On the Motion of Planets:

Prepared by: Robin Radomski

"Let no one without skill in geometry enter"

Man has always looked to the heavens and wondered about his place in the universe. One such brave man who took the task literally was:

Ptolemy

He tried to explain the movements of celestial bodies across the heavens. Using tables filled with all sorts of numbers, he developed a very successful model that could predict the locations of all the planets between Mercury and Saturn, and the Sun.

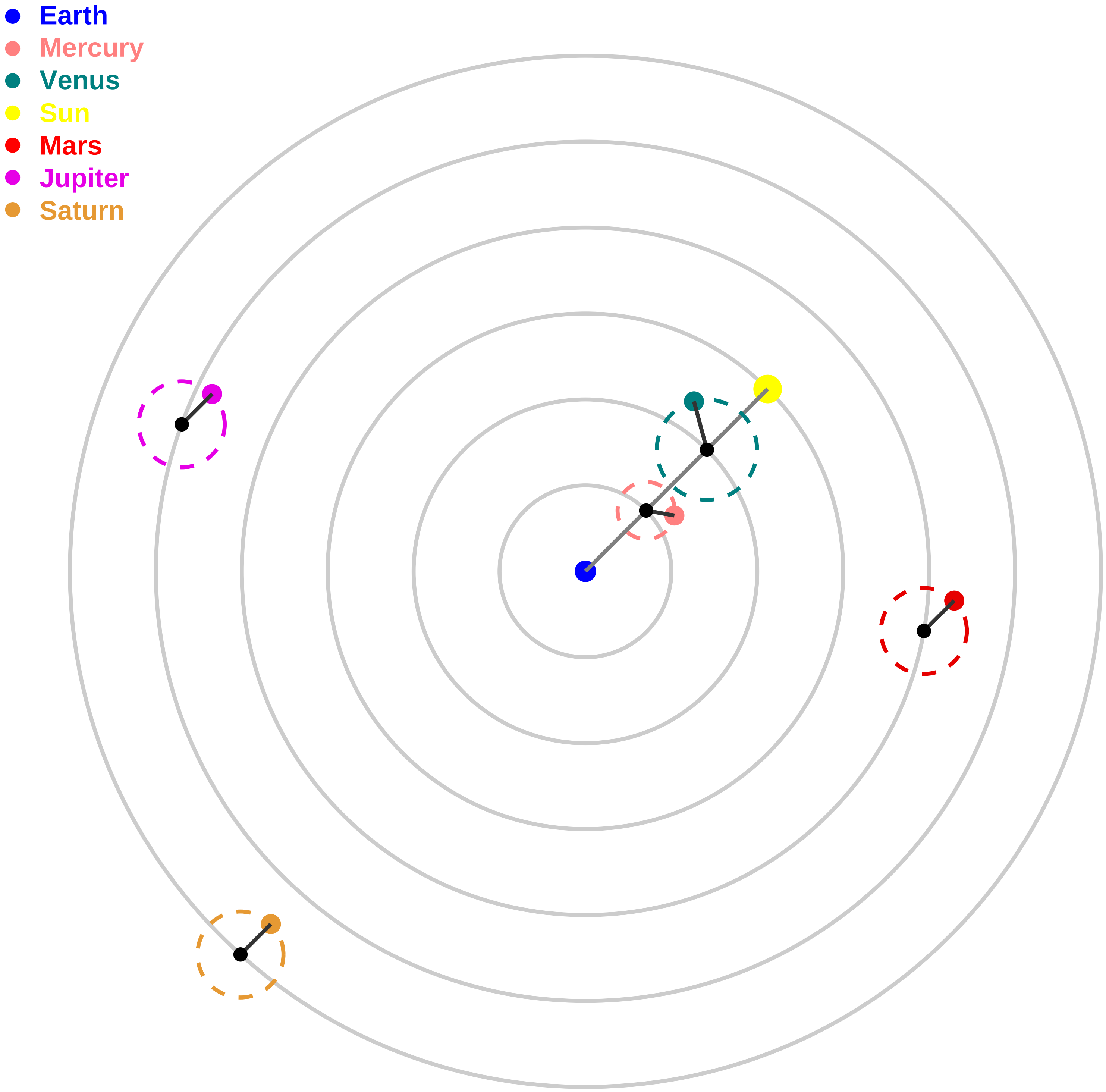
GEOMETRY MADE IT POSSIBLE !

In fact, Ptolemy states that an elementary knowledge of geometry is necessary to understand the work.

Let’s learn some basic astronomy...

The Ptolemaic System

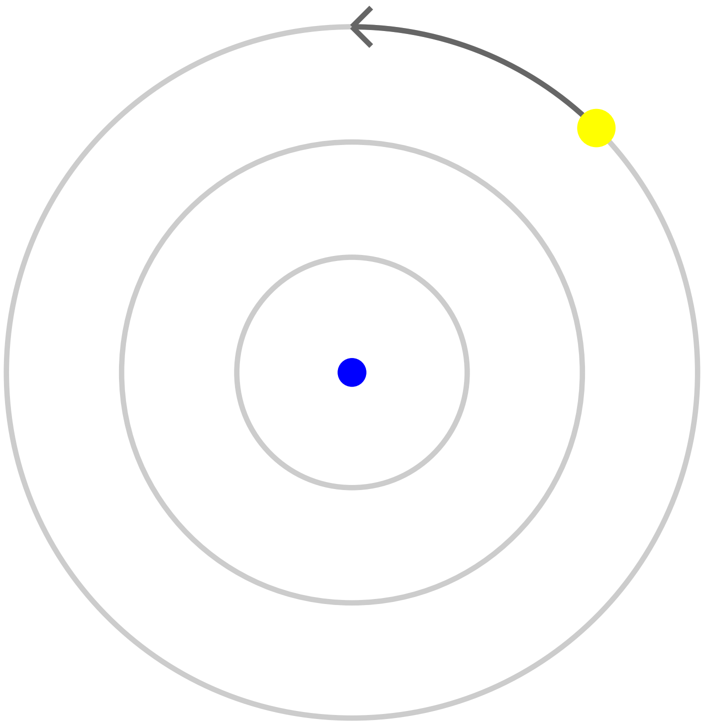
Written in about 150 AD by Ptolemy, the Almagest is a treatise of what the Greeks understood about mathematical astronomy. It consists of 13 books. The first two books are devoted almost entirely to geometry.
Let’s look at some of the key components...



The Ptolemaic System - Key Components

**It is "geocentric" - the Earth is in the middle. NOT the Sun!
The Sun is in Earth’s position - namely 3rd from the center.**

It is important to note that distances between planets had not been established but were inferred from how fast they moved across the sky. This is the big difference between Ptolemy’s and Copernicus’ systems. This is how Ptolemy placed the Sun third. We travel around the Sun as fast as it travels around us.

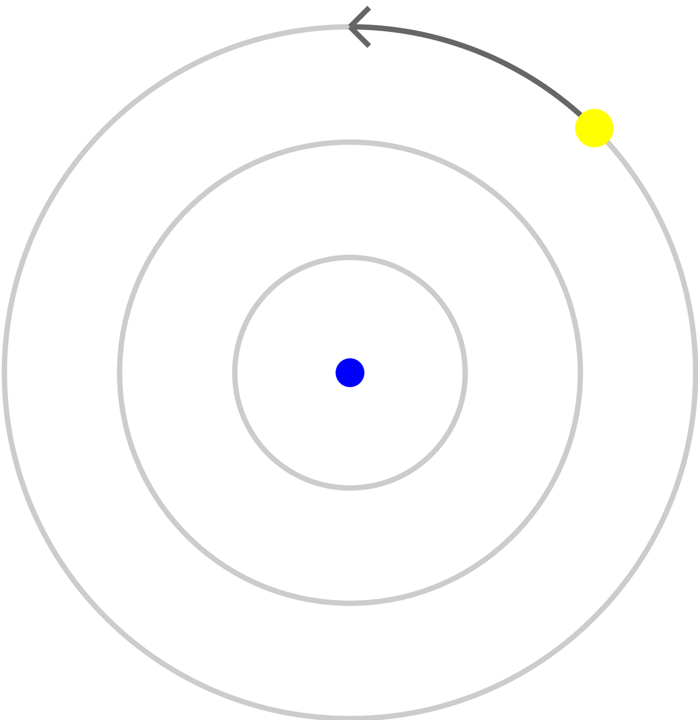


The Ptolemaic System - Key Components

Notice the Sun's orbit - the gray circle.
Ptolemy called Earth-centered orbits "deferents".

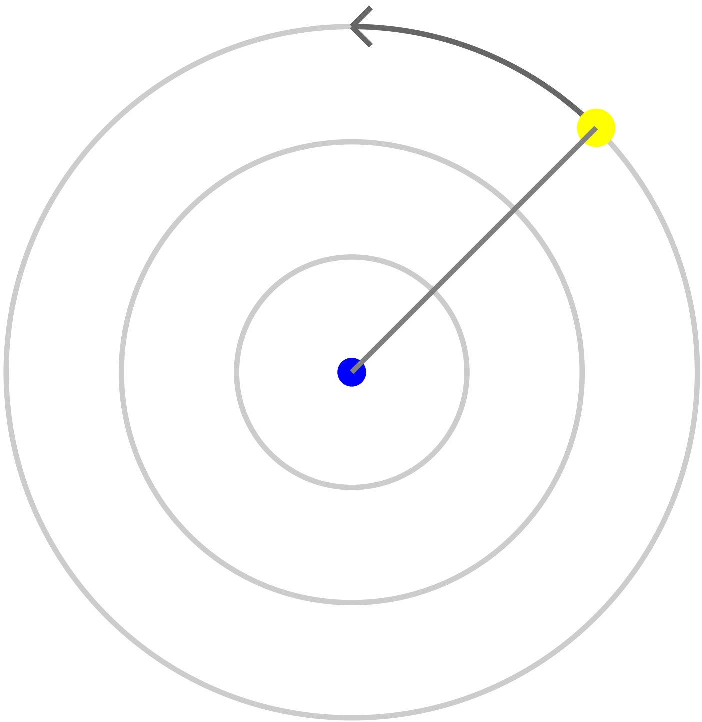
A second type of orbit was called an epicycle.
Notice the Sun doesn't have an epicycle.

We'll see an epicycle in a moment...



The Ptolemaic System - Key Components

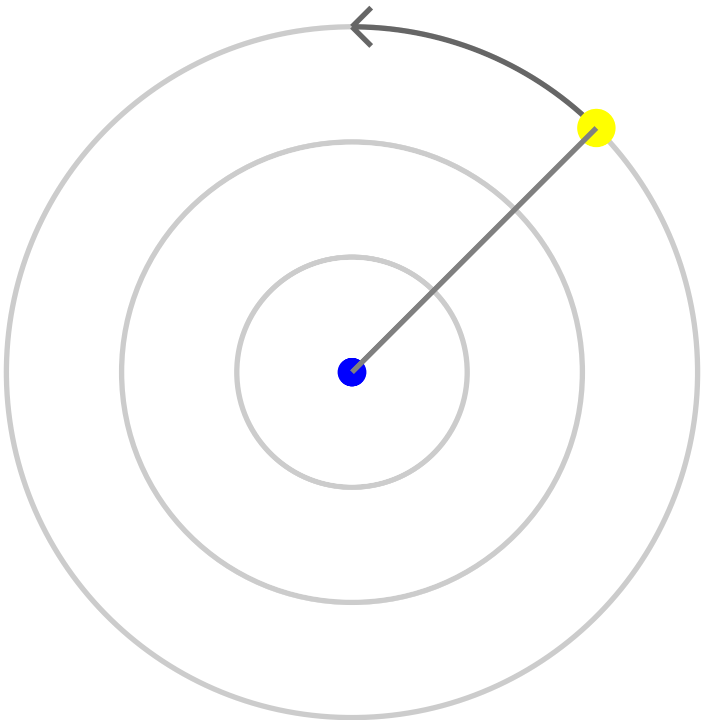
Ptolemy then connects the Earth and the Sun.
This line will allow us to establish several components.



The Ptolemaic System - Key Components

Working outwards we'll establish the positions of the planets. First, using this new line we can establish Mercury's position.

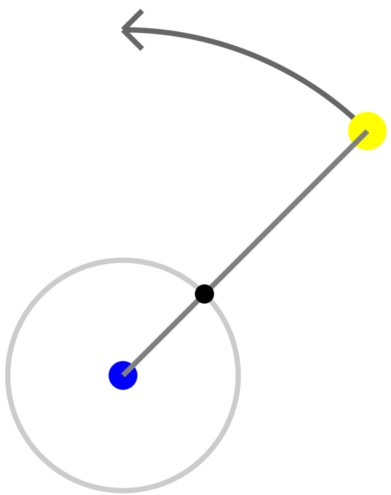
Ptolemy adjusted the sizes of his deferents and epicycles to match what was seen in the sky.



The Ptolemaic System - Key Components

Mercury’s Orbit

First, a dot is placed where the deferent and line intersect. This is the location of the center of the epicycle.



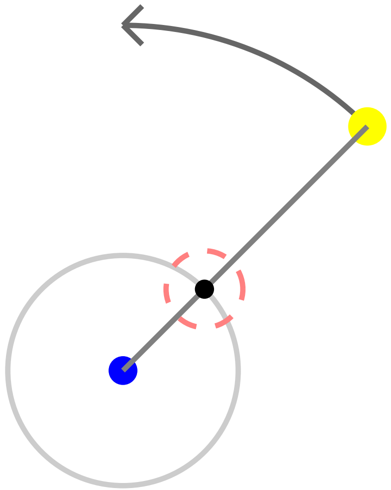
The Ptolemaic System - Key Components

Mercury’s Orbit

First, a dot is placed where the deferent and line intersect. This is the location of the center of the epicycle.

Drawing a circle around this center establishes Mercury’s epicycle. Note that the size of the epicycle and deferent must match the data Ptolemy had acquired.

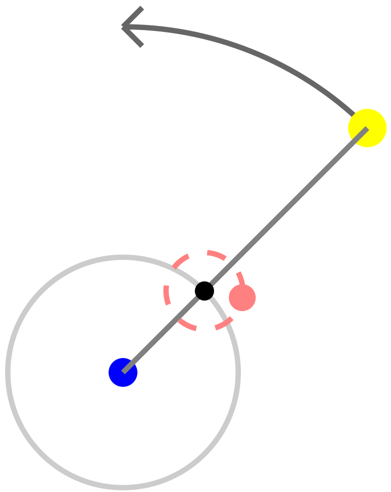
Also, keep in mind Mercury is placed first only because it moves the greatest distance in the sky when compared to other planets.



The Ptolemaic System - Key Components

Mercury’s Orbit

Mercury is then placed on this epicycle.

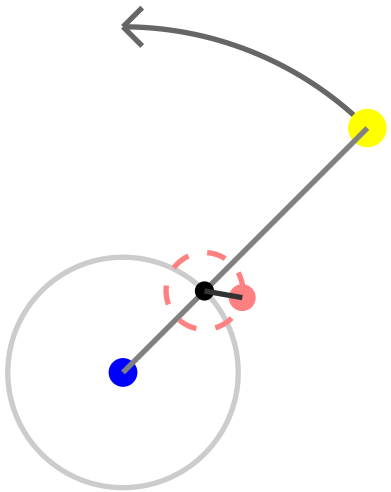


The Ptolemaic System - Key Components

Mercury's Orbit

Mercury is then placed on this epicycle.

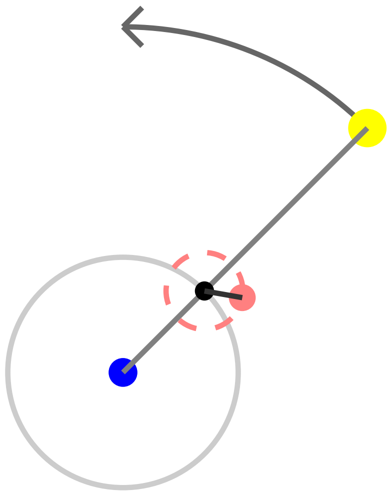
Notice that the line joining the center of the epicycle to the location of the planet is along a different direction than the line joining the Earth to the Sun.



The Ptolemaic System - Key Components

Venus’s Orbit

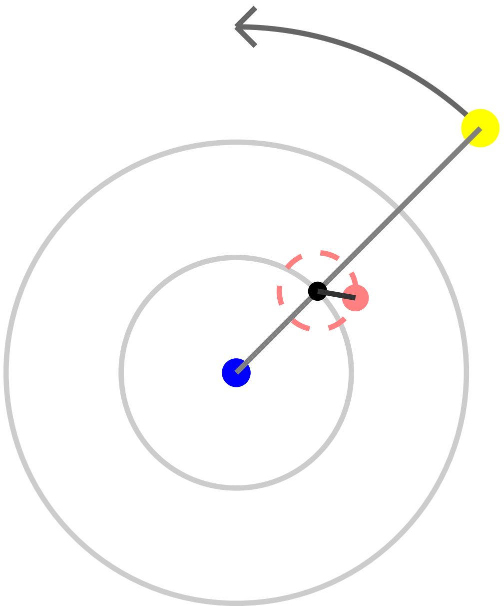
Now let’s place Venus similarly...



The Ptolemaic System - Key Components

Venus's Orbit

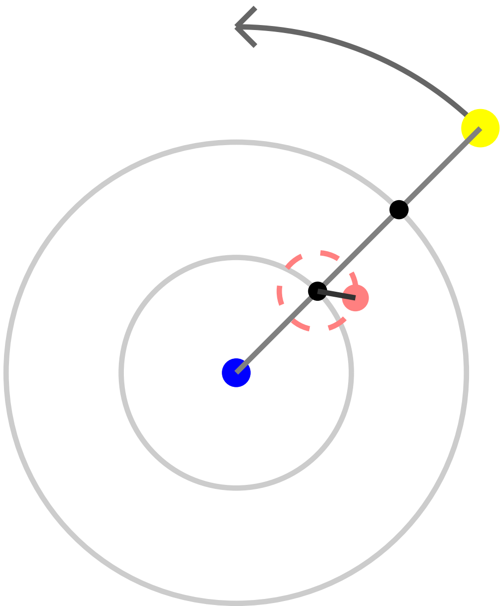
Draw out the deferent...



The Ptolemaic System - Key Components

Venus’s Orbit

Place the center of the epicycle...

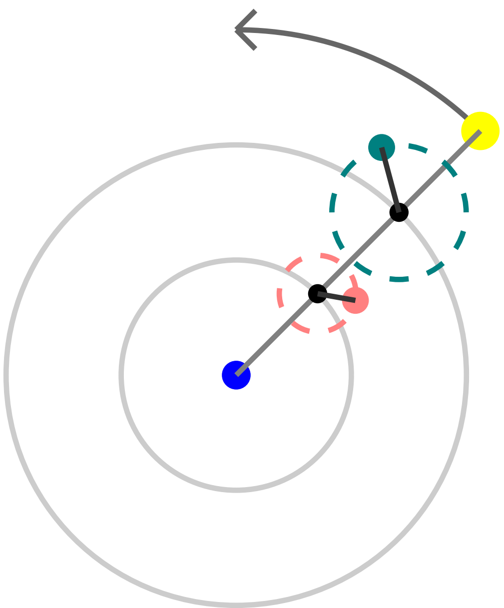


The Ptolemaic System - Key Components

Venus's Orbit

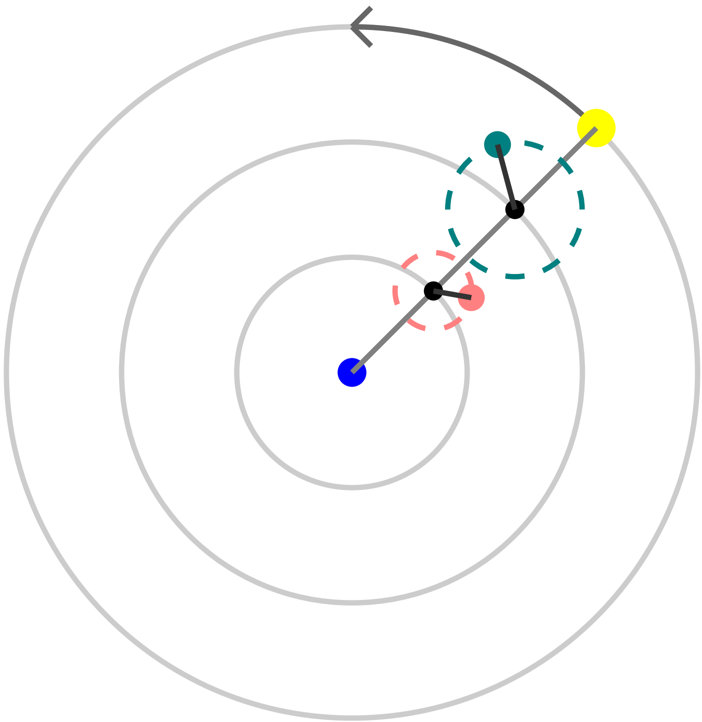
Draw the epicycle with Venus on it...

Notice again that the line joining the center of the epicycle to the location of the planet is along a different direction than the line joining the Earth to the Sun.



The Ptolemaic System - Key Components

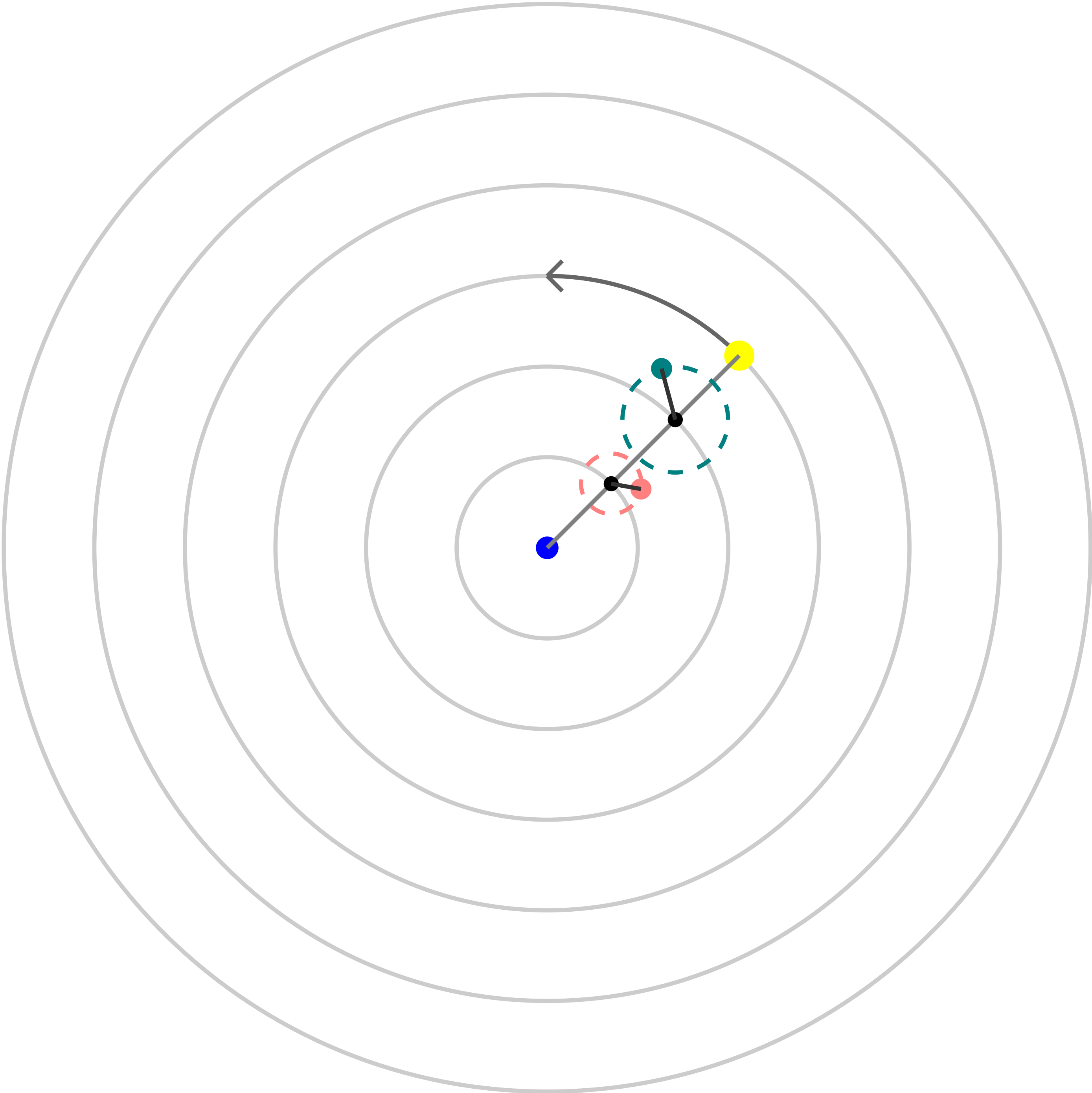
The Inner Solar System is complete.



The Ptolemaic System - Key Components

The outer planets are placed similarly but have slightly different restrictions.

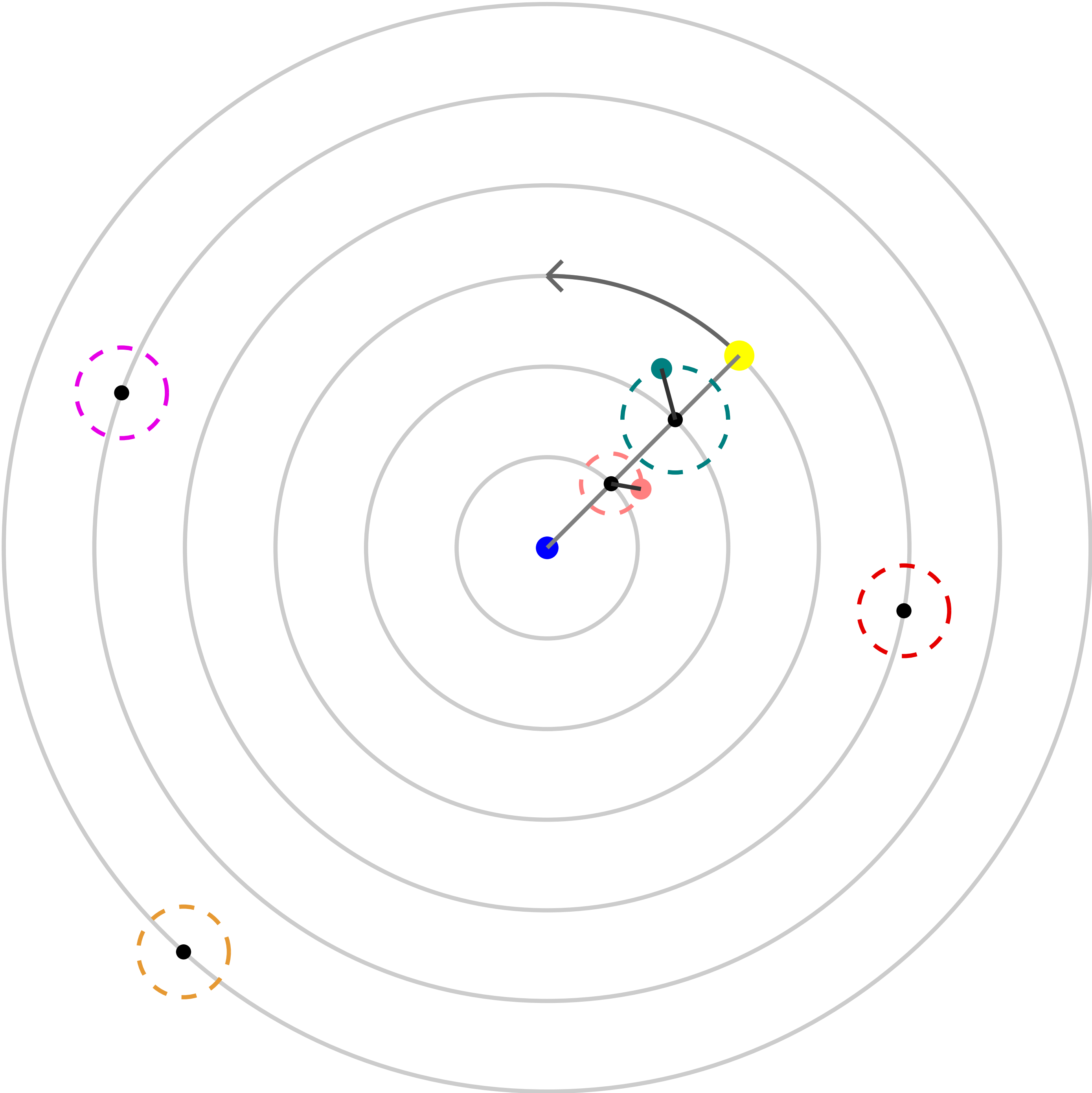
The deferents to the outer planets are shown below.



The Ptolemaic System - Key Components

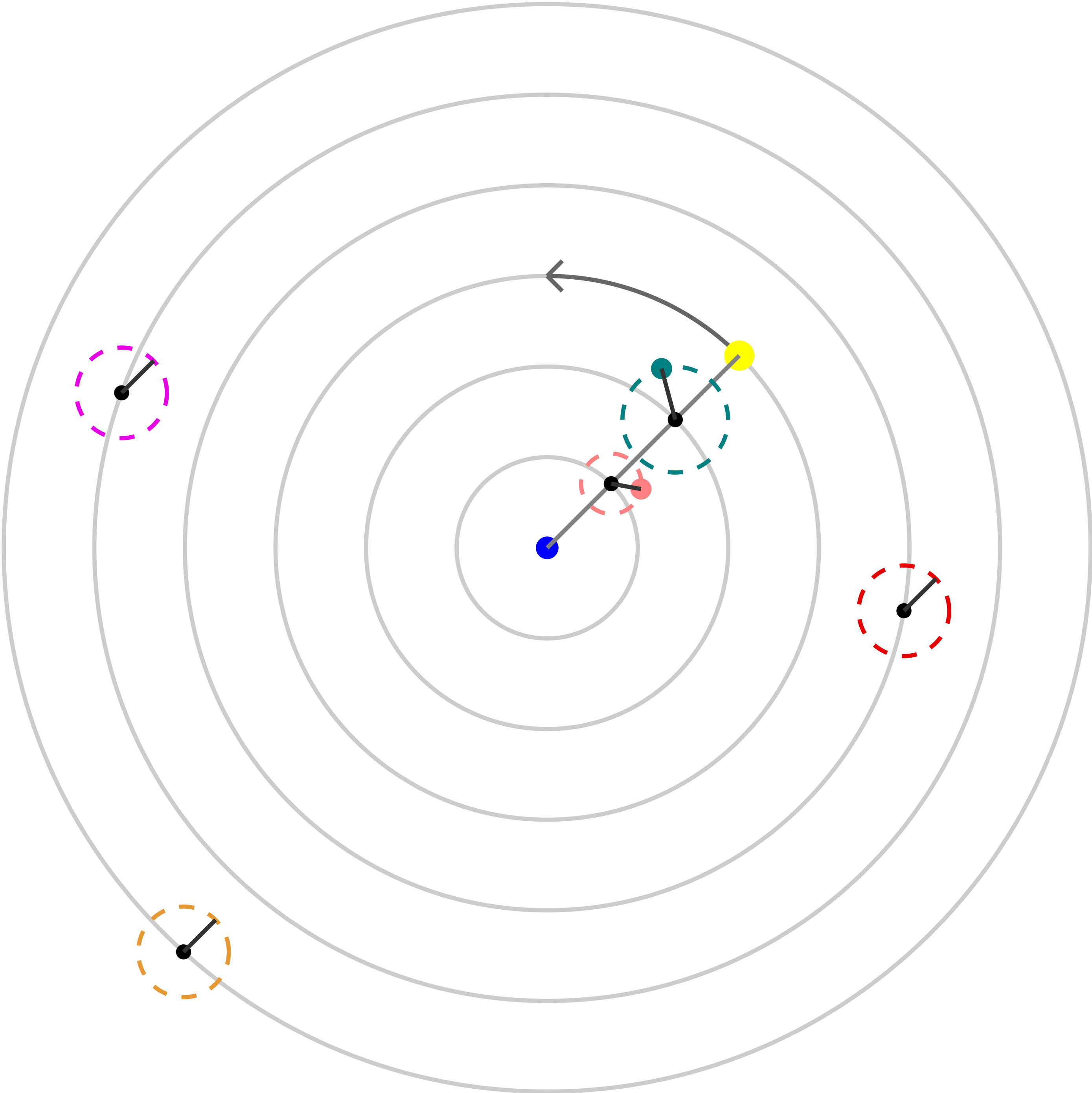
The outer planets are placed similarly but have slightly different restrictions.

First the centers of the epicycles to the outer planets are not arranged on any Earth to Sun line.
Also the size of an epicycle has no correlation with distance.



The Ptolemaic System - Key Components

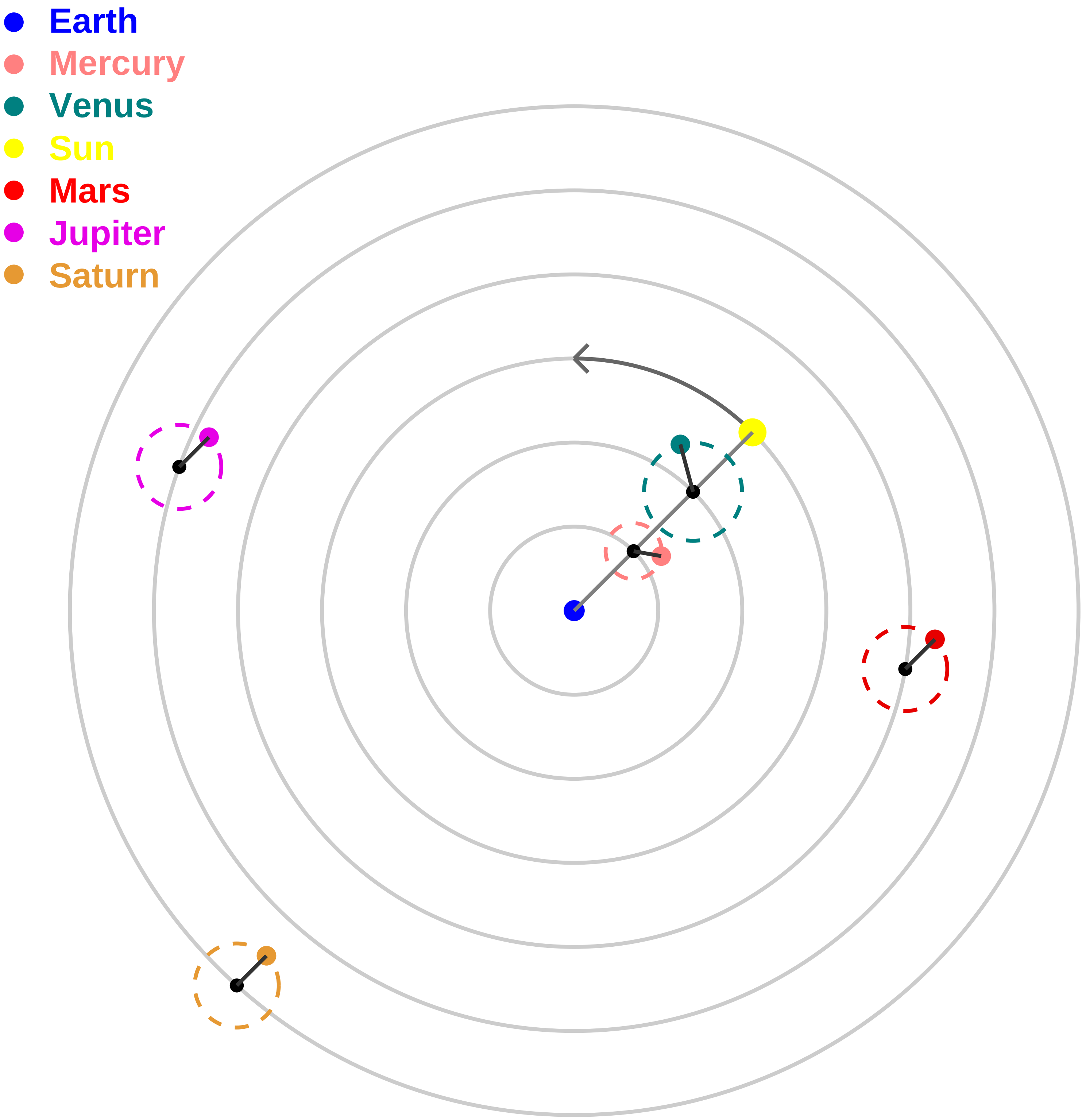
The other restriction is that the line joining the center of an epicycle to an outer planet must be parallel to the Earth to Sun line.



The Ptolemaic System - Key Components

The outer planets are placed at the tips of these parallel lines.

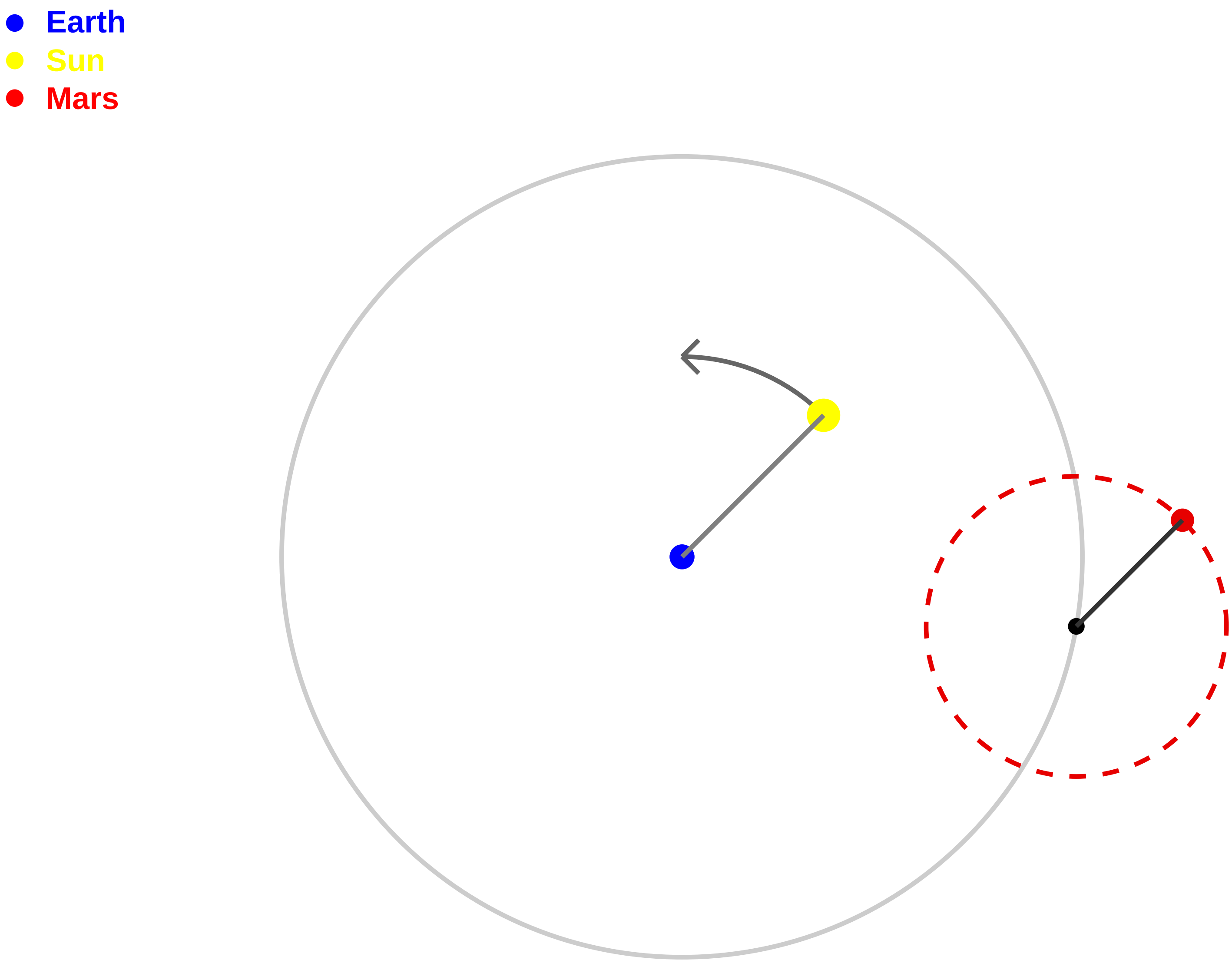
Though there are other refinements, this is the basic Ptolemaic system.



The Ptolemaic System - Mars' Retrograde Motion

Let's use the Earth-Sun-Mars System;
although any 3-body system can be used.

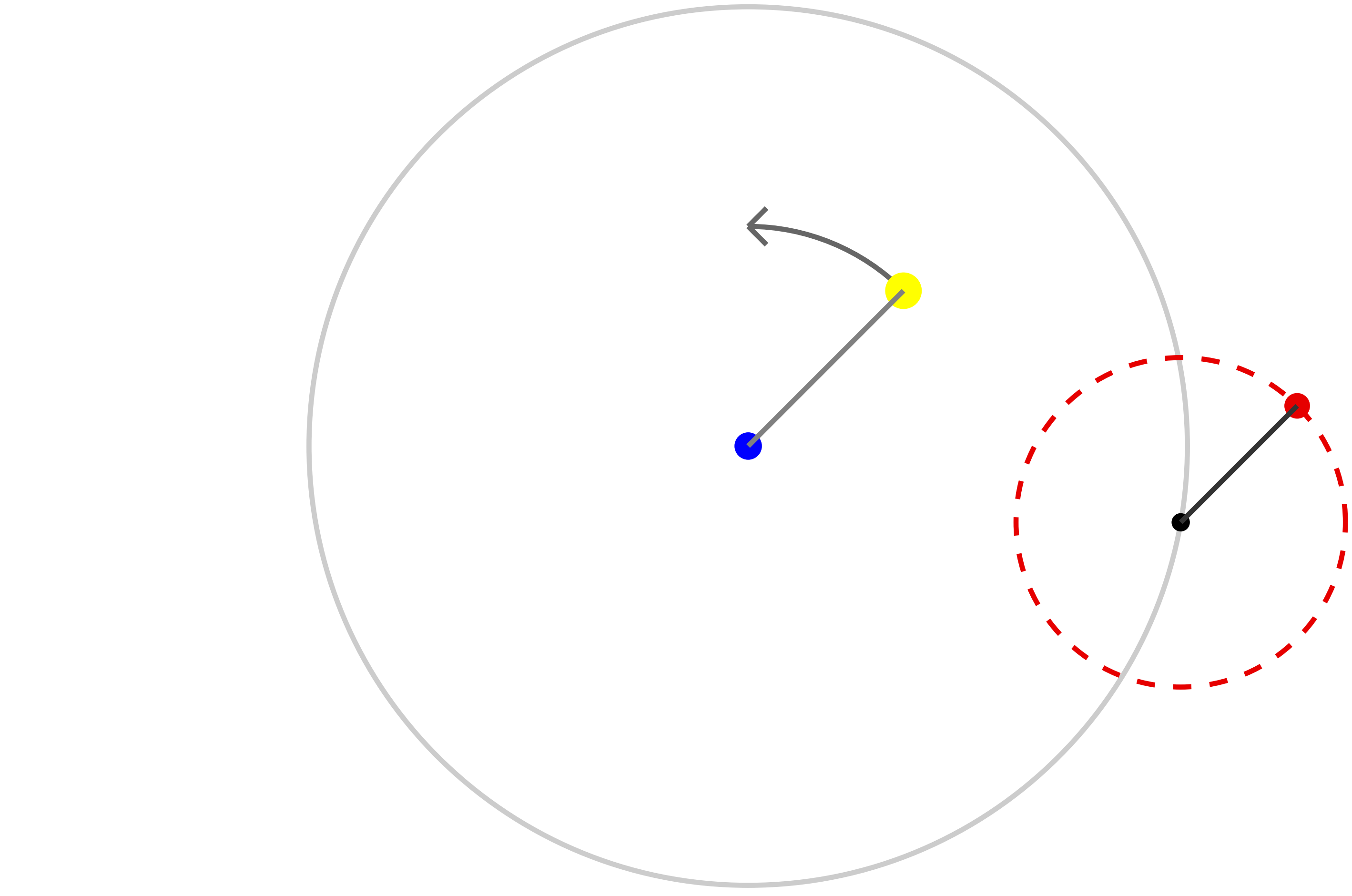
By allowing Mars and the Sun to move according to the rules and restrictions mentioned earlier, we should still be able to observe Mars move backwards. as it does in the night sky about every two years.



The Ptolemaic System - Mars' Retrograde Motion

Tracing Mars' path shows...

forward motion as expected.



The Ptolemaic System - Mars' Retrograde Motion

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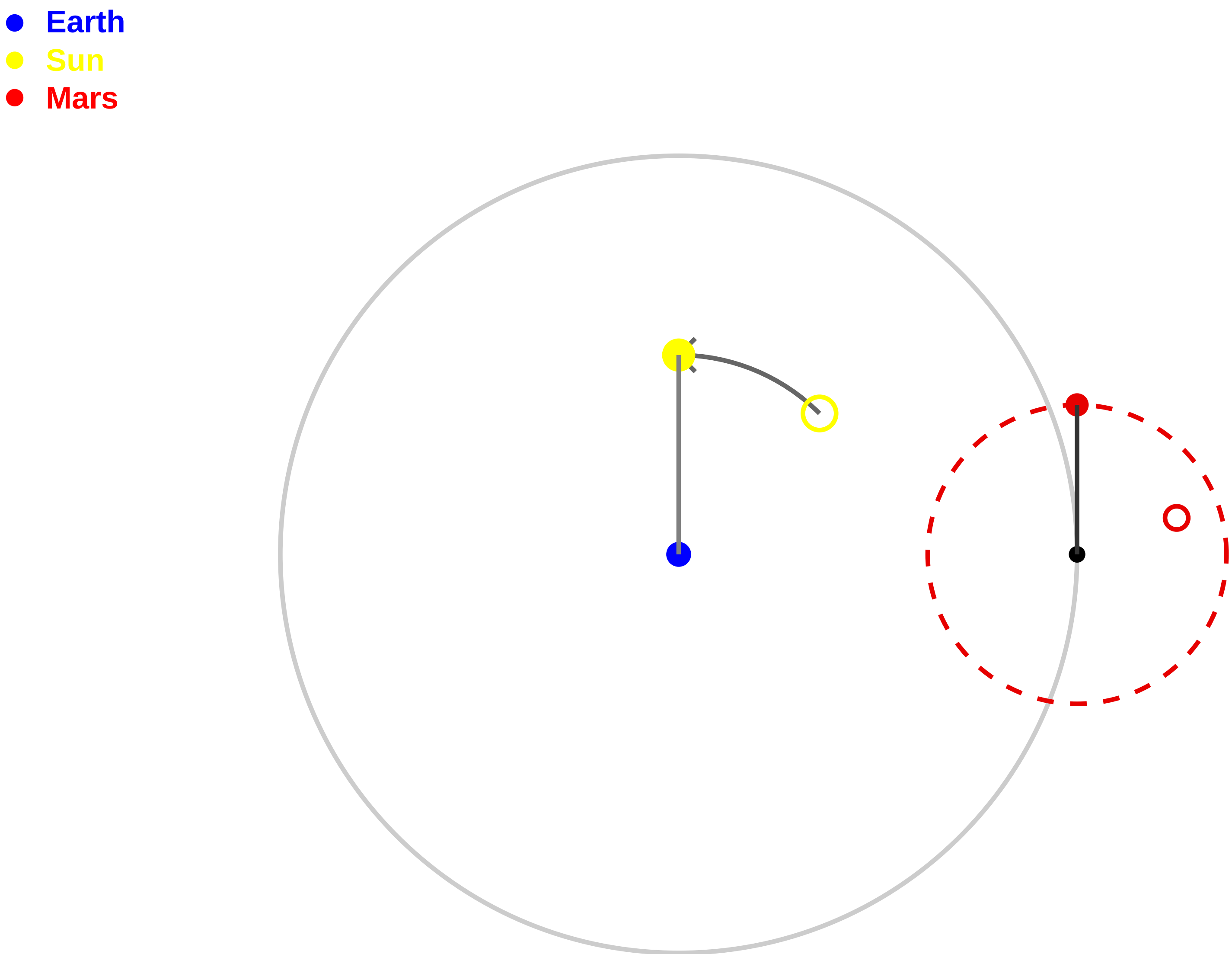
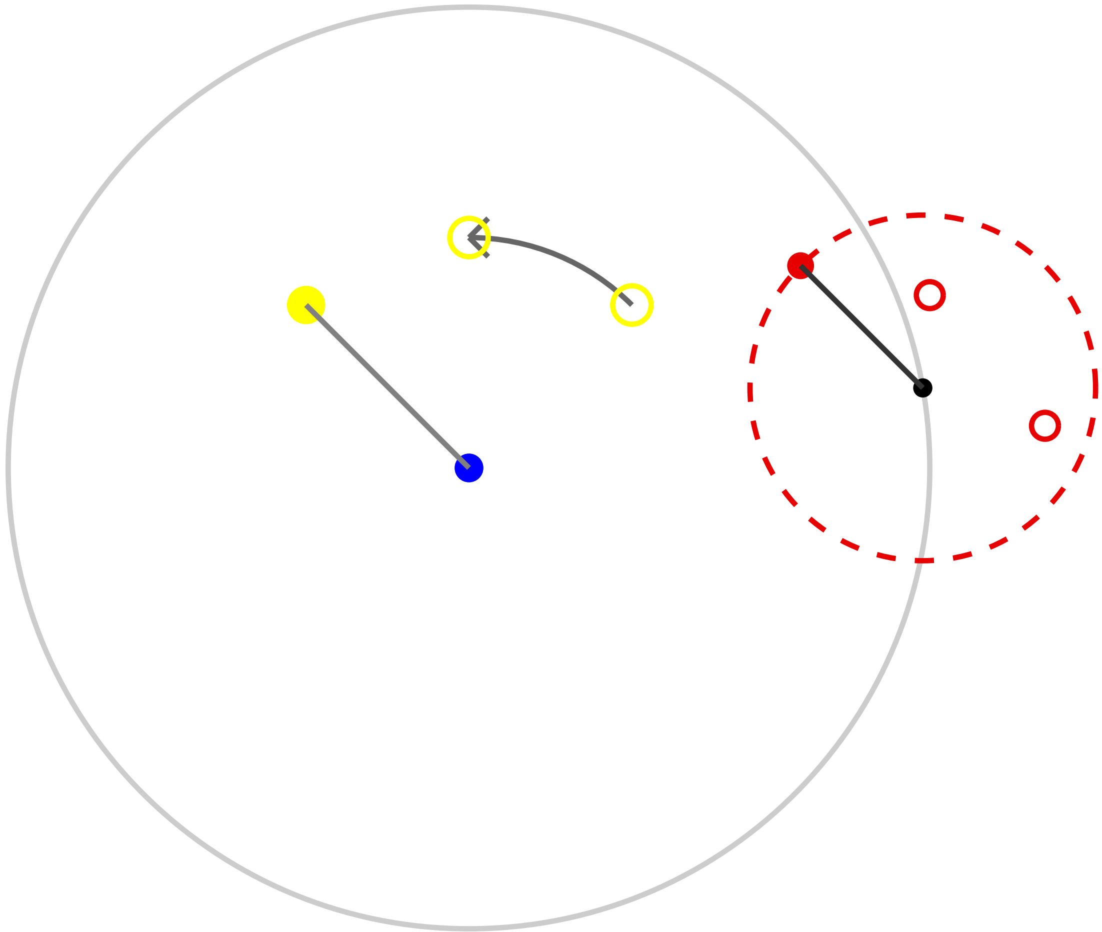


The Ptolemaic System - Mars’ Retrograde Motion

Tracing Mars’ path shows...

forward motion as expected.

- **Earth**
- **Sun**
- **Mars**

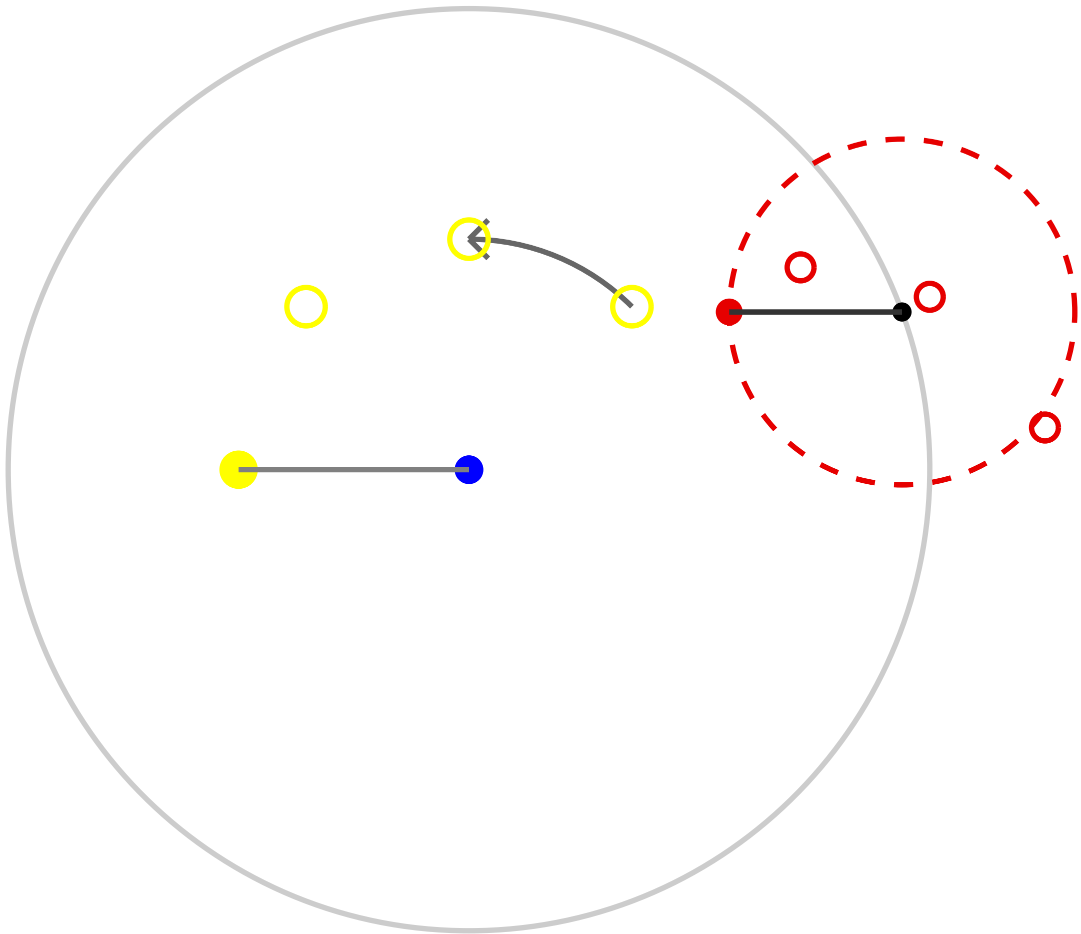


The Ptolemaic System - Mars' Retrograde Motion

Tracing Mars' path shows...

Mars stationary in sky!

- Earth
- Sun
- Mars

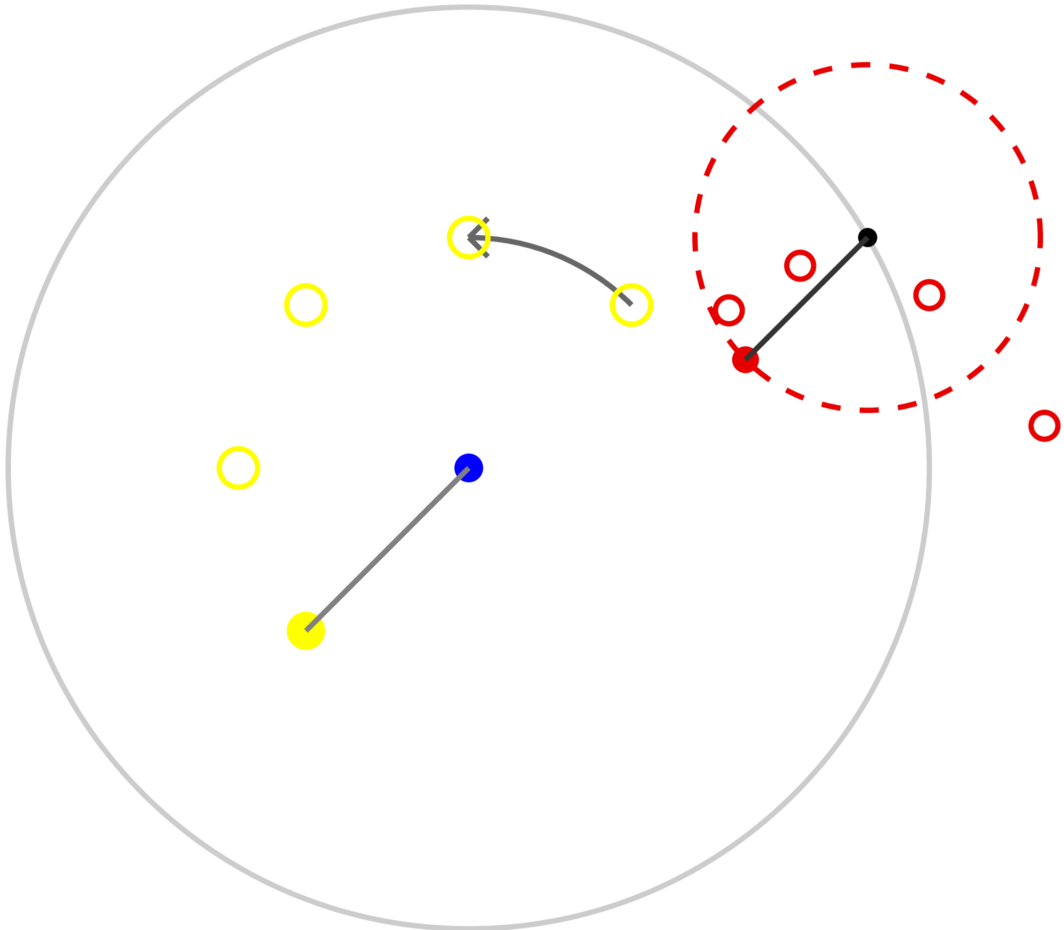


The Ptolemaic System - Mars' Retrograde Motion

Tracing Mars' path shows...

Retrograde motion!

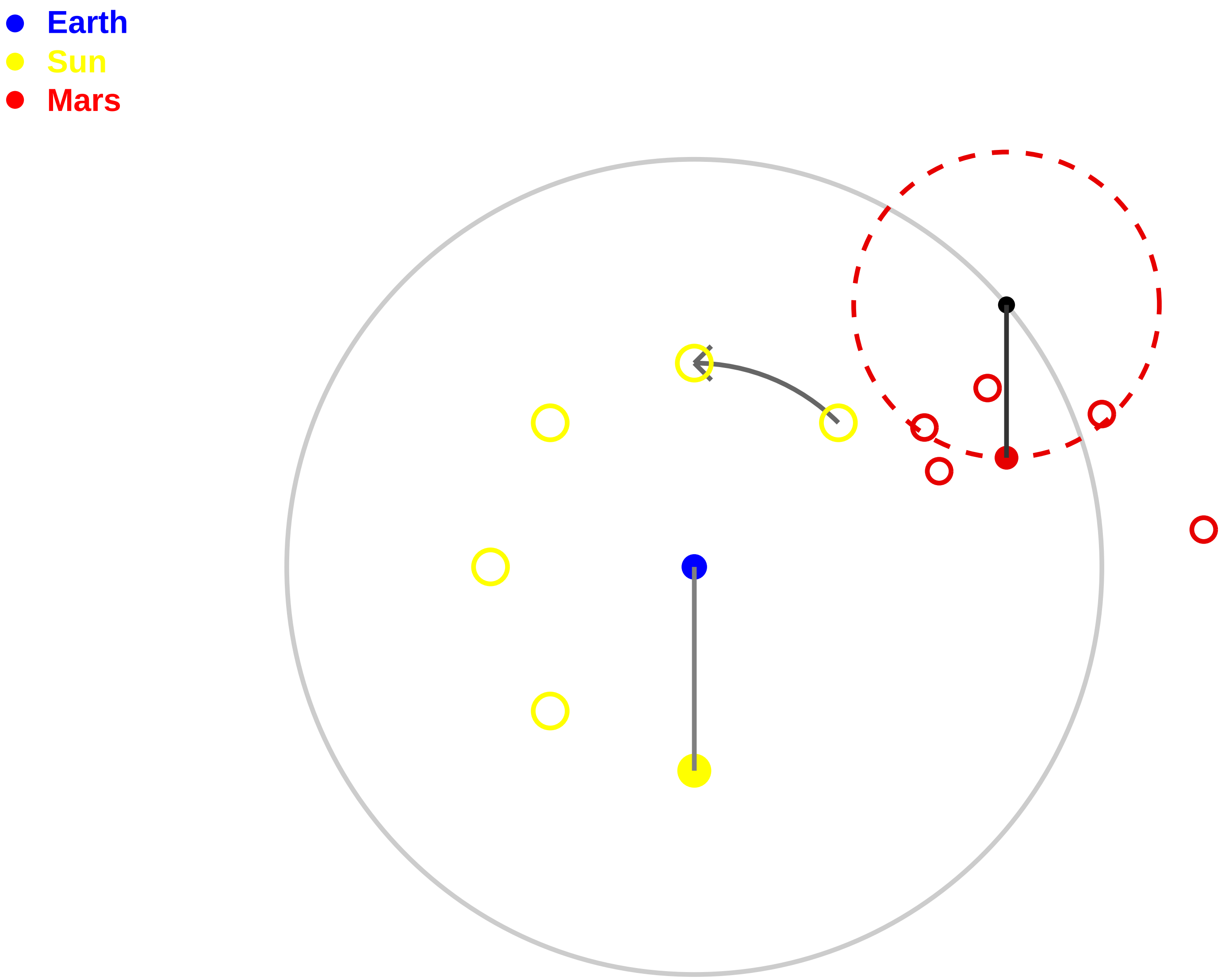
- Earth
- Sun
- Mars



The Ptolemaic System - Mars’ Retrograde Motion

Tracing Mars’ path shows...

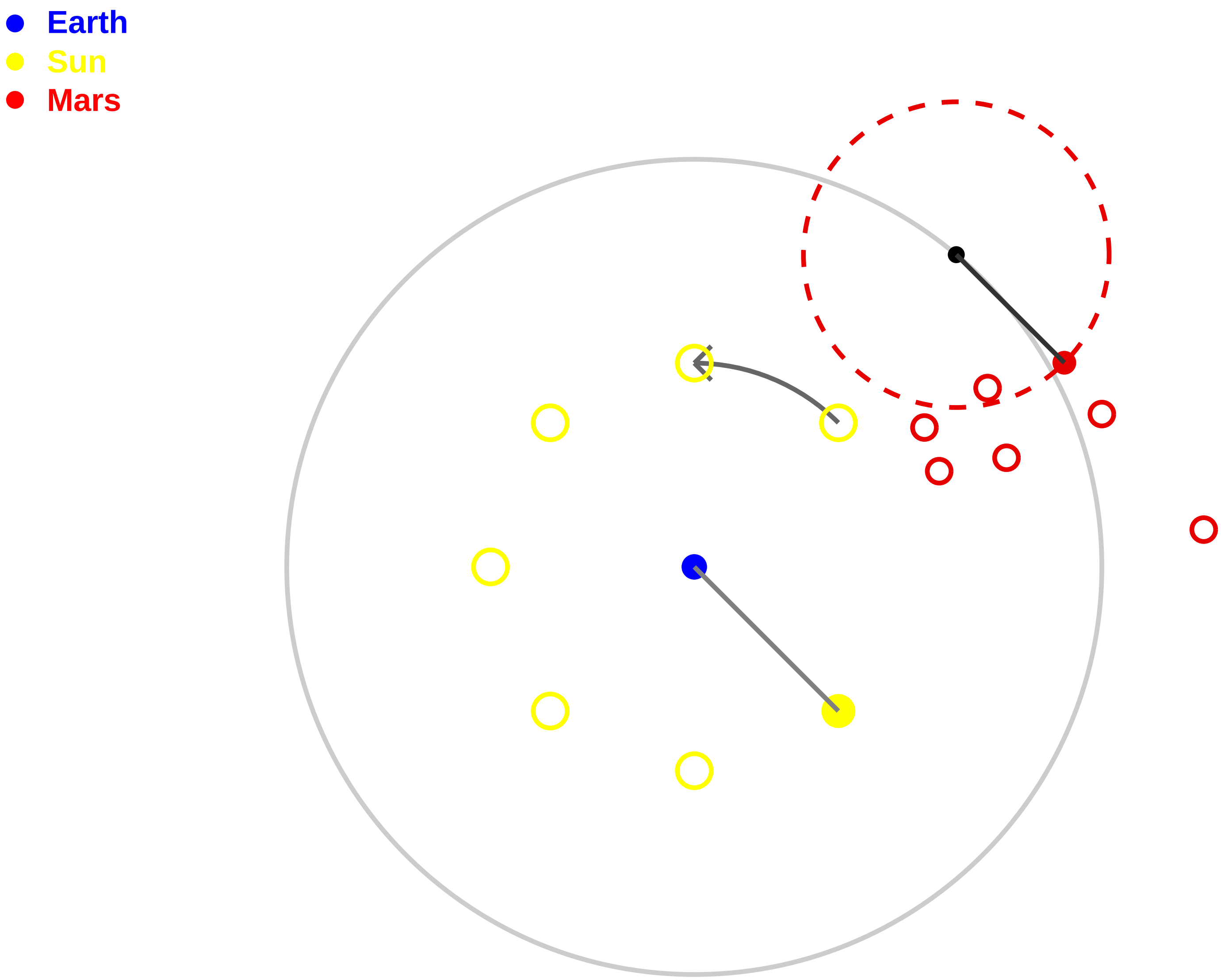
Retrograde motion!



The Ptolemaic System - Mars' Retrograde Motion

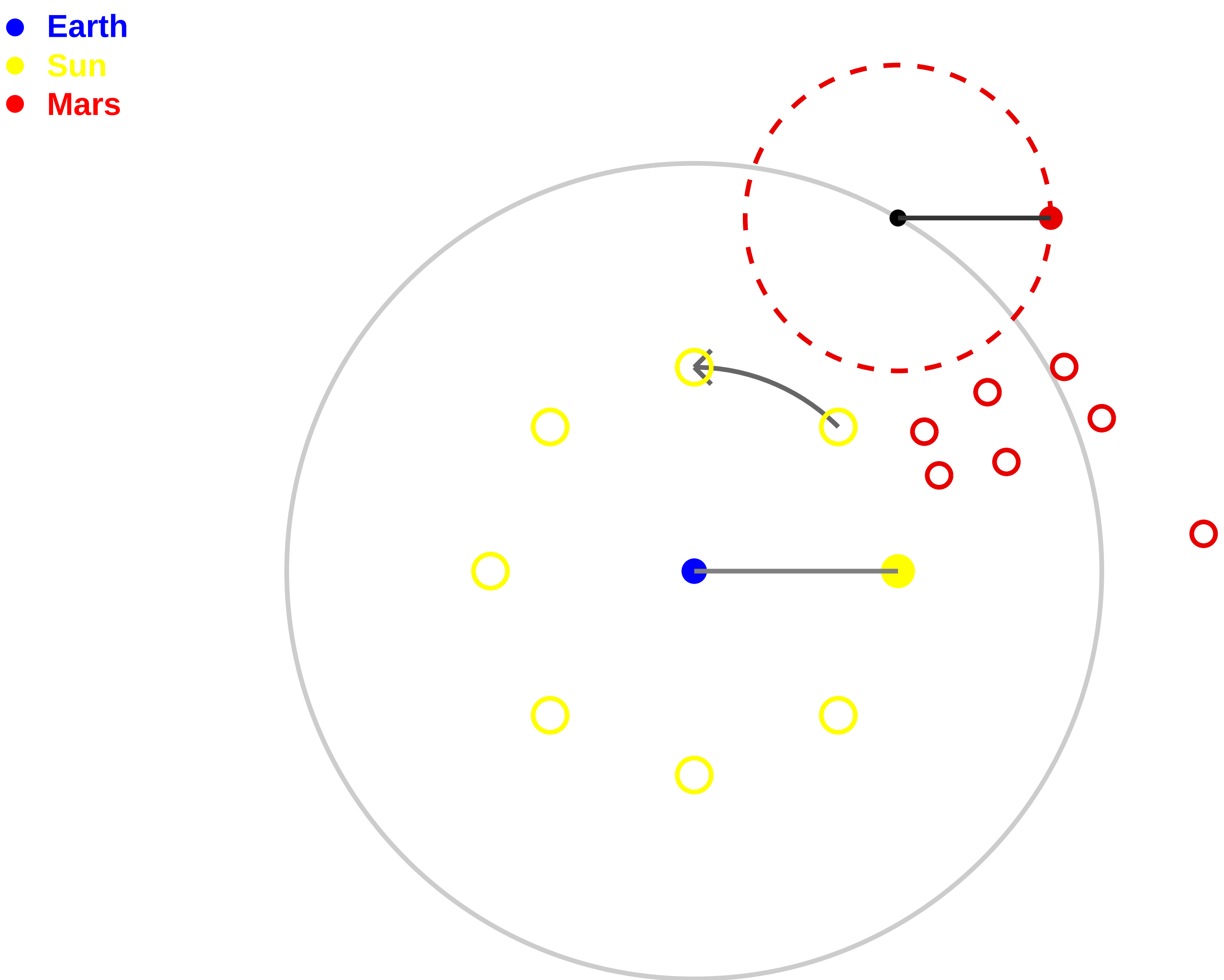
Tracing Mars' path shows...

Mars resumes forward motion.



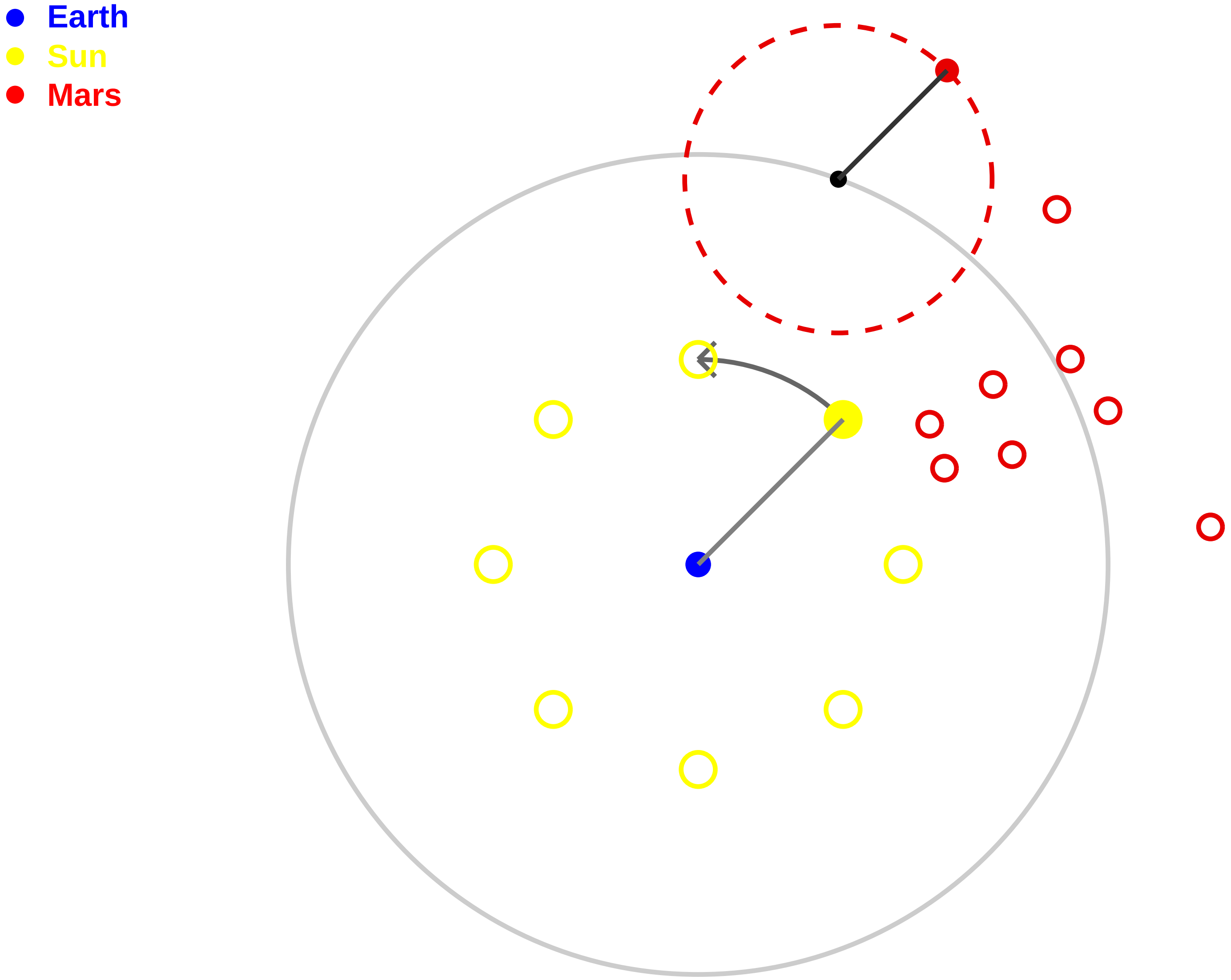
The Ptolemaic System - Mars' Retrograde Motion

Tracing Mars' path shows...
forward motion as expected.



The Ptolemaic System - Mars’ Retrograde Motion

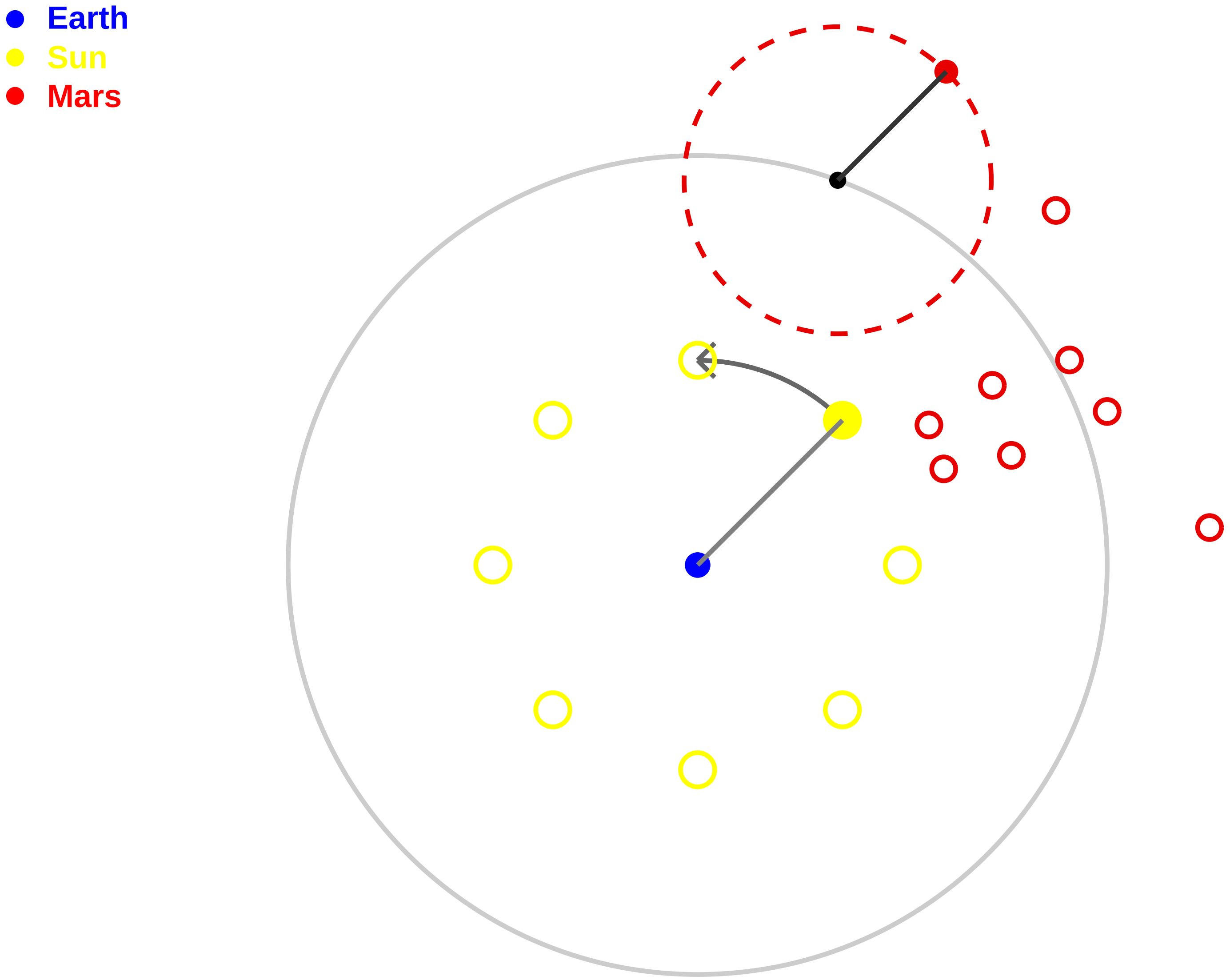
Tracing Mars’ path shows...
forward motion as expected.



The Ptolemaic System - Mars' Retrograde Motion

So Ptolemy's model was able to explain the retrograde motion of the planets. Though in this case the epicycles were a bit exaggerated the principle remains the same.

The "loop" created by Mars also allowed Ptolemy to explain how the brightness of planets could change as well. Notice how Mars gets closer to Earth as it retrogrades. The shorter distance causes Mars to appear brighter in the sky.



The Ptolemaic System

In fact, adding another layer of epicycles and another point Ptolemy called an "equant" allowed him to also explain the eccentricities of orbits and refine his predictions.

His model was so successful that it would take about 1400 years for the next astronomical revolution to occur!

Sources

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