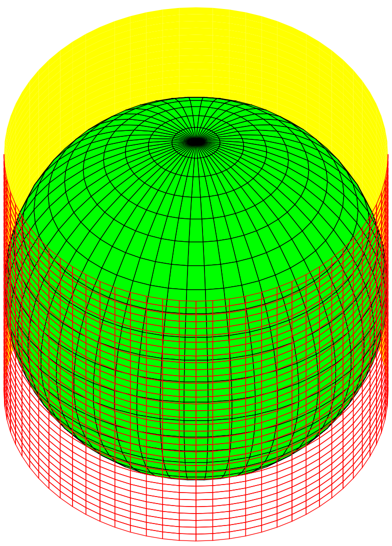
Archimedes :

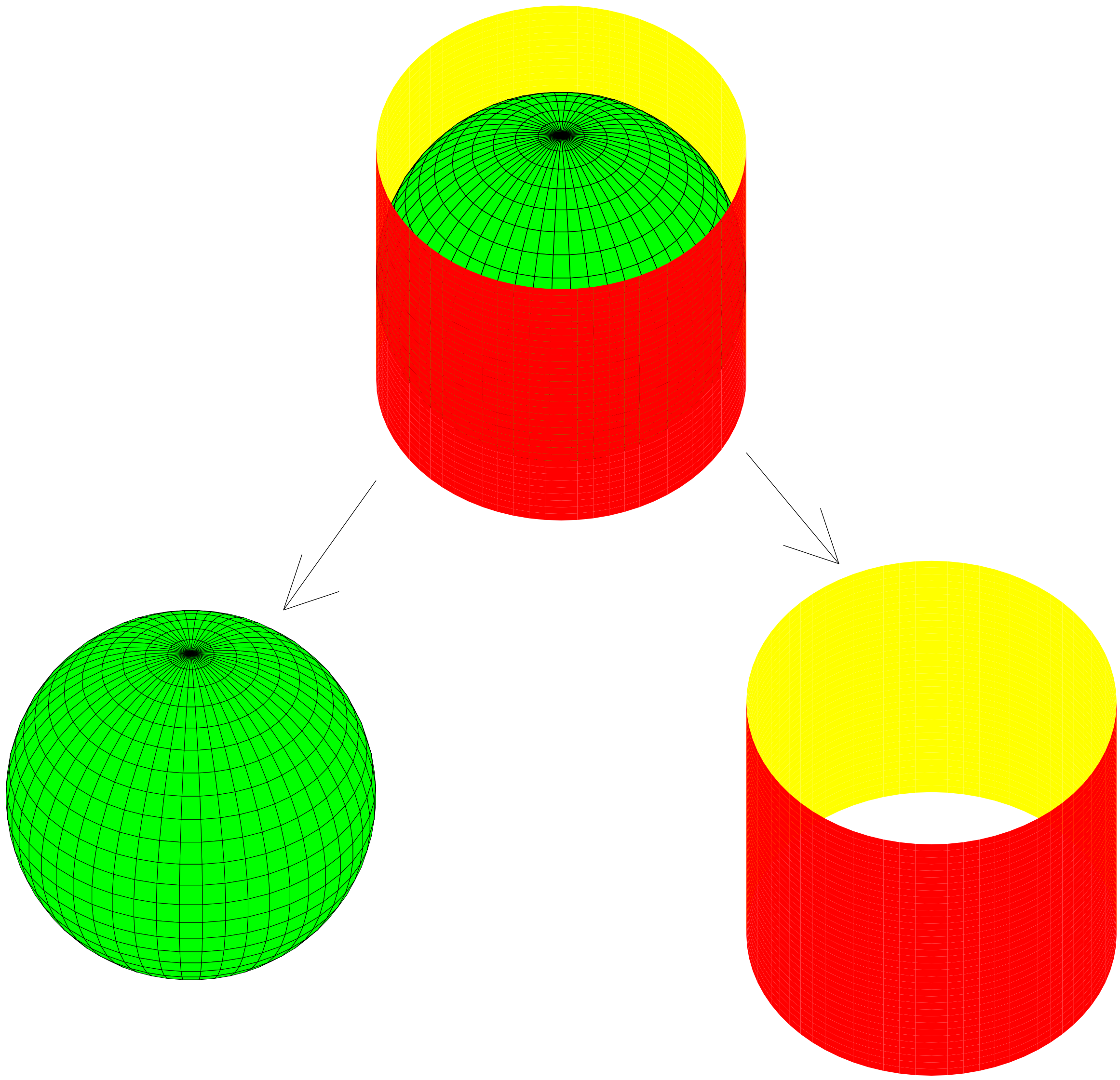
A Sphere and A Cylinder



Archimedes said: The surface area of a sphere equales the surface

area of a cylinder whose height from the base to the top and

the diameter of the base are the same as the diamiter of the sphere.



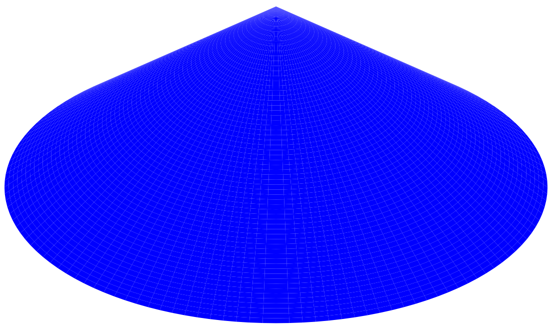
Before going to the details of what Archimedes did,

we first must review how to calculate the area of the

conical frustum.

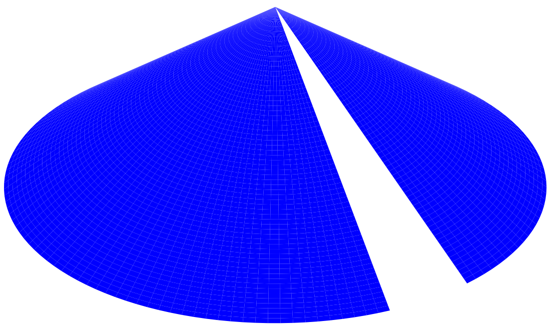
First, let us consider the area of a cone.

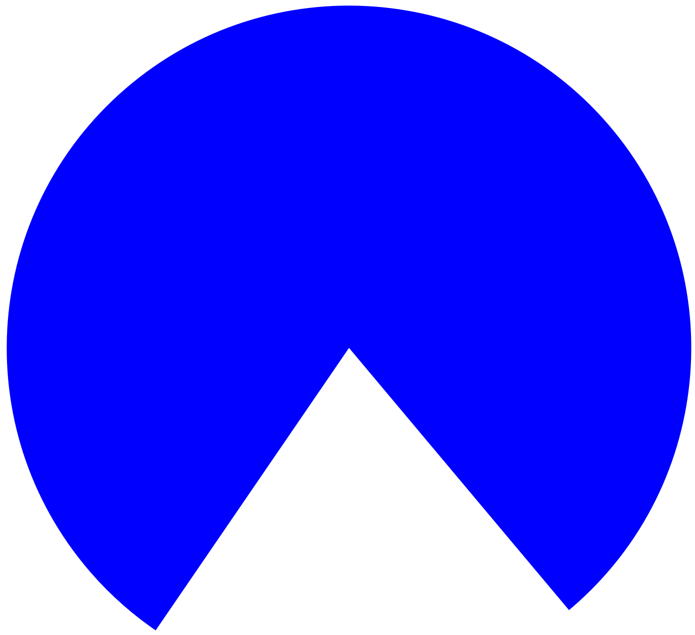
The Area of Cone



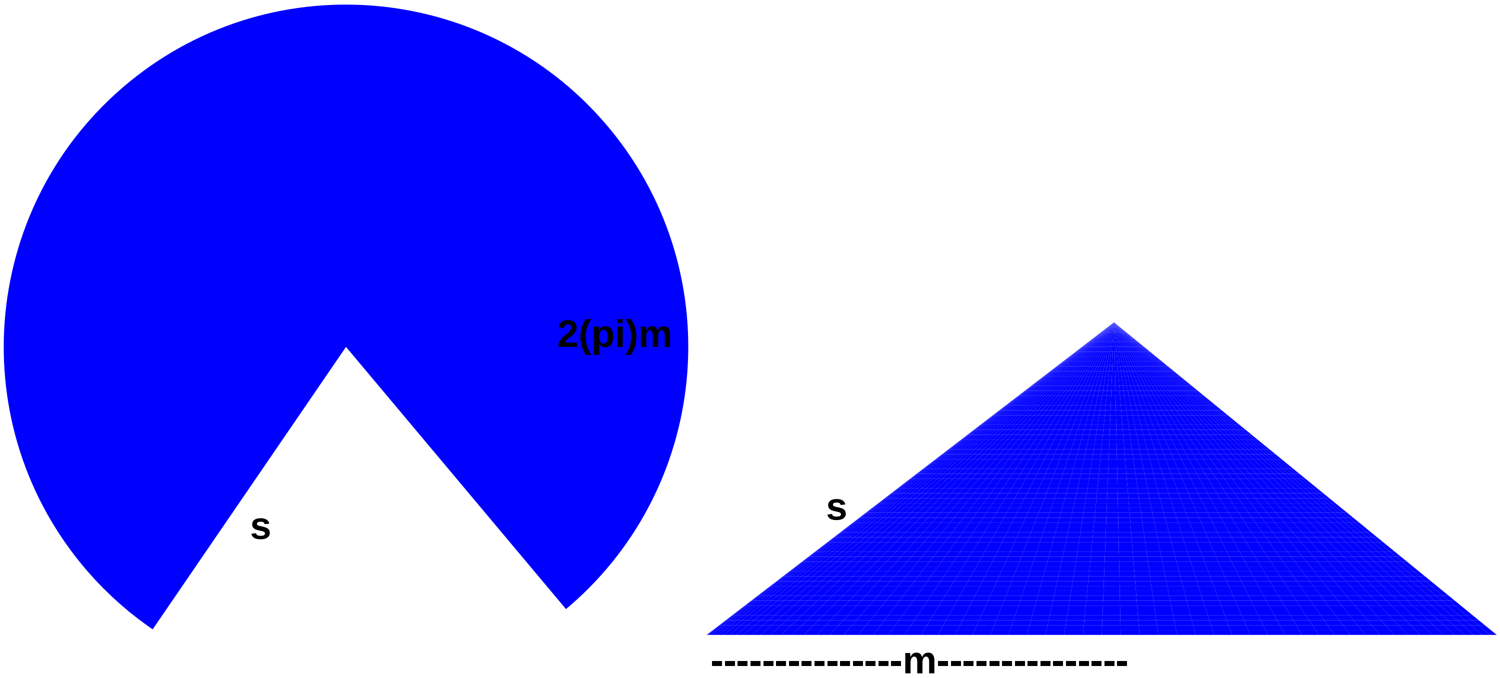
As we know, we can cut a cone

and make it 2D.

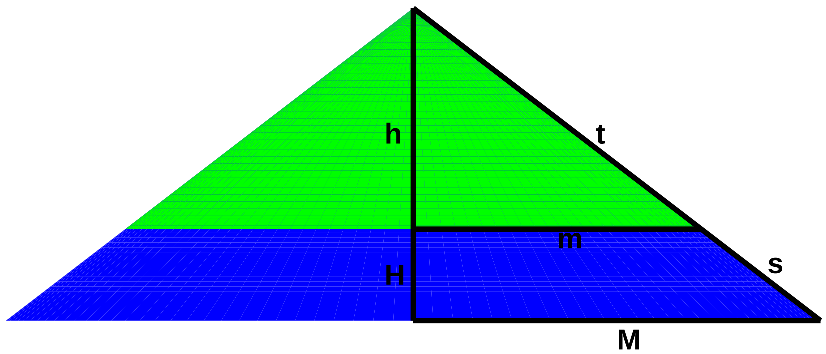




Area of Cone = (1/2) x 2(pi)m x s



Now,consider the area of a conical frustum.



from the side view of a cone, we get

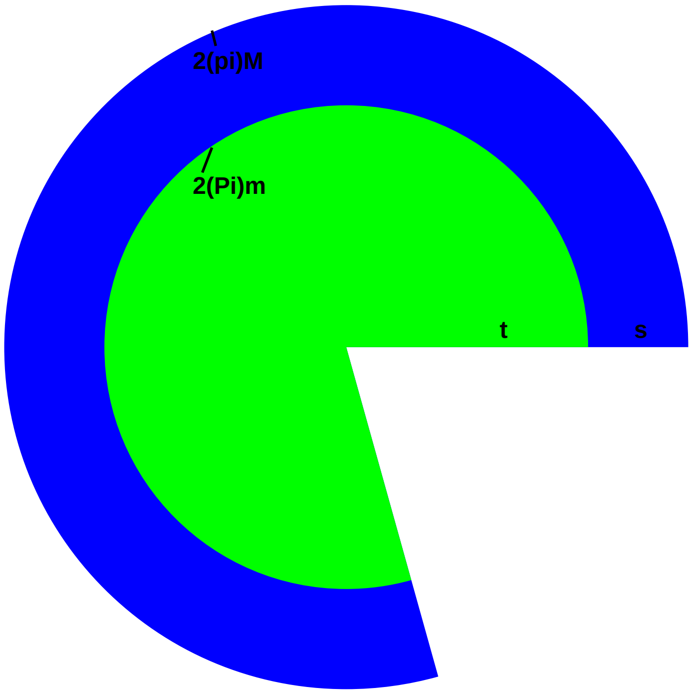
$t:m = (t+s) : M$

$tM = (t+s)m$

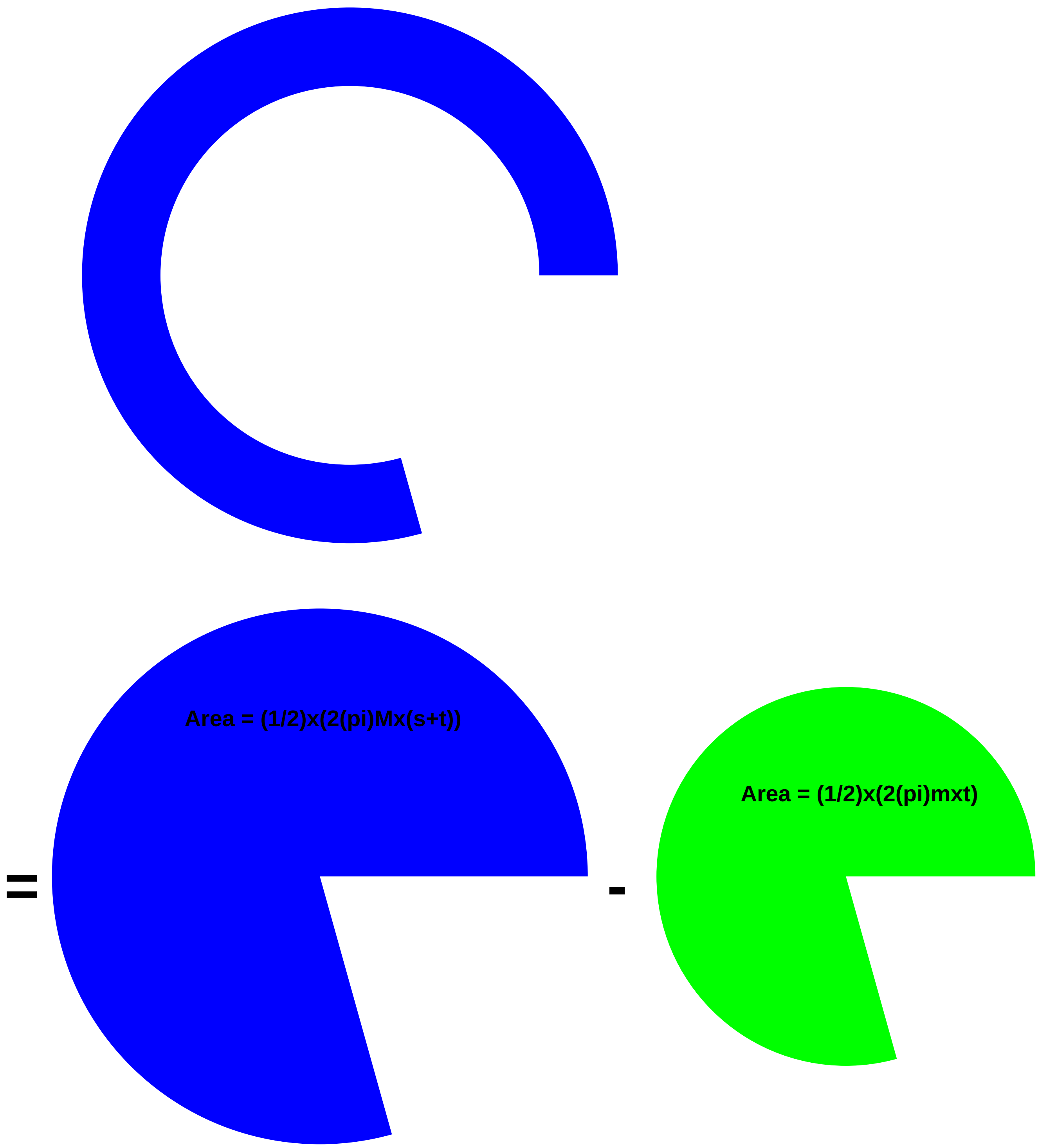
$t(M-m)=sm$

Top View of The Area of

The Conical Frusrum in 2D



The Area of Conical Frustum



The Area of above Conicol Frustum

= {(1/2)x(2(pi)Mx(s+t))}- {(1/2)x(2(pi)mxt)}

=(pi)M x (s+t)-(pi)m x t = (pi)(Ms+Mt-mt)

From the side vies of Conicol Frustum: t(M-m) =sr-----(a)

substitute (a) in this equation,

Area of Conicol Frustum

=(pi)(Ms-ms)=(pi)(M-m)s

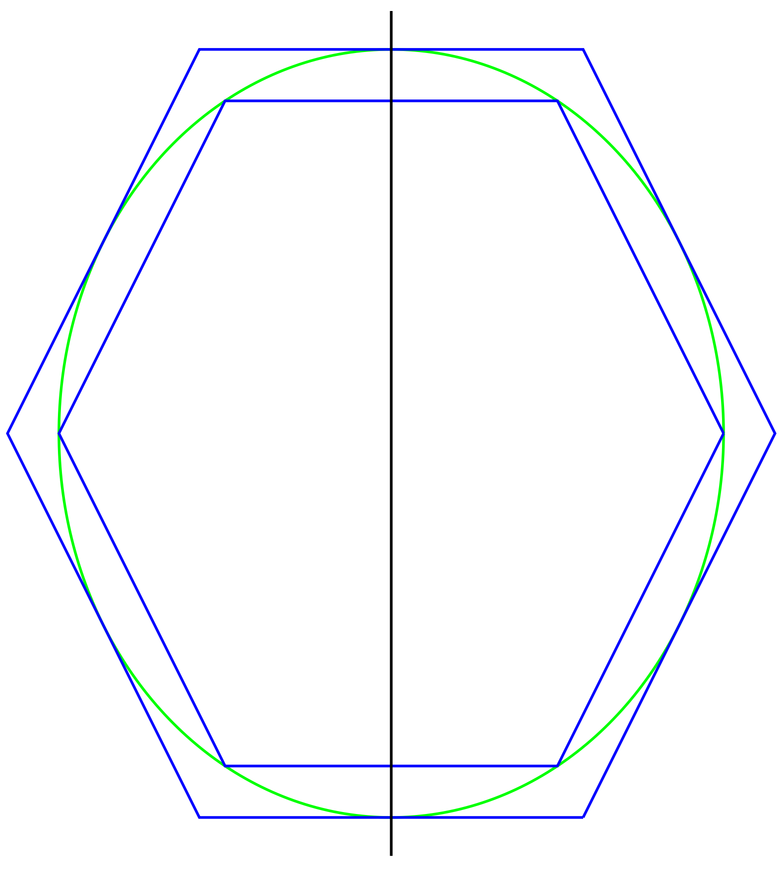
The surface area of a sphere:

When a polygon is inscribed inside the great circle

of a sphere and rotated around an axis,the surface area

of polyhedron gets very close to the surface area of

sphere as n gets larger and larger.



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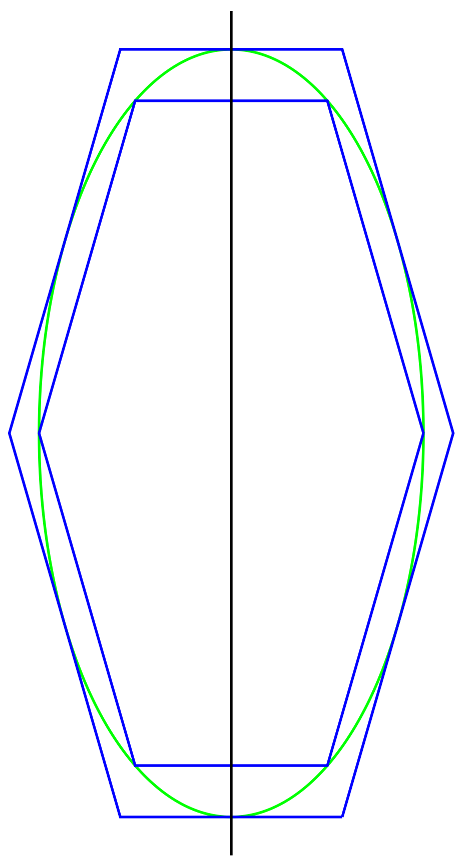

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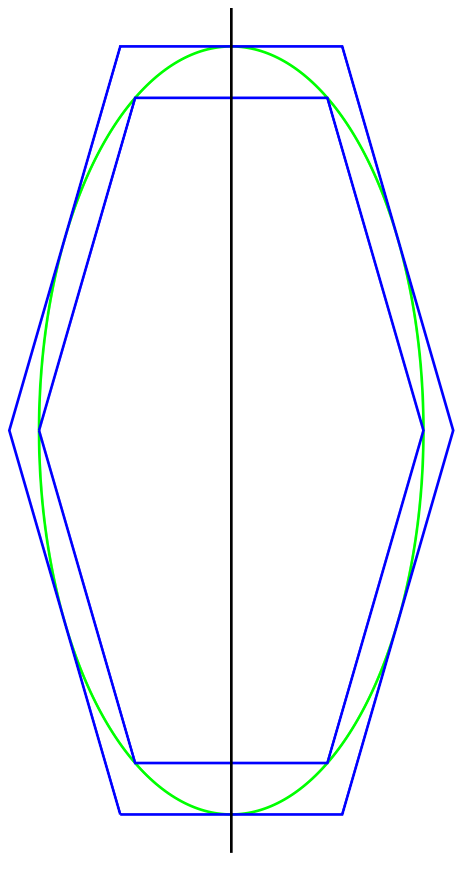

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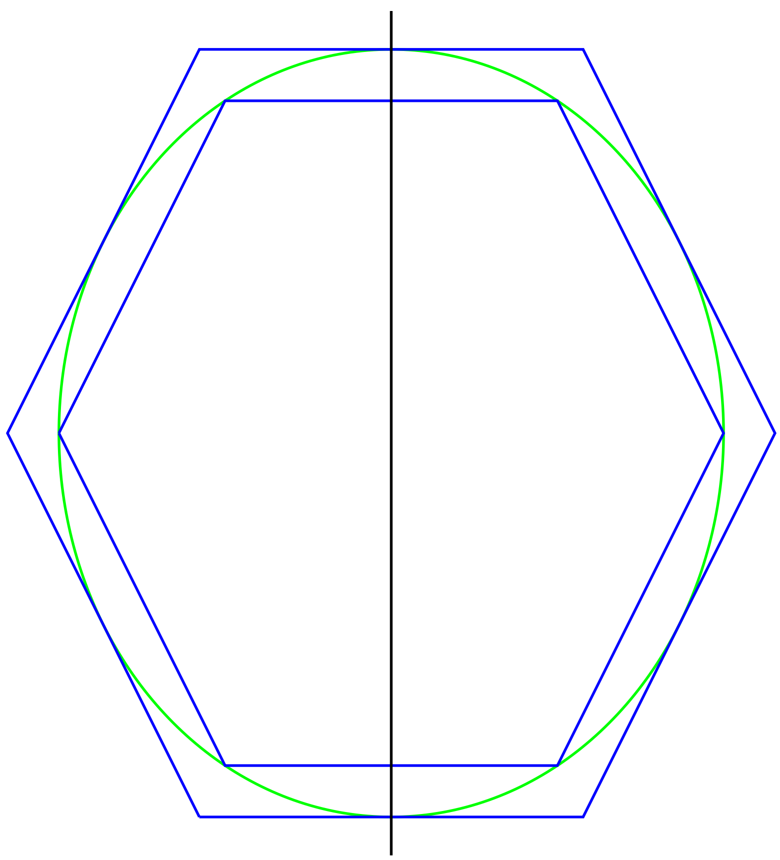

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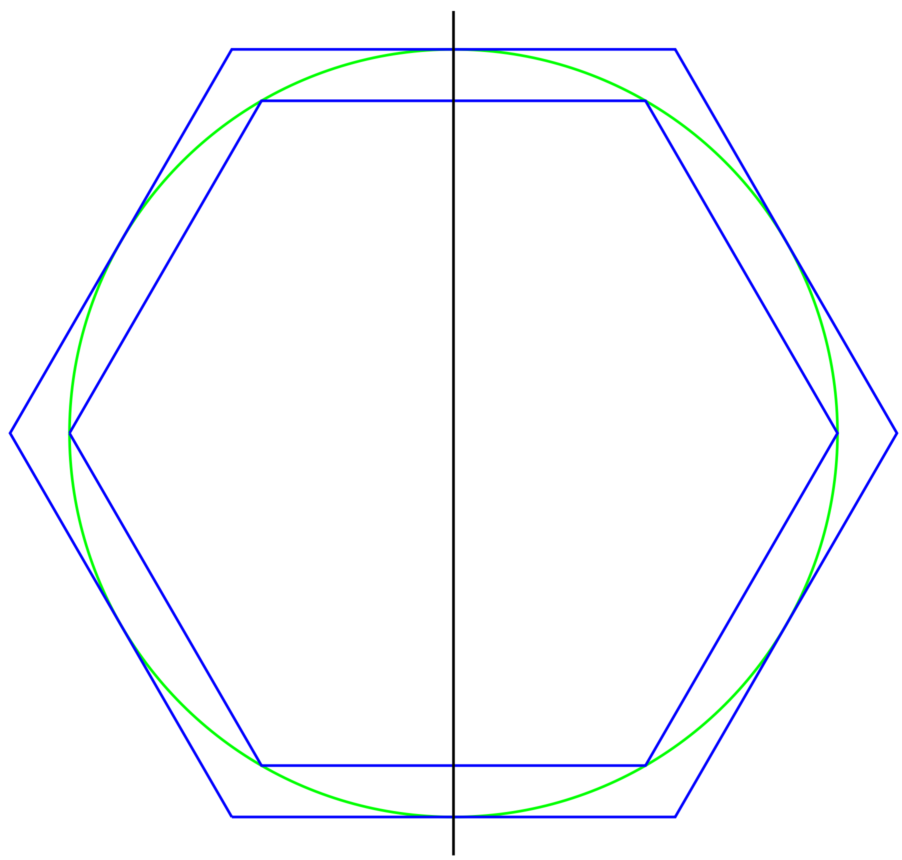
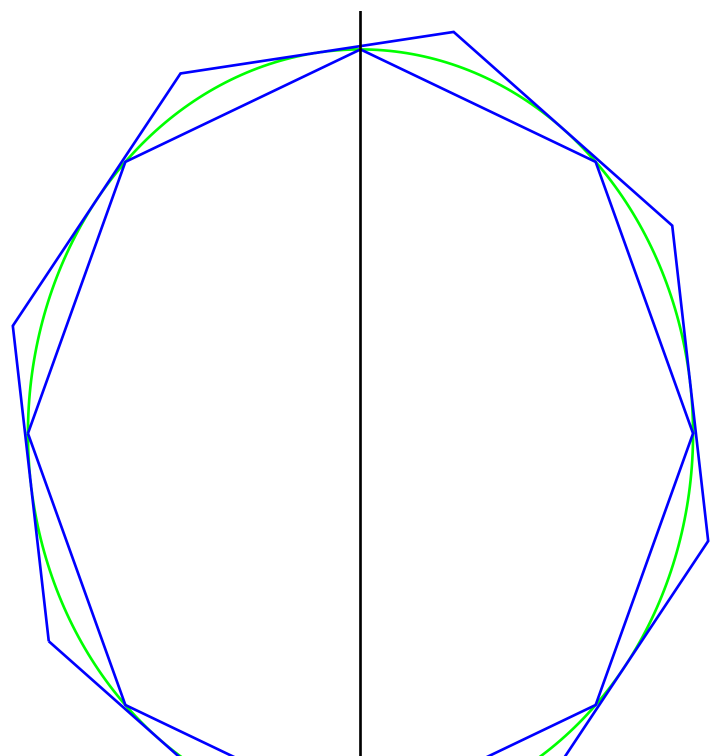
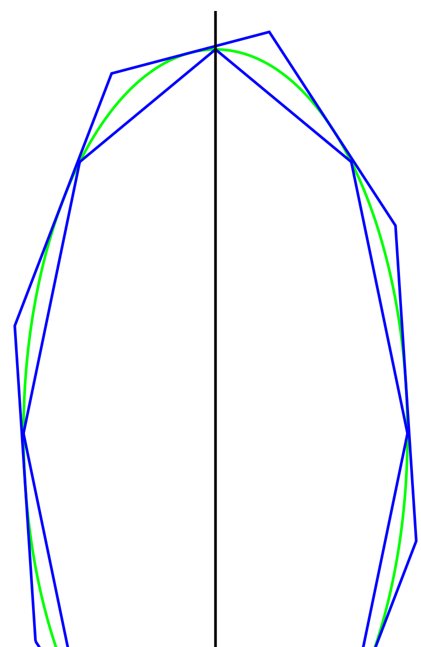
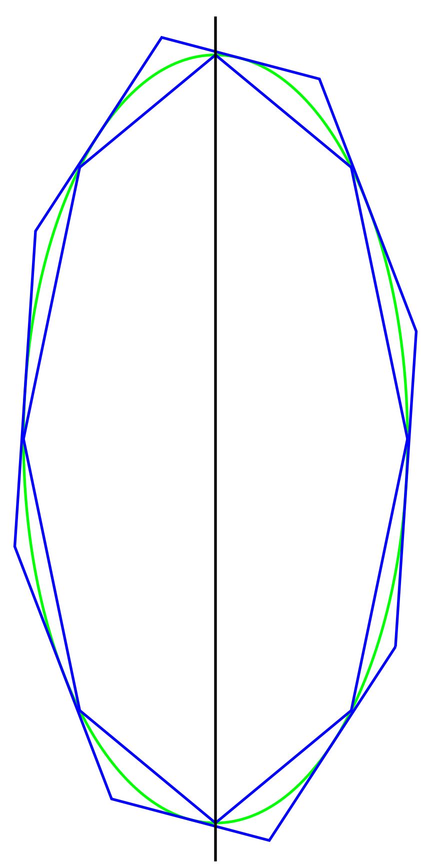
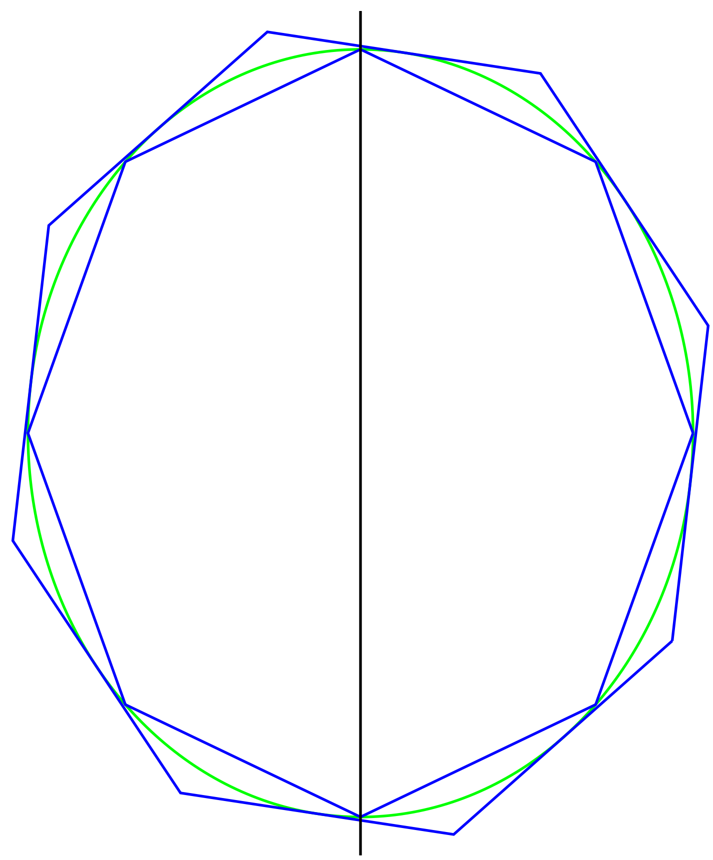
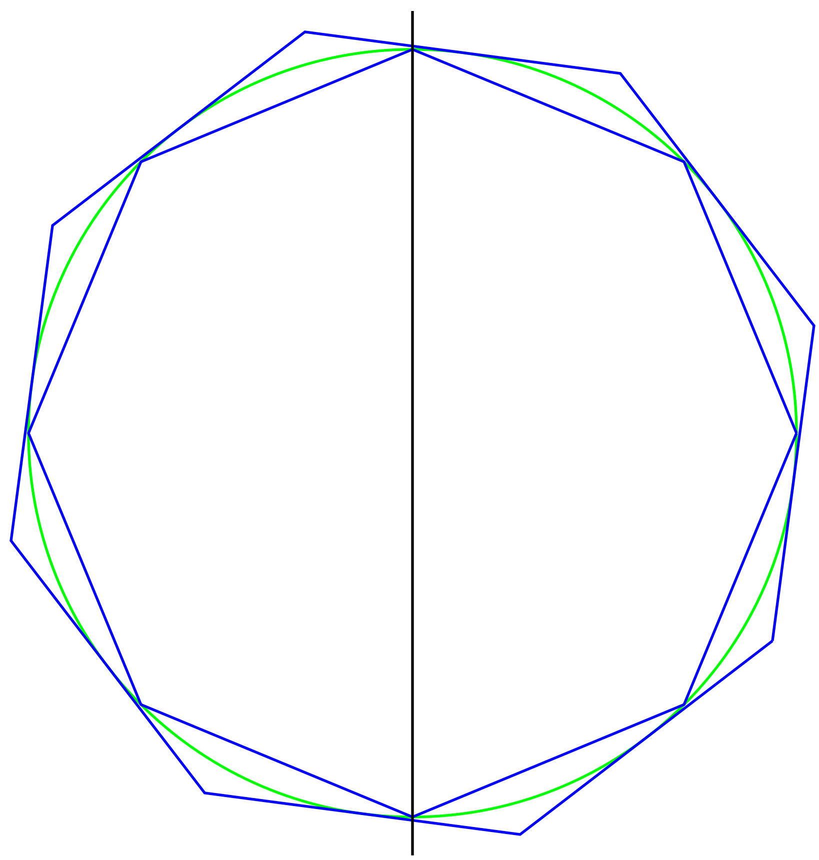


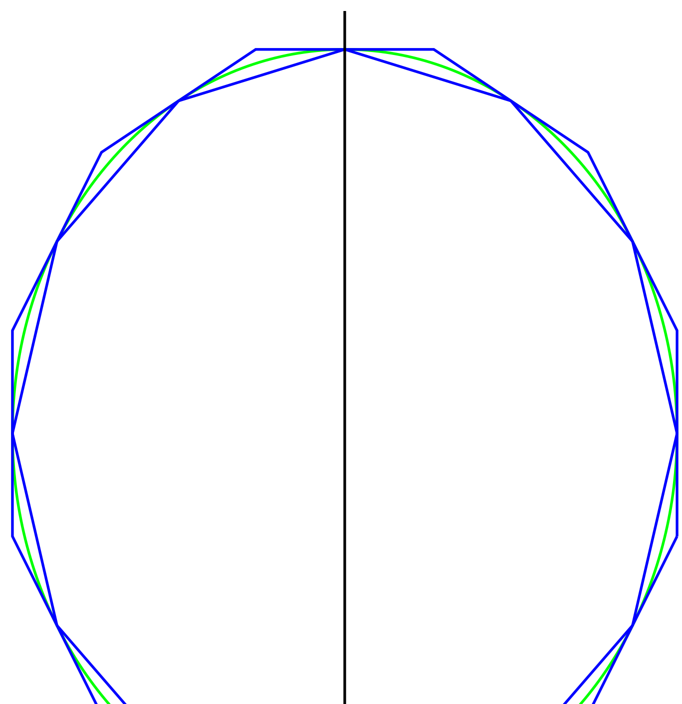


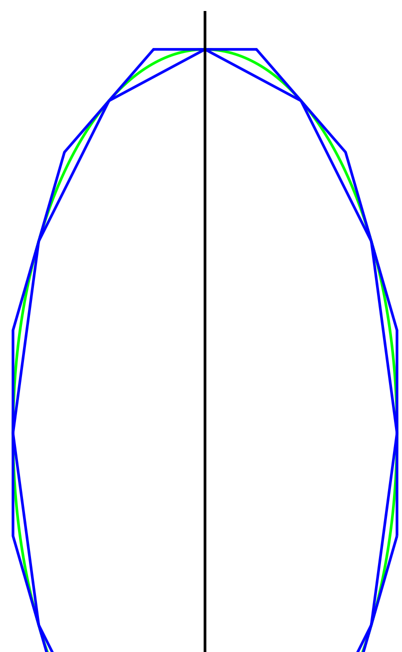


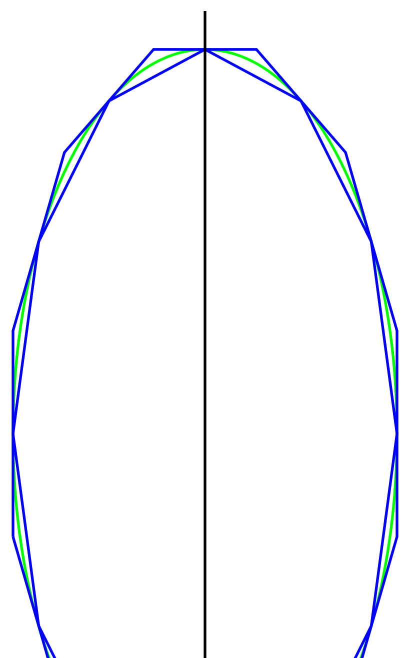


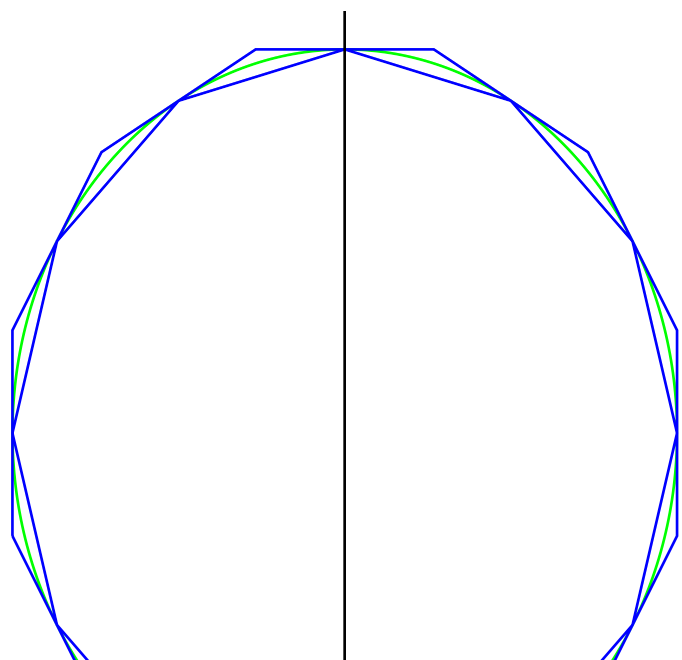


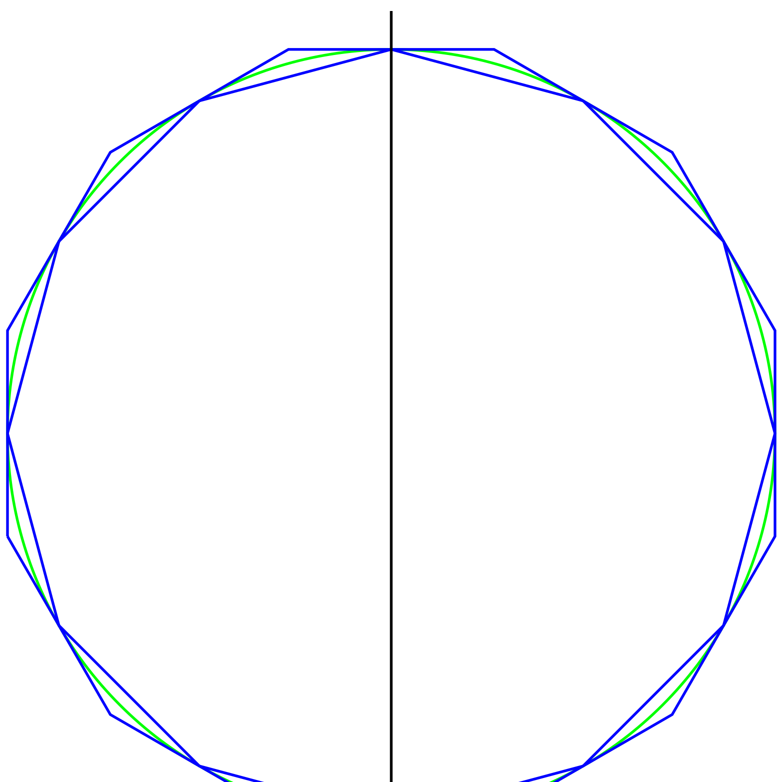


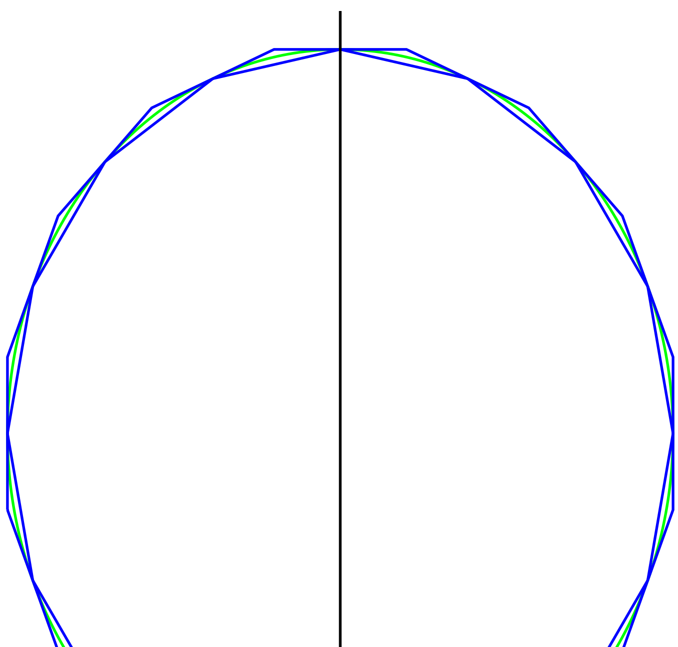


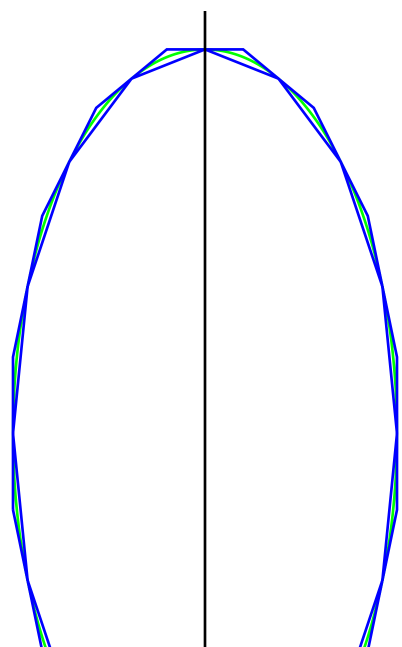


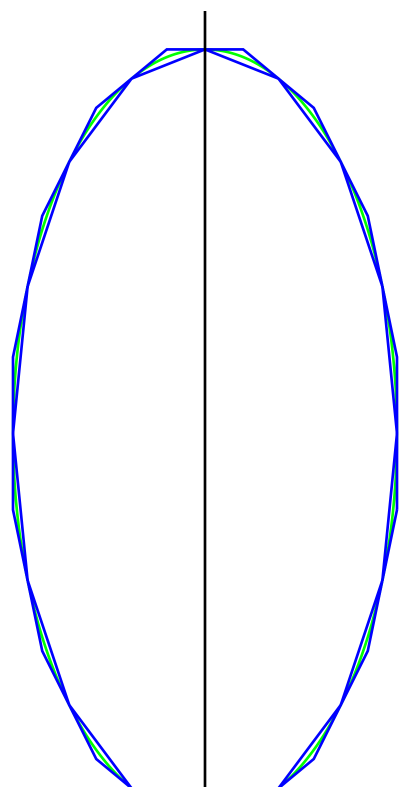


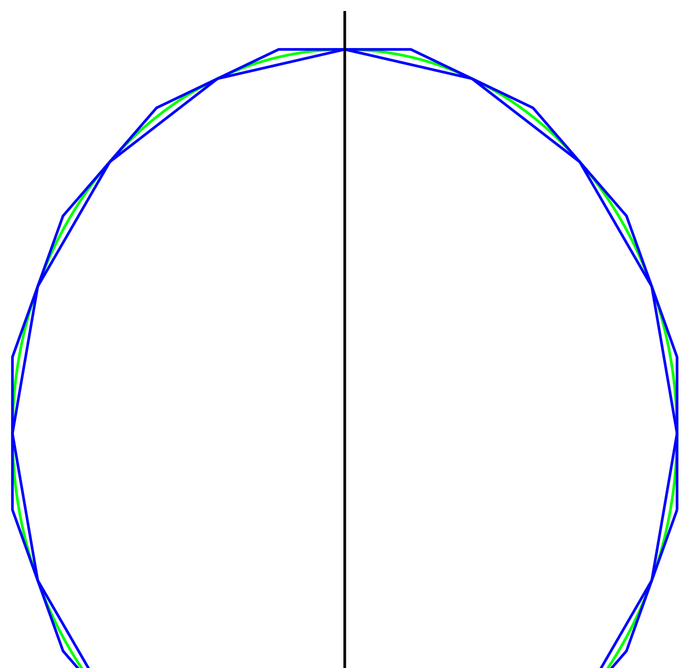


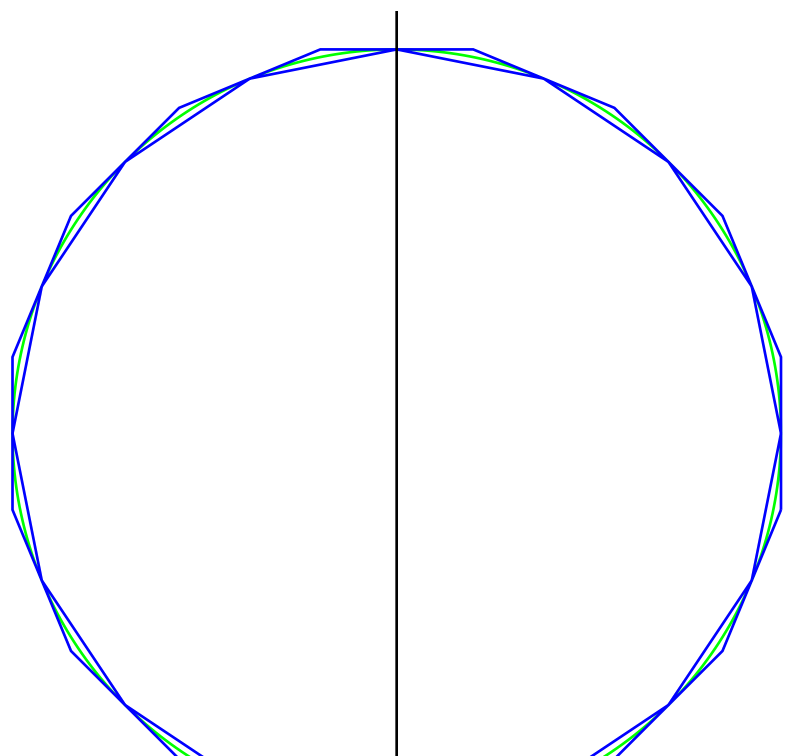








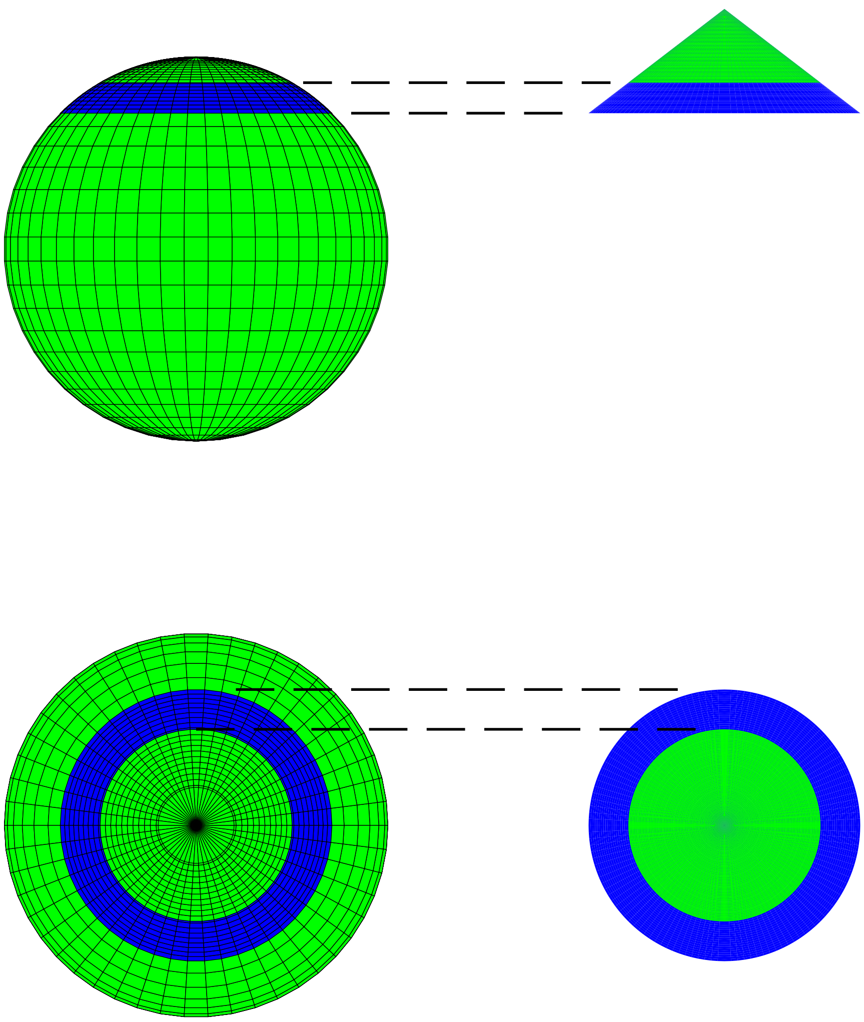




Archimedes method:

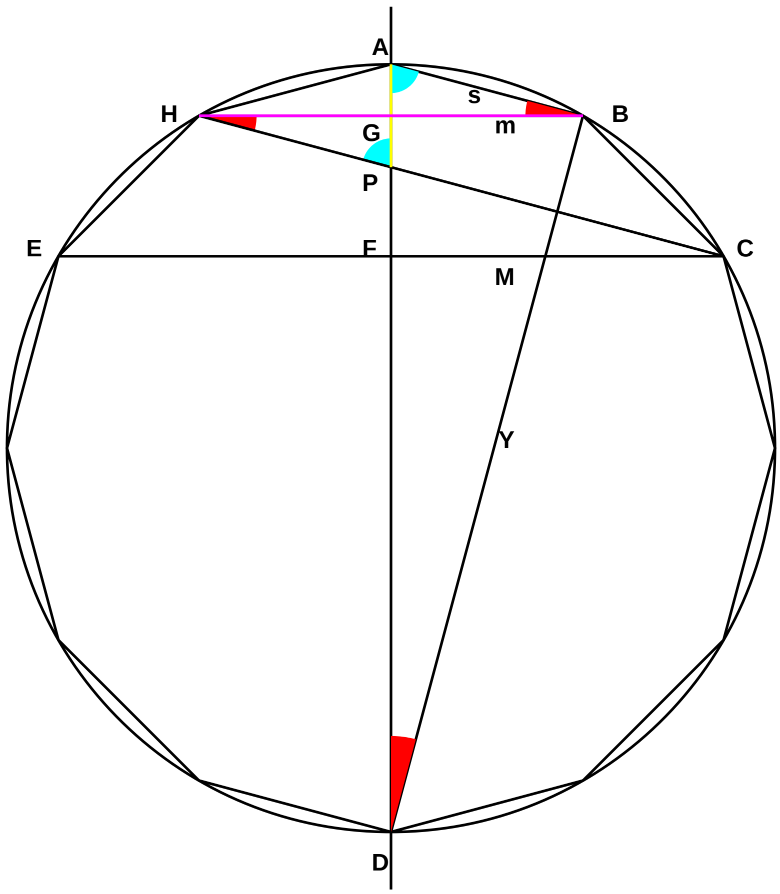
Use a cone to calculate

the area of a sphere

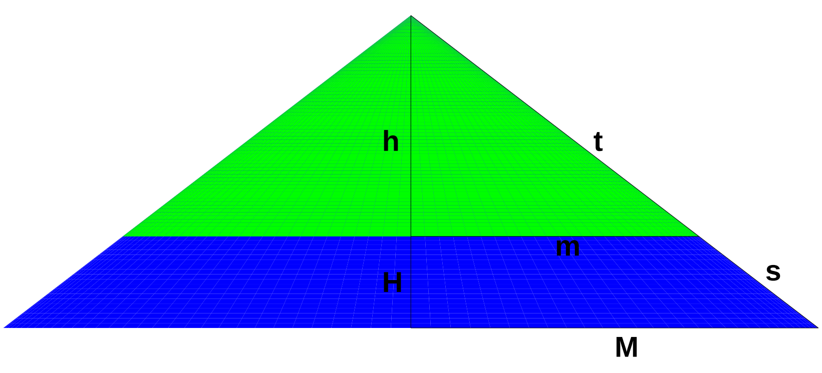


Archimedes rotated the circle below

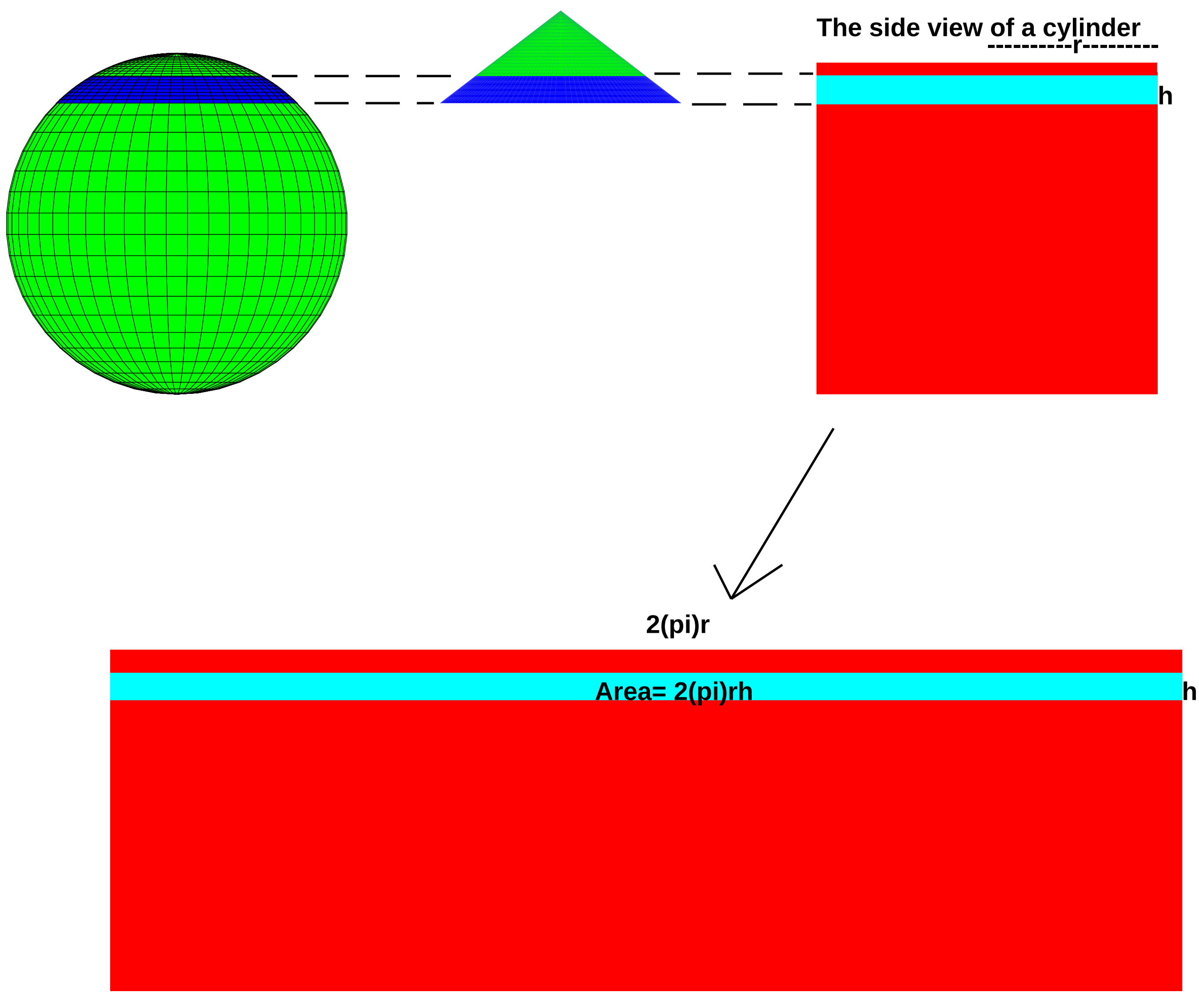
along the axis AD.



Since the triangle ABG
and the triangle ABD are
similar, m(=GB) : AG = Y : s.
so, ms = (AG)Y.
Since the triangle AGB is
similar to the triangl HGP,
m : AG = m :Gp = Y :s.
so, ms = (GP)Y.
Similarly, M : PF = m : GP = Y : s.
So, Ms = (PF)Y.



Area of Conical Frustum (blue)
= pi(m+M)s
= pi(ms + Ms)
= pi{(GP)Y + (PF)Y}
= pi{(GP +PF)Y}
= pi(GF)Y
As n goes to infinity, y becomes increasingly
to the diameter of the
great circle. Therefore,
Area of the Conical Frustum
= 2(pi)r(GF), where r is a radius
of the great circle of a sphere.


The side view of a cylinder
----------r----------
h
2(pi)r
Area= 2(pi)rh
h