Mathematics 308 — eighth homework, partial solutions

1. *A cube of side* 1 *is placed with centre at* $(-1,-1,-2)$ *with edges parallel to coordinates axes. The eye is located at* $(0,0,5)$. *Find exact expressions for the projections of its vertices onto the view plane.*

If $(0,0,e,1)$ is the eye and (x,y,z) a point in 3D, the point corresponding to it on the viewing plane is (*Manual*, §13.2)

$$\left(\frac{xe}{e-z}, \frac{ye}{e-z}\right)$$

We apply this to the vertices of the cube, which are the points $(-1,-1,-2)+[\pm 1/2, \pm 1/2, \pm 1/2]$.

2. *An regular octahedron has* 8 *equilateral triangular faces. If its radius (distance from centre to vertex) is* 1, *what is the length of the edges?*

If we orient the octahedron correctly, two of the vertices are $(1,0,0)$ and $(0,1,0)$. Distance bewteen them is $\sqrt{2}$.

3. *What is the cosine of the angle between two radii of a regular tetrahedron (exact expression)? The angle itself?*

From the last assignment, two vertices are $(0,0,1)$ and $(0,1,-1/3)$. Use the formula for the angle between two vectors.

4. *What are the normal functions for the faces of the tetrahedron from the last assignment (exact)?*

Say the face is that containing $(0,1,0)$, $(1,0,0)$, and $(0,0,1)$. In this order, the points go in the positive direction around the face, so the outward normal vector is $[1,1,1]$. The equation $x+y+z-1=0$.

5. *(a) Find the matrix of rotation of* 30° *around the* x*-axis. (b) The inverse of a rotation matrix* A *is its transpose* ${}^tA$. *What is the inverse of the* 4×4 *matrix*

$$\begin{bmatrix} A & 0 \\ v & 1 \end{bmatrix}$$

where A *is a rotation matrix,* v *a 3D row vector? (c) Find the inverse of the matrix in (b) if* A *is as in (a) and* $v=(1,1,1)$.

The matrix in (a) is

$$\begin{bmatrix} 1 & & \\ 0 & \cos 30^\circ & \sin 30^\circ \\ 0 & -\sin 30^\circ & \cos 30^\circ \end{bmatrix}$$

if we think of it as acting on the right on row vectors. This is essentially a 2D rotation, but in this case with y and z replacing x and y. To see that the orientation is right, a mental picture helps: the $+y$-axis rotates towards the $+z$-axis.

For (b) and (c) write the inverse matrix as a block, so we want to solve

$$\begin{bmatrix} A & 0 \\ v & 1 \end{bmatrix}\begin{bmatrix} B & 0 \\ u & 1 \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & 1 \end{bmatrix}$$