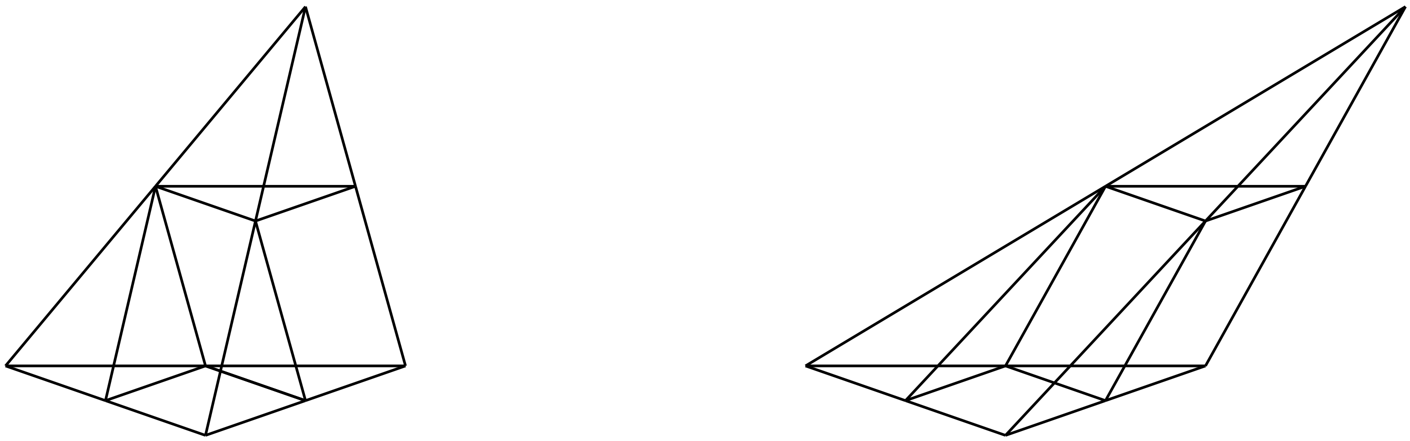
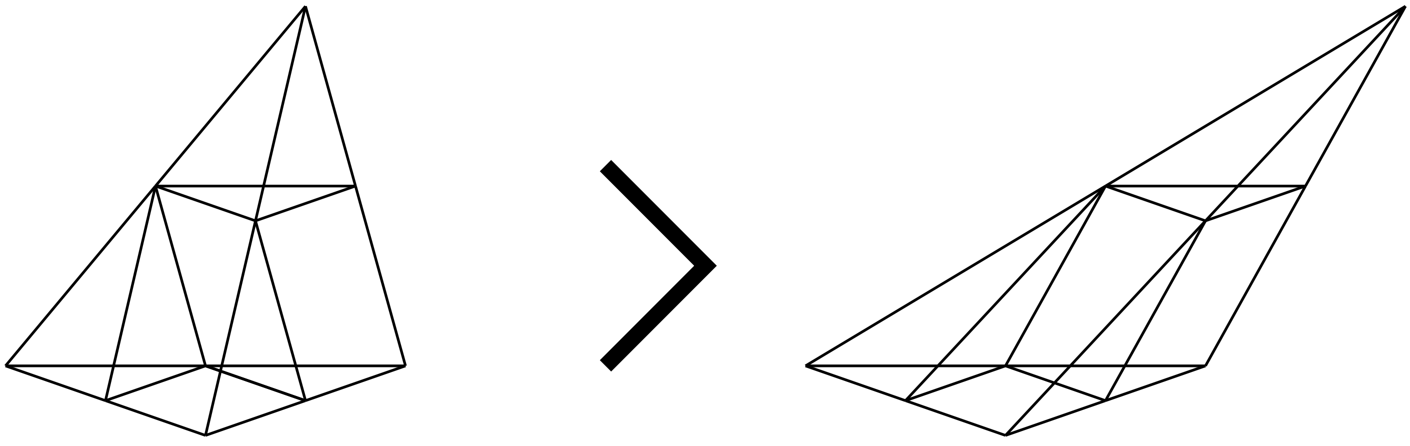
The next proof shows that when two pyramids have equal heights with bases that are equal and similar, then the pyramids are equal also. This is a specific case of Euclid XII prop. #5

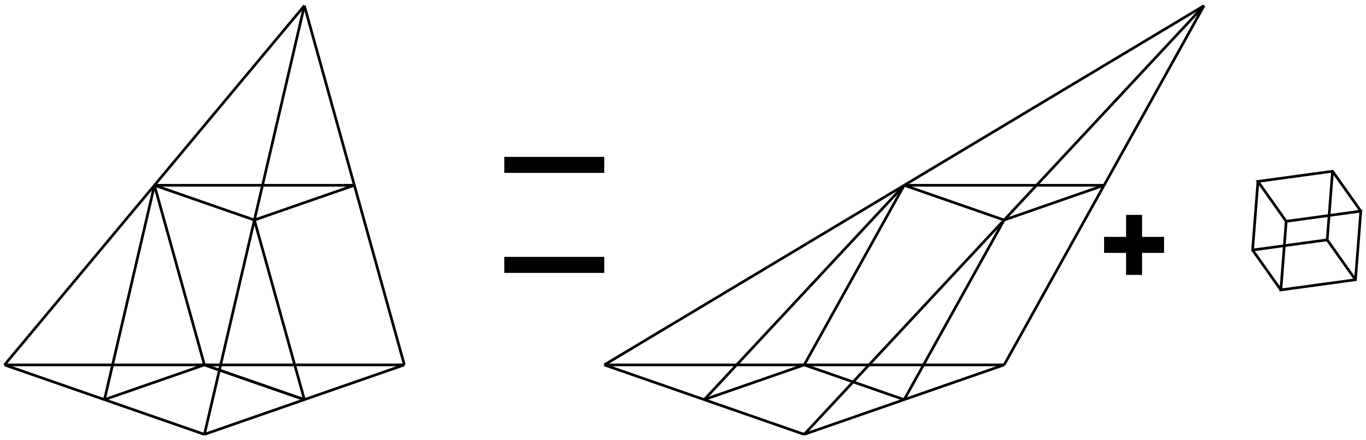
Let two pyramids with equal heights and equal and similar triangular bases, with each corresponding side of the bases equal, be divided up according to Euclid XII prop. #3, so that they fit the criteria of having two equal and similar pyramids, and two equal prisms



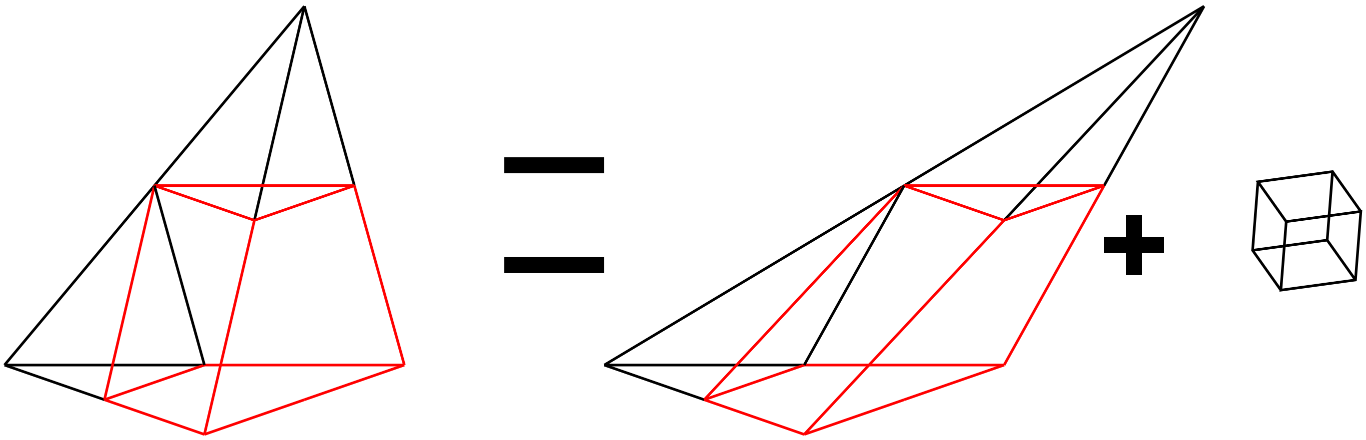
These two pyramids are equal. This will be proved via a contradiction. Assume they are not equal. Then one is greater than the other. Suppose it’s the left-most



Then the left pyramid is equal to the right pyramid,
plus some extra volume, represented as a cube



But we know from the specific case of Euclid XII prop#4 shown earlier, that the two prisms in the left pyramid are equal to the two prisms in the right pyramid, hence:red = red

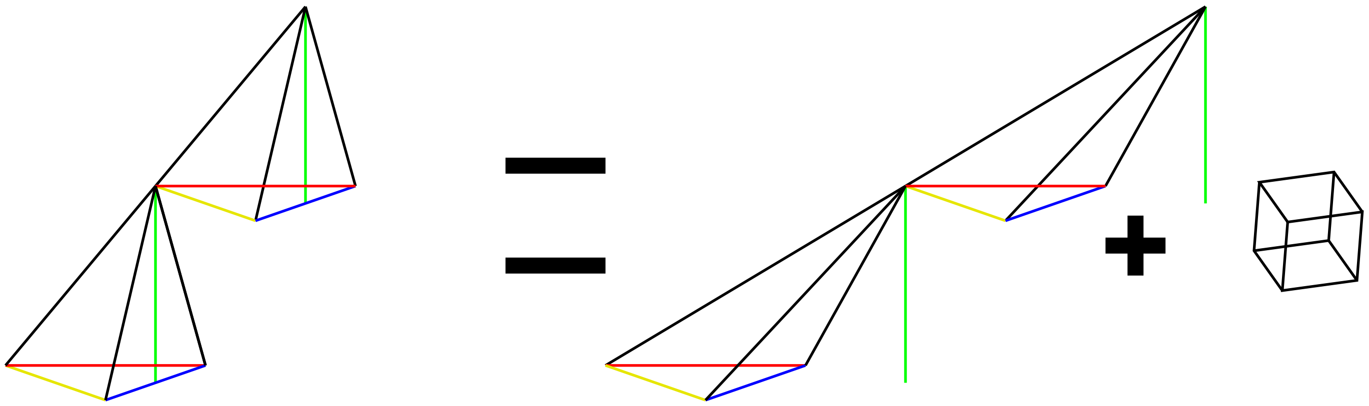


Since they are equal, and on opposite sides of the equality,
they can both be subtracted from the equation...

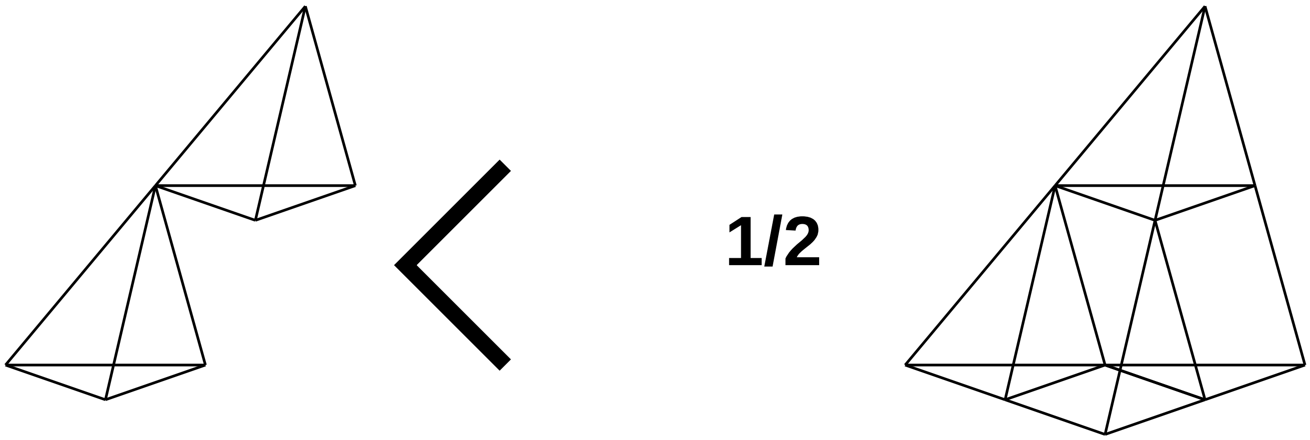
...so we are left with the following, where the bases of the remaining remaining pyramids are equal and similar, and the heights of the pyramids are equal. This is all shown in the proof of our specific case of Euclid XI prop. #4

red = red   blue = blue

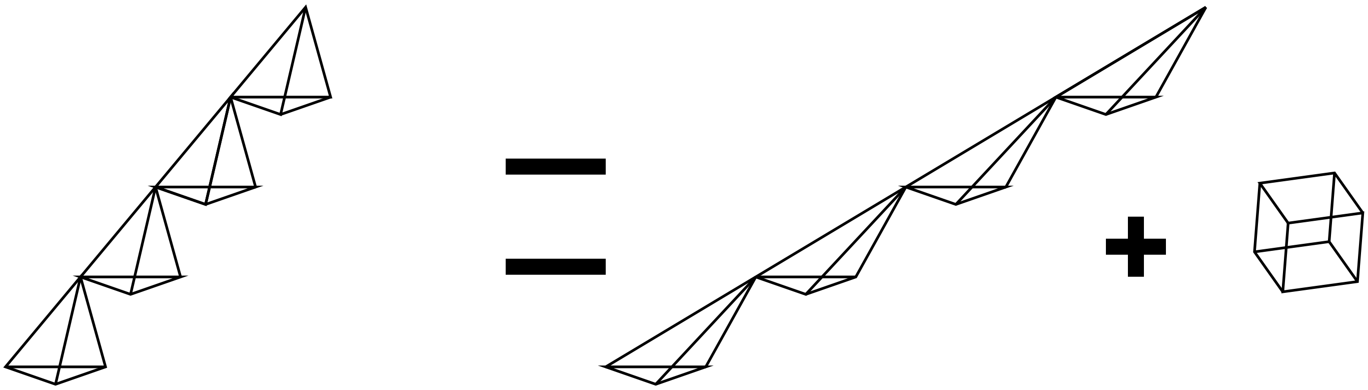
green = green   yellow = yellow



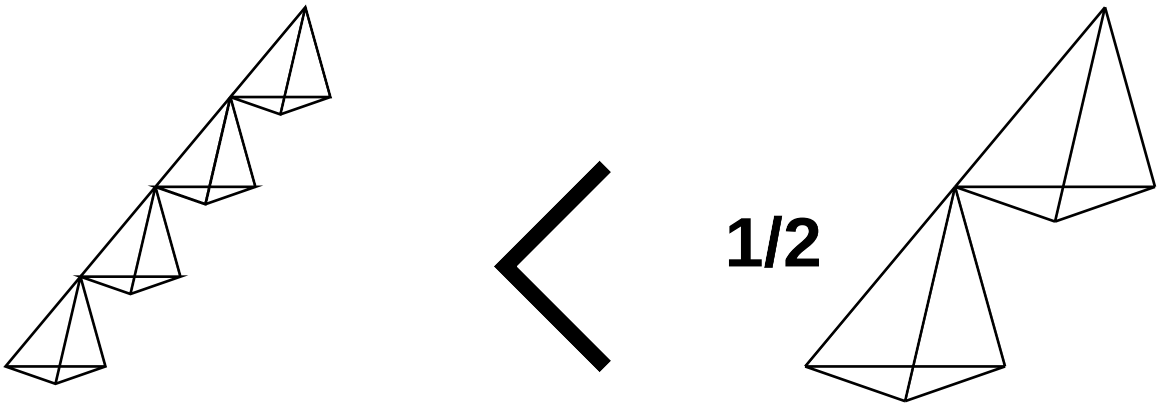
Euclid XII prop.#3 tells us that those two prisms that we just subtracted from both sides were greater than half of the pyramid on the left, hence:



Because these 4 pyramids are have equal and similar bases, and equal heights, we can subtract equal prisms as before, and are left with the following equation:

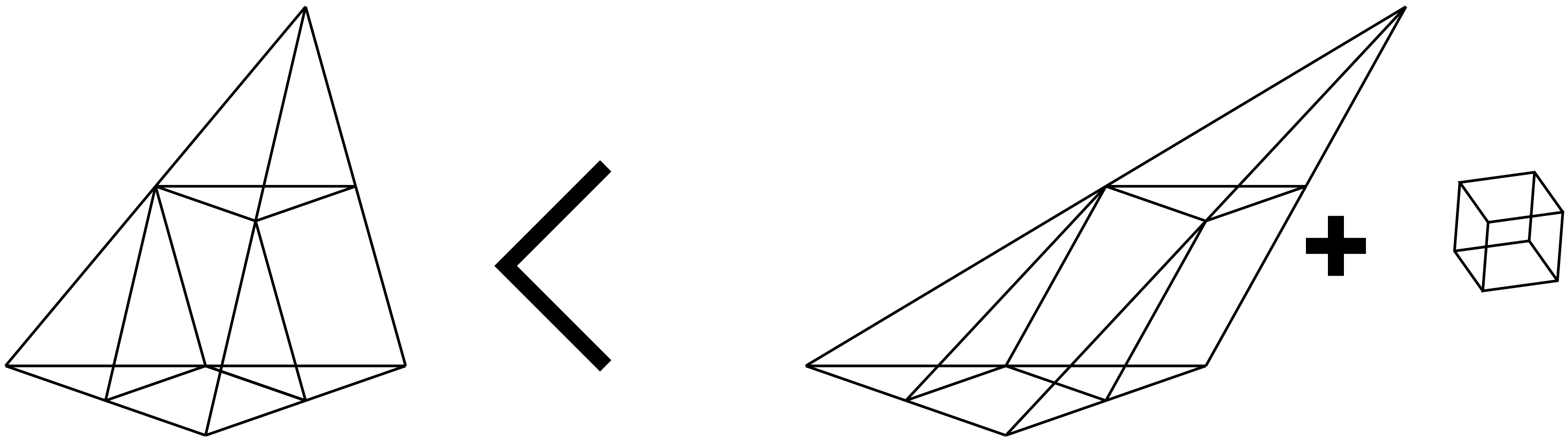


and...



If we continue to repeat this process of subtracting equal prisms from both sides...

...eventually, according to Euclid X prop. #1, our volume on the left will be less than the cube on the right, therefore:



A contradiction!
so...