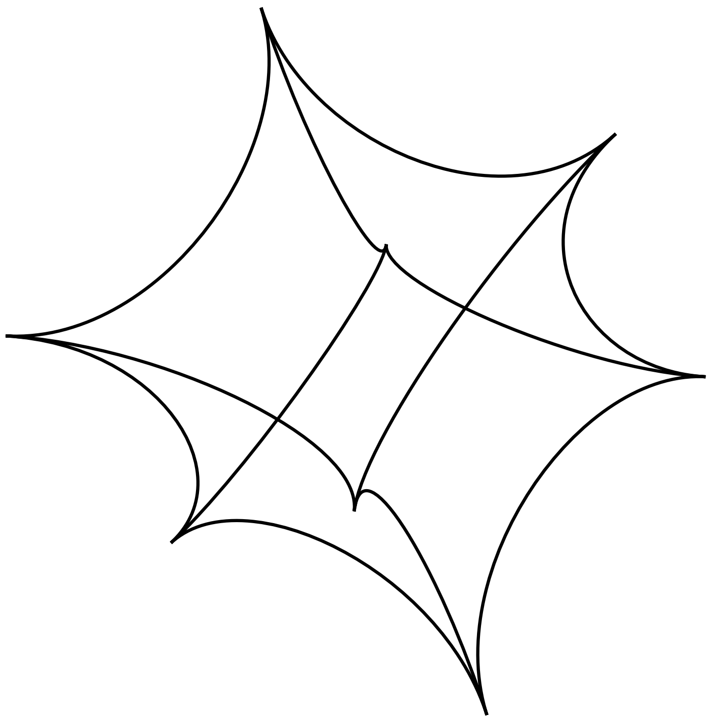

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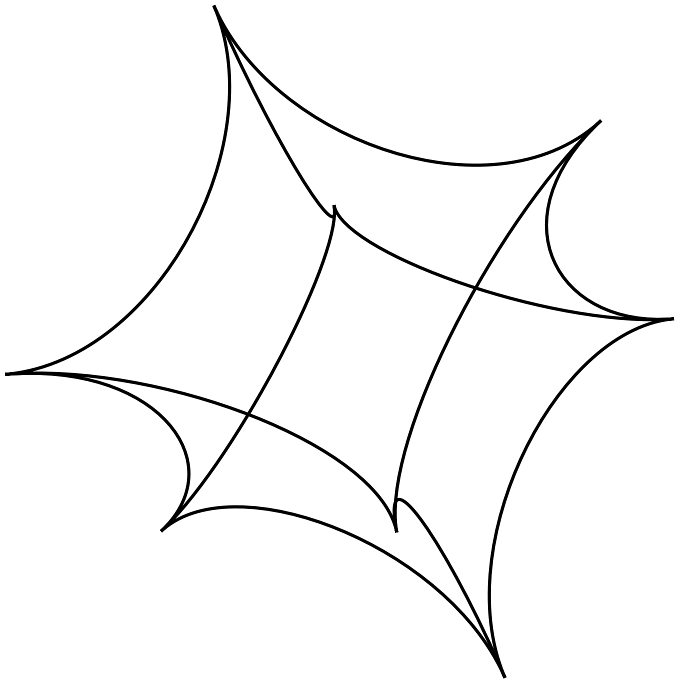

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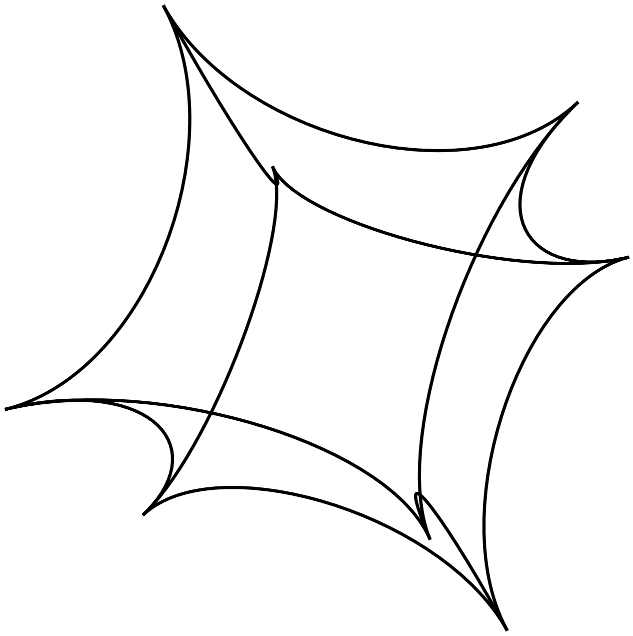

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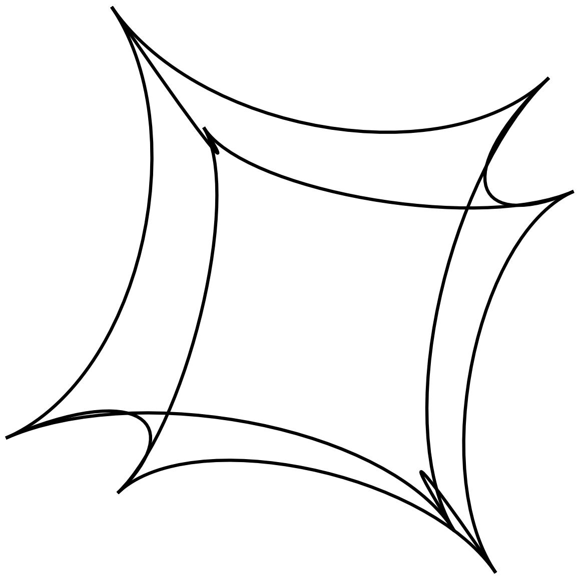

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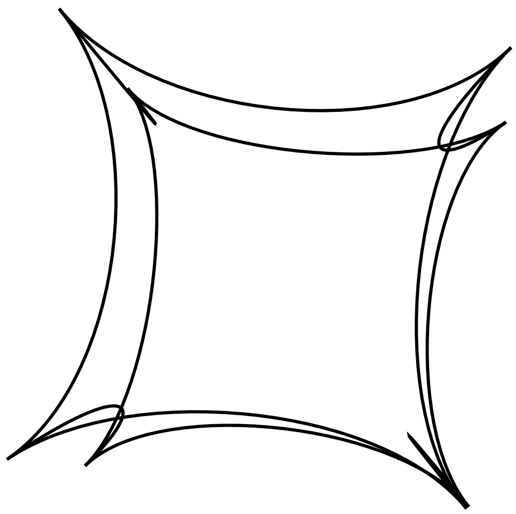

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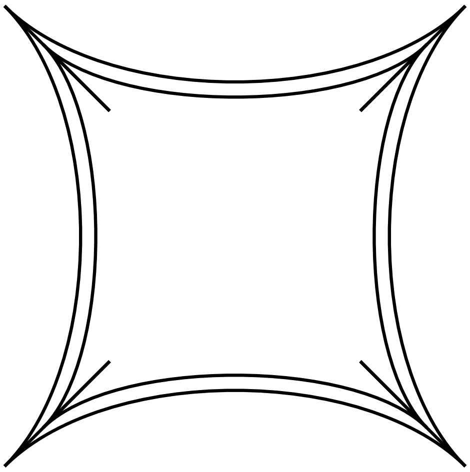

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