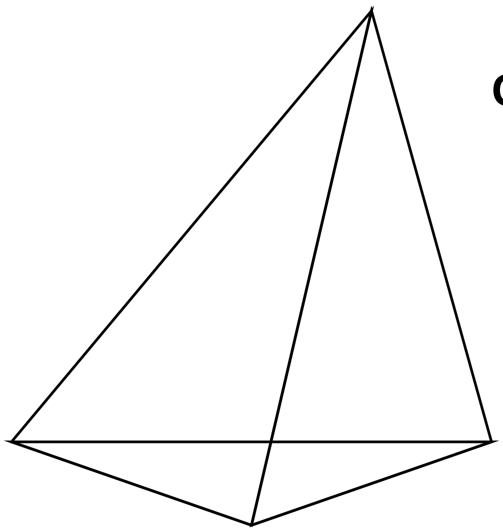
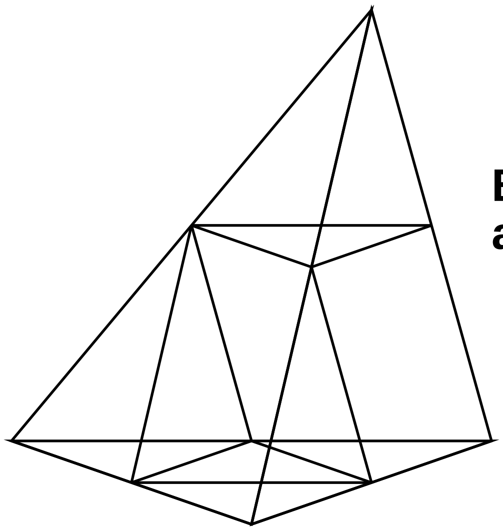
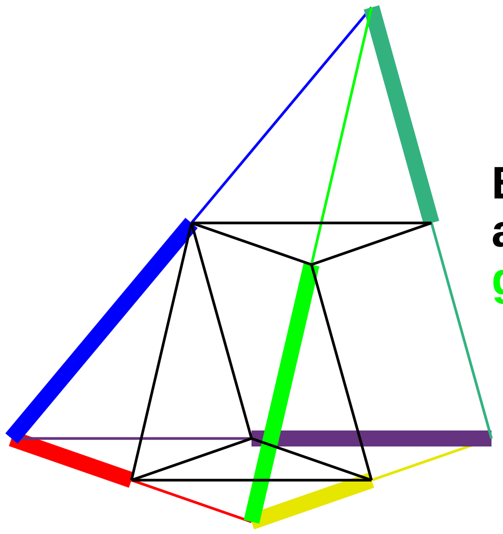
Euclid’s XII prop. #3 :
A pyramid can be divided into two pyramids, equal and similar to one another, and each similar to the whole, and into two equal prisms, and the two prisms are greater than half of the whole pyramid.



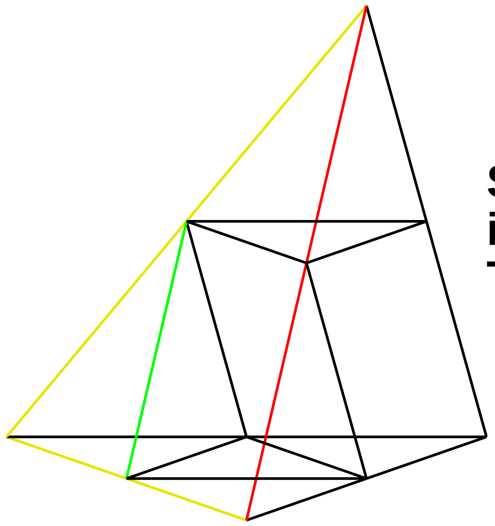
Construct a pyramid:



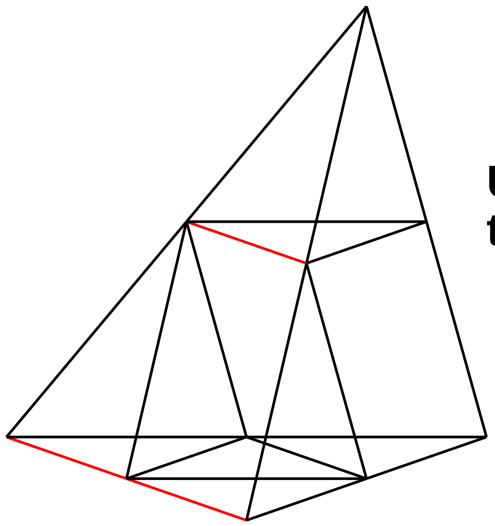
Bisect each side of the pyramid
and connect the bisected points as shown:



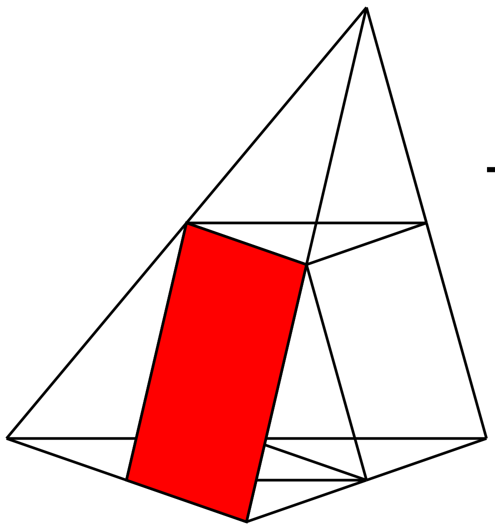
By Definition of bisection, the two red lines are equal, as well as the two blue, yellow, green, purple, and teal lines.



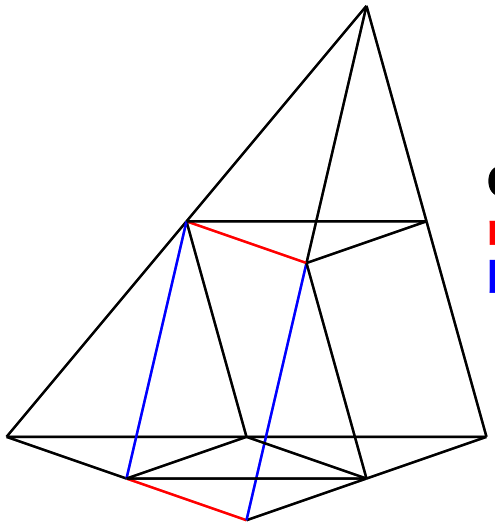
Since the green line cuts both yellow lines in half, it is parallel to the red line
This is proved true via Euclid VI. prop. #2



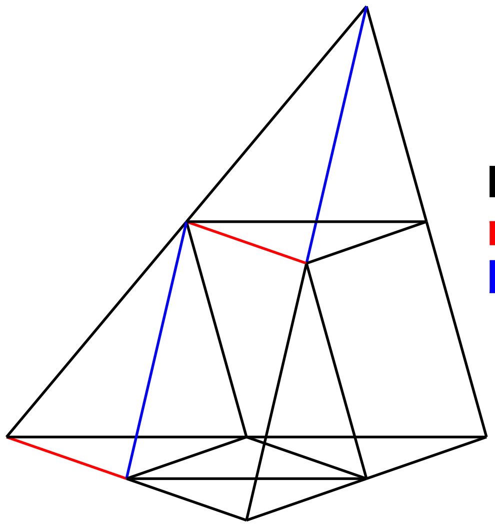
Using identical logic, it is seen that the red lines are parallel.



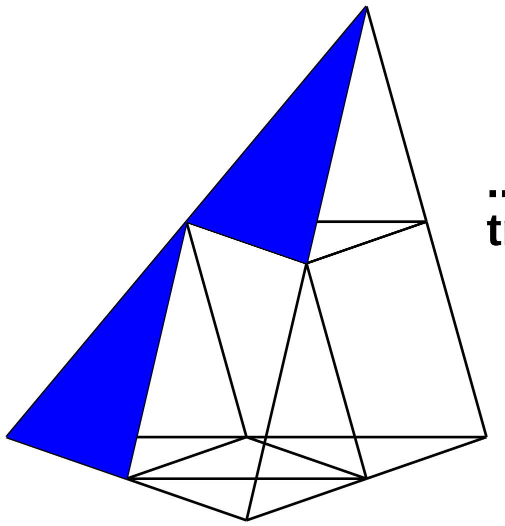
Therefore, we have the following parallelogram



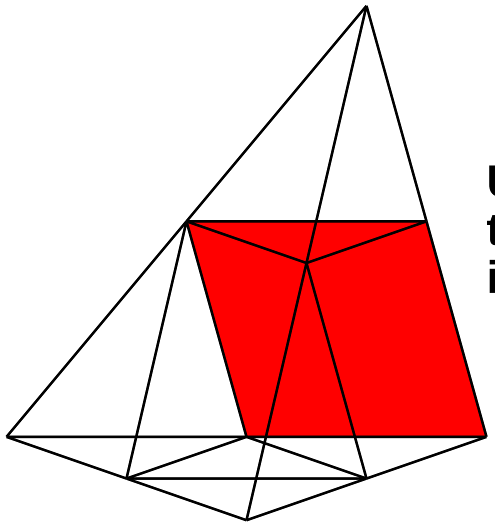
Opposite sides of parrallelograms are equal:
red = red
blue = blue



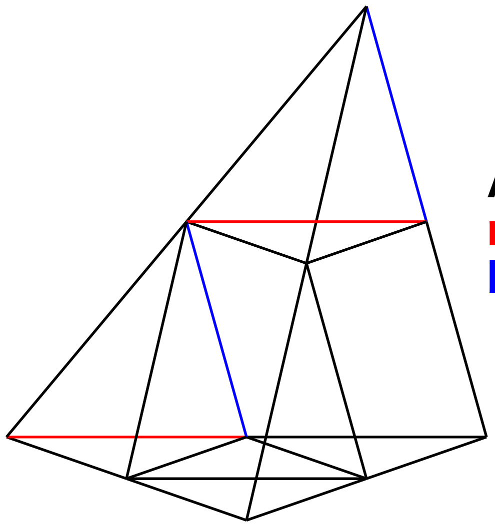
Due to equalities shown earlier,
red = red
blue = blue...



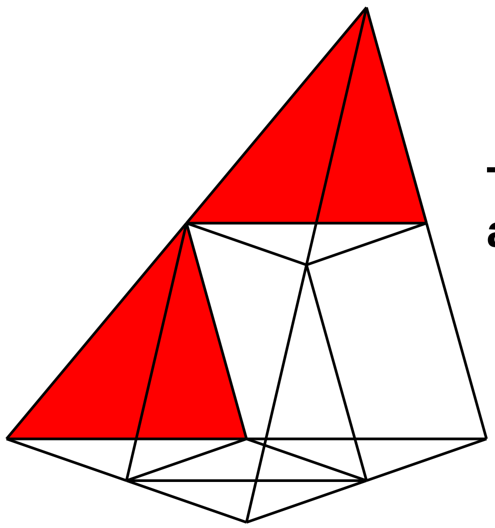
... so therefore the two blue
triangles are equal (since all sides equal)



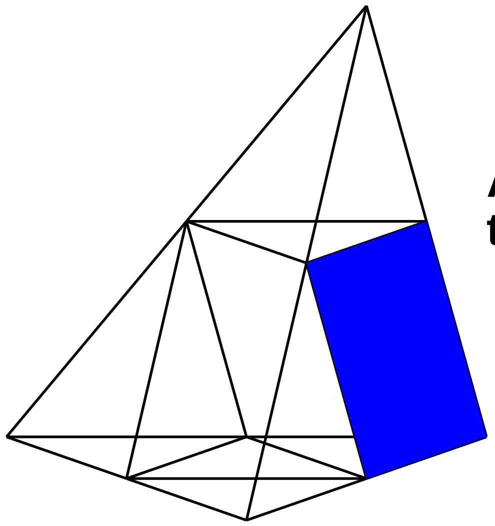
Using the same logic already seen,
the following red polygon
is a parallelogram



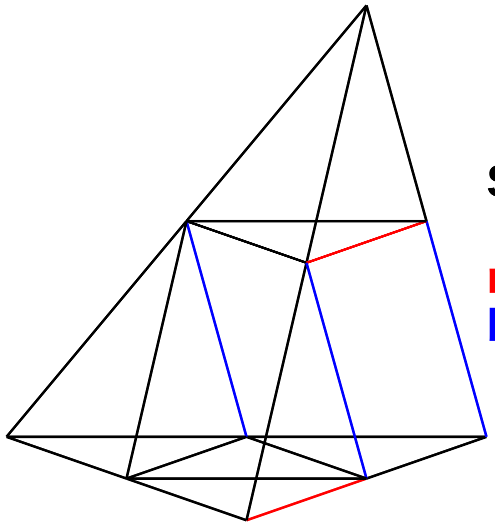
And therefore the following lines are equal:
red = red
blue = blue



Therefore the red triangles
are equal (all sides equal, as before)

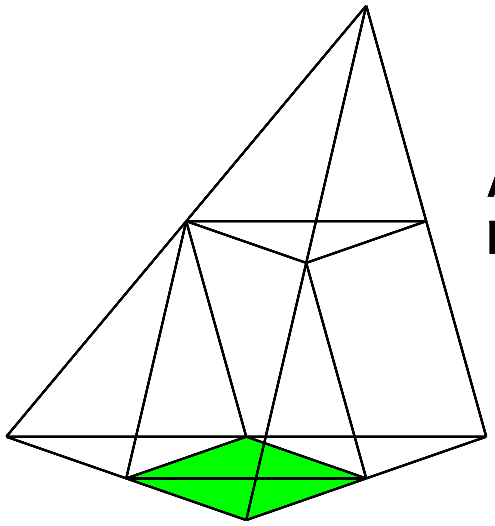


Again note that following earlier logic, the blue polygon is a parallelogram

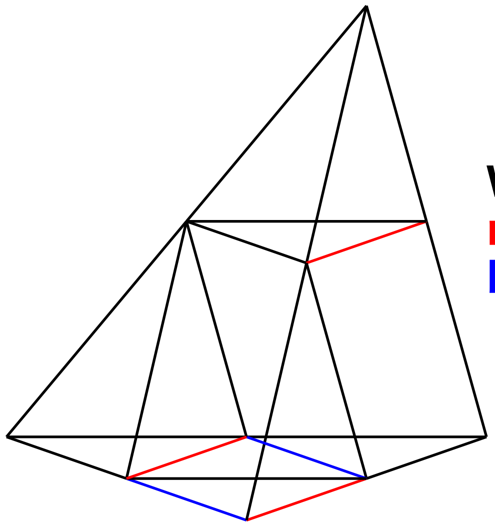


So the following lines are equal, once again

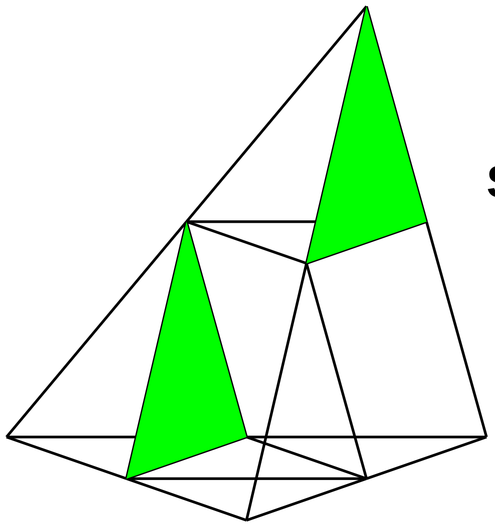
red = red
blue = blue



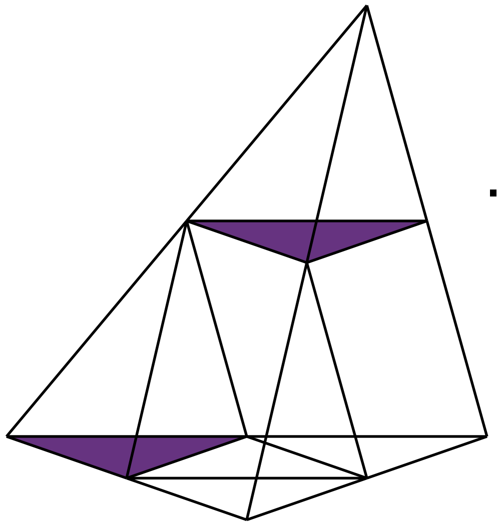
And yet again, we find the following green polygon to be a parallelogram



Which gives us the important equalities:
red = red
blue = blue



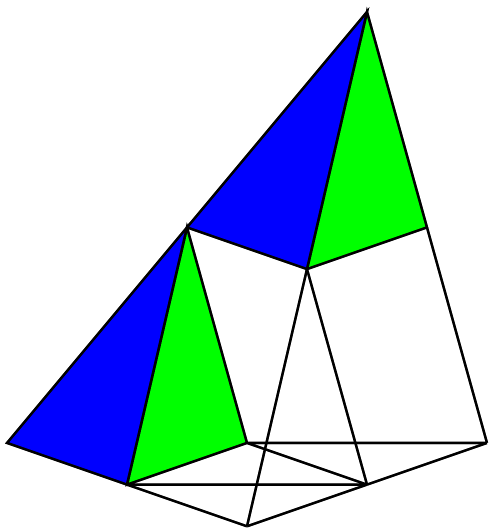
So the two green triangles are equal...



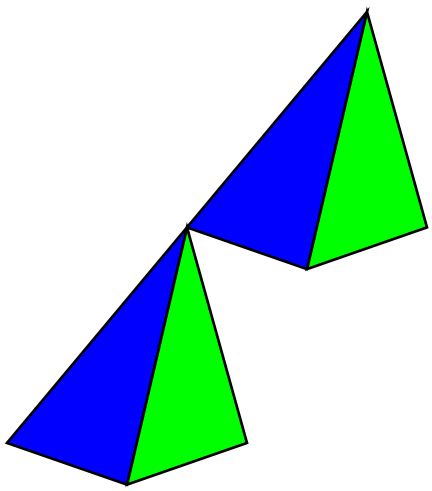
...And the two purple triangles are also equal.

This is because for both sets of equal triangles, we've already seen that they're corresponding sides are equal
(same logic as earlier)

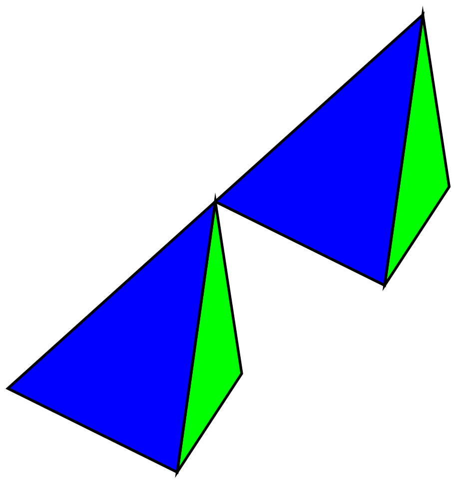
Therefore, these two pyramids are equal and similar:



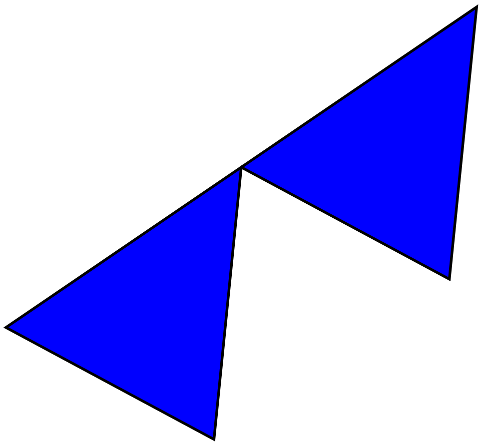
Therefore, these two pyramids are equal and similar:



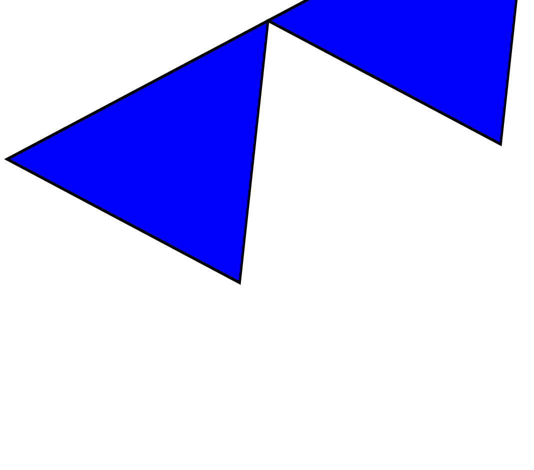
Therefore, these two pyramids are equal and similar:



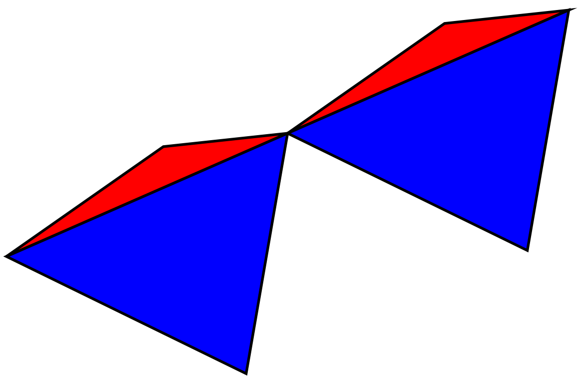
Therefore, these two pyramids are equal and similar:



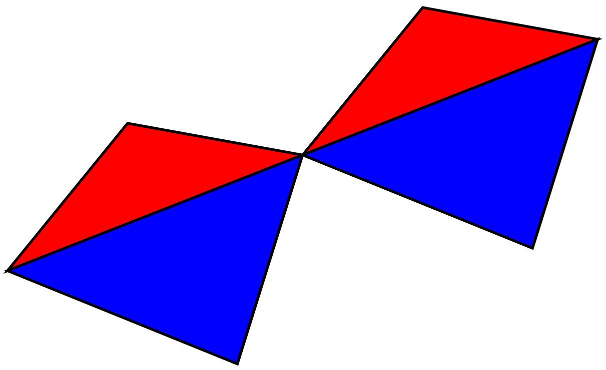
Therefore, these two pyramids are equal and similar:



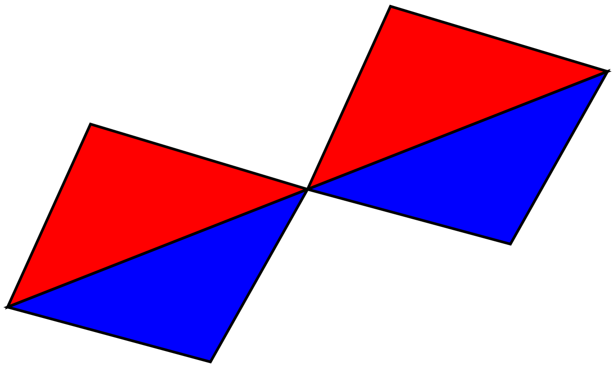
Therefore, these two pyramids are equal and similar:



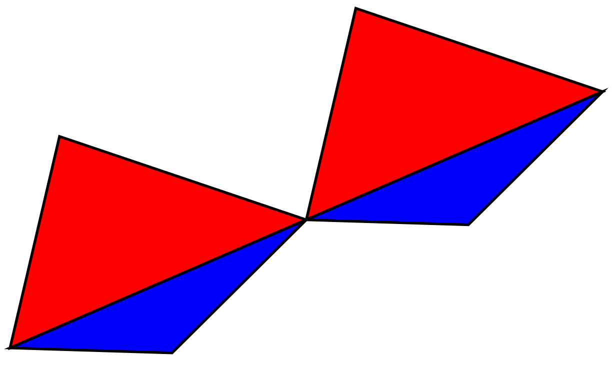
Therefore, these two pyramids are equal and similar:



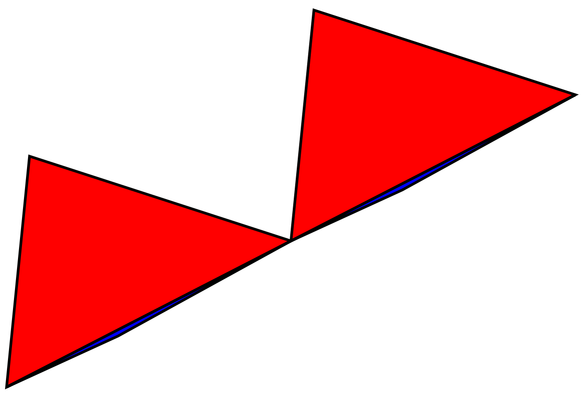
Therefore, these two pyramids are equal and similar:



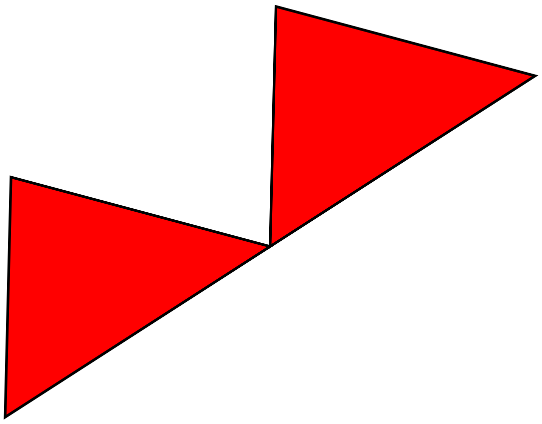
Therefore, these two pyramids are equal and similar:



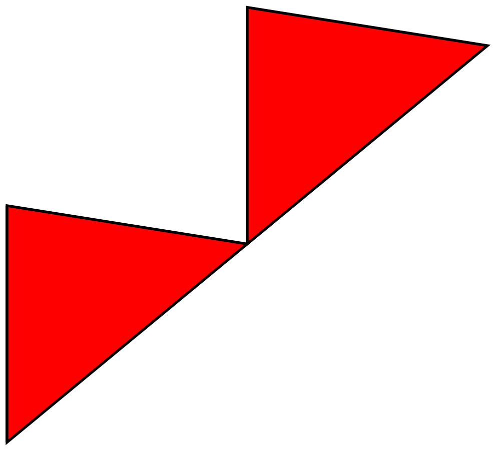
Therefore, these two pyramids are equal and similar:



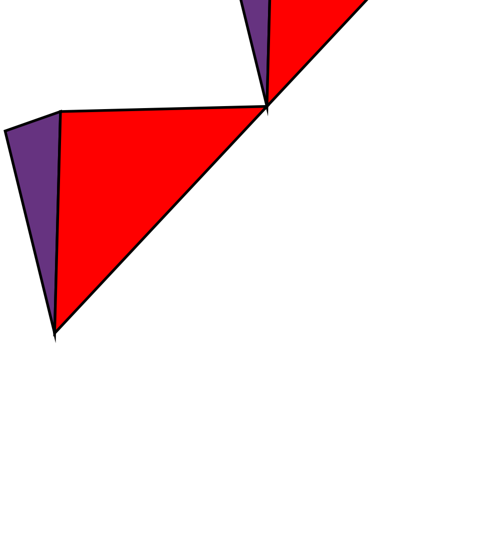
Therefore, these two pyramids are equal and similar:



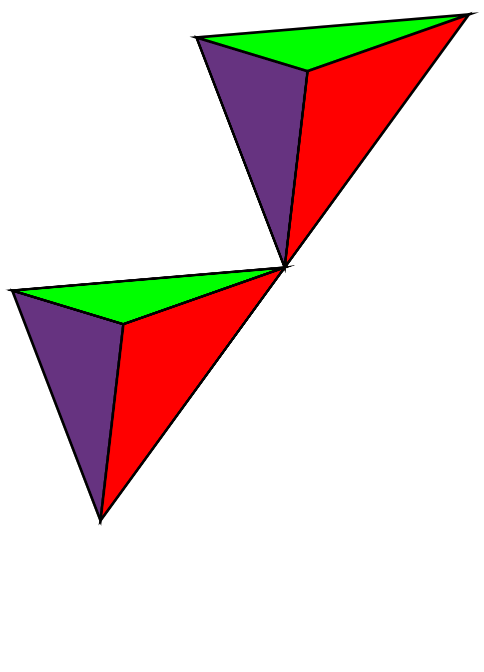
Therefore, these two pyramids are equal and similar:



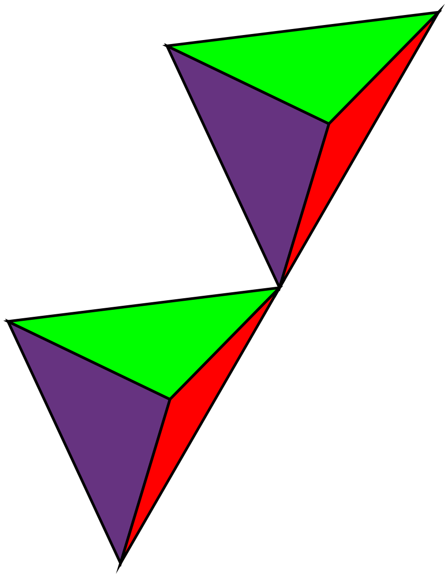
Therefore, these two pyramids are equal and similar:



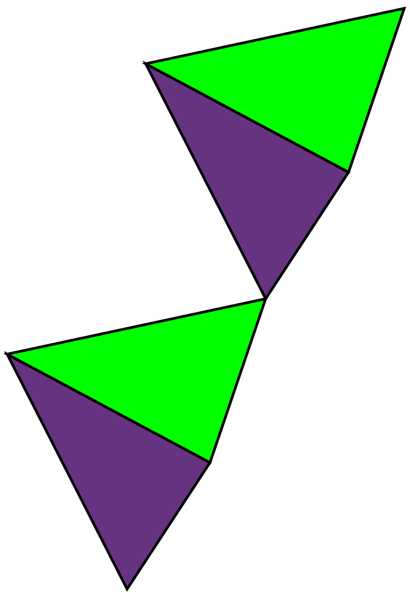
Therefore, these two pyramids are equal and similar:



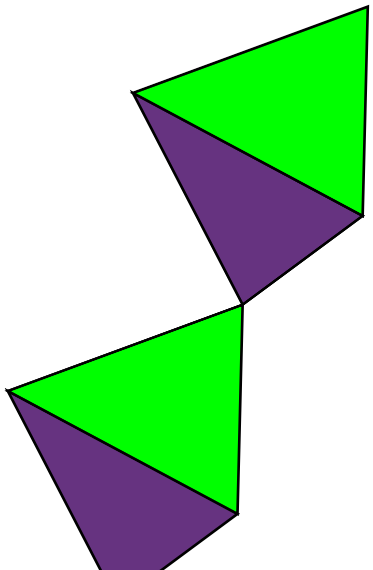
Therefore, these two pyramids are equal and similar:



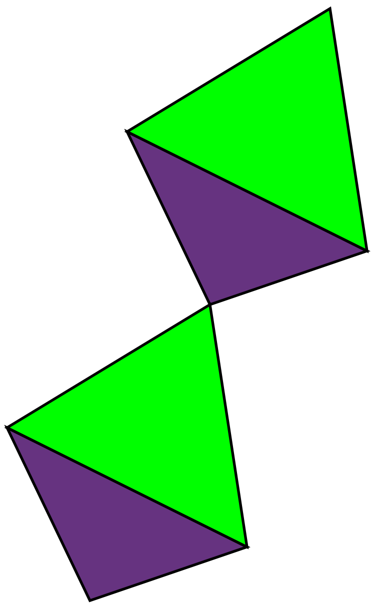
Therefore, these two pyramids are equal and similar:



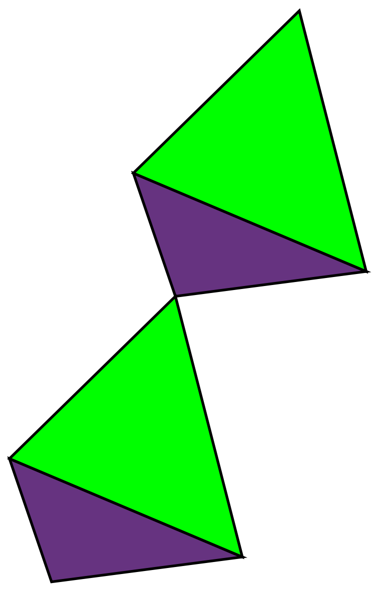
Therefore, these two pyramids are equal and similar:



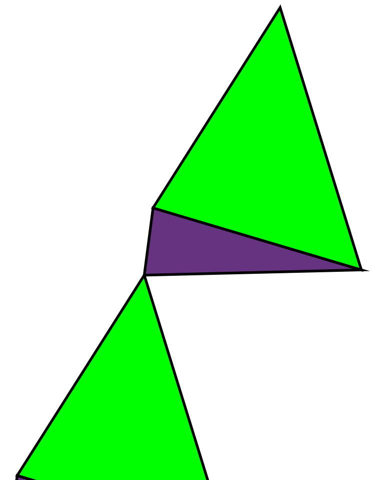
Therefore, these two pyramids are equal and similar:



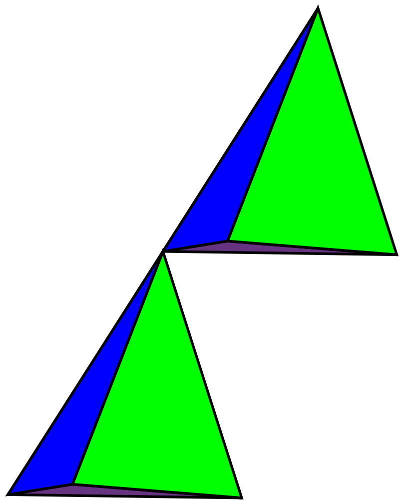
Therefore, these two pyramids are equal and similar:



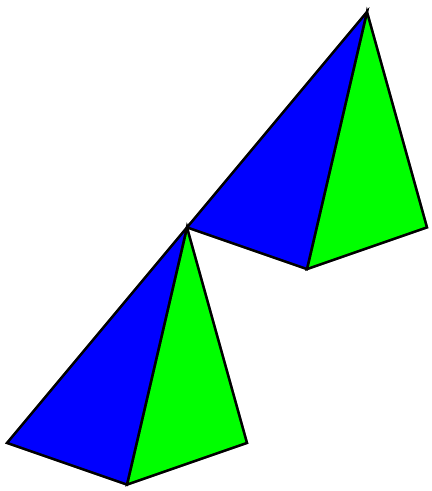
Therefore, these two pyramids are equal and similar:



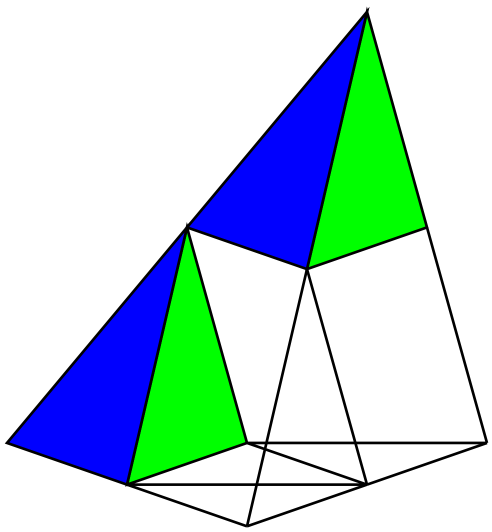
Therefore, these two pyramids are equal and similar:



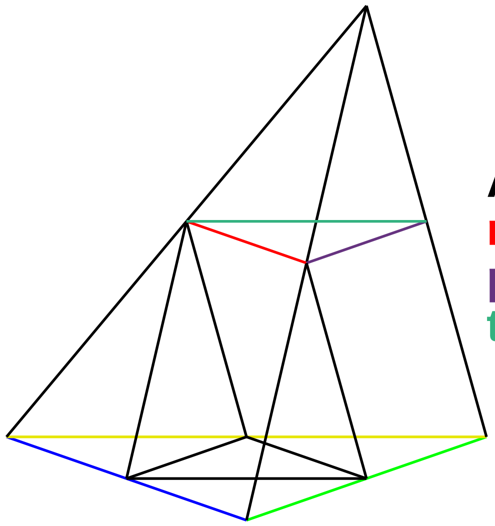
Therefore, these two pyramids are equal and similar:



Therefore, these two pyramids are equal and similar:

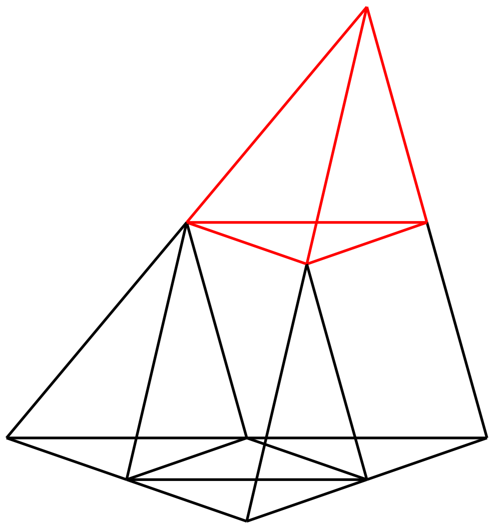


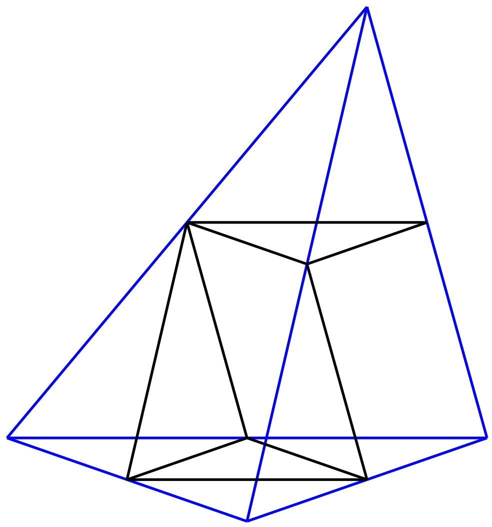
Since they are defined be equal planes (Euclid XI def. #10)



Also, we've seen that:
red = 1/2 blue
purple = 1/2 green
teal = 1/2 yellow

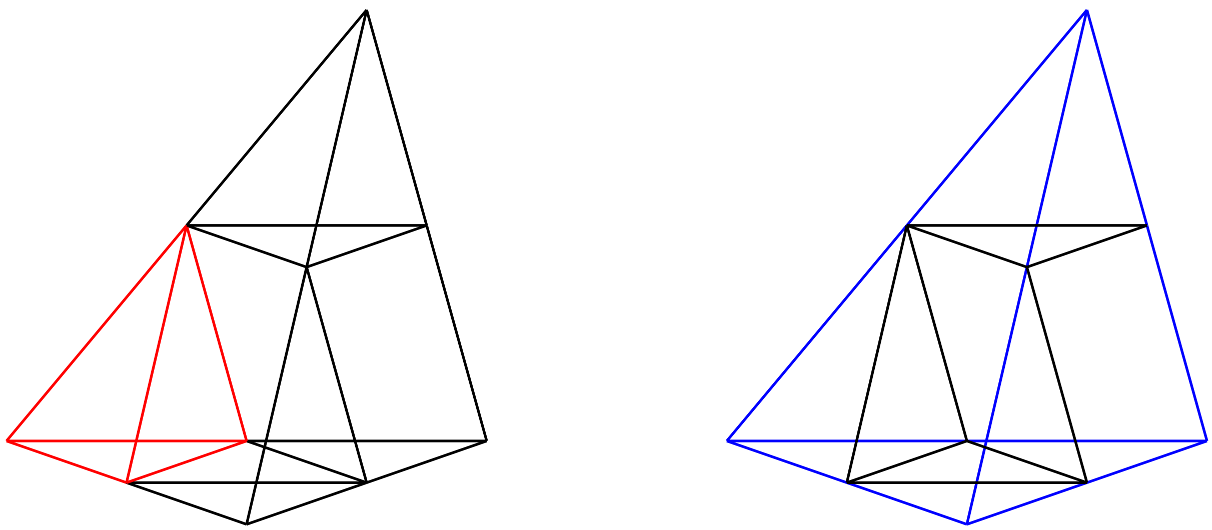
So we see that each line defining the pyramid outlined in red is 1/2 of the corresponding line defining the greater pyramid outlined in blue

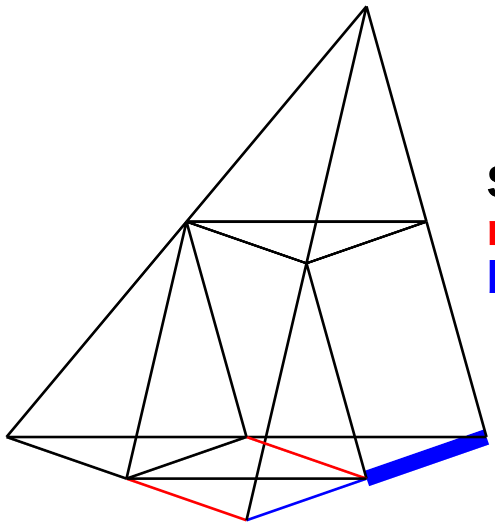




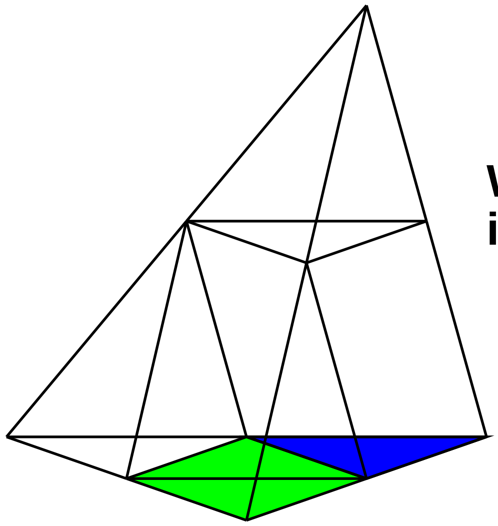
Because corresponding faces have equally proportional sides they're corresponding angles are equal (Euclid VI prop. #5)
Therefore, each corresponding face is similar
Therefore, the two pyramids are similar (Euclid XI def. #9)

Due to the earlier similarity (and equality) shown,
The pyramid outlined in red
and the pyramid outlined in blue
are similar also.



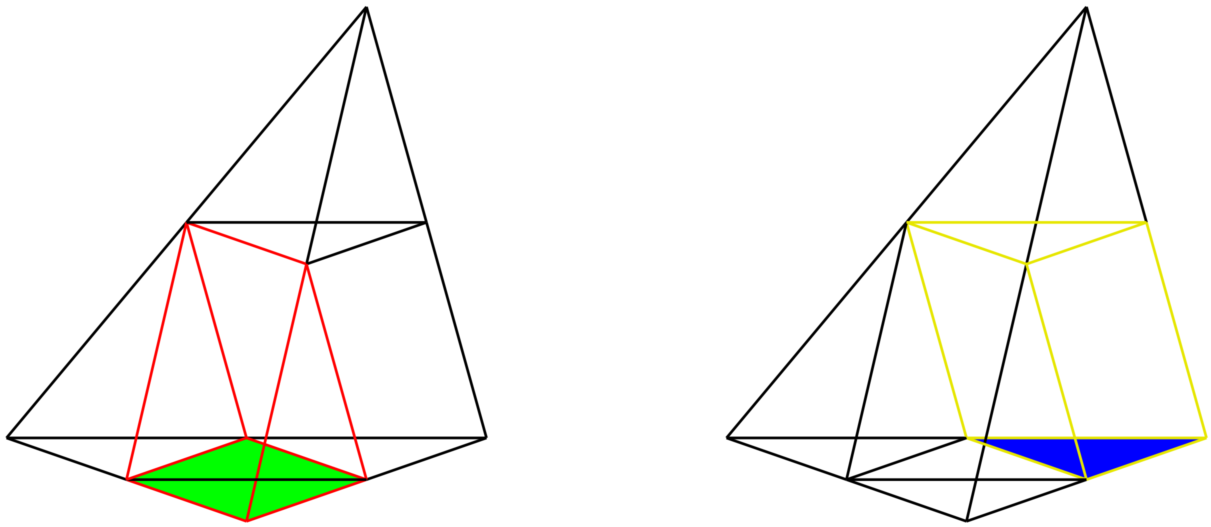


Since we have the following equalities:
red = red
blue = blue



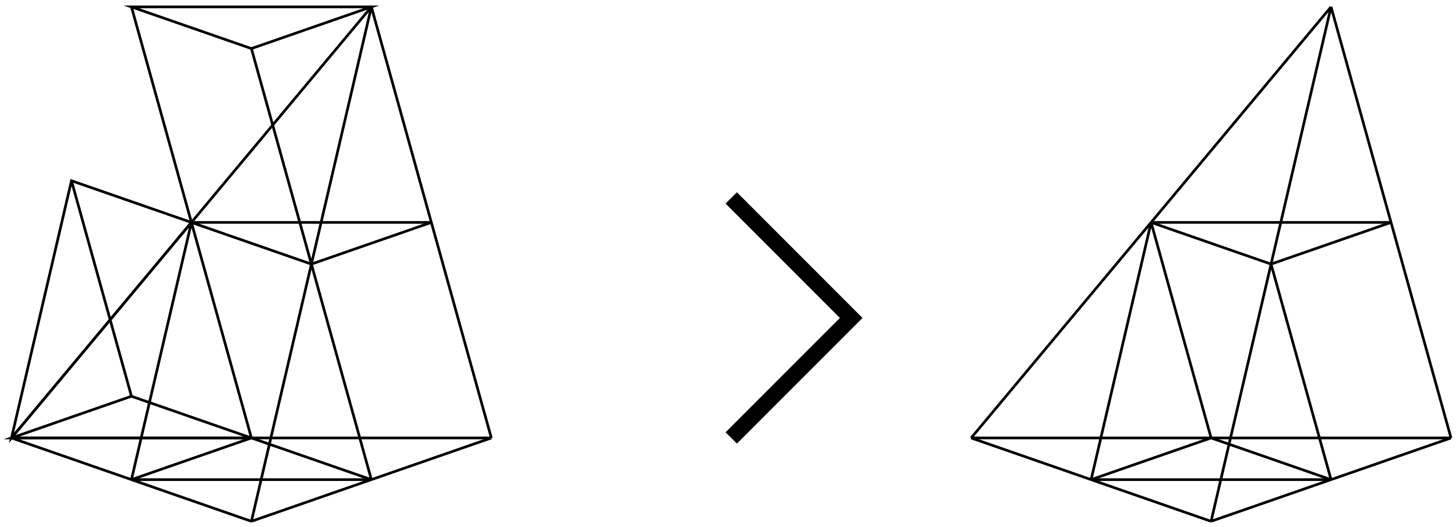
We see that the green parallogram is double the blue triangle

So the prism with the green parallelogram as base, outlined in red, is equal to the prism with the blue triangle as base, outlined in yellow...



...since they are prisms of equal height, with one having a parrallelogram as a base, the other having a triangle as base, and the parralelogram is double the triangle (Euclid XI prop. #39)

**If we add two more prisms, equal to the existing prisms,
We see that the resulting volume is greater than our pyramid**



Since doubling the volume of our prisms produces a volume greater than our pyramid, the area of the prisms combined is greater than half of the volume of the pyramid