Mathematics 308—February 12, 1996

Second homework—due Monday, February 12

Exercise 1. If P, Q, R are three non-collinear points in the plane and P', Q', R' are three others, there exists a unique affine transformation taking the first triple to the second. Find the one which takes $(1,1)$, $(-1,2)$, $(1,3)$ to $(1,-1)$, $(2,3)$, $(2,-2)$. Express it in PostScript form (as an array of 6 elements, acting on the right). Also write down the 3×3 matrix corresponding to it.

Exercise 2. Draw this picture of a Bezier curve:

The bottom points are $(-1,0)$, $(1,0)$. The top points are (x_1, y_1) and (x_2, y_2) where $y_1 = y_2 = 1$ and $x_2 = -x_1 = 0.5$. The control lines are gray, and the 'pixels' are $0.05''$ on a side, centred at the control points.

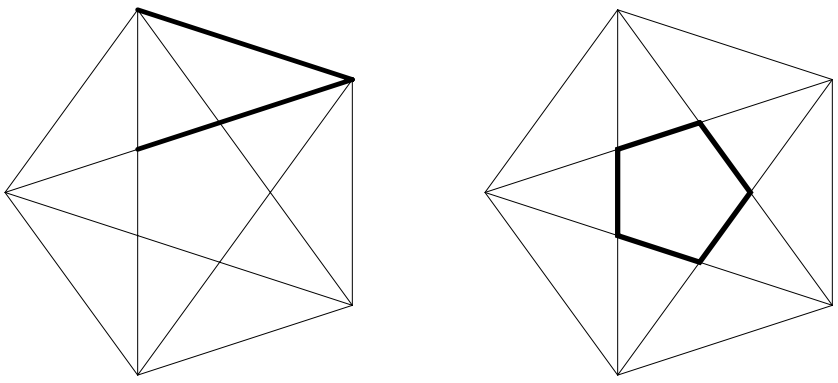
Write your program using variables for x_1, x_2, y_1, y_2 so that by changing only their assignments a new figure can be drawn. You should draw the pixels by means of a subroutine with a pair `x y` as argument.

Exercise 3. Draw the hyperbola

$$x^2 - y^2 = 1$$

with PostScript, using Bezier curves. Use a coordinate on the page where the origin is at the centre, units are inches, and the boundary is clipped to a $0.25''$ margin. Draw also the asymptotes in gray.

Exercise 4. Draw the following figure in PostScript:



If the polygon is centred at the origin and has radius 1, find an **exact** and completely explicit formula for the length of a side; of the heavy segments in the first pentagon; of those in the second.

Exercise 5. If you draw the graph of $y = \sin x$ in the range $[0, \pi]$ by means of one Bezier curve, how inaccurate is it? With 2 Bezier curves?

Exercise 6. Write a PostScript procedure with two parameters, a 2×2 matrix A and a time interval Δ, and draws on successive pages the effect of applying the corresponding infinitesimal motion to the unit square at times separated by the interval.