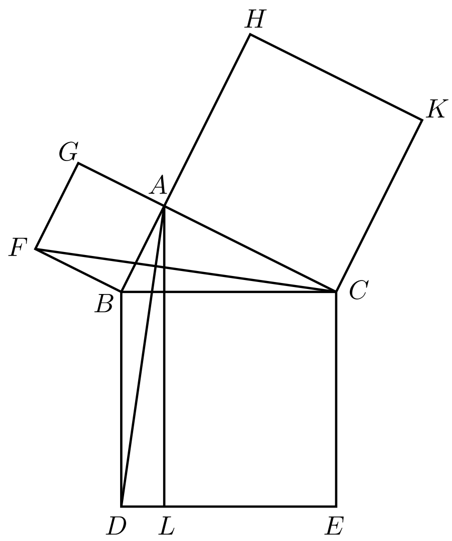
[40] 1. Assume that the files which are included in the following program do what they have to do to define the procedures used, and that the procedures behave as expected. In particular assume that the file *cube.inc* defines the faces of a cube of side 1, centred at the origin and aligned with the axes. Draw by hand below what the following program produces.

```
%!

72 dup scale
4 5 translate
/d 5 def
0.01 d div setlinewidth
d dup scale

(matrix3d.inc) run
(draw3d.inc) run
(cube.inc) run

/A [[0 1 0] 45 rotation-matrix-3d [0 -2 d neg] ] def

0 1 5 –
  /i exch def
  /C A cube surface-transform def
  /f C i get def
  f is-visible-in-perspective –
    /p f 0 get def
    newpath
    p 0 get perspective aload pop moveto
    /n p length 1 sub def
    1 1 n –
      /i exch def
      p i get perspective aload pop lineto
    ″ for
    stroke
  ″ if

″ for
```

[40] 2. I include below Euclid's proof of Pythagoras' Theorem. Read it. Then: (1) Finish labelling the points in the diagram just below. (2) On the next page, draw pictures of your own to illustrate what is going on. Use as few labels and words as possible. More credit for more colour. You do not have to follow Euclid exactly, but just explain the main lines of the argument. **Think before writing!**



Since each of the angles BAC and BAG is right, it follows that with a straight line BA, and at the point A on it, the two straight lines AC and AG not lying on the same side make the adjacent angles equal to two right angles, therefore CA is in a straight line with AG.

For the same reason BA is also in a straight line with AH.

Since the angle DBC equals the angle FBA, for each is right, add the angle ABC to each, therefore the whole angle DBA equals the whole angle FBC.

Since DB equals BC, and FB equals BA, the two sides AB and BD equal the two sides FB and BC respectively, and the angle ABD equals the angle FBC, therefore the base AD equals the base FC, and the triangle ABD equals the triangle FBC.

Now the parallelogram BL is double the triangle ABD, for they have the same base BD and are in the same parallels BD and AL. And the square GB is double the triangle FBC, for they again have the same base FB and are in the same parallels FB and GC.

Therefore the parallelogram BL also equals the square GB.

Similarly, if AE and BK are joined, the parallelogram CL can also be proved equal to the square HC. Therefore the whole square $BDEC$ equals the sum of the two squares GB and HC.

And the square $BDEC$ is described on BC, and the squares GB and HC on BA and AC.

Therefore the square on BC equals the sum of the squares on BA and AC.

Therefore in right-angled triangles the square on the side opposite the right angle equals the sum of the squares on the sides containing the right angle.

Q.E.D.

[40] 3. Write a **complete** PostScript program to draw the graph of the function $y = x^3 - 1$ for $x = -1$ to $x = 1$. Use only basic PostScript commands.

[30] 4. (1) What is the matrix of rotation through 45° around the x-axis, oriented in the direction of positive x? (2) Of 90° around the axis through the origin and $P = (1, 1, 0)$, oriented towards P?

Please check your work.

The University of British Columbia
November 3, 1997

Mathematics 308
Section 101
Instructor: Dr. Casselman

Duration: 50 minutes

- Be sure that you have 9 pages in addition to this one.
- Put your name below and on the **back** of the other pages.
- In all questions, **you must show work** — i.e. display intermediate results — to get full credit.
- This is an open book exam. You may also use a calculator.
- Be neat! I will not attempt to decipher messy calculations or interpret messy pictures.
- *All work you wish to be graded must be placed on these sheets. Work books should be used for scrap work only, and must not be handed in. I will simply throw away any other papers I receive.*

First Name ____________________ **Last Name** ____________________

Signature ____________________ **Student Number** ____________________

1.	2.	3.	4.	Total
40	40	40	30	150